Multiaxial fatigue and deformation including non-proportional hardening and variable amplitude loading effects

Nima Shamsaei
The University of Toledo

Follow this and additional works at: http://utdr.utoledo.edu/theses-dissertations

Recommended Citation
Shamsaei, Nima, "Multiaxial fatigue and deformation including non-proportional hardening and variable amplitude loading effects" (2010). Theses and Dissertations. 974.
http://utdr.utoledo.edu/theses-dissertations/974

This Dissertation is brought to you for free and open access by The University of Toledo Digital Repository. It has been accepted for inclusion in Theses and Dissertations by an authorized administrator of The University of Toledo Digital Repository. For more information, please see the repository's About page.
A Dissertation

entitled

Multiaxial Fatigue and Deformation Including Non-proportional Hardening and Variable Amplitude Loading Effects

by

Nima Shamsaei

Submitted to the Graduate Faculty as partial fulfillment of the requirements for the Doctor of Philosophy Degree in Engineering

Dr. Ali Fatemi, Committee Chair

Dr. Yong Gan, Committee Member

Dr. Mohamed Hefzy, Committee Member

Dr. Efstratios Nikolaidis, Committee Member

Dr. Douglas Nims, Committee Member

Dr. Darrell Socie, Committee Member
University of Illinois at Urbana-Champaign

Dr. Patricia R. Komuniecki
College of Graduate Studies

The University of Toledo
August 2010
An Abstract of

Multiaxial Fatigue and Deformation Including Non-proportional Hardening and Variable Amplitude Loading Effects

by

Nima Shamsaei

Submitted to the Graduate Faculty as partial fulfillment of the requirements for the Doctor of Philosophy Degree in Engineering

The University of Toledo

August 2010

This study investigates fatigue damage and deformation behavior under multiaxial loading conditions, with the aim of evaluating reliable predictive models for life predictions. Life prediction for multiaxial variable amplitude loading involves a variety of issues to be considered. These include cyclic plasticity, material properties and variations with hardness and microstructure, fatigue damage evolution, fatigue damage quantification parameters, cycle counting procedure, damage accumulation rule, and effects of load non-proportionality.

To evaluate the effect of hardness and microstructure on additional non-proportional hardening and fatigue behaviors, 1050 steel in normalized, quenched and tempered (QT), and induction hardened conditions as well as 304L stainless steel were utilized. Reduction in the non-proportional cyclic hardening was observed as the 1050 steel changed from low hardness to higher hardness. Significant non-proportional cyclic hardening was observed for 304L stainless steel. Multiaxial data generated in this study
as well as from literature suggest non-proportional cyclic hardening can be related to uniaxial cyclic hardening. Non-proportional hardening coefficients predicted from a proposed equation based on this observation were found to be in very good agreement with the experimental values in this study and from literature.

Multiaxial fatigue data for all hardness levels were satisfactorily correlated with the Fatemi-Socie parameter. In order to predict multiaxial fatigue life of steels in the absence of any fatigue data, the Roessle-Fatemi hardness method was extended to multiaxial loading. The applicability of the prediction method based on hardness was examined for several steels under a wide range of loading conditions. The great majority of the observed fatigue lives were found to be in good agreement with predicted lives.

Several discriminating multiaxial cyclic strain paths with incremental and random sequences were used to investigate fatigue and cyclic deformation behaviors of materials with low and high additional hardening resulting from non-proportional loadings. Tubular specimens made of 1050 QT steel and 304L stainless steel were utilized for this purpose. The 1050 QT steel was found to exhibit very similar stress response under various multiaxial loading paths, whereas significant effects of loading sequence were observed on stress response of 304L stainless steel. In-phase cycles with a random sequence of axial-torsional cycles on an equivalent strain circle caused cyclic hardening levels similar to 90° out-of-phase loading of 304L stainless steel. In contrast, straining with a gradual sequence resulted in much lower stress than for 90° out-of-phase loading. Tanaka’s non-proportionality parameter coupled with a Fredrick-Armstrong incremental plasticity model resulted in accurate prediction of the stabilized stress response. Kanazawa et al.’s empirical formulation as a representative of such empirical models
could not distinguish between strain paths with random and incremental sequences of straining, resulting in significant over-prediction of stress for 304L stainless steel.

Contrary to common expectations, fatigue lives for 1050 QT steel with no non-proportional hardening were found to be more sensitive to non-proportionality of loadings as compared to 304L stainless steel with significant non-proportional hardening. In-phase loading cycles with random sequences of axial-torsion strain ratio within an equivalent strain circle did not significantly affect fatigue life for either material. Experimentally observed failure planes were in good agreements with predicted failure planes based on the Fatemi-Socie critical plane parameter. Bannantine-Socie and Wang-Brown cycle counting methods were utilized to identify loading cycles for variable amplitude strain paths. Fatigue damage for each counted cycle was evaluated using Fatemi-Socie damage parameter and linear cumulative fatigue damage was then employed to account for accumulation of damage. Fatigue lives for both materials under these discriminating strain paths were predicted satisfactorily employing this approach and either Bannantine-Socie or Wang-Brown cycle counting method.

Cracking behavior was analyzed for different materials investigated and under various loading conditions. Micro-cracks were observed to be around the maximum shear or critical plane. The ratio of crack initiation life to total fatigue life as well as the crack growth rate depended on a variety of factors including strain amplitude, load non-proportionality, material ductility, and specimen geometry. Crack growth rates for in-phase and 90° out-of-phase loading were correlated well by Reddy-Fatemi effective strain-based intensity factor.
This dissertation is dedicated to my parents, Maryam and Freidoun.
Acknowledgements

I would like to sincerely thank my advisor Dr. Ali Fatemi who has supported me throughout this long journey with his knowledge and patience. This dissertation could not be done without his guidance, encouragement and support during long meetings, almost daily, and hundreds of emails. Dr. Yong Gan, Dr. Mohammad Samir Hefzy, Dr. Efstratios Nikolaidis, and Dr. Douglas Nims are greatly acknowledged for serving on my Ph.D. committee. I also would like to offer my sincere gratitude to Professor Darrell Socie from University of Illinois at Urbana-Champaign for his time and advice through his frequent traveling to Toledo and numerous emails.

I would also like to thank Dr. Abdollah Afjeh, Chair of MIME Department, for all his administrative supports during the past few years. Chrysler Group LLC and specifically Dr. Yung-Li Lee and Mr. Peter Bauerle are acknowledged for the partial funding of this research study. The time and effort of Mr. John Jaegly, Mr. Tim Grivanos, and Mr. Randall Reihing from the machine shop of the MIME Department for troubleshooting of the multiaxial fatigue machine is really appreciated. I would like to thank my colleagues at the fatigue and fracture research laboratory of the University of Toledo for making the long hours of hard work enjoyable.

Finally, I would like to thank my parents and family for all their support and encouragement throughout my education from primary school to doctorate degree.
# Contents

Abstract iii

Acknowledgments vii

Contents viii

List of Tables xi

List of Figures xiv

List of Nomenclature xxiv

List of Abbreviations xxx

1 Introduction 1

1.1 Historical preview 1

1.2 Motivation for the study and objectives 3

1.3 Outline of dissertation 6

2 Literature Survey 9

2.1 Introduction 9

2.2 Constitutive behavior and hardening models 12

2.3 Multiaxial fatigue models 25

2.4 Cycle counting and application to variable amplitude loading 28

2.4.1 Bannantine and Socie 34

2.4.2 Wang and Brown 35

2.5 Discussion of the methods and relevant works 38

3 Experimental Program 60

3.1 Introduction 60
3.2 Material and specimen fabrication 60  
3.3 Testing equipment 62  
3.4 Test methods and procedures 63  
  3.4.1 Monotonic tests 63  
  3.4.2 Constant amplitude multiaxial fatigue tests 64  
  3.4.3 Variable amplitude multiaxial fatigue tests 66  
3.5 Replication technique 69  
3.6 Experimental results 71  
  3.6.1 Monotonic experimental results 71  
  3.6.2 In-phase and 90° out-of-phase experimental results 75  
  3.6.3 Variable amplitude experimental results 80  

4 Deformation Behavior and Predictions 124  
  4.1 Introduction 124  
  4.2 Constant amplitude deformation behavior and predictions 125  
    4.2.1 Constant amplitude deformation behavior 125  
    4.2.2 Effects of microstructure on non-proportional cyclic behavior 129  
    4.2.3 Non-proportional cyclic hardening coefficient prediction 133  
  4.3 Variable amplitude deformation behavior and predictions 138  
    4.3.1 Variable amplitude deformation behavior 138  
    4.3.2 Predictions of the non-proportional cyclic deformation 141  
  4.4 Conclusions 148  

5 Fatigue Behavior and Life Predictions 170  
  5.1 Introduction 170  
  5.2 Constant amplitude fatigue behavior and predictions 172  
    5.2.1 Constant amplitude fatigue behavior 172  
    5.2.2 Constant amplitude fatigue life predictions 176  
    5.2.3 Fatigue life predictions based only on hardness 181
5.3 Variable amplitude fatigue behavior and predictions 184
  5.3.1 Variable amplitude fatigue behavior 185
  5.3.2 Cycle counting and fatigue life predictions 190
  5.3.3 Some discussions on variable amplitude fatigue analysis 195

5.4 Conclusions 200

6 Cracking Behavior 241
  6.1 Introduction 241
  6.2 Parameter definitions and cracking analysis 242
    6.2.1 Effects of strain amplitude level on cracking behavior 244
    6.2.2 Effects of load non-proportionality on cracking behavior 247
    6.2.3 Effects of material on cracking behavior 249
    6.2.4 Effects of specimen geometry on cracking behavior 250
    6.2.5 Cracking behaviors under variable amplitude strain paths 251
  6.3 Correlation of crack growth rate data 252
  6.4 Conclusions 254

7 Summary and Possible Future Research 272
  7.1 Summary 273
  7.2 Possible future research 278

References 282
List of Tables

Table 3.1 Chemical compositions in weight percent of 1050 steel and 304L stainless steel used in this study. 86

Table 3.2 Monotonic axial and torsional properties for 1050 steel in normalized, quenched and tempered, and induction hardened conditions as well as 304L stainless steel. 86

Table 3.3 Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) test information and results for tubular and solid specimens of 1050 N steel. 87

Table 3.4 Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) test information and results for tubular specimens of 1050 QT steel. 88

Table 3.5 Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) test information and results for tubular specimens of 1050 IH steel. 89

Table 3.6 Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) test information and results for tubular specimens of 304L stainless steel. 90

Table 3.7 Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) axial-torsion stable cycle stress-strain analysis results for tubular and solid specimens of 1050 N steel. 91

Table 3.8 Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) axial-torsion stable cycle stress-strain analysis results for tubular specimens of 1050 QT steel. 92

Table 3.9 Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) axial-torsion stable cycle stress-strain analysis results for tubular specimens of 1050 IH steel. 93
Table 3.10  Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) axial-torsion stable cycle stress-strain analysis results for 304L stainless steel.

Table 3.11  Axial-torsion variable amplitude test information and results for tubular specimens of 1050 QT steel.

Table 3.12  Axial-torsion variable amplitude test information and results for tubular specimens of 304L stainless steel.

Table 3.13  Axial-torsion variable amplitude stable block stress-strain analysis results for tubular specimens of 1050 QT steel.

Table 3.14  Axial-torsion variable amplitude stable block stress-strain analysis results for tubular specimens of 304L stainless steel.

Table 4.1  Midsection equivalent stress and strain components for tubular specimens of 1050 N steel under in-phase and 90° out-of-phase loading.

Table 4.2  Midsection equivalent stress and strain components for tubular specimens of 1050 QT steel under in-phase and 90° out-of-phase loading.

Table 4.3  Midsection equivalent stress and strain components for tubular specimens of 1050 IH steel under in-phase and 90° out-of-phase loading.

Table 4.4  Midsection equivalent stress and strain components for tubular specimens of 304L stainless steel under in-phase and 90° out-of-phase loading.

Table 4.5  In-phase and 90° out-of-phase cyclic deformation properties and coefficients for 1050 steel and 304L stainless steel.

Table 4.6  Midsection equivalent stress and strain components for tubular specimens of 1050 QT steel under variable amplitude star shape strain paths.

Table 4.7  Midsection equivalent stress and strain components for tubular specimens of 304L stainless steel under variable amplitude star shape strain paths.

Table 5.1  Surface stress-strain components for stable cycle in axial-torsion constant amplitude fatigue experiments of 1050 N steel under in-phase and 90° out-of-phase loading.
<table>
<thead>
<tr>
<th>Table 5.2</th>
<th>Surface stress-strain components for stable cycle in axial-torsion constant amplitude fatigue experiments of 1050 QT steel under in-phase and 90° out-of-phase loading.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 5.3</td>
<td>Surface stress-strain components for stable cycle in axial-torsion constant amplitude fatigue experiments of 1050 IH steel under in-phase and 90° out-of-phase loading.</td>
</tr>
<tr>
<td>Table 5.4</td>
<td>Surface stress-strain components for stable cycle in axial-torsion constant amplitude fatigue experiments of 304L stainless steel under in-phase and 90° out-of-phase loading.</td>
</tr>
<tr>
<td>Table 5.5</td>
<td>In-phase and 90° out-of-phase fatigue properties for 1050 steel and 304L stainless steel.</td>
</tr>
<tr>
<td>Table 5.6</td>
<td>Surface stress-strain components for stable cycle of axial-torsion variable amplitude fatigue experiments of 1050 QT steel.</td>
</tr>
<tr>
<td>Table 5.7</td>
<td>Surface stress-strain components for stable cycle of axial-torsion variable amplitude fatigue experiments of 304L stainless steel.</td>
</tr>
<tr>
<td>Table 5.8</td>
<td>Comparison of fatigue life predictions employing BS and WB cycle counting methods using eFatigue software.</td>
</tr>
<tr>
<td>Table 6.1</td>
<td>Summary of cracking observations for tubular specimens of 1050 N steel.</td>
</tr>
<tr>
<td>Table 6.2</td>
<td>Summary of cracking observations for tubular specimens of 1050 QT steel.</td>
</tr>
<tr>
<td>Table 6.3</td>
<td>Summary of cracking observations for tubular specimens of 304L stainless steel.</td>
</tr>
</tbody>
</table>
List of Figures

Figure 2.1 Stress-time spectrum in critical element of anti-roll-bar of a vehicle (Shamsaei, 2001).

Figure 2.2 Illustration of in-phase and 90º out-of-phase loading and the effect of out-of-phase loading on Mohr’s circle and rotation of principal axes (Stephens et al., 2000).

Figure 2.3 Isotropic hardening representing the expansion of yield surface (Socie and Marquis, 2000).

Figure 2.4 Kinematic hardening representing the translation of yield surface (Socie and Marquis, 2000).

Figure 2.5 Bauschinger effect on yield points.

Figure 2.6 (a) Isotropic hardening and (b) kinematic hardening during non-proportional cyclic loading (Socie and Marquis, 2000).

Figure 2.7 Cyclic stress-strain curve for proportional and non-proportional loading (Socie and Marquis, 2000).

Figure 2.8 Illustration of the factor of non-proportionality based on Kanazawa et al. (1979) for three load paths: (a) proportional, (b) 90º out-of-phase, and (c) random (Socie and Marquis, 2000).

Figure 2.9 (a) Definition of $\xi(t)$ (Itoh et al., 2004), (b) polar figure of $\Delta \varepsilon_i$ (Itoh et al., 2004), and (c) descriptions of the maximum circumscribing circle (MCC) and the maximum circumscribing ellipse (MCE) (Li et al., 2002).

Figure 2.10 Crack mechanisms for ductile behaving materials (left) showing shear damage mechanism and brittle behaving materials (right) showing tensile damage mechanism (Socie and Marquis, 2000).

Figure 2.11 Rainflow cycle counting method based on the rain falling down on pagoda roof (Wright, 1993).
Figure 2.12 Rainflow cycle counting based on hysteresis loops, counting one cycle for a closed hysteresis loop and half a cycle for an open hysteresis loop (Collins, 1993).

Figure 2.13 Rainflow counting method (ASTM Standard E 1049-85, 2007).

Figure 2.14 Simplified rainflow counting method (ASTM Standard E 1049-85, 2007).

Figure 2.15 (a) Observed fatigue lives for several strain block loadings of pure Ti and its alloy. (b) Comparison of observed and calculated fatigue lives by LDR and FS damage parameter (Shamsaei et al., 2010b).

Figure 2.16 Different failure planes in Bannantaine and Socie variable amplitude multiaxial fatigue prediction method (Socie and Marquis, 2000).

Figure 2.17 An example of Bannantaine and Socie cycle counting method on the critical plane.

Figure 2.18 An example of WB reversal counting method for cruciform axial-torsion strain path. Point A on the axial cycle is considered as the first reference point.

Figure 2.19 Two methods for defining shear stress in Dang Van model (Socie and Marquis, 2000).

Figure 2.20 Schematic of axial-torsion strain paths used in Kim’s studies (Han et al., 2002).

Figure 3.1 The microstructure of 1050 steel in (a) normalized condition as pearlite and ferrite, (b) quenched and tempered condition as tempered martensite, and (c) induction-hardened condition as tempered martensite.

Figure 3.2 Tubular specimen configuration and dimensions in mm.

Figure 3.3 Instron closed-loop servo hydraulic axial-torsional fatigue testing frame.

Figure 3.4 Biaxial extensometer used to measure axial-torsion strains and control strain-controlled tests.

Figure 3.5 In-phase (proportional) and 90° out-of-phase (non-proportional) constant amplitude axial-torsional multiaxial loadings used in this study.
Figure 3.6 Axial-torsional star shape strain paths utilized in this study. Strain-time diagrams shown are for one complete strain block, except for the FRR path for which one quarter of the strain cycles in the block are shown for clarity.

Figure 3.7 Axial-torsional cruciform strain paths utilized in this study.

Figure 3.8 Axial monotonic test data points and curve for 1050 steel in normalized and quenched and tempered conditions.

Figure 3.9 Torsional monotonic test data points and curve for 1050 steel in normalized, quenched and tempered, and induction hardened conditions.

Figure 3.10 Stable cycle hysteresis loops for in-phase axial-torsion loading of 1050 N steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops.

Figure 3.11 Stable cycle hysteresis loops for 90° out-of-phase axial-torsion loading of 1050 N steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops. (c) Midsection shear strain versus axial strain. (d) Midsection shear stress versus axial stress.

Figure 3.12 Stable cycle hysteresis loops for in-phase axial-torsion loading of 1050 QT steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops.

Figure 3.13 Stable cycle hysteresis loops for 90° out-of-phase axial-torsion loading of 1050 QT steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops. (c) Midsection shear strain versus axial strain. (d) Midsection shear stress versus axial stress.

Figure 3.14 Stable cycle hysteresis loops for in-phase axial-torsion loading of 1050 IH steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops.

Figure 3.15 Stable cycle hysteresis loops for 90° out-of-phase axial-torsion loading of 1050 IH steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops. (c) Midsection shear strain versus axial strain. (d) Midsection shear stress versus axial stress.

Figure 3.16 Stable cycle hysteresis loops for in-phase axial-torsion loading of 304L stainless steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops.

Figure 3.17 Stable cycle hysteresis loops for 90° out-of-phase axial-torsion loading of 304L stainless steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops. (c) Midsection shear strain versus axial strain. (d) Midsection shear stress versus axial stress.
Figure 3.18 Axial stress amplitudes for in-phase axial-torsion loading of 304L stainless steel versus (a) number of cycles, and (b) normalized number of cycles.

Figure 3.19 Midsection shear stress amplitudes for in-phase axial-torsion loading of 304L stainless steel versus (a) number of cycles, and (b) normalized number of cycles.

Figure 3.20 Axial stress amplitudes for 90° out-of-phase axial-torsion loading of 304L stainless steel versus (a) number of cycles, and (b) normalized number of cycles.

Figure 3.21 Midsection shear stress amplitudes for 90° out-of-phase axial-torsion loading of 304L stainless steel versus (a) number of cycles, and (b) normalized number of cycles.

Figure 3.22 Transient behaviors and mean stress relaxations for 1050 QT steel and 304L stainless steel (304L SS) under similar strain amplitude levels ($\varepsilon_a = 0.0034$ and $\gamma_a = 0.0058$) of FRI and PI strain paths in (a) axial, and (b) shear channels. The axial data plotted is for the pure axial loading cycle during the block load, while the shear data plotted is for the pure torsion cycle during the block load.

Figure 3.23 Maximum axial stress amplitudes for FRI and FRR axial-torsion strain paths of 1050 QT steel in versus (a) number of blocks and (b) normalized number of blocks. The axial and shear data plotted are respectively from the pure axial and pure torsion cycle during the block load.

Figure 3.24 Maximum midsection shear stress amplitudes for FRI and FRR axial-torsion strain paths of 1050 QT steel versus (a) number of blocks, and (b) normalized number of blocks. The axial and shear data plotted are respectively from the pure axial and pure torsion cycle during the block load.

Figure 3.25 Change in stress response over several load blocks for 1050 QT steel ($\varepsilon_a = 0.007$ and $\gamma_a = 0.012$) under (a) FRI strain path (10FRI1QT), and (b) FRR strain path (10FRR1QT).

Figure 3.26 Maximum axial stress amplitudes for FRI, FRR, and FRI15 axial-torsion strain paths of 304L stainless steel versus (a) number of blocks, and (b) normalized number of blocks. The axial and shear data plotted are respectively from the pure axial and pure torsion cycle during the block load.
Figure 3.27   Maximum midsection shear stress amplitudes for FRI, FRR, and FRI15 axial-torsion strain paths of 304L stainless steel versus (a) number of blocks and (b) normalized number of blocks. The axial and shear data plotted are respectively from the pure axial and pure torsion cycle during the block load.

Figure 3.28   Change in stress response over several load blocks for 304L stainless steel ($\varepsilon_a = 0.007$ and $\gamma_a = 0.012$) under (a) FRI strain path (10FRI1SS), and (b) FRR strain path (10FRR1SS).

Figure 4.1   Equivalent stress-strain data and generated stress-strain curves for 90° out-of-phase tests of 1050 N steel at three strain levels.

Figure 4.2   Midsection equivalent stress amplitude versus midsection equivalent plastic strain amplitude data and fit for in-phase loading of (a) 1050 N steel, (b) 1050 QT steel, (c) 1050 IH steel, and (d) 304L stainless steel.

Figure 4.3   Midsection equivalent stress amplitude versus midsection equivalent plastic strain amplitude data and fit for 90° out-of-phase loading of (a) 1050 N steel, (b) 1050 QT steel, (c) 1050 IH steel, and (d) 304L stainless steel.

Figure 4.4   Comparison of cyclic deformation behavior of 1050 steel at different hardness levels as well as 304L stainless steel under (a) in-phase, and (b) 90° out-of-phase loadings.

Figure 4.5   Comparison of cyclic in-phase, cyclic 90° out-of-phase, and monotonic stress-strain curves for (a) 1050 N steel, (b) 1050 QT steel, (c) 1050 IH steel, and (d) 304L stainless steel.

Figure 4.6   Comparison of the non-proportional cyclic hardening coefficient, $\alpha$, observed from experiments with the predicted values based on Borodii-Shukaev equation.

Figure 4.7   Definition of cyclic hardening coefficient, $h$.

Figure 4.8   Correlation of non-proportional hardening coefficient, $\alpha$, and cyclic hardening coefficient, $h$, for steels used in this study.

Figure 4.9   Comparison of experimental non-proportional cyclic hardening coefficient, $\alpha$, with predicted values based on Equation (4.11) proposed in this study.
Figure 4.10 (a) Experimental non-proportional cyclic hardening coefficient, $\alpha$, and (b) experimental cyclic hardening coefficient, $h$, versus equivalent strain amplitude.

Figure 4.11 Comparison of experimental 90° out-of-phase equivalent stress amplitudes with the predicted values based on Equations (4.11).

Figure 4.12 Cyclic deformation behavior under various strain paths for (a) 1050 QT steel, and (b) 304L stainless steel.

Figure 4.13 Equivalent stress amplitude responses for various strain paths under equivalent strain amplitudes of 0.7% and 0.34% for (a) 1050 QT steel, and (b) 304L stainless steel.

Figure 4.14 Comparison of observed and predicted equivalent stress amplitudes for 1050 QT steel and 304L stainless steel (304L SS) using (a) Kanazawa et al.’s empirical method, and (b) Tanaka’s non-proportionality parameter.

Figure 4.15 Comparison of the experimental midlife stress boundaries and predicted ones using the Tanaka’s non-proportionality parameter for 304L stainless steel. y-axis represents $\sqrt{3}\tau$ and x-axis represents $\sigma$, both in MPa. There are two experimental boundaries for each strain path.

Figure 4.16 Experimental and predicted stress boundaries for 304L stainless steel for FRI strain path at $\varepsilon_\sigma = 0.7\%$ by Kanazawa et al’s empirical method and Tanaka’s non-proportionality parameter.

Figure 4.17 Comparison of the experimental axial and shear stress boundaries and the predicted axial and shear stresses for PI strain path at strain level of $\varepsilon_\sigma = 0.35\%$ using Tanaka’s non-proportionality parameter for (a) 1050 QT steel, and (b) 304L stainless steel.

Figure 5.1 Equivalent surface strain amplitude versus reversals to failure for (a) in-phase and (b) 90° out-of-phase loadings of 1050 N steel.

Figure 5.2 Equivalent surface strain amplitude versus reversals to failure for (a) in-phase and (b) 90° out-of-phase loadings of 1050 QT steel.

Figure 5.3 Equivalent surface strain amplitude versus reversals to failure for (a) in-phase and (b) 90° out-of-phase loadings of 1050 IH steel.

Figure 5.4 Equivalent surface strain amplitude versus reversals to failure for (a) in-phase and (b) 90° out-of-phase loadings of 304L stainless steel.
Figure 5.5 Comparison of in-phase and 90° out-of-phase equivalent strain amplitudes versus fatigue life for (a) 1050 N steel, (b) 1050 QT steel, (c) 1050 IH steel, and (d) 304L stainless steel.

Figure 5.6 Equivalent surface strain amplitude versus reversals to failure for (a) in-phase and (b) 90° out-of-phase axial-torsion loading of 1050 steel.

Figure 5.7 Correlation of in-phase and 90° out-of-phase fatigue data by von Mises criterion for (a) 1050 N steel, (b) 1050 QT steel, (c) 1050 IH steel, and (d) 304L stainless steel.

Figure 5.8 Cracks observed under in-phase (IP) and 90º out-of-phase (OP) loading for (a) 1050 N steel, (b) 1050 QT steel, and (c) 304L stainless steel. Maximum shear planes for IP (20º and 110º) and OP (0º) loading are also presented.

Figure 5.9 Correlation of in-phase and 90° out-of-phase fatigue data by the Fatemi-Socie critical plane parameter for (a) 1050 N steel, (b) 1050 QT steel, (c) 1050 IH steel, and (d) 304L stainless steel.

Figure 5.10 Variation of the FS damage parameter with plane orientation for in-phase (IP) and 90º out-of-phase (OP) loadings at $\varepsilon_a = 0.7\%$ for (a) 1050 QT steel, and (b) 304L stainless steel.

Figure 5.11 Predicted fatigue lives based on only hardness versus observed fatigue lives for different materials and hardneses under various multiaxial loading conditions.

Figure 5.12 Variation of $k$ in the Fatemi-Socie parameter with fatigue life and Brinell hardness for steels.

Figure 5.13 Fatigue lives (a) and equivalent stress amplitude (b) for various strain paths under equivalent strain amplitudes of 0.7% and 0.34% for 1050 QT steel. Values are normalized by in-phase (IP) data at each strain level and represent average values for duplicate tests.

Figure 5.14 Fatigue lives (a) and equivalent stress amplitude (b) for various strain paths under equivalent strain amplitudes of 0.7% and 0.34% for 304L stainless steel. Values are normalized by in-phase (IP) data at each strain level and represent average values for duplicate tests.

Figure 5.15 Experimental and predicted fatigue lives based on von Mises criterion for 1050 QT steel and 304L stainless steel.
Figure 5.16 Variation of the cumulative damage based on FS parameter with plane orientation for in-phase (IP), 90° out-of-phase (OP), and FRI strain paths at $\bar{\varepsilon}_d = 0.7\%$ for (a) 1050 QT steel, and (b) 304L stainless steel. Values are normalized for one cycle for FRI strain path.

Figure 5.17 Examples of cracks observed on the surface of 304L stainless steel specimens at $\bar{\varepsilon}_a = 0.34\%$ under (a) IP, (b) OP, (c) FRI, (d) FRR, (e) FRI15, and (d) PI strain paths. Maximum damage plane is also presented for all strain paths.

Figure 5.18 (a) Uniaxial variable amplitude strain block used to verify the capability of the linear cumulative damage rule for similar loadings used in the multiaxial tests. Every 5th cycle is presented for clarity. (b) Comparison of the predicted and experimental fatigue lives using FS parameter and linear cumulative damage rule.

Figure 5.19 Experimental and predicted fatigue lives based on BS cycle counting method, FS critical plane approach, and linear cumulative damage rule using eFatigue for 1050 QT steel and 304L stainless steel.

Figure 5.20 (a) Linear cumulative fatigue damage, and (b) fatigue damage caused by each cycle of one block of FRI strain path based on the FS critical plane approach for 304L stainless steel at $\bar{\varepsilon}_d = 0.7\%$ on 0° plane.

Figure 5.21 Experimental and predicted fatigue lives based on WB reversal counting method, FS critical plane approach, and linear cumulative damage rule using eFatigue for 1050 QT steel and 304L stainless steel.

Figure 5.22 (a) Shear strain and normal stress time histories on the loading plane, and (b) rainflow cycle counting for a fully-reversed incremental strain path with 45° increments.

Figure 5.23 (a) Shear strain and normal stress time histories on the loading plane, and (b) BS cycle counting and results for PI90 strain path.

Figure 5.24 (a) Shear strain and normal stress time histories on the loading plane, and (b) BS cycle counting and results for FRI90 strain path.

Figure 5.25 BS cycle counting results on the modified PI90 strain path.

Figure 5.26 BS cycle counting results on the modified FRI90 strain path.
Figure 5.27  WB reversal counting on FRI90 strain path. Point A on the axial cycle is considered as the first reference point.  

Figure 5.28  WB reversal counting on FRI90 strain path. Point C on the shear cycle is considered as the first reference point.  

Figure 5.29  WB reversal counting on PI90 strain path. Point A on the axial cycle is considered as the first reference point.  

Figure 5.30  WB reversal counting on PI90 strain path. Point D on the shear cycle is considered as the first reference point.  

Figure 6.1  Different stages of crack initiation and growth during the fatigue process and the approach used for analysis.  

Figure 6.2  (a) Definition of crack parameters. (b) Definition of the plane angle.  

Figure 6.3  Examples of cracks observed for tubular specimens of 1050 N steel under in-phase tests at (a) high amplitude ($\varepsilon_\alpha = 0.0100$), (b) intermediate amplitude ($\varepsilon_\alpha = 0.0048$), and (c) low amplitude ($\varepsilon_\alpha = 0.0022$) levels. The orientation of the critical planes on the specimen is also presented.  

Figure 6.4  Crack length and orientation observed at different stages of fatigue life for the high strain level test ($\varepsilon_\alpha = 0.0100$) of 1050 N steel solid specimen under in-phase loading (i.e. NS 3). The orientation of the critical planes on the specimen is also presented.  

Figure 6.5  Crack length and orientation observed in different stages of fatigue life for the intermediate strain level test ($\varepsilon_\alpha = 0.0049$) of 1050 N steel solid specimen under in-phase loading (i.e. NS 7). The orientation of the critical planes on the specimen is also presented.  

Figure 6.6  Crack length and orientation observed at different stages of fatigue life for the low strain level test ($\varepsilon_\alpha = 0.0023$) of 1050 N steel solid specimen under in-phase loading (i.e. NS 5). The orientation of the critical planes on the specimen is also presented.  

Figure 6.7  Effect of strain amplitude level on cracking behavior for solid specimens of 1050 N steel under (a) in-phase, and (b) 90° out-of-phase loadings.
Figure 6.8 Effects of load non-proportionality on cracking behavior for solid specimens of 1050 N steel under high and intermediate strain amplitude levels.

Figure 6.9 Effects of load non-proportionality on crack growth rate of 1050 N steel solid specimens under (a) high strain amplitude ($\varepsilon_a = 0.0069$ and $\gamma_a = 0.0120$), and (b) intermediate strain amplitude level tests ($\varepsilon_a = 0.0033$ and $\gamma_a = 0.0057$).

Figure 6.10 Some cracks observed for the highest strain amplitude level tests ($\varepsilon_a = 0.0069$ and $\gamma_a = 0.0120$) of 304L stainless steel under (a) in-phase, and (b) 90° out-of-phase loadings.

Figure 6.11 Effects of material ductility on the cracking behavior at $\bar{\varepsilon}_a = 0.0100$. Crack length versus (a) normalized number of cycles, and (b) number of cycles.

Figure 6.12 Effects of solid and tubular geometries on cracking behavior for 1050 N steel at $\bar{\varepsilon}_a = 0.0100$. Crack length versus (a) normalized number of cycles, and (b) number of cycles.

Figure 6.13 Crack growth rate versus Reddy-Fatemi effective strain-stress-based intensity factor for (a) in-phase (IP) and 90° out-of-phase (OP) loading of 1050 N steel solid specimens, and (b) in-phase loading of 1050 N steel and 1050 QT steel tubular specimens at $\bar{\varepsilon}_a = 0.0100$.

Figure 7.1 Some specific load paths designed to challenge FS critical plane approach; (a) load path “I”, (b) load path “P”, and (c) load path “D”.

Figure 7.2 Variation of shear strain amplitude, maximum normal stress, and FS parameter for 1050 QT steel under (a) load path “I”, (b) load path “P”, and (c) load path “D”.
List of Nomenclature

A  
Tanaka’s non-proportionality parameter

b  
axial fatigue strength exponent

de  
shear fatigue strength exponent

\bar{b}  
equivalent fatigue strength exponent

c  
axial fatigue ductility exponent

c_e  
shear fatigue ductility exponent

c_T  
material constants in evolution of the Tanaka’s non-proportional tensor

c_i  
material constants in Armstrong-Fredrick model

\overline{c}  
equivalent fatigue ductility exponent

C  
Tanaka’s non-proportionality tensor

d  
specimens’ cross-section diameter

d_i  
inner diameter of tubular cross-section

d_o  
outer diameter of tubular cross-section

dp  
equivalent plastic strain increment

D  
cumulative damage

e  
engineering strain

E  
modulus of elasticity

F  
loading function, and also factor of non-proportionality

G  
shear modulus
\( h \)    cyclic hardening coefficient
\( h_p \)    plastic modulus function
\( HB \)    Brinell hardness
\( I_1 \)    the first invariant of the stress tensor
\( k \)    yield stress in simple shear, and material constant in the Fatemi-Socie parameter
\( k_s \)    initial yield strength in simple shear
\( K \)    strength coefficient
\( K_s \)    shear strength coefficient
\( K' \)    cyclic strength coefficient
\( K'_p \)    proportional cyclic strength coefficient
\( \bar{K}' \)    equivalent cyclic strength coefficient
\( L_f \)    number of blocks to failure
\( n \)    strain hardening exponent
\( n_s \)    shear strain hardening exponent
\( n_i \)    number of cycles at a given strain level
\( n' \)    cyclic strength exponent
\( n'_p \)    proportional cyclic strength exponent
\( \bar{n}' \)    equivalent cyclic strain hardening exponent
\( n \)    exterior unit normal vector on the yield surface at the stress state
\( N_f \)    cycles to failure
\( 2N_f \)    reversals to failure
\( P \)    axial load
$P_{ij}$ components of spherical stress tensor

$r$ radius

$r'$ material constants in Armstrong-Fredrick model

$r_{mid.}$ midsection radius

$r_{surf.}$ surface radius

$R$ isotropic hardening variable

$R_a$ circumscribed ellipse major axis/circumscribed circle radius

$R_a$ circumscribed ellipse minor axis

$s_{ij}$ components of deviator stress tensor

$S$ engineering stress

$S_y$ engineering yield strength

$S_U$ engineering ultimate tensile strength

$\bar{S}$ deviatoric stress tensor

$t$ time

$T$ torque

$T_c$ time for cycle

$T_U$ ultimate torque

$\alpha$ additional non-proportional hardening coefficient

$\alpha$ total backstress tensor

$\alpha_{ij}$ components of back stress tensor

$\beta$ static hardening parameter, and a constant for non-proportional hardening rate

$\chi'$ scalar functions for ratcheting rate

$\delta_{ij}$ Kronecker delta components
\( \varepsilon \)  
axial strain

\( \varepsilon_1, \varepsilon_2, \varepsilon_3 \)  
principal strains

\( \varepsilon_e \)  
axial elastic strain

\( \varepsilon_{ij} \)  
components of strain tensor

\( \varepsilon_i(t) \)  
principal strain at time \( t \)

\( \varepsilon_{\text{Imax}} \)  
the maximum absolute value of \( \varepsilon_i(t) \)

\( \varepsilon^m_{ij}(t) \)  
extreme value corresponding to each stress component

\( \varepsilon_p \)  
axial plastic strain

\( (\varepsilon_p)^c \)  
cyclic plastic strain

\( (\varepsilon_p)_M \)  
monotonic plastic strain

\( \varepsilon_{tr} \)  
transverse strain

\( \bar{\varepsilon} \)  
equivalent strain

\( \bar{\varepsilon}_e \)  
equivalent elastic strain

\( \bar{\varepsilon}_p \)  
equivalent plastic strain

\( \varepsilon'_f \)  
axial fatigue ductility coefficient

\( \bar{\varepsilon}'_f \)  
equivalent fatigue ductility coefficient

\( \varepsilon^p \)  
plastic strain tensor

\( \gamma \)  
shear strain

\( \gamma_e \)  
elastic shear strain

\( \gamma_{\text{max}} \)  
maximum shear strain

\( \gamma_{\text{mid}} \)  
midsection shear strain

\( \gamma_p \)  
plastic shear strain
\[ \gamma_{\text{surf.}} \] surface shear strain
\[ \gamma'_f \] shear fatigue ductility coefficient
\[ \eta_i \] set of constant in Marko and Starkey cumulative damage model
\[ \lambda \] shear to axial strain ratio
\[ \lambda' \] positive scalar of proportionality equal to zero in the elastic domain
\[ \nu_e \] elastic Poisson’s ratio
\[ \nu_p \] plastic Poisson’s ratio
\[ \bar{\nu} \] equivalent Poisson’s ratio
\[ \sigma \] axial stress
\[ \sigma_1, \sigma_2, \sigma_3 \] principal stresses
\[ \sigma_c \] cyclic uniaxial stress
\[ \sigma_{ij} \] components of stress tensor
\[ \sigma_M \] monotonic stress
\[ \sigma_{m_{ij}} (t) \] extreme value corresponding to each stress component
\[ \sigma_{n, \text{max}} \] maximum normal stress on the maximum shear strain plane
\[ \sigma_y \] yield strength
\[ \sigma_U \] ultimate tensile strength
\[ \bar{\sigma} \] equivalent stress
\[ \sigma_{\text{IP}} \] in-phase equivalent stress
\[ \sigma_{\text{OP}} \] 90° out-of-phase equivalent stress
\[ \sigma'_f \] axial fatigue strength coefficient
\[ \sigma'_y \] cyclic yield strength
\[ \sigma'_f \] equivalent fatigue strength coefficient
τ shear stress

τ_{mid.} midsection shear stress

τ_{f} shear fracture strength

τ_{y} shear yield strength

τ_{U} shear ultimate strength

τ'_{f} shear fatigue strength coefficient

ξ(t) the angle between ε_{f}(t) and ε_{\text{max}}
# List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM</td>
<td>American Society for Testing and Materials</td>
</tr>
<tr>
<td>BM</td>
<td>Brown-Miller</td>
</tr>
<tr>
<td>BS</td>
<td>Bannantine-Socie</td>
</tr>
<tr>
<td>fcc</td>
<td>face-center-cubic</td>
</tr>
<tr>
<td>FRI</td>
<td>Fully-Reversed with 1° Increments</td>
</tr>
<tr>
<td>FRI15</td>
<td>Fully-Reversed with 15° Increments</td>
</tr>
<tr>
<td>FRR</td>
<td>Fully-Reversed Random</td>
</tr>
<tr>
<td>FS</td>
<td>Fatemi-Socie</td>
</tr>
<tr>
<td>hcp</td>
<td>hexagonal-close-pack</td>
</tr>
<tr>
<td>HCF</td>
<td>High Cycle Fatigue</td>
</tr>
<tr>
<td>IH</td>
<td>Induction-Hardened</td>
</tr>
<tr>
<td>IP</td>
<td>In-Phase</td>
</tr>
<tr>
<td>LCF</td>
<td>Low Cycle Fatigue</td>
</tr>
<tr>
<td>LDR</td>
<td>Linear Damage Rule</td>
</tr>
<tr>
<td>MCC</td>
<td>Minimum Circumstantial Circle</td>
</tr>
<tr>
<td>MCE</td>
<td>Minimum Circumstantial Ellipse</td>
</tr>
</tbody>
</table>

xxx
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Normalized</td>
</tr>
<tr>
<td>OP</td>
<td>90° Out-of-Phase</td>
</tr>
<tr>
<td>PI</td>
<td>Pulsating with 1° Increments</td>
</tr>
<tr>
<td>QT</td>
<td>Quenched and Tempered</td>
</tr>
<tr>
<td>SAE</td>
<td>Society of Automotive Engineers</td>
</tr>
<tr>
<td>SFE</td>
<td>Stacking Fault Energy</td>
</tr>
<tr>
<td>SS</td>
<td>Stainless Steel</td>
</tr>
<tr>
<td>SWT</td>
<td>Smith-Watson-Topper</td>
</tr>
<tr>
<td>WB</td>
<td>Wang-Brown</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Historical preview

Fatigue is the progressive and localized structural damage which occurs when a material is subjected to cyclic loading. The process starts with dislocation movements which eventually form persistent slip bands and leading to nucleation of cracks.

The first article about fatigue was published in 1837 by Wilhelm Albert devising a test machine for conveyor chains used in the Clausthal mines (Schutz, 1996). However, the first reported accident due to fatigue failure causing numerous victims happened in 1842 in Paris-Versailles railway (Schutz, 1996). William Rankine (1842) expressed gradual process of the unexpected fracture which was called fatigue first by Poncelet from France and Fredrick Braithwaite from England. After that, many engineers worked on designing for fatigue in the nineteenth century. This includes Wohler, a German railway engineer, who established S-N approach for long life fatigue as the most significant work in this area. Early in the twentieth century Basquin (1910) formulated Wohler’s S-N approach by proposing an exponential law for fatigue life.

Most of the work done by the middle of the twentieth century belongs to high cycle fatigue and stress-based analysis where the plastic deformation is negligible. By
development of aerospace and nuclear power structures that work under severe cyclic loadings, the development of strain-based design models taking into account plastic deformation became a necessity. As a result, Tavernelli and Coffin (1962) and Manson (1962) proposed a strain-based fatigue model as a power law relationship between plastic strain and fatigue life for low cycle regime.

Realistic service load histories are not always constant amplitude. Therefore, in 1960s and 1970s, some cycle counting methods, which decompose a variable loading history to a series of constant amplitude load levels, were proposed. Furthermore, to estimate total damage under variable amplitude loading, cumulative damage models have been developed since Palmgren (1924) and later Miner (1945) proposed the linear damage rule.

Nevertheless, many service loading histories are multiaxial rather than uniaxial. This fact was the motivation for the works done by Lame and Tresca in the late nineteenth century and von Mises in the early twentieth century to propose classical strength hypotheses (Zenner, 2004). These hypotheses which were originally proposed for yielding and under static or monotonic loading were later extended to cyclic loading and fatigue strength. However, these models may only work for proportional or in-phase loading, where different loading axes reach their extreme values at the same time. For the case of non-proportional or out-of-phase loadings, where different axes do not reach their extreme values simultaneously, using classical models often leads to significant errors.

Therefore, critical plane models that consider a few specific planes with the maximum damage have been studied over the last 50 years (Socie and Marquis, 2000; Zenner, 2004). These models can be used for both proportional and non-proportional
loading conditions. Another important aspect of critical plane approaches is the ability for crack angle prediction. These models are categorized as stress-based models for high cycle fatigue, strain-based models for both low and high cycle fatigue, and energy-based models. Energy based models are based on elastic energy for high cycle regime, plastic energy for low cycle regimes, or sum of elastic and plastic energies.

In addition to a fatigue damage model, a proper multiaxial cycle counting method, a constitutive model taking into account the non-proportionality of loading, and a cumulative damage rule are other requirements for variable amplitude multiaxial fatigue analysis. The cycle counting method is to identify loading cycles in a variable amplitude multiaxial load history. To use strain-stress multiaxial fatigue models, an empirical formulation relating the non-proportional hardening to the strain path or a plasticity model, which reflects the material response under variable amplitude multiaxial loading is needed. Linear or non-linear cumulative damage rules can be used to take into account the accumulated fatigue damage from each counted cycle of the variable amplitude loading history.

Even though most of the service loads are multiaxial and have variable amplitudes, the number of works dealing with these types of loadings is small in the literature. Therefore, the topic of this dissertation is on multiaxial fatigue with an emphasis on the non-proportional hardening and variable amplitude loading effects.

1.2 Motivation for the study and objectives

The state of stress at critical location of most components and structures is multiaxial. Such components and structures are typically subjected to variable amplitude or random loading conditions. Therefore, the study of multiaxial fatigue and deformation
under variable amplitude loading is of great practical significance. While commonly accepted life prediction methodologies for uniaxial loading have been developed and used for many years, such methodologies for more complex case of multiaxial variable amplitude loading are not yet well-established, particularly when the loads are out-of-phase or non-proportional. Only a few proposals can be found in the literature for fatigue life estimation for non-proportional multiaxial loading under variable amplitudes. Therefore, a study to find the effective parameters for practical loading conditions under variable amplitude multiaxial loading is necessary.

On the other hand, analysis and testing are two essential tools of fatigue design. Nevertheless, the desire to produce inexpensive and high quality products requires reducing the portion of testing by more accurate analysis. In order to evaluate fatigue life more accurately, a simple fatigue analysis procedure should be developed for variable amplitude non-proportional multiaxial loadings. Such loading is the realistic loading in most applications including vehicle design.

One of the complicated characteristics of non-proportional loading situations is the additional cyclic hardening due to the interaction of dislocations in different directions. Because of the change in direction of principal stresses and strains in the material, dislocations move in different directions, interact with each other and generate an additional hardening which is called non-proportional hardening. This additional hardening depends on loading conditions, temperature, and material hardness and microstructures. Therefore, one of the very first steps in analysis of fatigue life under non-proportional variable amplitude multiaxial loading is to evaluate the material response to the rotation of principal axes.
Medium carbon steel is not only one of the most commonly used materials in many applications, especially automotive industry, but it is also a good material for studying the effects of microstructure and hardness on non-proportional hardening. Some materials such as many grades of aluminum alloys are not sensitive to non-proportional loadings, whereas some others show very significant additional hardening, such as stainless steels. The response of medium carbon steels to non-proportional loading is something in between and suitable to be studied. Hence, medium carbon 1050 steel with three hardness levels of normalized ($HB = 198$), quenched and tempered ($HB = 360$), and induction hardened ($HB = 565$) were utilized in this study.

Since stainless steels exhibit more then 50% additional non-proportional hardening, they are challenging materials to be investigated under multiaxial loading conditions. Stainless steels are used in many applications from aeronautics to pressure vessels and nuclear power plants. Therefore, 304L stainless steel is another material used in this study.

In-phase and 90° out-of-phase constant amplitude multiaxial tests on 1050 steel with three different hardness levels and 304L stainless steel were conducted to observe the effect of hardness levels on additional hardening due to non-proportionality of loading. These observations are also used to evaluate the capability of well-known multiaxial fatigue models in fatigue life prediction and considering the additional non-proportional hardening. A second series of testing were conducted to distinguish between different models and proposals regarding their suitability to work for deformation and fatigue life predictions in multiaxial variable amplitude loadings. Specimens made of
1050 QT steel and 304L stainless steel were used in this study for variable amplitudes multiaxial loading tests.

The specific objectives of this research were, therefore:

1) To study the effect of microstructure and hardness level on additional non-proportional hardening and develop a model relating the non-proportional hardening coefficient to simple available or easily obtainable material properties.

2) To evaluate the capability of empirical methods, as compared to a plasticity model, to predict material stress response under non-proportional variable amplitude multiaxial strain paths.

3) To evaluate effective parameter(s) for fatigue damage and life assessment for variable amplitudes multiaxial loading.

4) To investigate the effect of hardness on multiaxial fatigue behavior of steels and investigate the possibility of developing a model to predict multiaxial fatigue life only based on hardness.

5) To evaluate the capability of available cycle counting procedures for life predictions under some discriminating strain paths.

1.3 Outline of dissertation

A good understanding of the key parameters in multiaxial fatigue life assessment and prior works done in this area are essential to this study. Therefore, this dissertation begins by a brief literature survey on multiaxial fatigue models and some related aspects, presented in Chapter 2. Literature surveys on constitutive and hardening models are first presented in this chapter. One needs to have a good understanding of constitutive behavior and non-proportional hardening to be able to use strain-stress based multiaxial
fatigue models for out-of-phase loading. Then, a brief review of multiaxial fatigue models are presented in Chapter 2. Cycle counting methods and cumulative damage rules are two necessary tools for dealing with variable amplitude loading; thus, cycle counting methods are reviewed and a brief discussion on cumulative damage models is also presented in this chapter. Proposals related to fatigue analysis for out-of-phase multiaxial loading with variable amplitudes as the main goal of this dissertation are then reviewed and discussed at the end of Chapter 2.

Experimental program including material and specimen fabrications, testing apparatus, and test methods used are presented in Chapter 3. Test methods and results are presented for monotonic axial and torsion tests as well as constant and variable amplitude multiaxial fatigue tests. Hysteresis loops for in-phase and 90° out-of-phase loading as well as softening/hardening behaviors observed for constant and variable amplitude multiaxial tests are also presented and discussed in this chapter.

The effect of hardness and microstructure on additional non-proportional hardening is presented in Chapter 4. A predictive equation for non-proportional cyclic hardening based on the experimental observations is developed in this chapter. Chapter 4 also includes a discussion on the effect of straining sequence on the stress response of materials with different constitutive behaviors. The capability of an empirical formulation as well as a plasticity model for prediction of stress response under some discrimination strain paths are also examined and discussed in Chapter 4.

Fatigue behavior and life predictions are presented in Chapter 5. First, the effect of hardness on fatigue behavior is presented. Then, the capability of some well-known multiaxial models in fatigue life prediction for different materials and hardness levels is
discusses. A multiaxial fatigue model only based on hardness level for steels is also presented in Chapter 5. Fatigue behaviors of materials with different constitutive behaviors under discriminating strain paths and the capability of available proposals for variable amplitude multiaxial fatigue life predictions are also discussed in this chapter.

Cracking behavior and analysis are presented in Chapter 6. Effects of strain amplitude level, load non-proportionality, material ductility, and specimen geometry on crack initiation and growth behaviors are discussed in this chapter. The capability of Reddy-Fatemi effective strain-based intensity factor in correlating in-phase and out-of-phase multiaxial data is also evaluated in Chapter 6. Finally, a summary of the dissertation is presented in Chapter 7. Some suggestions for further studies in multiaxial fatigue are also presented in this chapter.
Chapter 2

Literature Survey

2.1 Introduction

Multiaxial fatigue life models fall into three categories: stress-based, strain-based and energy-based models (Zenner, 2004). The stress-based models may be appropriate for application in the high cycle fatigue regime where the behavior is mainly elastic, whereas strain-based models can be used also for the low cycle fatigue regime where the behavior can be mainly inelastic. Energy-based models are proposed based on three applications; elastic energy for high cycle fatigue, plastic energy for low cycle fatigue, and criteria based on the sum of elastic and plastic energies (Macha, 2001).

In order to use multiaxial fatigue models, with the advantage of the sensitivity to the stress response under general variable amplitude multiaxial loading, one should have a good understanding of material constitutive behavior and non-proportional hardening models as well as cycle counting and cumulative damage proposals. Therefore, at the beginning of this section, fatigue loadings and its characteristics are defined. Then, some stress and strain relations are introduced. Aforementioned parts are necessary for introducing the main parts including constitutive and non-proportional hardening
proposals, multiaxial fatigue models, cycle counting methods, and cumulative damage rules.

The cyclic loading for fatigue can be categorized as constant or variable amplitude. Constant amplitude loading is usually used for material characterization. The cyclical parameters used to define constant amplitude signals are the minimum, maximum, mean, and amplitude values. The minimum and the maximum values are corresponding to extreme values of the signal. The mean value is the average of the minimum and the maximum values and the amplitude is half of the range value.

However, most component and structures are not subjected to constant amplitude cyclic loadings. As an example, Figure 2.1 shows the stress-time spectrum in the critical element of an anti-roll-bar of a vehicle. To analyze complex variable amplitude loadings, one typically decomposes loading into constant amplitude loads at different stress or strain levels using a cycle counting method. Then, damage caused by each cycle at different levels is taken into account by the means of a cumulative damage model. Cycle counting methods and cumulative damage models are discussed later in this chapter.

In multiaxial loading, it is important to have a proper definition of an equivalent stress or strain. This is an equivalent uniaxial stress or strain taking into account components of stresses and strains in all six directions. Since the most widely used equivalent stress criterion is based on the shear stress on the octahedral plane, this criterion will be defined here. This criterion, which is equivalent to von Mises or distortion energy, considers an oblique plane with equal intersect with principal axes. Shear stress and strain on the aforementioned plane are as follows:
\[
\tau_{oct} = \frac{1}{3} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)} 
\]  
(2.1)

\[
\gamma_{oct} = \frac{2}{3} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + \frac{3}{2}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2)} 
\]  
(2.2)

Based on shear stress and strain on this plane (i.e. the octahedral plane), equivalent stress and strain are defined, respectively, as follows:

\[
\sigma = \frac{1}{\sqrt{2}} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)} 
\]  
(2.3)

\[
\bar{\varepsilon} = \frac{1}{\sqrt{2}(1+\nu)} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + \frac{3}{2}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2)} 
\]  
(2.4)

Here $\nu$ is the equivalent Poisson’s ratio calculated as:

\[
\nu = \frac{\nu_e \bar{\varepsilon}_e + \nu_p \bar{\varepsilon}_p}{\bar{\varepsilon}_i} 
\]  
(2.5)

where $\bar{\varepsilon}_e$, $\bar{\varepsilon}_p$, and $\bar{\varepsilon}_i$ are equivalent or uniaxial elastic, plastic and total strains and $\nu_e$ and $\nu_p$ are elastic and plastic Poisson’s ratios, respectively. $\nu_p$ is usually considered equal to 0.5.

Equivalent stress and strain can also be defined based on principal stresses and strains as follows:

\[
\sigma = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} 
\]  
(2.6)

\[
\bar{\varepsilon} = \frac{1}{\sqrt{2}(1+\nu)} \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2} 
\]  
(2.7)

Another important consideration for studying multiaxial fatigue is proportionality or non-proportionality of cyclic loading. Any load path resulting in fixed cyclic principal stress/strain directions is proportional or in-phase. In contrast, any load path resulting in
rotation of cyclic principal stress/strain in time is considered as non-proportional load. Figure 2.2 illustrates these definitions for axial-torsion loading. Figure 2.2(a) shows the stress element, Figure 2.2(b) shows the in-phase loading signal, Figure 2.2(c) shows the stress path for in-phase and 90° out-of-phase loadings, Figure 2.2(d) shows 90° out-of-phase loading signal, and Figures 2.2(e) and 2.2(f) show Mohr’s circle of stress at different times for in-phase and 90° out-of-phase loadings, respectively.

Although, there are several definitions for proportional and non-proportional loadings, the definition often used is based on the additional non-proportional hardening due to the interaction of multiple slip systems. Therefore, any cyclic strain path resulting in the rotation of principal axes in time and consequently interaction of multiple slip systems in different directions is considered as a non-proportional loading (Socie and Marquis, 2000). If the principal directions remain fixed, then the corresponding loading history is considered as proportional. As can be seen in parts (e) and (f) of Figure 2.2, in the case of proportional loading, the size of Mohr’s circle changes during cyclic loading, but the orientation of principal stresses remains fixed, whereas for non-proportional loading, orientation of principal stresses rotates in time. This last definition is identical to the one used by Lamba and Sidebottom (1978), Kanazawa et al. (1979), McDowell (1985), and Jiang and Kurath (1997) defining the proportional loading as the loading for which the total strain is radial in a fixed direction in total strain space.

2.2 Constitutive behavior and hardening models

Plastic comes from a Greek word which means to shape. Fatigue analysis, specially in low cycle fatigue regime, requires an understanding of plasticity due to the existence of considerable cyclic plastic deformation. Therefore, many attempts have been
made to present a proper hardening model since Prager proposed a model in 1956. In this section of the literature review, basic concepts of plasticity are defined and some incremental plasticity rules and non-proportional hardening models are briefly reviewed.

An important concept in plasticity is the concept of yield surface. The yield surface is a separator convex surface between elastic and plastic regions. For the perfect plastic materials, the yield surface does not change after yielding. In contrast, for strain-hardened materials, this surface changes for the values of stress beyond the initial yield point according to a hardening rule. For instance, if \( F(\sigma_{ij}) \) is the loading function and \( k \) is a yield function depending on previous stress and strain history of the material, yielding occurs as \( F \) becomes equal to \( k \), and the yield function is defined as follows:

\[
F(\sigma_{ij}) = k \tag{2.8}
\]

For the points on the yield surface in this case three different conditions may occur: unloading, neutral loading, and loading. In the case of unloading the state of stress will move back to the elastic domain; thus, plasticity will not occur and the following relation represents this condition:

\[
dF = \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij} < 0 \tag{2.9}
\]

For the neutral loading, the state of stress will move on the yield surface and plasticity will not occur. The following relation represents the neutral condition:

\[
dF = \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij} = 0 \tag{2.10}
\]

Finally, for the loading situation, the state of stress will move outside of the yield surface and plasticity will occur and the following relation represents this condition:
\[ dF = \frac{\partial F}{\partial \sigma_{ij}} d\sigma_{ij} > 0 \] (2.11)

Another essential component of a constitutive model is a flow rule. The flow rule represents the relationship between stresses and plastic strains during plastic deformation. The total strain increment tensor is the sum of two parts: elastic, \( \varepsilon_e \), and plastic, \( \varepsilon_p \), increments:

\[ (d\varepsilon_i)_{ij} = (d\varepsilon_e)_{ij} + (d\varepsilon_p)_{ij} \] (2.12)

The plastic strain increment vector is normal to the yield surface during plastic deformation and is referred to as the postulation of Drucker (1952), as follows:

\[ (d\varepsilon_p)_{ij} = d\lambda' \frac{\partial F}{\partial \sigma_{ij}} \] (2.13)

where \( F \) is the yield function and \( \lambda' \) is a positive scalar of proportionality equal to zero in the elastic domain.

A hardening rule which describes the changes of yield function by plastic straining is also required as a component of plasticity theory. In the case of loading, two types of hardening generally occur: isotropic and kinematic. Increase in material strength because of plastic strain is the isotropic hardening. In other words, isotropic hardening can be described as the expansion of the yield surface. As shown in Figure 2.3, if a specimen is uniaxially loaded in tension beyond the yield stress and then unloaded, followed by loading in uniaxial compression, a new yield stress is obtained in tension.

When the yield surface translates in deviatoric stress space without any change in size or shape, kinematic hardening occurs. As shown in Figure 2.4, if a specimen is uniaxially loaded beyond the yield stress and then it is unloaded and reloaded in uniaxial
compression, the new yield stress in compression will be smaller than the original one. This is known as Bauschinger effect (Bauschinger, 1886), as presented in Figure 2.5.

A significant difference between isotropic and kinematic hardening is observed when the stress path changes direction. For the case of torsion from point A to point B, and again unloading to point A, then tension to point C, the differences in hardening rules are presented in Figure 2.6 (Socie and Marquis, 2000). In reality, both isotropic and kinematic hardenings occur in a material to become stabilized, while after the stabilization only kinematic hardening occurs (Ziegler, 1959). Considering two types of hardening, the von Mises yield function is expressed as follows:

\[
\sqrt{\frac{3}{2}}(s_{ij} - \alpha_{ij})(s_{ij} - \alpha_{ij}) - \sigma_y - R = 0
\]  

(2.14)

where \( \alpha_{ij} \) is the tensor which defines the center of the yield surface or back stress tensor, \( \sigma_y \) is initial yield point, and \( R \) is the isotropic hardening variable.

Cyclic behaviors of some materials such as 1100 aluminum are identical under both proportional and non-proportional loadings, whereas some materials such as stainless steels exhibit significant additional strain hardening under non-proportional loading, similar to that presented in Figure 2.7. This phenomenon was first observed by Taira et al. (1968), and then explained by Lamba and Sidebottom (1978) and Kanazawa et al. (1979) in the late 1970s.

Kanazawa et al. (1979) related the additional non-proportional hardening phenomenon to the change in slip plane from one crystallographic slip system to another due to the rotation of maximum shear plane under non-proportional loading. This can prevent the development of stable dislocation structures associated with proportional
cycling, and therefore one would expect the stress-strain curve for out-of-phase loading to rise above the in-phase stress-strain curve. Such hardening can be attributed to dislocation jogs and to intersection of active slip planes, that arise from complex dislocation movements on the many slip systems of each grain (Kanazawa et al., 1979).

Kanazawa et al. (1977) also reported shorter fatigue life due to the additional hardening under non-proportional loading as compared to the proportional loading with the same amplitude level. Subsequently, the mathematical modeling of non-proportional cyclic loading has been a topic of interest in solid mechanics (McDowell, 1985; Benallal and Marquis, 1987; Doong and Socie, 1991; Fan and Peng, 1991; Jiang and Kurath, 1997) and presents a challenging issue for design engineers due to the effect of the non-proportional hardening on fatigue life (Jordan et al., 1985; Wu and Yang, 1987; Socie, 1987; Itoh et al., 1995; Kurath et al. 1999). As a result, multiaxial fatigue models reflecting the constitutive behavior of material, such as those in Fatemi and Socie (1988) and Park and Nelson (2000) have been proposed over the last few decades.

Studies on non-proportional hardening can be categorized into three groups consisting of experimental studies, constitutive modeling, and empirical methods. Although this dissertation is not intended to provide a comprehensive review of such studies, several studies are reviewed to complement this investigation. A number of studies have investigated the effect of strain path shape, load amplitude, and load sequence on additional non-proportional hardening behavior by experimental observations. For instance, Tanaka et al. (1985a, 1985b) conducted a series of plastic strain-controlled proportional and non-proportional axial-torsional tests on 316 stainless steel. They observed that the stress response of a 90° out-of-phase path is only affected
by prior cycles of the larger amplitude (Tanaka et al., 1985a). On the other hand, the stress response of torsional path was observed to be independent of the prior cycles regardless of the amplitude level, indicating a non-hardening strain region, as previously proposed by Ohno (1982). Tanaka et al. also reported that the deformation memory effect of the prior larger amplitude cycles was erased in presence of sufficient accumulated plastic strain for non-proportional loading. However, they observed the memory effect not to vanish for non-proportional loading, if the prior cycles have very large amplitudes, which is similar to the observations of Chaboche et al. (1979).

Tanaka et al. (1985b) also conducted tests with several combinations of proportional and non-proportional strain paths on 316 stainless steel. Results indicate that proportional paths with a fixed principal direction resulted in the least cyclic strain hardening. Strain paths with intersecting proportional segments such as cruciform and stellate paths resulted in more strain hardening due to the interaction of dislocation structures in different directions. They suggest that the intersection of the stable dislocation structures explains the significant discontinuous cyclic hardening observed in cross hardening, such as when a proportional loading suddenly changes direction. The concept of cross hardening was also discussed by Krempl and Lu (1984).

Microstructural investigations have been recently done by Taleb and Hauet (2009) to explain significant cross hardening observed in 304L stainless steel under alternating axial and shear cycles. They explained the cross hardening phenomenon of this stainless steel by generation of high defects density caused by multiple slip systems, intersecting stacking faults and twins, formation of dislocation heterogeneous structures, and nucleation of martensite structure.
Tanaka et al. (1985b), however, observed the highest level of cyclic hardening for square and circular strain paths. They explained this significant cyclic strain hardening by the requirement that the plastic strain rate vector and the corresponding plastic strain vector are not in the same direction, therefore, activating additional slip systems within the material. They reported that the saturated stress level is independent of previous less significant hardening cycles due to the tendency of materials to rearrange the dislocation structures under the larger strain hardening cycle. Similar behaviors were also observed by Ohashi et al. (1985) for the same material.

Another group of non-proportional hardening cyclic plasticity studies focus on developing constitutive models, relating stress to strain or plastic strain by means of continuum mechanics. These models are mostly based on either the Mroz multiple surface (1967, 1969) or the Armstrong-Frederick (1966) plasticity formulations. Mroz multiple surface plasticity model consists of several surfaces, similar to the initial yield surface, in deviatoric stress spaces. The translation of the yield surface in Mroz model is defined in the direction of the vector connecting the current stress point with the stress point on the next surface in a way that both stress points have the same outward normal. Armstrong and Frederick considered the movement of the yield surface in the deviatoric stress space by a nonlinear kinematic hardening rule, taking into account the strain memory effect by a recovery term. Due to considerable contributions and accomplishments to Armstrong-Frederick model by Chaboche (Chaboche et al., 1979; Chaboche, 1987, 1991), this incremental plasticity model is sometimes called Armstrong-Frederick-Chaboche model (Ohno, 2008).
McDowell (1985) proposed a non-proportionality parameter based on the time derivative of the principal strain on a two-surface Mroz type model, which uses the kinematic hardening rules of Mroz and Prager for the yield and limit surfaces, respectively. Benallal and Marquis (1987) considered the non-proportionality parameter to be the angle between the stress and the plastic strain rate. Fan and Peng (1991) employed two parameters based on the Benallal and Marquis’ non-proportionality factor to distinguish lateral hardening due to sudden change in loading direction and the hardening associated with the non-proportional loading. They also addressed the different load history effects for wavy slip and planar slip materials by introducing deformation memory parameters. Doong and Socie (1991) proposed a constitutive model to address the anisotropic deformation behavior of metals after plastic deformation, based on a two-surface form of Mroz plasticity model. They considered the non-proportionality parameter to be a function of the plastic strain history.

Chaboche and Rousselier (1983) modified the Armstrong-Frederick model by decomposing the total backstress parameter into several parts, each of which independently satisfies the Armstrong-Frederick relation. Tanaka (1994) incorporated a non-proportionality parameter based on the state of internal dislocation structures into Chaboche’s viscoplastic model to explain the experimental observations from several studies (Tanaka et al., 1985a, 1985b; Benallal et al., 1989; Nishino et al., 1986), investigating load path shape and amplitude effects on non-proportional hardening. Tanaka considered the non-proportionality parameter to be a function of the normalized plastic strain rate vector and the internal microstructure of material, which is represented by a fourth rank tensor in a 5-D plastic strain vector space (Tanaka, 1984). Based on this
model, he reported consistent results between experimental observations and predictions for a wide variety of load paths involving 316 stainless steel (Tanaka, 1994).

Jiang and Kurath (1996a, 1996b) comprehensively evaluated the Armstrong-Frederick and Mroz multiple surface plasticity models and reported the Armstrong-Frederick model to be a proper foundation for modeling material behaviors such as cyclic softening and hardening as well as the additional non-proportional hardening. Plasticity theories including modified Armstrong-Frederick rules as well as two surface and multiple surface models are also comprehensively reviewed by Chaboche (2008).

Nevertheless, cyclic plasticity models often require a large number of material constants, making them difficult for use in many industrial applications. Estimation of non-proportional hardening based on the amplitude strain path using a phenomenological approach is the emphasis of another group of studies. Most of these methods have been proposed to be used for fatigue life predictions under non-proportional loading conditions. One such method to evaluate the factor of non-proportionality, $F$, was proposed by Kanazawa et al. (1979). This method is based on the interaction of slip bands on different planes due to the change in the maximum shear plane within the material. It is defined as the fraction of shear strain at 45° to maximum shear strain range. The factor of non-proportionality, $F$, in this model is a factor of ellipticity of the strain path in the $\gamma/2-\varepsilon$ plot, as shown in Figure 2.9, defining the ratio of the minor to major axes of the circumstantial ellipse (Socie and Marquis, 2000). Then, the additional non-proportional hardening can be assumed as the following correction in the cyclic behavior:

$$\frac{\Delta \overline{\sigma}}{2} = K'(1 + \alpha F) \left( \frac{\Delta \overline{\varepsilon}_p}{2} \right)^\nu$$  \hspace{1cm} (2.15)
where $\Delta \bar{\sigma} / 2$ is the equivalent stress amplitude, $\Delta \overline{E_p} / 2$ is the equivalent plastic strain amplitude, $K'_p$ is the proportional (or uniaxial) cyclic strength coefficient, $n'_p$ is the proportional (uniaxial) cyclic strength exponent, and $\alpha$ is the material dependent non-proportional hardening coefficient.

Itoh et al. (1995) and Kida et al. (1997) proposed a formulation for the factor of non-proportionality computed from the strain path that allows evaluating the stress response directly from the strain history. They considered the non-proportionality parameter as the angle change of principal strain direction and strain path length after changing direction on a $\gamma/\sqrt{3} - \varepsilon$ plot as follows:

$$F = \frac{1.57}{T_c\varepsilon_{\text{Imax}}} \int_0^T (\sin \xi(t) |\varepsilon_f(t)|) dt$$ (2.16)

where $\varepsilon_f(t)$ is the principal strain at time $t$, $\varepsilon_{\text{Imax}}$ is the maximum absolute value of $\varepsilon_f(t)$, and $\xi(t)$ is the angle between $\varepsilon_f(t)$ and $\varepsilon_{\text{Imax}}$, and $T_c$ is the time for cycle. These quantities are defined in Figure 2.9. The strain path length is defined as a straight line connecting two points with the utmost distance on the strain path. Itoh et al. (2004) incorporated this factor of non-proportionality into an incremental multiple surface plasticity model, requiring only six material properties, and reported satisfactory predictions for 304 stainless steel and 6061 aluminum under various loading paths. The multiple surface plasticity model, strata, had been proposed by Obataya et al. (Obataya and Kato, 1998; 2001) to consider the activation state of slip system within polycrystalline metals under control of shear loading. The activation state of a slip system and the direction of the incremental plastic shear strain in this model are defined by a second order tensor.
Another method which directly extracts the maximum shear stress amplitude from the loading path is the Minimum Circumstantial Circle (MCC) proposed by Dang Van et al. (1989b) and Papadopoulos (1998) and later used by Li et al. (2002). This method estimates the shear stress amplitude as the radius of the minimum circle circumscribing to the load path in the deviatoric stress space. The Minimum Circumstantial Ellipse (MCE) (Li et al., 2002) is a similar method considering the shear stress as \( \sqrt{R_a^2 + R_b^2} \), where \( R_a \) and \( R_b \) are the length of the axes of the minimum circumscribed ellipse around the whole loading path in deviatoric stress space. Figure 2.9(c) compares MCE and MCC together.

Nevertheless, the level of strain hardening in Equation (2.15) is a function of both the factor of non-proportionality, \( F \), and the non-proportional cyclic hardening coefficient, \( \alpha \). Maximum level of non-proportional hardening occurs under 90° out-of-phase strain path, which results in complete rotation of principal axes and activation of slip systems in all directions. Therefore, non-proportional cyclic hardening coefficient is defined as the ratio of the subtraction of the equivalent stress under proportional (in-phase) loading from the equivalent stress under 90° out-of-phase loading to the equivalent stress under in-phase loading (see Figure 2.7). While this ratio is often obtained at high plastic strains where the stress-strain curve is relatively flat (Socie and Marquis, 2000), this coefficient may be defined at any plastic strain level as follows:

\[
\alpha = \frac{\sigma_{op}}{\sigma_{ip}} - 1
\]  

(2.17)

where \( \sigma_{op} \) is 90° out-of-phase equivalent stress and \( \sigma_{ip} \) is in-phase equivalent stress at the same strain level.
The 90° out-of-phase cyclic curves of materials are not commonly available. This is mainly due to the fact that multiaxial fatigue testing facilities are not widely available and preparation of thin-walled tube specimens needed for such testing is much more costly and time consuming as compared to standard uniaxial specimens. Therefore, a prediction method for non-proportional hardening based on simple and commonly available uniaxial properties of metallic materials is very useful for industrial applications. This may be done by relating the non-proportional cyclic hardening phenomenon to the uniaxial monotonic and cyclic deformations by investigating the effects of microstructure on deformation aspects of metals.

Doong et al. (1990) utilized polycrystalline 1100 aluminum, OFHC copper, and types 304 and 310 stainless steel, with no sensitivity for aluminum, medium sensitivity for copper, and high sensitivity for stainless steels to the non-proportionality of loading, to study the effect of microstructure and slip character on non-proportional hardening. They observed that stainless steels with low Stacking Fault Energy (SFE) exhibit planar slip, whereas aluminum with high SFE has wavy slip. OFHC copper with medium degree of non-proportional hardening and SFE represents a combination of wavy and planar slip modes. They conclude that the level of strain hardening decreases by an increase in SFE.

Liu et al. (2009) reported exhibition of labyrinth substructure of dislocations for 6063 aluminum under 90° out-of-phase loading, and therefore, a decrease in dislocation movement, resulting in severe cyclic strain hardening. Although aluminums usually have high SFE levels and do not exhibit non-proportional cyclic hardening, the SFE of this alloy was reduced by addition of Mg (Wang et al., 2008). Similar effects of alloying elements for face-center-cubic (fcc) materials are also reported in (Ohkubo et al., 1994;
Ebrahimi et al., 2004; Dumay et al., 2008). Increase in the SFE by adding elements to the base material can result in a decrease in strain hardening. However, strain hardening rate increases by additive elements which decrease the SFE of fcc materials such as austenitic stainless steels (Ohkubo et al., 1994; Dumay et al., 2008) and nano-crystalline nickel (Ebrahimi et al., 2004).

Li and Almazouri (2009) studied deformation and microstructure of stainless steels with different SFE and reported a linear banded dislocation microstructure for the lower SFE alloy. This causes formation of twins and martensitic transformation and, therefore, an increase in strain hardening. However, the relation of SFE and the level of monotonic and cyclic strain hardening proposed in the aforementioned studies are only qualitative.

Borodii and Shukaev (2007) noted that the monotonic strain hardening exponent decreases by an increase in SFE. They ranked different materials in term of decrease in SFE and proposed an approximate relation for the non-proportional cyclic hardening coefficient. This relation is based on monotonic strength properties of the material as follows:

$$\log|\alpha| = 0.705\beta - 1.22$$  \hspace{1cm} (2.18)

where the static hardening parameter, $\beta$, is a dimensionless parameter related to strain hardening exponent and defined as the ratio of the material ultimate tensile strength, $\sigma_u$, to the yield strength, $\sigma_y$, as:

$$\beta = \frac{\sigma_u}{\sigma_y} - 1$$  \hspace{1cm} (2.19)
This relation is limited to the cyclic strain amplitudes from $\Delta \varepsilon_p/2 > 0.02\%$ to $\Delta \varepsilon/2 < 1\%$.

Although stress response is calculated from the load and torque measured experimentally from the test, the capability of a Frederick-Armstrong plasticity model coupled with the Tanaka’s non-proportionality parameter (1994) in predicting stress response of materials with different sensitivities to non-proportional loading under various loading paths is evaluated in this study. Kanazawa et al. (1979) empirical non-proportionality method, as the most commonly used empirical method, is also employed in this study to evaluate its capability in stress response predictions under various discriminating strain paths for 1050 QT steel and 304L stainless steel. A model relating the non-proportional cyclic hardening coefficient to available or easily obtained material properties is also developed.

### 2.3 Multiaxial fatigue models

Multiaxial fatigue models can be categorized into classical and critical plane approaches. Classical strength theories were developed in the late nineteenth up to middle of twentieth century (Zenner, 2004). These classical hypotheses usually do not reflect the damage mechanism (Socie and Marquis, 2000). von Mises as the distortion energy criterion is the most commonly used classical strength theory. However, this criterion can not correlate in-phase and 90° out-of-phase test data (Fatemi and Socie, 1989) and it does not correlate data from different ratios of axial-torsional fatigue tests (Fatemi and Stephens, 1989; Fatemi and Socie, 1989). Therefore, using these models results in significant error in case of non-proportional biaxial loading conditions. In addition, these
models are not sensitive to deformation memory effects. Gough models (Gough and Pollard, 1935; Gough, 1950) as an ellipse taking into account both axial and shear fatigue strength for brittle or ductile behaving materials and Sines models (1955, 1959) based on distortion energy criterion for ductile behaving materials are two more examples of classical strength theories.

In order to overcome the classical fatigue models shortcoming to work for non-proportional multiaxial loadings, critical plane approaches which consider specific plane(s) with the maximum damage have been proposed over the last few decades. Critical plane models can be used for both proportional and non-proportional loading conditions. These models take into account localized phenomenon of crack initiation and can also predict the crack initiation angle within the material.

Critical plane approaches which are typically based on either the maximum principal plane or the maximum shear plane failure mode can be divided into three groups. The first group are stress-based models which may be appropriate in high cycle regime where the plastic deformation is negligible. Findley (Findley, 1953, 1956, 1959; Findley et al., 1956) and McDiarmid (1991, 1994) models are two well-known stress-based critical plane models for materials with shear failure mode. The second group of critical plane models are the strain-based models, such as Brown and Miller (BM) model (Brown and Miller, 1973, 1979; Kandil et al., 1982; Wang and Brown, 1993) for materials which fail on the plane of maximum shear. These models may be suitable in presence of plastic deformation; however, they cannot reflect the constitutive behavior of material including additional hardening due to non-proportionality of loadings and deformation memory effects.
Models with both stress and strain terms are another group of critical plane approaches, such as Smith-Watson-Topper (SWT) (1970) and Fatemi-Socie (FS) (1988) models for materials with tensile failure mode and shear failure mode, respectively. These models reflect the constitutive behavior of material including mean stress effect and non-proportional hardening (Socie 1987; Fatemi and Kurath, 1988). It has been shown over the last two decades that FS critical plane model works well for various materials and loading paths (Park and Nelson, 2000; Kim and Park, 1999; Han et al., 2002; Chen et al., 2004a). Since critical plane energy-based models such as Liu (1993), Chu et al. (1993), and Glinka et al. (1995) include both stress and strain terms, they may also be appropriate for such loadings.

In general, a good multiaxial fatigue model should be physically-based, sensitive to material deformation behavior, robust, and applicable to variable amplitude loading. A physically-based model reflects damage mechanism including tensile versus shear failure modes, as shown in Figure 2.10. Shear stress or strain usually initiates cracks in ductile behaving materials, where tensile stress or strain typically causes crack initiation for brittle behaving materials. A physically-based multiaxial fatigue model should also reflect the fact that tension loading opens the crack and intensifies damage, whereas compressive loading closes the crack and decreases the fatigue damage driving force.

In addition, a multiaxial fatigue model should take into account plastic deformation in low cycle fatigue and reflect the constitutive behavior of material including non-proportional hardening and deformation history effects. A good constitutive model considers both yield surface expansion (isotropic hardening) and translation (kinematic hardening) during yielding of material. Another important
consideration is additional hardening due to the non-proportional loading which may cause higher stress and lower fatigue life. This character is very important for more accurate stress analysis and fatigue life assessment under non-proportional multiaxial loading conditions.

Furthermore, a multiaxial fatigue model should reflect the facts that normal mean stress affects both tension and torsion fatigue and shear mean stress affects neither tension nor torsion fatigue as long as shear mean stress is below yielding (Sines and Ohgi, 1981; Davoli et al., 2003; Bernasconi et al., 2004). Moreover, a proper multiaxial fatigue model takes into account the fact that tensile mean stress is detrimental for fatigue life, whereas compressive mean stress is beneficial.

A multiaxial fatigue model should work for wide variety of stress states. During bending-torsion loading, principal stresses are in the second or fourth quadrants of 2-D stress space. In contrast, principal stresses are in the first and third quadrants for tension-tension loadings (Socie and Marquis, 2000). A good multiaxial fatigue model is not limited to a specific loading condition and should work for all stress quadrants and loading conditions. In addition, such a model should be able to predict fatigue life for variable amplitude loadings. As an example, for predictions under variable amplitude multiaxial loading, a proper model should not accumulate fatigue damage from holding times with non-cyclic loads (such as static loads) during the variable amplitude load history.

2.4 Cycle counting and application to variable amplitude loading

Socie and Marquis (2000) note that “Variable amplitude multiaxial loading is essentially an issue of how cycles will be identified and damage computed for each cycle
or reversal in a complex loading history." Indeed, for variable amplitude proportional and non-proportional multiaxial loading, a significant step is to identify the relation between multiple stresses and strains and fatigue damage parameter. Therefore, there are four necessities for life estimation in the case of variable amplitude multiaxial loading: a constitutive model, a cycle counting method, a multiaxial fatigue damage model, and a cumulative damage rule. Constitutive models and multiaxial damage models were reviewed in previous sections. Cycles counting methods as well as cumulative damage rules are presented in this section of the literature review.

In the case of uniaxial loading, many cycle counting methods have been proposed. However, the most popular cycle counting method is the rainflow proposed by Matsuishi and Endo (1968). This model is based on rain falling down on pagoda roof represented by the history of peaks and valleys as shown in Figure 2.11. Figure 2.12 represents the fact that in this method for a closed hysteresis loop, one cycle is counted, and for an open hysteresis loop, half a cycle is taken into account. Figure 2.13 shows an example for cycle counting by rainflow method. Detailed procedure of the rainflow cycle counting method can be found in ASTM standard (ASTM Standard E 1049-85, 2007). A variation of rainflow cycle counting method for repeating load blocks is the simplified rainflow. Figure 2.14 presents an example of cycle counting using this method. In this method only full cycles are counted. Detailed procedure of the simplified rainflow cycle counting method can also be found in the ASTM standard (ASTM Standard E 1049-85, 2007).

In the case of proportional multiaxial loading, one may use different proposals for signed equivalent stresses or strains (SAE fatigue design handbook, 1997) to convert multiaxial stresses or strains to an equivalent stress or strain. Then, aforementioned cycle
counting methods may be used to identify cycle ranges and numbers. Nevertheless, for non-proportional loading situations, there is not a consistent method to define equivalent strain or stress ranges (Socie and Marquis, 2000). Scalar damage models such as Sines or equivalent stress or strain criteria, as well as classical energy-based methods, which are scalar quantities, are limited to the case of variable amplitude proportional multiaxial loading. Therefore, only critical plane approaches can be used for a general variable amplitude multiaxial loading histories.

As mentioned before, critical plane stress-strain approaches have the advantage of considering constitutive behavior of the material including additional non-proportional hardening and deformation memory effects. However, due to the stress term in such models, a cyclic plasticity model is required to compute stress-strain histories. Then, a cycles counting method such as rainflow can be employed for counting variable amplitude non-proportional loading cycles. Then, fatigue damage for each counted cycle may be calculated using the proper multiaxial fatigue parameter. Finally, total damage can be found by employing an appropriate cumulative damage rule.

Critical plane models which consider the plane corresponding to the maximum damage parameter as the critical plane, rather than maximum shear or normal strain plane, are more reliable (Socie and Marquis, 2000; Chen et al., 2006a). For strain-stress-based critical plane approaches, after identifying strain cycles using a cycle counting method and finding stress responses corresponding to each counted strain cycle, damage parameter can be calculated. However, due to the fact that the critical plane is not easily identified under complex variable amplitude non-proportional multiaxial loading, damage parameter should be calculated for many planes within the material.
In order to account for accumulation of damage, depending on material and loading condition, one may use Palmgren (1924)-Miner (1945) linear damage rule (LDR). This model is simple and no additional material constant is needed. Linear damage rule is expressed as:

\[
D = \sum \frac{n_i}{N_{\beta}} = \frac{n_1}{N_{f_1}} + \frac{n_2}{N_{f_2}} + \cdots 
\]  

(2.20)

where \( n_i \) is the number of cycles at a given stress or strain level and \( N_{\beta} \) is the number of cycles to failure associated with the same stress or strain level. This model predicts failure when the summation of cycle ratios approaches 1, which is not always true (Stephens et al., 2000; Agreskov, 2000; Shen et al., 2000) and can be expressed with a log-normal distribution with the mean value of 1 (Agreskov, 2000; Shen et al., 2000).

In case of sequence and interaction of load events not accounted for by linear damage rule, one may use a nonlinear damage accumulation rule. Cumulative fatigue damage rules were reviewed and comprehensively discussed by Fatemi and Yang (1998). These nonlinear cumulative damage rules may consider an exponential form of linear damage rule, such as Marko and Starkey (1954) model, as:

\[
D = \sum \left( \frac{n_i}{N_{\beta}} \right)^{\eta_i} 
\]  

(2.21)

where \( \eta_i \)'s depend on the load level to take into account the sequential effect of loadings.

Based on the Grover (1960) proposal to accumulate two stages of linear damage for crack initiation and propagation separately, Manson proposed the double linear damage rule to consider two load level effects by determining the knee point as the intersection of two linear damage lines.
Although many nonlinear damage rules have been proposed, linear damage rule still remains as the most commonly used rule due to its simplicity. Some of the shortcomings attributed to LDR in computing fatigue damage in the literature are not necessarily due to LDR, but due to the use of an unsuitable damage parameter. For example, rather than using stress-life or strain-life curve for fatigue life calculation, it is possible to calculate number of cycles to failure employing one of the strain-stress based critical plane approaches such as SWT and FS parameters which take into account the load sequence and strain history effects. However, while this approach addresses the deformation history dependence aspect of material fatigue damage with regards to material constitutive response in load sequence effects, it does not account for load amplitude dependence of load sequence effects which also usually exists. High amplitude cycles followed by low amplitude cycles are usually more damaging than the reverse sequence, even for materials without constitutive behavior sensitivity to load sequence.

In multiaxial fatigue, load path alteration can also affect fatigue life. For example, torsion followed by tension has been found to be more damaging than tension followed by torsion (Robillard and Cailletaud, 1991; Harada and Endo, 1991). This has been explained by torsion cycles nucleating small cracks on planes where subsequent tensile cycles can lead to their growth, while tensile cycles do not nucleate cracks on planes which can grow by torsion cycles (Socie and Marquis, 2000). In LCF of Ti and its alloy presented in Figure 2.15(a), however, tension-torsion sequence (B1 block) was observed to be slightly more damaging than torsion-tension sequence (B2 block) (Shamsaei et al., 2010b).
In addition to axial (i.e. tension)-torsion and torsion-axial sequences, Fig. 2.15(a) also includes sequences of axial or torsion with 90° out-of-phase straining. Each part of the sequence for each material in this figure has identical effective strain amplitude (0.9% for Ti and 1% for its alloy) and all strain blocks are fully-reversed (i.e. no mean stress). As seen from Figure 2.15(a), shorter fatigue lives are observed by 90° out-of-phase loading followed by axial loading (B6 strain block), as compared to axial loading followed by 90° out-of-phase loading (B3 strain block). In addition, 90° out-of-phase followed by axial loading (B6 strain block) is more damaging than 90° out-of-phase followed by torsion loading (B4 strain block). It should be noted that as Ti and its alloy are not sensitive to non-proportional hardening (Shamsaei et al., 2010b), this is not a factor in the observed differences, or their explanations.

Shamsaei et al. (2010b) satisfactorily predicted fatigue lives of pure and alloyed titanium in short life regime under a variety of multiaxial loadings including step loading and sequential loading employing the FS critical plane approach and linear cumulative damage rule, as presented in Figure 2.15(b). This figure includes the loading blocks from Figure 2.15(a), as well as other blocks with several segments of axial, torsion, and 90° out-of-phase strain paths, all with the same equivalent strain, with each block repeated to failure.

There are only two proposals in the literature for variable amplitude multiaxial fatigue predictions addressing to the concept of critical plane approaches; Bannantine-Socie and Wang-Brown. These methods are briefly reviewed in the following sections.
2.4.1 Bannantine and Socie (1991a; 1991b)

Bannantine and Socie (BS) proposed a method based on the critical plane concept and rainflow cycle counting. Their method contains two failure mode possibilities; tensile and shear failures. The cycle counting is with the strain history, as the main or primary channel, mapped on the candidate failure planes within the material to determine the critical plane. Rainflow cycle counting method is applied on the shear strain history for shear failure mode materials and on the normal strain history for tensile failure mode materials. Other components of the damage parameter, auxiliary channels, such as maximum normal stress for FS and SWT damage parameters, are then determined for each counted cycle or reversal as their maximum value during the primary cycle. Since the magnitude of strain and the shape of hysteresis loop are changed for each considered plane and also they are functions of time for each single plane, the critical plane is unknown and must be found by analyzing all possible failure planes.

Since stresses and strains usually get their extreme values on free surface, critical plane should be searched on the free surface. However, for the free surface there are only four possible damaging stress systems, as shown in Figure 2.16. The following facts can be considered for free surface analysis (Socie and Marquis, 2000):

- First of all, for the plane perpendicular to the free surface, parts (a) and (c), damage is possible by normal stress or in-plane shear stress. Indeed, the out-of-plane shear stress on this plane is always smaller than in-plane shear stress; hence, the out-of-plane shear stress can be ignored.
• Normal stress on any plane other than $\phi = 0^\circ$ (parts (b) and (d)) will be smaller than normal stress on plane corresponding to $\phi = 0^\circ$ (parts (a) and (c)); consequently, there is no need to account for it.

• For plane of $\phi = 45^\circ$ (parts (b) and (d)), one should consider in-plane and out-of-plane shear stresses, where as mentioned before normal stress can be ignored.

Damage accumulation is considered using linear damage rule and the plane experiencing the maximum damage is considered as the critical plane. An example of cycle counting by BS method on a specific plane within the material is shown in Figure 2.17. Shear strain as the main component and normal stress as the auxiliary components of the FS critical plane approach are mapped on this specific plane. Rainflow cycle counting is performed on the shear strain channel and corresponding normal stress of each counted cycle is reported in the table. Bannantine and Socie (1991b) reported satisfactory predictions with their cycle counting technique for SAE notch shaft specimens made from 1045 steel under proportional and non-proportional bending-torsion SAE transmission history loading.

2.4.2 Wang and Brown (1996a)

Wang and Brown (WB) proposed a reversal counting method based on a modified von Mises equivalent strain theory. To overcome the sign problem in the von Mises equivalent strain and to be able to count out-of-phase cycles, they proposed a modified strain history by assigning a reference point as $\varepsilon_{ij}^{\text{max}}$ associated with the greatest value of the equivalent strain in the loading history. Then, the components of the modified strain
history, $\varepsilon_{ij}'$, is generated by subtracting the strain components of the turning point from the strain components at each time as follows:

$$\varepsilon_{ij}' = \varepsilon_{ij} - \varepsilon_{ij}^{\text{max}}$$  \hspace{1cm} (2.22)

This method counts reversals and discards them from the strain history. A new reference point is then defined and the process continues until all reversals are counted.

Wang and Brown method is summarized as follows:

1. Arrange data using relative equivalent strain method.
2. Consider relative equivalent strain increasing from zero to maximum as one reversal.
3. Eliminate the counted reversal. If there is any remaining cycles, go to step (1).
4. Stop counting.

Figure 2.18 presents WB reversal counting method for a cruciform fully-reversed strain path, assuming point A on the axial strain as the first reference point. Since point B is the highest point in the modified von Mises strain history and strain is always increasing from point A to point B, the first counted reversal on the modified von Mises history is AOB, which is on the axial strain cycle, shown in Figure 2.18(a). Then, by selecting point B as the second reference point, reversal BOC is counted on the modified von Mises strain history. This counted reversal is associated with reversals BO and OC on axial and shear strain histories, respectively. The same process is continued until all eight reversals are counted, as presented in Figure 2.18. The WB method is explained on a strain time history using a general non-proportional multiaxial variable amplitude loading example by Socie and Marquis (2000).
Critical plane is then identified for each counted reversal using one of the damage parameters. Wang and Brown used Kandil, Brown and Miller (1982) parameter for fatigue prediction of proportional and non-proportional variable amplitude multiaxial loading of En15R steel, stainless steel, and medium carbon steel (Wang and Brown, 1996b) and reported acceptable fatigue life predictions.

It should be mentioned that for each counted reversal, critical plane may be different; therefore, critical plane should be identified for each counted reversal. As a result, the critical plane changes for each reversal and should be an upper bound for fatigue damage. Kim et al. (1999) employed the WB cycle counting method, linear cumulative damage rule, and Kandil, Brown and Miller (1982) fatigue damage parameter for some variable amplitude loading paths on S45C steel. They employed a modified form of this strain-based damage parameter proposed by Wang and Brown (1993) and reported satisfactory predictions using either the maximum shear strain range plane or the maximum damage plane as the critical plane. Kim et al. (1999) also reported acceptable results by damage computation for each reversal or for each cycle. Damage computation for each reversal resulted in non-conservative predictions for non-proportional loading. However, this method accumulates damage from all counted reversals, even though the critical planes associated with these reversals might not be identical. Therefore, WB reversal counting method does not identify a unique critical plane for the whole loading block and cannot predict the cracking orientation within the material. In fact, this method considers a random crack orientation, which is not in agreement with experimental observations.
2.5 Discussion of the methods and relevant works

The steps for a variable amplitude multiaxial critical plane analysis can be summarized as:

1) Transform the stress and strain to different candidate planes
2) Count stress and strain cycles on the candidate planes by using a cycle counting method
3) Evaluate the fatigue damage due to each counted cycle for each plane using a critical plane damage model
4) Accumulate damage for each single considered plane using a cumulative damage rule
5) Choose the plane with the greatest value of the cumulative damage as the critical plane and calculate fatigue life on this plane

ASME boiler and pressure vessel code (1986) provides a life prediction procedure for non-proportional multiaxial loading condition. For high cycle fatigue, it is based on maximum equivalent shear stress, while for low cycle fatigue it considers equivalent octahedral strain. Although this method is easily applicable due to its simplicity, Socie and Marquis (2000) criticize this method as follows:

- This method is only usable for simple load path, since it is not accompanied with any cycle counting method.
- This method only uses two extreme values during a cycle and it ignores the load path between these two points.
- This method only suggests an average shear value over many slip systems; therefore, it considers a random crack orientation, which is not correct.
• ASME code does not consider any mean stress effect.

• Interaction among complex stress and strain components are not taken into consideration in this method.

Furthermore, Jordan et al. (1985), Andrews and Brown (1989), and Ohnami and Sakane (1992) examined this method for non-proportional loading situations and reported that the ASME design code significantly underestimates fatigue damage.

Dang Van (Dang Van et al., 1989a, 1989b; 1993, 1999) proposed an endurance limit criterion based on the concept of micro-stress within a critical volume of material. The logic of this model is based on the fact that fatigue cracks usually nucleate from intragranular slip bands. Figure 2.19 presents two methods for defining the shear stress. The left side picture shows the longest chord considered as shear stress range. Mean shear stress in this method is defined by the magnitude of the stress vector from the origin to the center of the chord. The second method proposed by Dang Van and improved later by Papadopoulos (1998) considers the radius of the minimum circumscribing circle (MCC) to the load path in deviatoric stress space as maximum shear stress amplitude. In this model, shear stress is considered as the distance between center of the circle and the corresponding point on the shear stress history on the plane. Mean shear stress in this method is considered as the magnitude of the vector connecting origin to the center of the circle (Socie and Marquis, 2000). Freitas et al. (2000) modified this approach to the minimum circumscribing ellipse (MCE) to the load path to address to the non-proportionality of loading as the ratio of the axes of the circumscribed ellipse. Freitas et al. (2000), Li and Freitas (2001), and Li et al. (2001) used this method for fatigue life prediction under multiaxial random loading. However, they refer to this method as a
simple direct approach for fast fatigue evaluation in long life regime (Li and Freitas, 2001). Since this model works for loadings where the plane of maximum shear does not significantly rotate during a loading cycle, Socie and Marquis (2000) explain that Dang Van model may lead to some error in the case of general non-proportional loading due to the change on the maximum shear plane. This approach is also limited to the long life regime, where the deformation is mostly elastic.

Another interesting study was performed by Langlais, Vogel, and Chase (2003). They argued that uniaxial cycle counting methods such as rainflow cannot be applied directly to multiaxial loading specifically in the case of out-of-phase loading. They pointed out the fact that the extreme values of strain and stress are not simultaneous for out-of-phase multiaxial loadings, whereas a reversal in strain corresponds to a reversal in stress for in-phase or uniaxial loading. Since in using rainflow cycle counting method only peaks and valleys are typically considered, non-reversal points which may contain some important information are ignored. They proposed a multichannel rainflow counting method which considers one channel as counting main channel, and all other channels as auxiliary channels. This method includes a minimum and maximum for each auxiliary channel. The algorithm associates information from removed samples with the proceeding sample to ensure that the correct minimum and maximum will be identified for each auxiliary channel for a given cycle. They examined FS damage parameter on axial-torsion fatigue experiments done by Itoh et al. (1995) on type 304 stainless steel. They demonstrated that the multichannel cycle counting method identifies the correct maximum stress, whereas the basic rainflow method usually underestimates the stress value. The maximum error was associated with the 90° out-of-phase loading.
Kim and coauthors (Han et al., 2002; Lee et al., 2003) conducted several tests with uniaxial, axial-torsion proportional and non-proportional constant amplitude and variable amplitude loads as shown in Figures 2.20 and 2.21. Their studies include a variety of steels and stainless steels such as S45C steel (Kim and Park, 1999; Lee et al., 2003), SNCM439 steel (Han et al., 2002), SNCM630 steel (Lee et al., 2003; Kim et al., 2000), stainless steel 304 (Kim et al., 2000; Lee et al., 2003; Chen et al., 2006a, 2006b), 1Cr-18Ni-9Ti stainless steel (Chen et al., 2003), and 63Sn-37Pb (Chen et al., 2006c). They used different types of loading to distinguish between multiaxial critical plane approaches such as Brown-Miller (1973 and 1979), Wang-Brown (1993), SWT, and FS as strain and strain-stress based models, as well as several energy-based models. Their life assessment procedure was based on cycle counting on the critical plane by rainflow method, then using the above-mentioned critical plane approaches to evaluate the damage for each counted cycle, and finally adding the damage by a cumulative damage model to estimate the fatigue life.

Wang-Brown (1993) and FS models, which predict failure on the maximum shear plane, were reported by Kim et al. to correlate the data within a factor of 2 on life (Lee et al., 2003; Chen et al., 2006a, 2006c). Nevertheless, only the FS model, which takes into consideration the non-proportional additional hardening, correlated data well for materials that exhibited additional hardening such as 1Cr-18Ni-9Ti stainless steel (Chen et al., 2003). Furthermore, Kim and coauthors (Chen et al., 2006a) utilized these two multiaxial fatigue damage models on the plane with maximum fatigue damage, rather than on the maximum shear plane, and reported some improvement for both models, but slightly more conservative results for the FS critical plane approach.
In order to accumulate damage of each counted cycle, Kim et al. used LDR for materials such as S45C steel and SNCM439 steel with no significant memory effect (Kim and Park, 1999; Han, et al., 2002; Lee et al., 2003), and Morrow’s plastic work non-linear damage rule (Kurath et al., 1983; Morrow, 1986) to take into account load sequence effects for materials with considerable memory effect such as 304 stainless steel (Chen et al., 2006a, 2006b).

Jiang (2000) proposed a general multiaxial fatigue model incorporating the concepts of critical plane approach and material memory based on an incremental cyclic plasticity model. Cycle counting is embedded in the material memory model, but it is only applicable in presence of considerable plastic deformation. Therefore, cycle counting using either BS or WB methods and employing critical plane fatigue damage parameters seem to be the only current general proposed methodologies applicable to different materials, life regimes, and loading conditions. However, neither BS nor WB multiaxial cycle counting method has been yet proven to work well for all types of loading conditions.

Although service loads are typically variable amplitude and multiaxial, the number of studies on this subject is small. Moreover, most of the works are not comprehensive enough to cover a complete analysis. Constructing a comprehensive and simple procedure addressing all elements of variable amplitude multiaxial fatigue analysis is still a topic of interest and a necessity in many industries dealing with components and structures working under such loadings. This procedure should consider cumulative damage on critical plane and take into account constitutive behavior and non-proportional hardening. In addition, it should be along with a multiaxial cycle counting
method reflecting mean stress effects as well as interaction of stresses and strains for complex load paths. Discriminating loading paths can be designed to distinguish among different models and verify robustness of the methodology.
Figure 2.1 Stress-time spectrum in critical element of anti-roll-bar of a vehicle (Shamsaei, 2001).
Figure 2.2 Illustration of in-phase and 90° out-of-phase loading and the effect of out-of-phase loading on Mohr’s circle and rotation of principal axes (Stephens et al., 2000).
Figure 2.3  Isotropic hardening representing the expansion of yield surface (Socie and Marquis, 2000).

Figure 2.4  Kinematic hardening representing the translation of yield surface (Socie and Marquis, 2000).
Figure 2.5 Bauschinger effect on yield points (Stephens et al., 2000).

Figure 2.6 (a) Isotropic hardening and (b) kinematic hardening during non-proportional cyclic loading (Socie and Marquis, 2000).
Figure 2.7  Cyclic stress-strain curve for proportional and non-proportional loading (Socie and Marquis, 2000).

Figure 2.8  Illustration of the factor of non-proportionality based on Kanazawa et al. (1979) for three load paths: (a) proportional, (b) 90° out-of-phase, and (c) random (Socie and Marquis, 2000).
Figure 2.9  (a) Definition of $\xi(t)$ (Itoh et al., 2004), (b) polar figure of $\Delta \varepsilon_I$ (Itoh et al., 2004), and (c) descriptions of the maximum circumscribing circle (MCC) and the maximum circumscribing ellipse (MCE) (Li et al., 2002).
Figure 2.10  Crack mechanisms for ductile behaving materials (left) showing shear damage mechanism and brittle behaving materials (right) showing tensile damage mechanism (Socie and Marquis, 2000).

Figure 2.11  Rainflow cycle counting method based on the rain falling down on pagoda roof (Wright, 1993).
Figure 2.12  Rainflow cycle counting based on hysteresis loops, counting one cycle for a closed hysteresis loop and half a cycle for an open hysteresis loop (Collins, 1993).
Figure 2.13  Rainflow counting method (ASTM Standard E 1049-85, 2007).
Figure 2.14  Simplified rainflow counting method (ASTM Standard E 1049-85, 2007).
Figure 2.15  (a) Observed fatigue lives for several strain block loadings of pure Ti and its alloy. (b) Comparison of observed and calculated fatigue lives by LDR and FS damage parameter (Shamsaei et al., 2010b).
Figure 2.16 Different failure planes in Bannantine and Socie variable amplitude multiaxial fatigue prediction method (Socie and Marquis, 2000).

Figure 2.17 An example of Bannantine and Socie cycle counting method on the critical plane.
Figure 2.18  An example of WB reversal counting method for cruciform axial-torsion strain path. Point A on the axial cycle is considered as the first reference point.
Figure 2.19 Two methods for defining shear stress in Dang Van model (Socie and Marquis, 2000).
Figure 2.20  Schematic of axial-torsion strain paths used in Kim’ studies (Han et al., 2002).
<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Strain amp. (%)</th>
<th>Strain path</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon/%$</td>
<td>$\gamma/%$</td>
</tr>
<tr>
<td>T4A101</td>
<td>0.3</td>
<td>0.98</td>
</tr>
<tr>
<td>T4A102</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>T4A103</td>
<td>0.4</td>
<td>0.7</td>
</tr>
<tr>
<td>T1A401</td>
<td>0.25</td>
<td>0.98</td>
</tr>
<tr>
<td>T1A402</td>
<td>0.4</td>
<td>0.48</td>
</tr>
<tr>
<td>T1A403</td>
<td>0.3</td>
<td>0.78</td>
</tr>
<tr>
<td>T2A301</td>
<td>0.4</td>
<td>0.48</td>
</tr>
<tr>
<td>T1A201</td>
<td>0.4</td>
<td>0.48</td>
</tr>
<tr>
<td>T1A601</td>
<td>0.4</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Figure 2.21  Axial-torsion load profiles used in Kim’s studies (Lee et al., 2003).
Chapter 3

Experimental Program

3.1 Introduction

This chapter presents the experimental program including material and specimen fabrication, testing equipment, and testing methods and procedures used. Materials and hardening conditions as well as specimen dimensions and configuration are reviewed in Section 3.2. Testing apparatus including testing load frame and extensometer are presented in Section 3.3. Section 3.4 contains testing methods and procedures for monotonic axial and torsional tests as well as constant and variable amplitudes multiaxial fatigue tests. Replication technique used to monitor crack formation and growth is explained in Section 3.5. Finally, in Section 3.6 experimental results and observations from these tests are presented.

3.2 Material and specimen fabrication

Normalized bars of 1050 steel with Brinell hardness ($HB$) level of $HB = 198$ and pearlite and ferrite microstructure (shown in Figure 3.1(a)) were machined to the tubular specimens with a gage section wall thickness of 1.25 mm based on ASTM standard E2207 (2007). Configuration and dimensions of tubular specimens are shown in Figure
3.2. Chemical composition of 1050 steel is presented in Table 3.1. Some of the normalized tubular specimens were induction-hardened to $HB = 565$ with a tempered martensite microstructure as shown in Figure 3.1(c). Some normalized solid bars were also first made tubular and then quenched and tempered to $HB = 360$ with a tempered martensite microstructure shown in Figure 3.1(b). After the hardening process, the quenched and tempered bars were then machined to tubular round specimen dimensions shown in Figure 3.2, except with an ID of 15.2 mm and OD of 17.8 mm.

Although most shafts under torsional or multiaxial loadings are made of solid sections, most of the multiaxial fatigue data are generated with thin-walled tubular geometries. Using both solid and tubular specimens reveals the effect of specimen geometry on multiaxial fatigue life. Therefore, some normalized solid specimens were also machined to the same outer configuration and dimensions of the tubular specimens in Figure 3.2, but with no hole.

In addition, 304L stainless steel with the chemical composition presented in Table 3.1 and austenitic microstructure was also utilized in this study. Plates with 30 cm ×50 cm × 3 cm dimensions were cut into identical 3 cm × 3 cm × 15 cm square cross section bars. Length of the bars, and therefore, the specimen longitudinal axis was along the rolling direction of the plate. Then, these bars were machined to the tubular specimens’ configuration and dimensions presented in Figure 3.2.

The inside of the tubular specimens were honed and three different aluminum oxide $30\mu m$, $12\mu m$, and $3\mu m$ films were used for both solid and tubular specimens to provide a near to mirror outer surface finish in the gage section surface of all specimens. The polished surfaces were carefully examined under magnification to ensure complete
removal of machining marks within the test section. It should be mentioned that for the induction-hardened specimens, specimen polishing including honing was done prior to the heat treatment.

### 3.3 Testing equipment

An Instron closed-loop servo-hydraulic axial-torsional load frame, as shown in Figure 3.3, in conjunction with a digital controller was used to conduct the tests. The capacity of the load cell was 1 kNm in torsion and 100 kN in axial direction. Hydraulically operated grips using universal tapered collets were employed to secure the specimens' ends in series with the load cell.

In order to control or monitor axial and torsional strains, an Epsilon axial-torsional extensometer, shown in Figure 3.4, was employed. The extensometer gage length is 25.4 mm; hence, it could be easily mounted on the uniform specimens’ gage section which is 28.6 mm. The calibration of the extensometer was verified using rotation and displacement apparatus containing two micrometer barrels in divisions of 0.00254 mm based on ASTM standard E83-06 (2007). The abovementioned extensometer was designed to be used for gage section diameters from 12.7 to 50.8 mm. Its axial capacity is ±10% strain and torsional capacity is ±5° angle of twist or ±4.3% torsional strain.

In order to protect the specimen surface from the extensometer contact points, three layers of transparent tape, as recommended by ASTM Standard E606 (2007), were used. Before running the test, each specimen was elastically loaded in axial and torsion and axial strain and angle of twist were measured. Proper extensometer setup was verified by calculating the modulus of elasticity, $E$, and shear modulus, $G$. 
Significant effort was put forth to align the load train (load cell, grips, specimen, and actuator). Misalignment may be resulted from both tilt and offset between the central lines of the load train components. ASTM Standard E1012 (2007), Type A, Method 1 was followed to verify specimen alignment. For this procedure, two arrays of four strain gages per array were arranged at the upper and lower ends of a 15.2 mm diameter solid specimen with a uniform gage section. For each array, gages were equally spaced around the circumference. The maximum bending strain determined from the gage specimen was within the allowable ASTM limit.

3.4 Test methods and procedures

All tests in this study were performed using test methods specified by ASTM standards, wherever applicable.

3.4.1 Monotonic tests

Two specimens for each hardness condition of 1050 steel were used for axial and torsional monotonic tests to obtain the monotonic properties. However, monotonic axial test for induction hardened condition was only limited to the elastic behavior due to inadequate axial capacity of the testing machine.

Axial monotonic tests were conducted in position control with the rate of 0.001 mm/s, which is equal to $4 \times 10^{-5}$ mm/mm/s strain rate. This strain rate was three-quarter of the maximum allowable rate specified by ASTM Standard E8 (2007) for the initial yield region. In order to protect the extensometer, the extensometer was used up to 9% axial strain. After this point, the extensometer was removed and the test was continued in
position control until fracture. Intron MAX software was used and torsional channel was kept in torque control at zero value.

For torsional monotonic tests, the controlled parameter was rotation with the rate of 0.025 degrees/s to produce a constant equivalent strain rate of the same order as in axial monotonic tests. Again, in order to protect the extensometer, the extensometer was used up to 4.5° angle of twist, about 4% shear strain, in torsional channel. After this point, the extensometer was removed and the test was continued in rotation control until specimen fractured. For monotonic torsion tests, as ASTM Standard E2207 (2007) recommends, the axial channel was in load control at zero value, allowing the specimen to change in length and avoiding any axial stress. Intron MAX software was used for all monotonic torsional tests.

3.4.2 Constant amplitude multiaxial fatigue tests

All constant amplitude multiaxial fatigue tests in this study were performed according to ASTM Standard E2207 (2007). In-phase and 90° out-of-phase axial-torsional loadings are shown in Figure 3.5. For this study, at least 12 thin-walled tubular specimens for each hardness level of 1050 steel under in-phase (IP) and 90° out-of-phase (OP) loading at three equivalent strain amplitudes with strain ratio of $\lambda = \gamma / \varepsilon = \sqrt{3}$ ranging from 0.225% to 1.5% were utilized. Furthermore, seven additional normalized 1050 steel solid specimens were used under IP and OP constant amplitude multiaxial loadings at the same strain amplitude levels as tubular specimens to study the effect of specimen geometry. Ten tubular specimens of 304L stainless steel were also used for
several equivalent strain amplitude levels for IP and OP constant amplitude multiaxial fatigue tests.

During each strain-controlled multiaxial test, axial strain and total angle of twist in the gage section, and therefore shear strain, were recorded using the extensometer output. Test data including load/torque, axial strain/angle of twist in the gage section, and total axial/torsional displacement of actuator, were automatically recorded throughout each test in logarithmic order with the base of 2.

There were two control modes used for constant amplitude multiaxial tests of 1050 steel. Strain control was used in all tests for about half the life using the extensometer. After ensuring the cycles become stable, in order to protect the extensometer from large crack or fracture of specimen as well as to be able to take surface replicas, gage section axial/shear strain control mode was switched to actuator axial/torsional displacement control at the same displacement as prior to the control mode switch. After reaching the axial/torsional strains the same as those in strain-controlled test, the test was stopped, the extensometer was taken off, and the test was continued in displacement control until failure. Strain control mode was used for the entire duration of all multiaxial fatigue tests of 304L stainless steel.

For the constant amplitude fatigue tests, the applied frequencies ranged from 0.1 Hz to 3 Hz in order to have a nearly constant plastic strain rate. Instron MAX software was used for both strain-controlled and displacement-controlled tests and sinusoidal waveform was applied in both channels.

Failure criterion was considered as 5% load or torque drop (whichever occurred first) compared with the stable value obtained at midlife. This load/torque drop
corresponded to final crack length(s) of at least 10 mm long on the specimen. The range of crack lengths observed at this load or torque drop was between 10 mm to few centimeters. The location and orientation of the crack was also monitored on each specimen.

3.4.3 Variable amplitude multiaxial fatigue tests

All variable amplitude multiaxial fatigue tests were performed according to ASTM Standard E2207 (2007). For this study, 15 quenched and tempered (QT) 1050 steel specimens and 14 specimens of 304L stainless steel for some specific blocks of multiaxial strain paths were utilized. The reason of choosing these two materials was the fact that 1050 QT steel is not sensitive to non-proportional loading, whereas 304L stainless steel exhibits more than 50% additional non-proportional hardening. Axial strain and total angle of twist in the gage section, and therefore shear strain, were recorded using the extensometer output. All the test data; load, torque, axial and torsional strain and displacements were automatically recorded throughout each test for specified loading blocks.

Similar to constant amplitude multiaxial tests, two control modes were used for some of these tests. First, strain-controlled was used for about half the life using the extensometer, and then, gage section axial/shear strain control mode was switched to actuator axial/torsional displacement control at the same displacement as prior to the control mode switch. As ASTM standard recommends, applied frequencies were chosen to have a nearly constant plastic strain rate. Instron WaveMatrix software was used for both strain-controlled and displacement-controlled tests and variable amplitude waveforms were applied in both channels. Failure criterion was considered as 5% load or
torque drop (whichever occurred first) compared with the stable values obtained at midlife block. The location and orientation of the crack was also monitored on each specimen.

Some specific star shape strain paths were utilized to investigate the capability of non-proportional hardening models in stress response prediction, as well as to assess variable amplitude multiaxial fatigue life estimation techniques, as follows:

- Fully-Reversed with 1° Increments (FRI): Proportional fully-reversed axial-torsion cycles ($\varepsilon_{\text{min}} / \varepsilon_{\text{max}} = \gamma_{\text{min}} / \gamma_{\text{max}} = -1$) with 1° increments starting from the pure axial cycle with circular boundary in $\gamma/\sqrt{3}$-axes, as shown in Figure 3.6(a). Even though this strain block includes 360 proportional cycles passing through origin, it also activates all the material planes due to the rotation of principal axes, similar to a 90° out-of-phase path. Therefore, fatigue life and additional non-proportional hardening caused by this strain path can be compared to 360 in-phase or 90° out-of-phase cycles. Furthermore, the ability of multiaxial fatigue models to work for such a loading paths which applies almost the same damage on all planes can be evaluated. Axial and shear strains versus time for a block of this load path are also presented in Figures 3.6(a).

- Fully-Reversed Random (FRR): Proportional fully-reversed axial-torsion cycles with random sequence and with circular boundary in $\gamma/\sqrt{3}$-axes, as shown in Figure 3.6(b). This strain block is the same as FRI strain path, but rather than 1° increment, the sequence of loading is random between 1° to 360°. Each sequence is repeated only once in each block. The strain block which includes 360 proportional strain cycles was used to find the effect of random load sequence with sudden change in straining direction on cyclic deformation and fatigue behaviors. Axial and shear strain time spectrums for a
quarter block of this strain path are also presented in Figures 3.6(b). Although FRI and FRR strain blocks look similar in normal strain-shear strain plot, significant difference in loading sequence is observed from strain-time plots, compared in Figures 3.6(a) and 3.6(b).

- Fully-Reversed with 15° Increments (FRI15): Proportional fully-reversed axial-torsion cycles with 15° increments starting from the pure axial cycle and with circular boundary in $\gamma/\sqrt{3}$ axes, as shown in Figure 3.6(c). This strain path is identical to FRI, except with 15° increment rather than 1°. The 15° increment is chosen to investigate the effect of incremental angle on deformation and fatigue behaviors. Although this load block including 24 proportional cycles is incremental, 15° is assumed to be a significant change in straining direction, so it may increase the interaction between slip systems, affecting more non-proportional hardening as compared to 1° increment strain path (i.e. FRI). This strain path was only used for 304L stainless steel as a sensitive material to non-proportional loading and change in straining direction.

- Pulsating with 1° Increments (PI): Proportional pulsating axial-torsion cycles ($\varepsilon_{\text{min}} / \varepsilon_{\text{max}} = \gamma_{\text{min}} / \gamma_{\text{max}} = 0$) with 1° increments starting from the pure axial cycle with circular boundary in $\gamma/\sqrt{3}-\varepsilon$ axes, as shown in Figure 3.6(d). Axial and shear strains versus time for a block of this strain path, are also presented in this figure. This strain path is similar to FRI path and includes 360 proportional cycles, which are zero to maximum cycles. Therefore, the material may exhibit stress relaxation as a result of plastic straining. Stress relaxation phenomenon is a challenging topic for both constitutive and fatigue models. In addition, the coupling of cycles from both sides of
strain path may affect fatigue life; hence, the capability of cycle counting methods to correctly identify this coupling can be also examined.

In addition to the aforementioned strain paths, two more simple cruciform paths were also utilized, FRI90 and PI90. These strain paths only include alternating pure axial and pure torsion cycles and, therefore, straining direction changes severely between axial and shear axes. FRI90 path, presented in Figure 3.7(a), is fully-reversed with 90° increments, with each block containing 4 cycles. PI90 path, presented in Figure 3.7(b), is pulsating with 90° increments, both starting from pure axial cycle with circular boundary in $\gamma/\sqrt{3}$- axes, with each block also containing 4 cycles. These strain paths were only used for quenched and tempered 1050 steel to minimize the effect of additional hardening and to check the robustness of BS and WB cycle counting methods.

Although axial and shear strain amplitudes vary from cycle to cycle in the above-mentioned strain blocks, all the strain cycles are within the same circular boundary in $\gamma/\sqrt{3}$- axes. Therefore, nearly equal equivalent strain amplitudes are applied for all cycles within a block of each strain path. These strain paths were used for at least two different levels of deformation; an intermediate strain level to induce a combination of elastic and plastic deformations and a higher strain level causing mostly plastic deformation in the materials under investigation.

### 3.5 Replication technique

Cellulose acetate sheet replication method was employed to monitor specimen surface crack development and growth for proportional (in-phase) and non-proportional (90° out-of-phase) fatigue tests. One of the duplicate constant amplitude tests of normalized and quenched and tempered tubular specimens at each strain level was used
for crack replication. Due to the fact that the ratio of crack growth to crack initiation is more dominant for solid specimens, six solid specimens were used for this purpose. Since high hardness causes fracture as soon as crack initiates, there was little crack growth for induction hardened specimens. Therefore, no replication was taken for the induction hardened specimens. Extensometer was kept on throughout all 304L stainless steel specimen tests; thus, no replication was taken for this material.

Once the axial load and torque as well as the axial and torsional displacements became stable, the test was stopped, and extensometer and transparent tape were removed. While the test was stopped at zero load and torque, replication was taken by using cellulose acetate sheets. First, the surface of the specimen was cleaned with reagent grade acetone. Next, cellulose acetate sheet was placed on the surface of specimen by spraying acetone between the acetate sheet and the surface of the specimen by a hypodermic needle. Acetate sheet melts by acetate drawn to the specimen surface, conforming to the surface topography. The cellulose sheet was marked by a permanent marker according to a mark on the specimen to record its location on the surface of the specimen for future reference. Once the acetate sheet dried after a few minutes, it was removed carefully and placed between two glass microscope slides. The same procedure was conducted for the other side of the specimen to cover the entire specimen gage section surface.

After midlife, at each 10% of the expected fatigue life, the test was stopped periodically and the aforementioned process for surface replication was repeated. This incremental crack growth replication technique gives the opportunity to go backward from the last replica to the first one and detect the length and angle of initial cracks.
causing the final failure. This information can be also used to study short crack growth behavior of materials under multiaxial axial-torsional loading.

3.6 Experimental results

In this section, first monotonic axial and torsional test results for 1050 steel in normalized (N), quenched and tempered (QT), and induction hardened (IH) conditions as well as 304L stainless steel are presented. Then, experimental results for constant amplitude in-phase (IP) and 90° out-of-phase (PI) multiaxial loadings for all materials and hardening conditions used in this study are presented. These results include axial and shear hysteresis loops as well as axial and shear stress variations over the fatigue life. Finally, test results of variable amplitude experiments are presented in terms of maximum axial and shear stress amplitude responses from the axial and torsion cycles of each strain block, respectively.

3.6.1 Monotonic experimental results

The properties determined from monotonic axial tests were: modulus of elasticity ($E$), yield strength ($\sigma_y$), ultimate tensile strength ($S_u$), strength coefficient ($K$), and strain hardening exponent ($n$). Table 3.2 contains monotonic properties obtained for the three different hardness conditions of 1050 steel as well as 304L stainless steel. Axial monotonic properties listed in this table for the stainless steel are from small solid specimen with 5 mm diameter tensile test (Colin, 2009). Due to the capacity limitation of the testing machine, axial monotonic test for induction hardened conditions could not be completed.
True stress ($\sigma$), true strain ($\varepsilon$), and true plastic strain ($\varepsilon_p$), were calculated from engineering stress ($S$), and engineering strain ($e$), according to the following relationships, which are based on constant volume assumption:

$$\sigma = S(1 + e)$$  
(3.1)

$$\varepsilon = \ln(1 + e)$$  
(3.2)

$$\varepsilon_p = \varepsilon - \varepsilon_e = \varepsilon - \frac{\sigma}{E}$$  
(3.3)

Yield strength ($\sigma_y$), is defined using 0.2% (or 0.002) strain offset of elastic portion on the stress-strain curve and engineering ultimate tensile strength ($S_U$) is defined as the maximum value of the engineering stress.

The true stress-strain ($\sigma$ - $\varepsilon$) plot is often represented by the Ramberg-Osgood equation:

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \frac{1}{K} \left( \frac{\sigma}{K} \right)^n$$  
(3.4)

The strength coefficient ($K$), and strain hardening exponent ($n$), are the intercept and slope of the best line fit to true stress ($\sigma$), versus true plastic strain ($\varepsilon_p$), data in log-log scale, represented by:

$$\sigma = K(\varepsilon_p)^n$$  
(3.5)

When performing the least square fit, the true plastic strain was the independent variable and the stress was the dependent variable. To generate the $K$ and $n$ values, the range of data used was chosen in accordance with ASTM Standard E646 (2007). Therefore, the valid data ranged between the yield point defined at 0.2% offset and the maximum strain measured by the extensometer. Figure 3.8 represents axial true stress-true strain
monotonic curve and data points for 1050 steel in normalized and quenched and tempered conditions. More details of axial monotonic test results and analyses can be found in (Shamsaei and Fatemi, 2008).

The properties determined from monotonic torsional tests were: monotonic shear modulus \( (G) \), shear yield strength \( (\tau_y) \), ultimate shear strength \( (\tau_U) \), shear strength coefficient \( (K_s) \), and shear strain hardening exponent \( (n_s) \). These properties are also presented in Table 3.2 for normalized, quenched and tempered, and induction hardened conditions of 1050 steel. No torsional monotonic test was performed for 304L stainless steel.

Shear stress \( (\tau) \), shear strain \( (\gamma) \), and shear plastic strain \( (\gamma_p) \), for the thin-walled tube specimen midsection were calculated based on measured torque \( (T) \), specimen radius \( (r) \), and cross-section area \( (A) \), as follows:

\[
\tau_{\text{mid.}} = \frac{T}{Ar_{\text{mid.}}} \tag{3.6}
\]

\[
\gamma_{\text{mid.}} = \gamma_{\text{surf.}} \left( \frac{r_{\text{mid.}}}{r_{\text{surf.}}} \right) \tag{3.7}
\]

\[
(\gamma_{\text{mid.}})_p = \gamma_{\text{mid.}} - \frac{\tau_{\text{mid.}}}{G} \tag{3.8}
\]

Since shear strain varies linearly with radius, Equation (3.7) was used to extrapolate the surface shear strain to the midsection shear strain. This linearity is valid for both elastic and plastic strain domains. Shear modulus \( (G) \), was obtained by calculating the slope of the elastic portion of shear stress-shear strain curve and using the following relation:
\[ G = \frac{\Delta \tau_{\text{mid.}}}{\Delta \gamma_{\text{mid.}}} \] (3.9)

Yield shear strength \((\tau_y)\) was defined using 0.002 shear strain offset of elastic portion on the shear stress-shear strain curve. Ultimate shear strength \((\tau_U)\), and shear fracture strength \((\tau_f)\), were calculated as follow:

\[ \tau_U = \frac{T_U}{A r_{\text{mid.}}} \] (3.10)

where \(T_U\) is the ultimate or maximum torque. \(\tau_f\) and \(\tau_U\) were found to be the same for all three material conditions, therefore, only one value is reported in Table 3.2.

Similar to axial stress-strain relationship, the shear stress-shear strain relation can also be presented a Ramberg-Osgood type equation, as follows:

\[ \gamma = \gamma_e + \gamma_p = \frac{\tau}{G} + \left( \frac{\tau}{K_s} \right)^{1/n_s} \] (3.11)

The shear strength coefficient \((K_s)\), and shear strain hardening exponent \((n_s)\), are the intercept and slope of the best line fit to shear stress versus plastic shear strain data in log-log scale:

\[ \tau = K_s (\gamma_p)^{n_s} \] (3.12)

When performing the least squares, the shear plastic strain was the independent variable and the shear stress was the dependent variable. Figure 3.9 represents torsional monotonic curve and data points for the three hardening conditions of 1050 steel used in this study. More details for torsional monotonic tests results and analyses can be found in (Shamsaei and Fatemi, 2008).
3.6.2 In-phase and 90° out-of-phase experimental results

Tables 3.3, 3.4, and 3.5 present the test details and results for tubular specimens made from 1050 steel normalized, quenched and tempered, and induction hardened conditions, respectively. Furthermore, Table 3.3 contains the same information for solid specimens made from 1050 steel in normalized condition. These tables present the test information including frequency, specimen dimensions, and load path, as well as test results including load, torque, axial and torsional strain amplitudes and mean values, and number of cycles to failure. These data for tubular specimens of 304L stainless steel are presented in Table 3.6.

Axial strain amplitude, $\varepsilon_a$, is measured directly from extensometer output and axial stress amplitude, $\sigma_a$, is uniform through the surface and is calculated as:

$$\sigma_a = \frac{\Delta \sigma}{2} = \frac{\Delta P}{2A}$$  \hspace{1cm} (3.13)

where $\Delta P / 2$ is axial load amplitude, and $A$ is specimen cross-section area. Having stress and strain amplitudes, elastic strain amplitude, $\Delta \varepsilon_e / 2$, and plastic strain amplitude, $\Delta \varepsilon_p / 2$, are then calculated as:

$$\frac{\Delta \varepsilon_e}{2} = \frac{\Delta \sigma}{2E}$$  \hspace{1cm} (3.14)

$$\frac{\Delta \varepsilon_p}{2} = \frac{\Delta \varepsilon}{2} - \frac{\Delta \varepsilon_e}{2}$$  \hspace{1cm} (3.15)

where $\Delta \varepsilon / 2$ is the total strain amplitude and $E$ is the first cycle modulus of elasticity.

Equivalent Poisson’s ratio is also calculated as:

$$\nu = \frac{\nu_e \Delta \varepsilon_e + \nu_p \Delta \varepsilon_p}{\Delta \varepsilon}$$  \hspace{1cm} (3.16)
where $\Delta \varepsilon_e$, $\Delta \varepsilon_p$, and $\Delta \varepsilon$ are axial elastic, plastic and total strain range and $\nu_e$ and $\nu_p$ are elastic and plastic Poisson’s ratios. $\nu_e$ is about 0.27 for materials in this study and $\nu_p$ is considered equal to 0.5. Equivalent Poisson’s ratio is needed for computing transverse strain resulting from axial component of the loading.

For solid specimens, surface shear strain amplitude, $\gamma_s$, was measured directly from extensometer output, and surface shear stress amplitude, $\tau_s$, was calculated from the following equation (Miller and Chandler, 1970):

$$
\tau_s = \frac{\Delta \tau}{2} = \frac{2}{\pi d^3} \left[ 3 \Delta T + (\Delta \gamma) \frac{d(\Delta T)}{d(\Delta \gamma)} \right]
$$

(3.17)
where $d$ is the diameter of the section, $\Delta T$ is the torque range, $\Delta \gamma$ is the shear strain range, and $\frac{d(\Delta T)}{d(\Delta \gamma)}$ is the average of the slopes of the tips at the end of the loading and unloading portions of the torque versus shear strain hysteresis loop.

For tubular specimens, cyclic deformation is considered based on midsection stress and strain amplitudes. Midsection shear stress amplitude, $(\tau_s)_{mid.}$, is calculated from:

$$
(\tau_s)_{mid.} = \frac{\Delta \tau}{2} = \frac{\Delta T}{2r_{mid.} A}
$$

(3.18)
where $r_{mid.}$ is the midsection radius. Because of the fact that shear strain varies linearly through the cross section, the following equation relates shear strain at any radius to the maximum shear strain which is on the surface:

$$
\gamma_r = \frac{2r}{d} \gamma_{surf.}
$$

(3.19)
where $\gamma_r$ is the shear strain at radius $r$, and $\gamma_{surf.}$ is the surface shear strain. Therefore, the midsection shear strain amplitude is calculated from:

$$\gamma_d^{\text{mid.}} = \frac{2r_d}{d} \gamma_d^{\text{surf.}}.$$  \hspace{1cm} (3.20)

Having shear stress and shear strain amplitudes, elastic shear strain amplitude, $\Delta\gamma_e / 2$, and plastic shear strain amplitude, $\Delta\gamma_p / 2$, are then calculated as:

$$\frac{\Delta\gamma_e}{2} = \frac{\Delta\tau}{2G}$$  \hspace{1cm} (3.21)

$$\frac{\Delta\gamma_p}{2} = \frac{\Delta\gamma}{2} - \frac{\Delta\gamma_e}{2}$$  \hspace{1cm} (3.22)

where $\Delta\gamma / 2$ is total shear strain amplitude and $G$ is the first cycle shear modulus.

Stable (i.e. near midlife) axial and shear hysteresis loops for proportional (in-phase) loading of 1050 N steel are presented in Figure 3.10. In addition, axial and shear hysteresis loops as well as strain paths and resulting stress paths are presented for non-proportional (90° out-of-phase) loading of 1050 N steel in Figure 3.11. The same hysteresis loops and plots for IP and OP loadings of 1050 QT steel are presented in Figures 3.12 and 3.13, and for 1050 IH steel in Figures 3.14 and 3.15. Obviously because of the higher hardness of induction hardened condition as compared to quenched and tempered and normalized conditions, less plastic deformation and consequently slimmer hysteresis loops are observed. High ductility in the normalized condition results in more plastic deformation and wider hysteresis loops compared to the other conditions. Figures 3.16 and 3.17 present similar plots for 304L stainless steel. Significant plastic deformation is observed for 304L stainless steel as a very ductile material. Comparison of IP and OP hysteresis loops shows that in the case of IP loading, maximum stress and
strain occur at the same time, whereas maximum stress and strain are not synchronous due to the rounding at the tips of OP hysteresis loops. This fact is more obvious for materials with more plastic deformation such as 1050 N steel and 304L stainless steel. As plastic deformation decreases, rounding of the hysteresis tips decreases; hence, the maximum stress and maximum strain data points become closer.

Tables 3.7, 3.8, and 3.9 present midsection axial and shear stress and strain component for tubular specimens of 1050 steel in normalized, quenched and tempered, and induction hardened conditions, respectively. Similar analysis results are presented in Table 3.7 for surface of solid specimens in normalized condition. In addition, Table 3.10 provides axial and shear midsection stress and strain analysis results for tubular specimens of 304L stainless steel. There are two sets of stress values reported in these tables for each channel; maximum stress amplitude and stress amplitude at the maximum strain. These values are identical for in-phase loading, but they are different for OP loading, especially at higher amplitude tests due to the rounding in hysteresis loops. There are also two different plastic strains reported in these tables, calculated and measured. Calculated plastic strain amplitudes were computed from Equations (3.15) and (3.22), whereas measured plastic strain ranges were obtained directly from hysteresis loops at zero axial or shear stress. All analyses in this study are based on calculated plastic strain; therefore, any further reference to plastic strain refers to the calculated value.

Some transient cyclic softening for normalized and quenched and tempered specimens in the first few cycles was observed. However, some gradual hardening occurred after the initial softening in normalized specimens until fracture. In the case of
induction hardened specimens, cyclic softening and hardening were negligible except for some high amplitude tests with some plastic deformation. Furthermore, observation of the axial and torsional mean stresses versus number of cycles showed that mean stresses were less than 10% of the corresponding stress amplitude.

Axial and shear stress amplitudes versus number of cycles for 304L stainless steel are presented in Figures 3.18 and 3.19 for IP loadings and Figures 3.20 and 3.21 for OP loadings. Some initial cyclic softening for IP loadings and some initial cyclic hardening for OP loadings especially in high amplitude tests are noticeable from these figures. Mean shear stresses for constant amplitude axial-torsion tests of 304L stainless steel were found to be about zero, but some compressive axial mean stresses, especially for lower amplitude IP and OP loadings, were observed. Stable stress values after initial softening or hardening were used for all calculations in this study. More details of constant amplitude multiaxial fatigue test results and analyses can be found in (Shamsaei and Fatemi, 2008).

Circumferential cracks were observed in random locations around the specimens and most of the time more than one crack was observed. This is an indication of good alignment of the testing apparatus. Surface crack replicas observed under optical microscope indicated crack orientations to be mostly within 110° to 170° for proportional (in-phase) loading, and 165° to 190° for non-proportional (90° out-of-phase) loading for 1050 steel in normalized and quenched and tempered conditions. Due to the high hardness level of induction hardened 1050 steel specimens, failure occurred as soon as crack initiated; therefore, no crack replication could be taken for these specimens. Since strain-controlled testing mode was used for the entire life of all constant amplitude
multiaxial tests of 304L stainless steel, no crack replication was taken for this material. Nevertheless, final crack orientations for in-phase and 90° out-of-phase of 304L stainless steel and 1050 IH steel specimens were found to be similar to the aforementioned ranges observed for 1050 normalized and quenched and tempered steel specimens.

### 3.6.3 Variable amplitude experimental results

Details and results of variable amplitude multiaxial tests of all strain paths for tubular specimens made from 1050 QT steel and 304L stainless steel are presented in Tables 3.11 and 3.12. This table includes specimens inner and outer gage section diameters, testing frequency, modulus of elasticity, shear modulus, stable cycle amplitude and mean of applied axial and shear strains, stable cycle amplitude and mean of responded load and torque, and number of blocks to failure for each test.

Axial strain amplitude, \( \varepsilon_a \), and surface shear strain amplitude, \( \gamma_s \), are measured directly from extensometer output and midsection shear strain amplitude is calculated from Equation (3.20). Axial stress amplitude, \( \sigma_a \), is calculated from Equation (3.13) and midsection shear stress amplitude, \( (\tau_a)_{mid} \), is calculated from Equation (3.18). Elastic and plastic strain amplitudes are calculated from Equations (3.14) and (3.15) for axial loading and from Equations (3.21) and (3.22) for shear loading. Equivalent Poisson’s ratio is also calculated from Equation (3.16).

Tables 3.13 and 3.14 present midsection axial and torsion stress and strain components for tubular specimens of 1050 QT steel and 304L stainless steel, respectively. The axial stress and strain amplitudes reported in these tables are the maximum values at the midlife block occurred when strain ratio \( \lambda = \gamma / \varepsilon \) is zero (i.e. for
the pure axial cycle). The shear stress and strain amplitudes reported in these tables are also the maximum values at the midlife block in the pure torsion cycle when strain ratio is infinity. Knowing these maximum axial and shear stress amplitudes, the whole block can be extrapolated as an ellipse in $(\sigma_a)_{\text{max}} - (\tau_a)_{\text{max}}$ axes. Only one set of plastic strains is reported in Tables 3.13 and 3.14 as calculated plastic strain computed from Equations (3.15) and (3.22) and using the maximum axial and shear stress and strain amplitudes.

Significant cyclic plastic strain caused mean stress relaxation for PI strain path tests of 304L stainless steel and at the higher strain levels of 1050 QT steel, as can be observed from the mean stress values listed in Tables 3.13 and 3.14. Although the PI strain path has mean strain with the same magnitude as strain amplitude in both axial and shear channels, mean stresses reduce to about 20% of the stress amplitude of 1050 QT steel and to about 10% of the corresponding stress amplitudes for 304L stainless steel at $\epsilon_a = 0.0035$ and $\gamma_a = 0.0060$. Due to higher plastic strain of stainless steel, however, more mean stress relaxation is observed, as compared to the 1050 QT steel.

Mean stress relaxation for both materials is presented in Figures 3.22(a) and 3.22(b) for the axial and shear cycles of the blocks of FRI and PI strain paths, respectively. Due to the considerable mean stress relaxation phenomenon observed, both materials exhibit identical stress responses under PI and FRI loading paths at the same strain amplitudes. Mean stress relaxation for 304L stainless steel occurs at the very beginning of the first block of the straining. Therefore, it can be concluded that the mean strain effects in the presence of plastic deformation is very minor on cyclic deformation behaviors, and consequently, on the fatigue life.
Axial and shear stress amplitudes for PI and FRI strain paths at the same equivalent strain amplitudes are also very similar, as observed from Figure 3.22. The stress amplitude displays a combination of kinematic and isotropic strain hardening behavior. The movement of the memory surface for mean stress relaxation is a kinematic hardening behavior, whereas the change in the size of the memory surface in case of cyclic softening/hardening is an isotropic behavior.

Variation of axial stress amplitudes versus number of blocks for 1050 QT steel for the pure axial cycle of the block of FRI and FRR strain paths is presented in Figure 3.23. Not a significant difference in stress response for FRI and FRR paths can be noticed. Significant initial cyclic softening was observed in the first loading block for the highest strain level (i.e. $\varepsilon_a = 0.007$ and $\gamma_a = 0.012$) of FRI and FRR tests of 1050 QT steel during the very first block, whereas the lower strain level tests of these strain paths exhibit just a small amount of cyclic softening. Not a noticeable difference in shear stress response can be observed between FRI and FRR strain paths presented in Figure 3.24, which represents the specimen midsection shear stress amplitude of the pure torsion cycle versus number of blocks for 1050 QT steel.

Stress response of the first, second, and fourth blocks of the highest strain level for 1050 QT steel under FRI strain path (10FRI1QT) is shown in Figure 3.25(a). Although cyclic softening is noticeable during the first block, a stable behavior is exhibited after the initial softening. At a lower strain amplitude level (i.e. $\varepsilon_a = 0.0034$ and $\gamma_a = 0.0058$), these strain paths exhibit small initial cyclic softening, as expected, since most of the deformation is elastic. The second and third blocks are presented by circumscribed ellipses passing through the maximum values. No distortion can be
observed in the boundaries and stress response of the stable block represents a perfect ellipse.

Stress response of the highest strain level for 1050 QT steel under FRR strain path (10FRR1QT) is presented in Figure 3.25(b). Data for the first block and circumscribed ellipses of the second and third blocks are presented in this figure. Similar to FRI load path, significant cyclic softening during the first block of loading is observed, but stable behavior is reached after the initial softening. Again, the boundaries of the stress responses can be presented by an ellipse. Comparing stress responses of the stable blocks in Figures 3.25(a) and 3.25(b), very similar stress levels are observed, pointing to the insensitivity of the stress response in 1050 QT steel to non-proportional loading.

Figure 3.26 shows variation of axial stress amplitudes of the pure axial cycle in FRI, FRI15, and FRR strain paths versus number of blocks for 304L stainless steel. Significant difference in stress responses between FRI and FRR strain paths, specifically at the higher strain amplitude level, is considerable in these figures. Higher stress responses resulted for random sequence strain path (FRR), as compared to incremental sequence strain path (FRI). Furthermore, higher degree of increments increases the hardening of material due to the sudden change in loading direction, as FRI15 path resulted in higher and lower stress response as compared to FRI and FRR paths, respectively (see Figure 3.26). Gradual cyclic hardening is noticeable for both FRI and FRR strain paths at the higher strain level, whereas the lower strain level tests of these strain paths exhibit a very stable behavior. All axial mean stresses were about zero for the FRI and FRR strain paths, except for some compressive axial mean stresses (less than 10% of the stress amplitude) at the lowest strain level tests of these paths.
Similar to axial stresses, significant differences in midsection shear stress amplitude responses of the pure torsion cycle in FRI, FRI15, and FRR strain paths are noticeable in Figure 3.27. Similar to the axial channel, FRR, FRI15, and FRI strain paths exhibit the highest, intermediate, and the lowest shear stress responses, respectively. Gradual cyclic hardening in shear channel is observed in Figure 3.27 for highest strain level tests for FRI and FRR strain paths.

While 304L stainless steel was found to stabilize after initial softening or hardening in in-phase and out-of-phase constant amplitude tests (see Figures 3.18 through 3.21), some gradual cyclic hardening continued until the end of the test for the highest strain level (i.e. $\varepsilon_a = 0.007$ and $\gamma_a = 0.012$) of FRI and FRR strain paths. Figures 3.26 and 3.27 also present the fact that stress response of 304L stainless steel is almost stable for lower strain levels of variable amplitude multiaxial strain paths utilized in this study.

Figure 3.28(a) presents the stress response of the first, second, and fifth blocks of the highest strain level (i.e. $\varepsilon_a = 0.007$ and $\gamma_a = 0.012$) of the FRI strain path for 304L stainless steel (10FRI1SS). For clarity, second and fifth blocks are shown by circumscribed ellipses passing through the maximum values. Cyclic hardening is observed in the first loading block, the rate of which decreases with increasing the number of applied blocks. The stress response in both channels is fully reversed and, similar to 1050 QT steel, stress response of blocks can be presented by concentric ellipses.

Stress response of the highest strain level of the FRR strain path for 304L stainless steel (10FRR1SS) is presented in Figure 3.28(b). This figure includes all data points of the first block and circumscribed ellipses of the second and fourth blocks.
Similar to the FRI test of this material, slight cyclic hardening is observed for the FRR load path till the final failure and the stress response can be presented by concentric ellipses. Nevertheless, a significant difference in stress response is noticeable between FRI and FRR strain paths shown in Figure 3.28. The stress response level for the FRR strain path is much higher, as compared to the FRI strain path, indicating strong sensitivity of this material to the sudden change in loading direction and, therefore, the interaction of slip systems. Both strain paths result in symmetric stress response.
Table 3.1  Chemical compositions in weight percent of 1050 steel and 304L stainless steel used in this study.

<table>
<thead>
<tr>
<th>Material</th>
<th>C</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Si</th>
<th>Cu</th>
<th>Ni</th>
<th>Cr</th>
<th>Mo</th>
<th>Sn</th>
<th>V</th>
<th>Al</th>
<th>Ca</th>
<th>N2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1050 Steel¹</td>
<td>0.48</td>
<td>0.91</td>
<td>0.011</td>
<td>0.034</td>
<td>0.22</td>
<td>0.13</td>
<td>0.02</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.003</td>
<td>0.04</td>
<td>0.0001</td>
<td>-</td>
</tr>
<tr>
<td>SS 304L²</td>
<td>0.029</td>
<td>1.86</td>
<td>0.029</td>
<td>0.004</td>
<td>0.37</td>
<td>0.02</td>
<td>10</td>
<td>18</td>
<td>0.04</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.056</td>
</tr>
</tbody>
</table>

¹ Supplied by Chrysler LLC.
² Supplied by Electricite de France.

Table 3.2  Monotonic axial and torsional properties for 1050 steel in normalized, quenched and tempered, and induction hardened conditions as well as 304L stainless steel.

<table>
<thead>
<tr>
<th>Material Condition</th>
<th>$E$ (GPa)</th>
<th>$\sigma_y$ (MPa)</th>
<th>$S_U$ (MPa)</th>
<th>$K$ (MPa)</th>
<th>$n$</th>
<th>$G$ (GPa)</th>
<th>$\tau_y$ (MPa)</th>
<th>$\tau_U$ (MPa)</th>
<th>$K_s$ (MPa)</th>
<th>$n_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1050 N Steel</td>
<td>206</td>
<td>421</td>
<td>709</td>
<td>1455</td>
<td>0.253</td>
<td>81</td>
<td>243</td>
<td>469</td>
<td>813</td>
<td>0.288</td>
</tr>
<tr>
<td>1050 QT Steel</td>
<td>203</td>
<td>1009</td>
<td>1164</td>
<td>1461</td>
<td>0.060</td>
<td>81</td>
<td>542</td>
<td>690</td>
<td>820</td>
<td>0.070</td>
</tr>
<tr>
<td>1050 IH Steel</td>
<td>198</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>79</td>
<td>1157</td>
<td>1564</td>
<td>2127</td>
<td>0.099</td>
<td></td>
</tr>
<tr>
<td>304L Stainless Steel¹</td>
<td>196</td>
<td>208</td>
<td>585</td>
<td>680</td>
<td>0.214</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

¹ Monotonic properties for 304L stainless steel are from small solid specimen (i.e. 5 mm diameter) tensile test (Colin, 2009).
² Average of the first cycles of fatigue tests.
Table 3.3  Multiaxial constant amplitude in-phase (IP) and 90º out-of-phase (OP) test information and results for tubular and solid specimens of 1050 N steel.

| Spec. ID | $d_i$ (mm) | $d_o$ (mm) | Load Path | Freq. (Hz) | $E$ (GPa) | $G$ (GPa) | $N_{stable}$ (Cycle) | $\varepsilon_a$ | $\varepsilon_m$ | Surface | $P_a$ (kN) | $P_m$ (kN) | $T_a$ (Nm) | $T_m$ (Nm) | $N_f$ (Cycle) |
|----------|-------------|-------------|------------|------------|------------|------------|----------------------|----------------|----------------|----------|--------------|-------------|-------------|-------------|-------------|----------------|
| **Tubular Specimen** |
| N 7 12.78 15.19 IP 0.2 201.3 79.3 1,024 0.0069 0.0000 0.0130 0 18.6 -0.5 75.1 0.3 1,561 |
| N 9 12.78 15.14 IP 0.2 210.5 83.5 600 0.0069 0.0000 0.0119 0 18.9 -0.4 70.8 0.3 1,667 |
| N 11 12.78 15.14 IP 0.5 204.7 80.0 4,152 0.0033 0.0000 0.0057 0 14.8 -0.2 58.1 -0.5 9,265 |
| N 5 12.78 15.19 IP 0.5 206.2 78.0 4,991 0.0033 0.0000 0.0057 0 15.0 -0.2 58.1 -0.2 11,353 |
| N 13 12.78 15.14 IP 1 210.6 83.6 94,813 0.0015 0.0000 0.0026 0 10.9 -0.3 45.0 -0.2 189,189 |
| N 3 12.78 15.16 IP 1 200.8 79.2 51,992 0.0014 -0.0006 0.0025 0.0008 10.5 -0.4 42.2 -0.3 387,373 |
| N 8 12.78 15.19 OP 0.2 NA NA 264 0.0069 0.0000 0.0119 0 25.3 -0.1 96.0 0.6 523 |
| N 10 12.70 15.14 OP 0.2 NA NA 200 0.0069 0.0000 0.0119 0 25.3 -0.1 97.0 0.5 573 |
| N 12 12.78 15.16 OP 0.5 NA NA 1,839 0.0033 0.0000 0.0057 0 21.0 0.0 79.4 0.2 3,827 |
| N 6 12.78 15.16 OP 0.5 NA NA 1,124 0.0033 0.0000 0.0057 0 20.4 0.0 78.2 0.1 6,023 |
| N 14 12.70 15.16 OP 1 NA NA 50,000 0.0015 0.0000 0.0026 0 14.9 -0.2 59.1 -0.4 102,570 |
| N 4 12.78 15.16 OP 1 NA NA 84,650 0.0015 0.0000 0.0026 0 14.6 -0.1 57.1 -0.9 133,419 |
| **Solid Specimen** |
| NS 3 0 15.16 IP 0.2 203.4 80.5 600 0.0069 0.0000 0.0120 0 66.3 -1.1 144.6 0.1 2,628 |
| NS 1 0 15.27 IP 0.5 203.9 78.8 2,840 0.0033 0.0000 0.0057 0 52.8 -0.3 117.5 0.3 18,205 |
| NS 7 0 15.19 IP 0.5 197.3 81.6 8,181 0.0033 0.0000 0.0057 0 53.8 -0.6 119.5 -4.3 18,312 |
| NS 5 0 15.28 IP 1 206.8 82.3 160,672 0.0015 0.0000 0.0026 0 39.6 0.1 95.6 -2.4 274,705 |
| NS 2 0 15.32 OP 0.2 208.7 82.1 200 0.0069 0.0000 0.0120 0 82.6 -0.3 224.8 2.0 1,059 |
| NS 4 0 15.20 OP 0.5 198.4 79.5 2,160 0.0033 0.0000 0.0057 0 67.8 -0.4 179.5 0.2 5,854 |
| NS 6 0 15.25 OP 1 201.0 79.9 41,097 0.0015 0.0000 0.0026 0 47.5 0.2 122.0 -8.7 141,153 |
Table 3.4  Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) test information and results for tubular specimens of 1050 QT steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>$d_i$ (mm)</th>
<th>$d_o$ (mm)</th>
<th>Load Path</th>
<th>Freq. (Hz)</th>
<th>$E$ (GPa)</th>
<th>$G$ (GPa)</th>
<th>$N_{stable}$ (Cycle)</th>
<th>$\epsilon_a$</th>
<th>$\epsilon_m$</th>
<th>$\gamma_a$</th>
<th>$\gamma_m$</th>
<th>$P_a$ (kN)</th>
<th>$P_m$ (kN)</th>
<th>$T_a$ (Nm)</th>
<th>$T_m$ (Nm)</th>
<th>$N_f$ (Cycle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q 7</td>
<td>15.52</td>
<td>17.84</td>
<td>IP</td>
<td>0.3</td>
<td>201.0</td>
<td>79.7</td>
<td>512</td>
<td>0.0069</td>
<td>0</td>
<td>0.0120</td>
<td>0</td>
<td>36.2</td>
<td>1.0</td>
<td>168.2</td>
<td>0.4</td>
<td>1,225</td>
</tr>
<tr>
<td>Q 1</td>
<td>15.52</td>
<td>17.86</td>
<td>IP</td>
<td>0.3</td>
<td>196.7</td>
<td>78.7</td>
<td>464</td>
<td>0.0068</td>
<td>0</td>
<td>0.0119</td>
<td>0.0001</td>
<td>36.5</td>
<td>0.8</td>
<td>167.6</td>
<td>0.9</td>
<td>1,532</td>
</tr>
<tr>
<td>Q 9</td>
<td>15.52</td>
<td>17.84</td>
<td>IP</td>
<td>0.75</td>
<td>206.2</td>
<td>80.9</td>
<td>10,204</td>
<td>0.0032</td>
<td>0</td>
<td>0.0055</td>
<td>0</td>
<td>29.0</td>
<td>0.8</td>
<td>140.7</td>
<td>0.6</td>
<td>12,048</td>
</tr>
<tr>
<td>Q 5</td>
<td>15.52</td>
<td>17.86</td>
<td>IP</td>
<td>0.75</td>
<td>203.5</td>
<td>80.6</td>
<td>3,239</td>
<td>0.0003</td>
<td>0</td>
<td>0.0057</td>
<td>0.0001</td>
<td>30.3</td>
<td>0.6</td>
<td>147.6</td>
<td>3.2</td>
<td>13,438</td>
</tr>
<tr>
<td>Q 11</td>
<td>15.52</td>
<td>17.84</td>
<td>IP</td>
<td>1.2</td>
<td>202.0</td>
<td>82.2</td>
<td>21,436</td>
<td>0.0024</td>
<td>0</td>
<td>0.0040</td>
<td>0.0001</td>
<td>27.8</td>
<td>3.2</td>
<td>138.4</td>
<td>7.7</td>
<td>61,034</td>
</tr>
<tr>
<td>Q 3</td>
<td>15.52</td>
<td>17.84</td>
<td>IP</td>
<td>1.5</td>
<td>204.3</td>
<td>81.6</td>
<td>86,561</td>
<td>0.0020</td>
<td>0</td>
<td>0.0035</td>
<td>0</td>
<td>24.6</td>
<td>2.6</td>
<td>131.1</td>
<td>12.8</td>
<td>208,572</td>
</tr>
<tr>
<td>Q 12</td>
<td>15.52</td>
<td>17.84</td>
<td>OP</td>
<td>0.15</td>
<td>203.7</td>
<td>80.3</td>
<td>30</td>
<td>0.0105</td>
<td>0</td>
<td>0.0182</td>
<td>0</td>
<td>49.6</td>
<td>0.7</td>
<td>232.2</td>
<td>3.3</td>
<td>115</td>
</tr>
<tr>
<td>Q 2</td>
<td>15.52</td>
<td>17.84</td>
<td>OP</td>
<td>0.3</td>
<td>NA</td>
<td>NA</td>
<td>200</td>
<td>0.0069</td>
<td>0</td>
<td>0.0119</td>
<td>0</td>
<td>44.0</td>
<td>0.5</td>
<td>205.8</td>
<td>1.7</td>
<td>354</td>
</tr>
<tr>
<td>Q 6</td>
<td>15.52</td>
<td>17.86</td>
<td>OP</td>
<td>0.3</td>
<td>NA</td>
<td>NA</td>
<td>200</td>
<td>0.0069</td>
<td>0</td>
<td>0.0120</td>
<td>0</td>
<td>45.3</td>
<td>0.5</td>
<td>213.7</td>
<td>1.4</td>
<td>406</td>
</tr>
<tr>
<td>Q 8</td>
<td>15.52</td>
<td>17.84</td>
<td>OP</td>
<td>0.75</td>
<td>NA</td>
<td>NA</td>
<td>2,257</td>
<td>0.0033</td>
<td>0</td>
<td>0.0056</td>
<td>0</td>
<td>35.4</td>
<td>0.3</td>
<td>168.1</td>
<td>1.0</td>
<td>2,351</td>
</tr>
<tr>
<td>Q 15</td>
<td>15.52</td>
<td>17.84</td>
<td>OP</td>
<td>0.75</td>
<td>NA</td>
<td>NA</td>
<td>1.965</td>
<td>0.0031</td>
<td>0</td>
<td>0.0053</td>
<td>0.0001</td>
<td>35.3</td>
<td>0.7</td>
<td>169.9</td>
<td>0.4</td>
<td>3,990</td>
</tr>
<tr>
<td>Q 4</td>
<td>15.52</td>
<td>17.84</td>
<td>OP</td>
<td>0.75</td>
<td>NA</td>
<td>NA</td>
<td>1.999</td>
<td>0.0031</td>
<td>0</td>
<td>0.0054</td>
<td>0.0001</td>
<td>36.1</td>
<td>1.1</td>
<td>176.4</td>
<td>2.1</td>
<td>6,697</td>
</tr>
<tr>
<td>Q 10</td>
<td>15.52</td>
<td>17.84</td>
<td>OP</td>
<td>1.2</td>
<td>202.0</td>
<td>79.3</td>
<td>12,952</td>
<td>0.0025</td>
<td>0</td>
<td>0.0043</td>
<td>0</td>
<td>29.7</td>
<td>0.3</td>
<td>147.1</td>
<td>0.4</td>
<td>22,293</td>
</tr>
<tr>
<td>Q 13</td>
<td>15.52</td>
<td>17.84</td>
<td>OP</td>
<td>3</td>
<td>207.9</td>
<td>82.8</td>
<td>23,667</td>
<td>0.0018</td>
<td>0</td>
<td>0.0031</td>
<td>0.0001</td>
<td>22.6</td>
<td>1.8</td>
<td>118.2</td>
<td>13.5</td>
<td>1,029,340</td>
</tr>
</tbody>
</table>
Table 3.5  Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) test information and results for tubular specimens of 1050 IH steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>$d_i$ (mm)</th>
<th>$d_o$ (mm)</th>
<th>Load Path</th>
<th>Freq. (Hz)</th>
<th>$E$ (GPa)</th>
<th>$G$ (GPa)</th>
<th>$N_{stable}$ (Cycle)</th>
<th>$\varepsilon_a$</th>
<th>$\varepsilon_m$</th>
<th>$\gamma_a$</th>
<th>$\gamma_m$</th>
<th>$P_a$ (kN)</th>
<th>$P_m$ (kN)</th>
<th>$T_a$ (Nm)</th>
<th>$T_m$ (Nm)</th>
<th>$N_f$ (Cycle)</th>
<th>$N_f$ (Cycle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I 13</td>
<td>12.73</td>
<td>15.19</td>
<td>IP</td>
<td>0.1</td>
<td>203.0</td>
<td>81.1</td>
<td>20</td>
<td>0.0100</td>
<td>0</td>
<td>0.0174</td>
<td>0.0001</td>
<td>80.6</td>
<td>-6.8</td>
<td>337.9</td>
<td>-3.4</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>I 15</td>
<td>12.70</td>
<td>15.14</td>
<td>IP</td>
<td>0.1</td>
<td>197.1</td>
<td>79.1</td>
<td>20</td>
<td>0.0100</td>
<td>0</td>
<td>0.0174</td>
<td>0.0001</td>
<td>77.0</td>
<td>-5.9</td>
<td>326.8</td>
<td>-5.4</td>
<td>117</td>
<td></td>
</tr>
<tr>
<td>I 19</td>
<td>12.75</td>
<td>15.10</td>
<td>IP</td>
<td>0.1</td>
<td>195.2</td>
<td>78.6</td>
<td>20</td>
<td>0.0085</td>
<td>0</td>
<td>0.0147</td>
<td>0.0001</td>
<td>70.4</td>
<td>-4.9</td>
<td>301.4</td>
<td>-6.7</td>
<td>288</td>
<td></td>
</tr>
<tr>
<td>I 5</td>
<td>12.78</td>
<td>15.08</td>
<td>IP</td>
<td>0.1</td>
<td>193.9</td>
<td>76.3</td>
<td>20</td>
<td>0.0085</td>
<td>0</td>
<td>0.0148</td>
<td>0.0001</td>
<td>66.8</td>
<td>-4.2</td>
<td>286.6</td>
<td>-7.6</td>
<td>304</td>
<td></td>
</tr>
<tr>
<td>I 1</td>
<td>12.73</td>
<td>15.09</td>
<td>IP</td>
<td>0.2</td>
<td>203.3</td>
<td>79.6</td>
<td>70</td>
<td>0.0069</td>
<td>0</td>
<td>0.0119</td>
<td>0</td>
<td>66.0</td>
<td>-3.5</td>
<td>283.6</td>
<td>-9.3</td>
<td>337</td>
<td></td>
</tr>
<tr>
<td>I 13</td>
<td>12.73</td>
<td>15.24</td>
<td>IP</td>
<td>0.2</td>
<td>201.4</td>
<td>80.1</td>
<td>50</td>
<td>0.0069</td>
<td>0</td>
<td>0.0120</td>
<td>0</td>
<td>70.4</td>
<td>-4.6</td>
<td>305.2</td>
<td>-9.1</td>
<td>358</td>
<td></td>
</tr>
<tr>
<td>I 123</td>
<td>12.73</td>
<td>15.10</td>
<td>IP</td>
<td>0.5</td>
<td>195.0</td>
<td>77.2</td>
<td>826</td>
<td>0.0033</td>
<td>0</td>
<td>0.0057</td>
<td>0</td>
<td>34.1</td>
<td>1.1</td>
<td>146.5</td>
<td>-12.9</td>
<td>6,991</td>
<td></td>
</tr>
<tr>
<td>I 17</td>
<td>12.70</td>
<td>15.06</td>
<td>IP</td>
<td>0.5</td>
<td>198.0</td>
<td>78.9</td>
<td>256</td>
<td>0.0033</td>
<td>0</td>
<td>0.0057</td>
<td>0</td>
<td>33.1</td>
<td>-0.7</td>
<td>146.8</td>
<td>-4.9</td>
<td>14,848</td>
<td></td>
</tr>
<tr>
<td>I 19</td>
<td>12.71</td>
<td>15.04</td>
<td>IP</td>
<td>2.5</td>
<td>206.3</td>
<td>80.5</td>
<td>1,000</td>
<td>0.0025</td>
<td>0</td>
<td>0.0043</td>
<td>0</td>
<td>26.0</td>
<td>0.1</td>
<td>112.9</td>
<td>-10.8</td>
<td>21,954</td>
<td></td>
</tr>
<tr>
<td>I 121</td>
<td>12.70</td>
<td>15.04</td>
<td>IP</td>
<td>2.5</td>
<td>195.5</td>
<td>77.2</td>
<td>5,000</td>
<td>0.0025</td>
<td>0</td>
<td>0.0043</td>
<td>0</td>
<td>25.9</td>
<td>-1.6</td>
<td>113.1</td>
<td>-12.0</td>
<td>890,054</td>
<td></td>
</tr>
<tr>
<td>I 110</td>
<td>12.70</td>
<td>15.05</td>
<td>OP</td>
<td>0.1</td>
<td>200.8</td>
<td>81.1</td>
<td>10</td>
<td>0.0095</td>
<td>0.0001</td>
<td>0.0166</td>
<td>0.0003</td>
<td>91.0</td>
<td>-4.3</td>
<td>371.4</td>
<td>-4.3</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>I 116</td>
<td>12.71</td>
<td>15.16</td>
<td>OP</td>
<td>0.1</td>
<td>194.4</td>
<td>78.1</td>
<td>20</td>
<td>0.0095</td>
<td>0</td>
<td>0.0165</td>
<td>0</td>
<td>91.4</td>
<td>-4.6</td>
<td>376.1</td>
<td>0.8</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>I 114</td>
<td>12.73</td>
<td>15.16</td>
<td>OP</td>
<td>0.1</td>
<td>190.7</td>
<td>76.5</td>
<td>30</td>
<td>0.0085</td>
<td>0</td>
<td>0.0147</td>
<td>0</td>
<td>81.0</td>
<td>-4.0</td>
<td>341.7</td>
<td>-0.1</td>
<td>243</td>
<td></td>
</tr>
<tr>
<td>I 12</td>
<td>12.70</td>
<td>15.14</td>
<td>OP</td>
<td>0.1</td>
<td>192.1</td>
<td>76.5</td>
<td>20</td>
<td>0.0085</td>
<td>0</td>
<td>0.0148</td>
<td>0</td>
<td>84.5</td>
<td>-4.4</td>
<td>344.5</td>
<td>0.1</td>
<td>317</td>
<td></td>
</tr>
<tr>
<td>I 16</td>
<td>12.73</td>
<td>15.09</td>
<td>OP</td>
<td>0.2</td>
<td>192.3</td>
<td>76.1</td>
<td>35</td>
<td>0.0069</td>
<td>0</td>
<td>0.0120</td>
<td>0</td>
<td>67.1</td>
<td>-2.2</td>
<td>286.0</td>
<td>-3.2</td>
<td>869</td>
<td></td>
</tr>
<tr>
<td>I 18</td>
<td>12.71</td>
<td>15.06</td>
<td>OP</td>
<td>0.5</td>
<td>197.9</td>
<td>79.3</td>
<td>200</td>
<td>0.0033</td>
<td>0</td>
<td>0.0057</td>
<td>0</td>
<td>34.3</td>
<td>-0.2</td>
<td>147.4</td>
<td>-9.3</td>
<td>14,560</td>
<td></td>
</tr>
<tr>
<td>I 112</td>
<td>12.70</td>
<td>15.15</td>
<td>OP</td>
<td>2.5</td>
<td>205.8</td>
<td>79.5</td>
<td>8,192</td>
<td>0.0025</td>
<td>0</td>
<td>0.0043</td>
<td>0</td>
<td>27.8</td>
<td>-1.0</td>
<td>118.3</td>
<td>-13.5</td>
<td>25,848</td>
<td></td>
</tr>
<tr>
<td>I 118</td>
<td>12.73</td>
<td>15.14</td>
<td>OP</td>
<td>2.5</td>
<td>199.5</td>
<td>80.7</td>
<td>3,046</td>
<td>0.0024</td>
<td>-0.0001</td>
<td>0.0042</td>
<td>0.0001</td>
<td>26.4</td>
<td>-1.3</td>
<td>113.5</td>
<td>0.0</td>
<td>137,328</td>
<td></td>
</tr>
<tr>
<td>I 122</td>
<td>12.70</td>
<td>15.11</td>
<td>OP</td>
<td>2.5</td>
<td>199.1</td>
<td>79.7</td>
<td>4,096</td>
<td>0.0025</td>
<td>0</td>
<td>0.0043</td>
<td>0</td>
<td>26.8</td>
<td>-0.5</td>
<td>114.6</td>
<td>-18.8</td>
<td>559,922</td>
<td></td>
</tr>
<tr>
<td>I 14</td>
<td>12.73</td>
<td>15.11</td>
<td>OP</td>
<td>3</td>
<td>197.9</td>
<td>79.0</td>
<td>10,000</td>
<td>0.0020</td>
<td>0</td>
<td>0.0035</td>
<td>0</td>
<td>21.7</td>
<td>0.0</td>
<td>91.7</td>
<td>-15.8</td>
<td>&gt;1,853,060</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.6  Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) test information and results for tubular specimens of 304L stainless steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>$d_i$ (mm)</th>
<th>$d_o$ (mm)</th>
<th>Load Path</th>
<th>Freq. (Hz)</th>
<th>$E$ (GPa)</th>
<th>$G$ (GPa)</th>
<th>$N_{stable}$ (Cycle)</th>
<th>$\varepsilon_a$</th>
<th>$\varepsilon_m$</th>
<th>$\gamma_a$</th>
<th>$\gamma_m$</th>
<th>$P_a$ (kN)</th>
<th>$P_m$ (kN)</th>
<th>$T_a$ (Nm)</th>
<th>$T_m$ (Nm)</th>
<th>$N_f$ (Cycle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSP 1</td>
<td>12.78</td>
<td>15.19</td>
<td>IP</td>
<td>0.2</td>
<td>188.21</td>
<td>74.83</td>
<td>256</td>
<td>0.0069</td>
<td>0</td>
<td>0.0120</td>
<td>0</td>
<td>17.7</td>
<td>-0.5</td>
<td>65.6</td>
<td>0.4</td>
<td>834</td>
</tr>
<tr>
<td>SSP 11</td>
<td>13.12</td>
<td>15.14</td>
<td>IP</td>
<td>0.4</td>
<td>194.1</td>
<td>79.50</td>
<td>1,024</td>
<td>0.0049</td>
<td>0</td>
<td>0.0084</td>
<td>0</td>
<td>12.1</td>
<td>-0.4</td>
<td>48.8</td>
<td>0.6</td>
<td>2,251</td>
</tr>
<tr>
<td>SSP 9</td>
<td>13.11</td>
<td>15.16</td>
<td>IP</td>
<td>0.5</td>
<td>194.73</td>
<td>77.19</td>
<td>2,048</td>
<td>0.0030</td>
<td>0</td>
<td>0.0057</td>
<td>0</td>
<td>11.4</td>
<td>-0.5</td>
<td>45.2</td>
<td>0.9</td>
<td>3,250</td>
</tr>
<tr>
<td>SSP 13</td>
<td>13.11</td>
<td>15.16</td>
<td>IP</td>
<td>1.3</td>
<td>198.3</td>
<td>78.83</td>
<td>8,464</td>
<td>0.0023</td>
<td>0</td>
<td>0.0040</td>
<td>0</td>
<td>8.7</td>
<td>-0.8</td>
<td>35.3</td>
<td>2.0</td>
<td>15,728</td>
</tr>
<tr>
<td>SSP 7</td>
<td>12.90</td>
<td>15.19</td>
<td>IP</td>
<td>1.5</td>
<td>191.00</td>
<td>77.96</td>
<td>50,990</td>
<td>0.0018</td>
<td>0</td>
<td>0.0030</td>
<td>0</td>
<td>8.4</td>
<td>-1.5</td>
<td>34.9</td>
<td>2.8</td>
<td>97,031</td>
</tr>
<tr>
<td>SSNP 10</td>
<td>13.11</td>
<td>15.16</td>
<td>OP</td>
<td>0.2</td>
<td>195.53</td>
<td>77.57</td>
<td>256</td>
<td>0.0069</td>
<td>0</td>
<td>0.0120</td>
<td>0</td>
<td>27.8</td>
<td>-0.4</td>
<td>113.4</td>
<td>-0.3</td>
<td>380</td>
</tr>
<tr>
<td>SSNP 2</td>
<td>12.90</td>
<td>15.19</td>
<td>OP</td>
<td>0.2</td>
<td>193.18</td>
<td>78.35</td>
<td>199</td>
<td>0.0069</td>
<td>0</td>
<td>0.0119</td>
<td>0</td>
<td>31.5</td>
<td>-0.6</td>
<td>127.2</td>
<td>0.6</td>
<td>400</td>
</tr>
<tr>
<td>SSNP 4</td>
<td>12.83</td>
<td>15.18</td>
<td>OP</td>
<td>0.5</td>
<td>195.27</td>
<td>77.20</td>
<td>2,048</td>
<td>0.0033</td>
<td>0</td>
<td>0.0057</td>
<td>0</td>
<td>21.8</td>
<td>-0.2</td>
<td>86.6</td>
<td>0.3</td>
<td>4,169</td>
</tr>
<tr>
<td>SSNP 8</td>
<td>13.11</td>
<td>15.15</td>
<td>OP</td>
<td>0.5</td>
<td>188.86</td>
<td>71.75</td>
<td>2,048</td>
<td>0.0033</td>
<td>0</td>
<td>0.0057</td>
<td>0</td>
<td>19.6</td>
<td>-0.2</td>
<td>78.9</td>
<td>0.1</td>
<td>5,850</td>
</tr>
<tr>
<td>SSNP 6</td>
<td>12.85</td>
<td>15.16</td>
<td>OP</td>
<td>2</td>
<td>189.94</td>
<td>76.88</td>
<td>69,166</td>
<td>0.0017</td>
<td>0</td>
<td>0.0030</td>
<td>0</td>
<td>13.7</td>
<td>-1.4</td>
<td>53.8</td>
<td>0.0</td>
<td>158,772</td>
</tr>
</tbody>
</table>
Table 3.7  Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) axial-torsion stable cycle stress-strain analysis results for tubular and solid specimens of 1050 N steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>Load Path</th>
<th>Maximum $\sigma_a$ (MPa)</th>
<th>$\nu$</th>
<th>$\sigma_{a@\epsilon_{Max}}$ (MPa)</th>
<th>$\epsilon_a$</th>
<th>$\Delta\epsilon_p/2$ Calculated</th>
<th>$\Delta\epsilon_p/2$ Measured</th>
<th>Maximum $\tau_a$ (MPa)</th>
<th>$\tau_{a@\epsilon_{Max}}$ (MPa)</th>
<th>$\gamma_a$ Calculated</th>
<th>$\gamma_a$ Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tubular Specimens</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N 7</td>
<td>IP</td>
<td>0.44</td>
<td>350</td>
<td>350</td>
<td>0.0069</td>
<td>0.0052</td>
<td>0.0050</td>
<td>203</td>
<td>203</td>
<td>0.0120</td>
<td>0.0095</td>
</tr>
<tr>
<td>N 9</td>
<td>IP</td>
<td>0.44</td>
<td>364</td>
<td>364</td>
<td>0.0069</td>
<td>0.0051</td>
<td>0.0049</td>
<td>196</td>
<td>196</td>
<td>0.0110</td>
<td>0.0086</td>
</tr>
<tr>
<td>N 11</td>
<td>IP</td>
<td>0.41</td>
<td>287</td>
<td>287</td>
<td>0.0033</td>
<td>0.0019</td>
<td>0.0018</td>
<td>161</td>
<td>161</td>
<td>0.0053</td>
<td>0.0033</td>
</tr>
<tr>
<td>N 5</td>
<td>IP</td>
<td>0.41</td>
<td>282</td>
<td>282</td>
<td>0.0033</td>
<td>0.0019</td>
<td>0.0018</td>
<td>156</td>
<td>156</td>
<td>0.0053</td>
<td>0.0033</td>
</tr>
<tr>
<td>N 13</td>
<td>IP</td>
<td>0.35</td>
<td>210</td>
<td>210</td>
<td>0.0015</td>
<td>0.0005</td>
<td>0.0004</td>
<td>125</td>
<td>125</td>
<td>0.0024</td>
<td>0.0008</td>
</tr>
<tr>
<td>N 3</td>
<td>IP</td>
<td>0.35</td>
<td>201</td>
<td>201</td>
<td>0.0014</td>
<td>0.0005</td>
<td>0.0004</td>
<td>115</td>
<td>115</td>
<td>0.0023</td>
<td>0.0008</td>
</tr>
<tr>
<td>N 8</td>
<td>OP</td>
<td>0.45</td>
<td>478</td>
<td>315</td>
<td>0.0069</td>
<td>0.0054</td>
<td>0.0052</td>
<td>259</td>
<td>152</td>
<td>0.0110</td>
<td>0.0091</td>
</tr>
<tr>
<td>N 10</td>
<td>OP</td>
<td>0.45</td>
<td>475</td>
<td>320</td>
<td>0.0069</td>
<td>0.0053</td>
<td>0.0053</td>
<td>261</td>
<td>154</td>
<td>0.0110</td>
<td>0.0091</td>
</tr>
<tr>
<td>N 12</td>
<td>OP</td>
<td>0.39</td>
<td>401</td>
<td>344</td>
<td>0.0033</td>
<td>0.0016</td>
<td>0.0018</td>
<td>217</td>
<td>172</td>
<td>0.0053</td>
<td>0.0031</td>
</tr>
<tr>
<td>N 6</td>
<td>OP</td>
<td>0.39</td>
<td>390</td>
<td>337</td>
<td>0.0033</td>
<td>0.0016</td>
<td>0.0017</td>
<td>214</td>
<td>174</td>
<td>0.0053</td>
<td>0.0031</td>
</tr>
<tr>
<td>N 14</td>
<td>OP</td>
<td>0.30</td>
<td>276</td>
<td>272</td>
<td>0.0015</td>
<td>0.0002</td>
<td>0.0002</td>
<td>157</td>
<td>155</td>
<td>0.0024</td>
<td>0.0005</td>
</tr>
<tr>
<td>N 4</td>
<td>OP</td>
<td>0.30</td>
<td>279</td>
<td>275</td>
<td>0.0015</td>
<td>0.0002</td>
<td>0.0002</td>
<td>156</td>
<td>152</td>
<td>0.0024</td>
<td>0.0005</td>
</tr>
<tr>
<td>Solid Specimens</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NS 3</td>
<td>IP</td>
<td>0.44</td>
<td>367</td>
<td>367</td>
<td>0.0069</td>
<td>0.0051</td>
<td>0.0048</td>
<td>198</td>
<td>198</td>
<td>0.0120</td>
<td>0.0095</td>
</tr>
<tr>
<td>NS 1</td>
<td>IP</td>
<td>0.40</td>
<td>288</td>
<td>288</td>
<td>0.0033</td>
<td>0.0019</td>
<td>0.0017</td>
<td>162</td>
<td>162</td>
<td>0.0057</td>
<td>0.0037</td>
</tr>
<tr>
<td>NS 7</td>
<td>IP</td>
<td>0.39</td>
<td>297</td>
<td>297</td>
<td>0.0033</td>
<td>0.0018</td>
<td>0.0016</td>
<td>162</td>
<td>162</td>
<td>0.0057</td>
<td>0.0037</td>
</tr>
<tr>
<td>NS 5</td>
<td>IP</td>
<td>0.33</td>
<td>216</td>
<td>216</td>
<td>0.0015</td>
<td>0.0004</td>
<td>0.0003</td>
<td>124</td>
<td>124</td>
<td>0.0026</td>
<td>0.0011</td>
</tr>
<tr>
<td>NS 2</td>
<td>OP</td>
<td>0.44</td>
<td>448</td>
<td>327</td>
<td>0.0069</td>
<td>0.0053</td>
<td>0.0051</td>
<td>258</td>
<td>153</td>
<td>0.0120</td>
<td>0.0101</td>
</tr>
<tr>
<td>NS 4</td>
<td>OP</td>
<td>0.38</td>
<td>374</td>
<td>336</td>
<td>0.0033</td>
<td>0.0016</td>
<td>0.0017</td>
<td>215</td>
<td>182</td>
<td>0.0057</td>
<td>0.0035</td>
</tr>
<tr>
<td>NS 6</td>
<td>OP</td>
<td>0.29</td>
<td>260</td>
<td>257</td>
<td>0.0015</td>
<td>0.0002</td>
<td>0.0002</td>
<td>162</td>
<td>157</td>
<td>0.0026</td>
<td>0.0007</td>
</tr>
</tbody>
</table>
Table 3.8  Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) axial-torsion stable cycle stress-strain analysis results for tubular specimens of 1050 QT steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>Load Path</th>
<th>Maximum</th>
<th>σa (MPa)</th>
<th>εa @ εMax</th>
<th>Δεp/2</th>
<th>Δγp/2</th>
<th>Maximum</th>
<th>τa (MPa)</th>
<th>εa @ εMax</th>
<th>Δγp/2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Calculated</td>
<td>Measured</td>
<td></td>
<td></td>
<td>Calculated</td>
<td>Measured</td>
</tr>
<tr>
<td>Q 7</td>
<td>IP</td>
<td>0.40</td>
<td>594</td>
<td>0.0069</td>
<td>0.0040</td>
<td>0.0036</td>
<td>331</td>
<td>331</td>
<td>0.0112</td>
<td>0.0071</td>
</tr>
<tr>
<td>Q 1</td>
<td>IP</td>
<td>0.40</td>
<td>596</td>
<td>0.0068</td>
<td>0.0039</td>
<td>0.0032</td>
<td>328</td>
<td>328</td>
<td>0.0112</td>
<td>0.0071</td>
</tr>
<tr>
<td>Q 9</td>
<td>IP</td>
<td>0.32</td>
<td>477</td>
<td>0.0032</td>
<td>0.0009</td>
<td>0.0007</td>
<td>277</td>
<td>277</td>
<td>0.0052</td>
<td>0.0017</td>
</tr>
<tr>
<td>Q 5</td>
<td>IP</td>
<td>0.32</td>
<td>495</td>
<td>0.0033</td>
<td>0.0009</td>
<td>0.0007</td>
<td>289</td>
<td>289</td>
<td>0.0054</td>
<td>0.0018</td>
</tr>
<tr>
<td>Q 11</td>
<td>IP</td>
<td>0.27</td>
<td>457</td>
<td>0.0024</td>
<td>0.0002</td>
<td>0.0001</td>
<td>273</td>
<td>273</td>
<td>0.0037</td>
<td>0.0004</td>
</tr>
<tr>
<td>Q 3</td>
<td>IP</td>
<td>0.26</td>
<td>403</td>
<td>0.0020</td>
<td>0</td>
<td>0</td>
<td>258</td>
<td>258</td>
<td>0.0032</td>
<td>0</td>
</tr>
<tr>
<td>Q 12</td>
<td>OP</td>
<td>0.43</td>
<td>814</td>
<td>0.0105</td>
<td>0.0076</td>
<td>0.0073</td>
<td>457</td>
<td>312</td>
<td>0.0170</td>
<td>0.0132</td>
</tr>
<tr>
<td>Q 2</td>
<td>OP</td>
<td>0.40</td>
<td>722</td>
<td>0.0069</td>
<td>0.0041</td>
<td>0.0040</td>
<td>405</td>
<td>328</td>
<td>0.0112</td>
<td>0.0071</td>
</tr>
<tr>
<td>Q 6</td>
<td>OP</td>
<td>0.39</td>
<td>740</td>
<td>0.0069</td>
<td>0.0039</td>
<td>0.0038</td>
<td>418</td>
<td>341</td>
<td>0.0112</td>
<td>0.0069</td>
</tr>
<tr>
<td>Q 8</td>
<td>OP</td>
<td>0.30</td>
<td>581</td>
<td>0.0033</td>
<td>0.0005</td>
<td>0.0006</td>
<td>331</td>
<td>325</td>
<td>0.0052</td>
<td>0.0012</td>
</tr>
<tr>
<td>Q 15</td>
<td>OP</td>
<td>0.28</td>
<td>582</td>
<td>0.0031</td>
<td>0.0003</td>
<td>0.0004</td>
<td>334</td>
<td>333</td>
<td>0.0050</td>
<td>0.0009</td>
</tr>
<tr>
<td>Q 4</td>
<td>OP</td>
<td>0.27</td>
<td>593</td>
<td>0.0031</td>
<td>0.0002</td>
<td>0.0003</td>
<td>347</td>
<td>344</td>
<td>0.0050</td>
<td>0.0008</td>
</tr>
<tr>
<td>Q 10</td>
<td>OP</td>
<td>0.27</td>
<td>488</td>
<td>0.0025</td>
<td>0.0001</td>
<td>0.0001</td>
<td>290</td>
<td>289</td>
<td>0.0040</td>
<td>0.0004</td>
</tr>
<tr>
<td>Q 13</td>
<td>OP</td>
<td>0.26</td>
<td>371</td>
<td>0.0018</td>
<td>0</td>
<td>0</td>
<td>233</td>
<td>233</td>
<td>0.0029</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 3.9  Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) axial-torsion stable cycle stress-strain analysis results for tubular specimens of 1050 IH steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>Load Path</th>
<th>Maximum</th>
<th>( \sigma_a ) @ ( \varepsilon_{\text{Max}} )</th>
<th>( \varepsilon_a )</th>
<th>( \Delta \varepsilon_p / 2 ) Calculated</th>
<th>( \Delta \varepsilon_p / 2 ) Measured</th>
<th>( \tau_a ) @ ( \varepsilon_{\text{Max}} )</th>
<th>( \gamma_a ) Calculated</th>
<th>( \Delta \gamma_p / 2 ) Calculated</th>
<th>( \Delta \gamma_p / 2 ) Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>113</td>
<td>IP</td>
<td>0.32</td>
<td>1493</td>
<td>1493</td>
<td>0.0100</td>
<td>0.0027</td>
<td>0.0023</td>
<td>896</td>
<td>896</td>
<td>0.0160</td>
</tr>
<tr>
<td>115</td>
<td>IP</td>
<td>0.32</td>
<td>1443</td>
<td>1443</td>
<td>0.0100</td>
<td>0.0027</td>
<td>0.0022</td>
<td>881</td>
<td>881</td>
<td>0.0160</td>
</tr>
<tr>
<td>119</td>
<td>IP</td>
<td>0.30</td>
<td>1369</td>
<td>1369</td>
<td>0.0085</td>
<td>0.0015</td>
<td>0.0009</td>
<td>842</td>
<td>842</td>
<td>0.0136</td>
</tr>
<tr>
<td>15</td>
<td>IP</td>
<td>0.30</td>
<td>1328</td>
<td>1328</td>
<td>0.0085</td>
<td>0.0017</td>
<td>0.0009</td>
<td>819</td>
<td>819</td>
<td>0.0136</td>
</tr>
<tr>
<td>11</td>
<td>IP</td>
<td>0.28</td>
<td>1279</td>
<td>1279</td>
<td>0.0069</td>
<td>0.0006</td>
<td>0.0005</td>
<td>791</td>
<td>791</td>
<td>0.0110</td>
</tr>
<tr>
<td>13</td>
<td>IP</td>
<td>0.28</td>
<td>1274</td>
<td>1274</td>
<td>0.0069</td>
<td>0.0006</td>
<td>0.0004</td>
<td>790</td>
<td>790</td>
<td>0.0110</td>
</tr>
<tr>
<td>123</td>
<td>IP</td>
<td>0.26</td>
<td>656</td>
<td>656</td>
<td>0.0033</td>
<td>0</td>
<td>0</td>
<td>406</td>
<td>406</td>
<td>0.0053</td>
</tr>
<tr>
<td>17</td>
<td>IP</td>
<td>0.26</td>
<td>643</td>
<td>643</td>
<td>0.0033</td>
<td>0.0001</td>
<td>0</td>
<td>411</td>
<td>411</td>
<td>0.0053</td>
</tr>
<tr>
<td>19</td>
<td>IP</td>
<td>0.26</td>
<td>512</td>
<td>512</td>
<td>0.0025</td>
<td>0</td>
<td>0</td>
<td>321</td>
<td>321</td>
<td>0.0040</td>
</tr>
<tr>
<td>121</td>
<td>IP</td>
<td>0.26</td>
<td>509</td>
<td>509</td>
<td>0.0025</td>
<td>0</td>
<td>0</td>
<td>321</td>
<td>321</td>
<td>0.0040</td>
</tr>
<tr>
<td>110</td>
<td>OP</td>
<td>0.28</td>
<td>1778</td>
<td>1758</td>
<td>0.0095</td>
<td>0.0008</td>
<td>0.0008</td>
<td>1045</td>
<td>1041</td>
<td>0.0153</td>
</tr>
<tr>
<td>116</td>
<td>OP</td>
<td>0.28</td>
<td>1702</td>
<td>1669</td>
<td>0.0095</td>
<td>0.0009</td>
<td>0.0007</td>
<td>1006</td>
<td>994</td>
<td>0.0152</td>
</tr>
<tr>
<td>114</td>
<td>OP</td>
<td>0.27</td>
<td>1517</td>
<td>1517</td>
<td>0.0085</td>
<td>0.0005</td>
<td>0.0001</td>
<td>918</td>
<td>918</td>
<td>0.0136</td>
</tr>
<tr>
<td>12</td>
<td>OP</td>
<td>0.26</td>
<td>1585</td>
<td>1585</td>
<td>0.0085</td>
<td>0.0003</td>
<td>0</td>
<td>928</td>
<td>928</td>
<td>0.0136</td>
</tr>
<tr>
<td>16</td>
<td>OP</td>
<td>0.26</td>
<td>1301</td>
<td>1301</td>
<td>0.0069</td>
<td>0.0001</td>
<td>0</td>
<td>797</td>
<td>797</td>
<td>0.0110</td>
</tr>
<tr>
<td>18</td>
<td>OP</td>
<td>0.26</td>
<td>670</td>
<td>670</td>
<td>0.0033</td>
<td>0</td>
<td>0</td>
<td>414</td>
<td>414</td>
<td>0.0053</td>
</tr>
<tr>
<td>112</td>
<td>OP</td>
<td>0.26</td>
<td>520</td>
<td>520</td>
<td>0.0025</td>
<td>0</td>
<td>0</td>
<td>318</td>
<td>318</td>
<td>0.0040</td>
</tr>
<tr>
<td>118</td>
<td>OP</td>
<td>0.26</td>
<td>501</td>
<td>501</td>
<td>0.0024</td>
<td>0</td>
<td>0</td>
<td>308</td>
<td>308</td>
<td>0.0038</td>
</tr>
<tr>
<td>122</td>
<td>OP</td>
<td>0.26</td>
<td>509</td>
<td>509</td>
<td>0.0025</td>
<td>0</td>
<td>0</td>
<td>313</td>
<td>313</td>
<td>0.0040</td>
</tr>
<tr>
<td>14</td>
<td>OP</td>
<td>0.26</td>
<td>416</td>
<td>416</td>
<td>0.0020</td>
<td>0</td>
<td>0</td>
<td>252</td>
<td>252</td>
<td>0.0032</td>
</tr>
</tbody>
</table>
Table 3.10  Multiaxial constant amplitude in-phase (IP) and 90° out-of-phase (OP) axial-torsion stable cycle stress-strain analysis results for 304L stainless steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>Load Path</th>
<th>Maximum $\sigma_a$</th>
<th>$\nu$</th>
<th>@ $\varepsilon_{Max}$</th>
<th>$\varepsilon_a$</th>
<th>$\Delta \varepsilon_p / 2$</th>
<th>$\Delta \varepsilon_p / 2$</th>
<th>Maximum $\tau_a$</th>
<th>@ $\varepsilon_{Max}$</th>
<th>$\gamma_a$</th>
<th>$\Delta \gamma_p / 2$</th>
<th>$\Delta \gamma_p / 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSP 1</td>
<td>IP</td>
<td>0.44</td>
<td>333</td>
<td>333</td>
<td>0.0069</td>
<td>0.0052</td>
<td>0.0049</td>
<td>177</td>
<td>177</td>
<td>0.0110</td>
<td>0.0087</td>
<td>0.0083</td>
</tr>
<tr>
<td>SSP 11</td>
<td>IP</td>
<td>0.43</td>
<td>269</td>
<td>269</td>
<td>0.0049</td>
<td>0.0035</td>
<td>0.0033</td>
<td>154</td>
<td>154</td>
<td>0.0079</td>
<td>0.0059</td>
<td>0.0058</td>
</tr>
<tr>
<td>SSP 9</td>
<td>IP</td>
<td>0.40</td>
<td>250</td>
<td>250</td>
<td>0.0030</td>
<td>0.0017</td>
<td>0.0019</td>
<td>140</td>
<td>140</td>
<td>0.0053</td>
<td>0.0035</td>
<td>0.0034</td>
</tr>
<tr>
<td>SSP 13</td>
<td>IP</td>
<td>0.40</td>
<td>191</td>
<td>191</td>
<td>0.0023</td>
<td>0.0013</td>
<td>0.0012</td>
<td>109</td>
<td>109</td>
<td>0.0038</td>
<td>0.0023</td>
<td>0.0023</td>
</tr>
<tr>
<td>SSP 7</td>
<td>IP</td>
<td>0.39</td>
<td>167</td>
<td>167</td>
<td>0.0018</td>
<td>0.0009</td>
<td>0.0007</td>
<td>98</td>
<td>98</td>
<td>0.0028</td>
<td>0.0015</td>
<td>0.0015</td>
</tr>
<tr>
<td>SSNP 10</td>
<td>OP</td>
<td>0.42</td>
<td>608</td>
<td>482</td>
<td>0.0069</td>
<td>0.0044</td>
<td>0.0042</td>
<td>351</td>
<td>280</td>
<td>0.0111</td>
<td>0.0075</td>
<td>0.0069</td>
</tr>
<tr>
<td>SSNP 2</td>
<td>OP</td>
<td>0.42</td>
<td>625</td>
<td>488</td>
<td>0.0069</td>
<td>0.0044</td>
<td>0.0043</td>
<td>359</td>
<td>279</td>
<td>0.0111</td>
<td>0.0074</td>
<td>0.0070</td>
</tr>
<tr>
<td>SSNP 4</td>
<td>OP</td>
<td>0.36</td>
<td>421</td>
<td>384</td>
<td>0.0033</td>
<td>0.0013</td>
<td>0.0014</td>
<td>239</td>
<td>217</td>
<td>0.0053</td>
<td>0.0024</td>
<td>0.0023</td>
</tr>
<tr>
<td>SSNP 8</td>
<td>OP</td>
<td>0.36</td>
<td>431</td>
<td>390</td>
<td>0.0033</td>
<td>0.0013</td>
<td>0.0014</td>
<td>246</td>
<td>223</td>
<td>0.0053</td>
<td>0.0024</td>
<td>0.0024</td>
</tr>
<tr>
<td>SSNP 6</td>
<td>OP</td>
<td>0.33</td>
<td>269</td>
<td>259</td>
<td>0.0017</td>
<td>0.0004</td>
<td>0.0005</td>
<td>151</td>
<td>148</td>
<td>0.0028</td>
<td>0.0009</td>
<td>0.0005</td>
</tr>
</tbody>
</table>
Table 3.11 Axial-torsion variable amplitude test information and results for tubular specimens of 1050 QT steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>$d_i$ (mm)</th>
<th>$d_o$ (mm)</th>
<th>Load Path</th>
<th>Freq. (Hz)</th>
<th>$E$ (GPa)</th>
<th>$G$ (GPa)</th>
<th>$(\varepsilon_a)_\text{max}$</th>
<th>$(\varepsilon_m)_\text{max}$</th>
<th>$(\gamma_a)_\text{max}$</th>
<th>$(\gamma_m)_\text{max}$</th>
<th>$(P_a)_\text{max}$ (kN)</th>
<th>$(P_m)_\text{max}$ (kN)</th>
<th>$(T_a)_\text{max}$ (Nm)</th>
<th>$(T_m)_\text{max}$ (Nm)</th>
<th>Failure $(L_f)$ (Block)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10FRI2QT</td>
<td>15.52</td>
<td>17.81</td>
<td>FRI</td>
<td>0.2</td>
<td>212.4</td>
<td>82.5</td>
<td>0.0069</td>
<td>0</td>
<td>0.0120</td>
<td>0</td>
<td>43.3</td>
<td>-0.6</td>
<td>204.0</td>
<td>0.0</td>
<td>4.5</td>
</tr>
<tr>
<td>10FRI1QT</td>
<td>15.52</td>
<td>17.83</td>
<td>FRI</td>
<td>0.2</td>
<td>208.7</td>
<td>NA</td>
<td>0.0070</td>
<td>0</td>
<td>0.0122</td>
<td>-0.0001</td>
<td>45.7</td>
<td>-0.7</td>
<td>216.1</td>
<td>-1.3</td>
<td>6.3</td>
</tr>
<tr>
<td>5FRI1QT</td>
<td>15.52</td>
<td>17.86</td>
<td>FRI</td>
<td>0.5</td>
<td>205.5</td>
<td>NA</td>
<td>0.0033</td>
<td>0</td>
<td>0.0057</td>
<td>0</td>
<td>38.7</td>
<td>-0.6</td>
<td>189.5</td>
<td>-3.5</td>
<td>41.0</td>
</tr>
<tr>
<td>5FRI2QT</td>
<td>15.37</td>
<td>17.81</td>
<td>FRI</td>
<td>0.5</td>
<td>210.8</td>
<td>82.4</td>
<td>0.0033</td>
<td>0</td>
<td>0.0058</td>
<td>0</td>
<td>39.4</td>
<td>-4.1</td>
<td>190.0</td>
<td>-10.0</td>
<td>90.0</td>
</tr>
<tr>
<td>10FRR1QT</td>
<td>15.52</td>
<td>17.83</td>
<td>FRR</td>
<td>0.2</td>
<td>209.8</td>
<td>84.4</td>
<td>0.0070</td>
<td>0</td>
<td>0.0121</td>
<td>0</td>
<td>48.5</td>
<td>-1.1</td>
<td>229.3</td>
<td>3.4</td>
<td>4.2</td>
</tr>
<tr>
<td>10FRR2QT</td>
<td>15.52</td>
<td>17.81</td>
<td>FRR</td>
<td>0.2</td>
<td>214.4</td>
<td>85.7</td>
<td>0.0070</td>
<td>0</td>
<td>0.0120</td>
<td>0</td>
<td>50.6</td>
<td>-0.4</td>
<td>231.5</td>
<td>0.5</td>
<td>6.5</td>
</tr>
<tr>
<td>5FRR3QT</td>
<td>15.34</td>
<td>17.82</td>
<td>FRR</td>
<td>0.5</td>
<td>206.5</td>
<td>83.3</td>
<td>0.0033</td>
<td>0</td>
<td>0.0059</td>
<td>-0.0001</td>
<td>39.9</td>
<td>-0.7</td>
<td>197.4</td>
<td>-3.4</td>
<td>94.0</td>
</tr>
<tr>
<td>5FRR1QT</td>
<td>15.34</td>
<td>17.83</td>
<td>FRR</td>
<td>0.5</td>
<td>206.5</td>
<td>83.3</td>
<td>0.0034</td>
<td>0</td>
<td>0.0059</td>
<td>-0.0001</td>
<td>40.6</td>
<td>-0.2</td>
<td>197.9</td>
<td>-8.3</td>
<td>118.0</td>
</tr>
<tr>
<td>5FRI1QT90</td>
<td>15.25</td>
<td>17.84</td>
<td>FRI90</td>
<td>0.5</td>
<td>NA</td>
<td>NA</td>
<td>0.0031</td>
<td>0</td>
<td>0.0058</td>
<td>0</td>
<td>40.2</td>
<td>-0.6</td>
<td>203.1</td>
<td>-5.6</td>
<td>4,974</td>
</tr>
<tr>
<td>5FRI2QT90</td>
<td>15.24</td>
<td>17.83</td>
<td>FRI90</td>
<td>0.5</td>
<td>NA</td>
<td>NA</td>
<td>0.0031</td>
<td>0</td>
<td>0.0058</td>
<td>0</td>
<td>40.0</td>
<td>-1.5</td>
<td>201.4</td>
<td>-7.2</td>
<td>7,325</td>
</tr>
<tr>
<td>10PI1QT</td>
<td>15.52</td>
<td>17.84</td>
<td>PI</td>
<td>0.5</td>
<td>203.4</td>
<td>81.6</td>
<td>0.0034</td>
<td>0.0034</td>
<td>0.0061</td>
<td>0.0061</td>
<td>35.6</td>
<td>8.0</td>
<td>172.0</td>
<td>37.6</td>
<td>42.0</td>
</tr>
<tr>
<td>10PI2QT</td>
<td>15.52</td>
<td>17.81</td>
<td>PI</td>
<td>0.5</td>
<td>205.4</td>
<td>81.2</td>
<td>0.0035</td>
<td>0.0035</td>
<td>0.0060</td>
<td>0.0060</td>
<td>37.1</td>
<td>10.0</td>
<td>180.6</td>
<td>43.5</td>
<td>54.0</td>
</tr>
<tr>
<td>5PI1QT</td>
<td>15.34</td>
<td>17.81</td>
<td>PI</td>
<td>1</td>
<td>203.4</td>
<td>81.6</td>
<td>0.017</td>
<td>0.017</td>
<td>0.0029</td>
<td>0.0029</td>
<td>22.0</td>
<td>21.6</td>
<td>114.3</td>
<td>103.8</td>
<td>NA</td>
</tr>
<tr>
<td>10PI1QT90</td>
<td>15.25</td>
<td>17.84</td>
<td>PI90</td>
<td>0.5</td>
<td>NA</td>
<td>NA</td>
<td>0.0034</td>
<td>0.0034</td>
<td>0.0061</td>
<td>0.0061</td>
<td>38.8</td>
<td>14.1</td>
<td>187.2</td>
<td>59.1</td>
<td>1,125</td>
</tr>
<tr>
<td>10PI2QT90</td>
<td>15.25</td>
<td>17.84</td>
<td>PI90</td>
<td>0.5</td>
<td>NA</td>
<td>NA</td>
<td>0.0034</td>
<td>0.0034</td>
<td>0.0060</td>
<td>0.0060</td>
<td>39.0</td>
<td>13.5</td>
<td>185.6</td>
<td>51.8</td>
<td>1,275</td>
</tr>
<tr>
<td>Spec. ID</td>
<td>$d_i$ (mm)</td>
<td>$d_o$ (mm)</td>
<td>Load Path</td>
<td>Freq. (Hz)</td>
<td>$E$ (GPa)</td>
<td>$G$ (GPa)</td>
<td>$\varepsilon_{a_{\max}}$</td>
<td>$\varepsilon_{m_{\max}}$</td>
<td>$\gamma_{a_{\max}}$</td>
<td>$\gamma_{m_{\max}}$</td>
<td>$P_{a_{\max}}$ (kN)</td>
<td>$P_{m_{\max}}$ (kN)</td>
<td>$T_{a_{\max}}$ (Nm)</td>
<td>$T_{m_{\max}}$ (Nm)</td>
<td>$L_f$ (Block)</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td>-------</td>
<td>----------</td>
<td>----------</td>
<td>--------</td>
<td>--------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>-------------</td>
</tr>
<tr>
<td>10FR12SS</td>
<td>13.11</td>
<td>15.15</td>
<td>FRI</td>
<td>0.2</td>
<td>190.6</td>
<td>74.2</td>
<td>0.0070</td>
<td>0</td>
<td>0.0120</td>
<td>0</td>
<td>21.7</td>
<td>-2.0</td>
<td>85.0</td>
<td>0.1</td>
<td>5.5</td>
</tr>
<tr>
<td>10FR11SS</td>
<td>12.88</td>
<td>15.16</td>
<td>FRI</td>
<td>0.2</td>
<td>NA</td>
<td>NA</td>
<td>0.0070</td>
<td>0</td>
<td>0.0120</td>
<td>0</td>
<td>23.2</td>
<td>-0.2</td>
<td>89.8</td>
<td>0.2</td>
<td>5.8</td>
</tr>
<tr>
<td>5FR11SS</td>
<td>12.92</td>
<td>15.18</td>
<td>FRI</td>
<td>0.5</td>
<td>193.5</td>
<td>76.3</td>
<td>0.0034</td>
<td>0</td>
<td>0.0057</td>
<td>0</td>
<td>19.5</td>
<td>-0.3</td>
<td>76.2</td>
<td>-0.2</td>
<td>28.0</td>
</tr>
<tr>
<td>5FR12SS</td>
<td>13.12</td>
<td>15.14</td>
<td>FRI</td>
<td>0.5</td>
<td>195.3</td>
<td>78.9</td>
<td>0.0034</td>
<td>0</td>
<td>0.0058</td>
<td>0</td>
<td>16.2</td>
<td>-0.2</td>
<td>64.7</td>
<td>0.0</td>
<td>53.0</td>
</tr>
<tr>
<td>2.5FR11SS</td>
<td>13.12</td>
<td>15.15</td>
<td>FRI</td>
<td>1.25</td>
<td>188.3</td>
<td>76.3</td>
<td>0.0018</td>
<td>0</td>
<td>0.0030</td>
<td>0</td>
<td>12.8</td>
<td>-1.2</td>
<td>51.6</td>
<td>-0.4</td>
<td>NA</td>
</tr>
<tr>
<td>10FRR1SS</td>
<td>12.95</td>
<td>15.16</td>
<td>FRR</td>
<td>0.2</td>
<td>NA</td>
<td>NA</td>
<td>0.0070</td>
<td>0</td>
<td>0.0120</td>
<td>0</td>
<td>29.5</td>
<td>-0.5</td>
<td>118.0</td>
<td>0.0</td>
<td>4.0</td>
</tr>
<tr>
<td>10FRR2SS</td>
<td>13.09</td>
<td>15.16</td>
<td>FRR</td>
<td>0.2</td>
<td>197.0</td>
<td>76.0</td>
<td>0.0070</td>
<td>0</td>
<td>0.0120</td>
<td>0</td>
<td>26.7</td>
<td>-0.4</td>
<td>108.2</td>
<td>0.0</td>
<td>4.0</td>
</tr>
<tr>
<td>5FRR1SS</td>
<td>13.08</td>
<td>15.16</td>
<td>FRR</td>
<td>0.5</td>
<td>192.0</td>
<td>85.0</td>
<td>0.0034</td>
<td>0</td>
<td>0.0058</td>
<td>0</td>
<td>20.0</td>
<td>-0.2</td>
<td>81.5</td>
<td>-0.2</td>
<td>39.0</td>
</tr>
<tr>
<td>5FRR2SS</td>
<td>13.12</td>
<td>15.14</td>
<td>FRR</td>
<td>0.5</td>
<td>187.7</td>
<td>73.2</td>
<td>0.0034</td>
<td>0</td>
<td>0.0058</td>
<td>0</td>
<td>19.1</td>
<td>-0.1</td>
<td>77.9</td>
<td>0.1</td>
<td>45.0</td>
</tr>
<tr>
<td>2.5FRR1SS</td>
<td>13.11</td>
<td>15.15</td>
<td>FRR</td>
<td>1.25</td>
<td>194.7</td>
<td>78.1</td>
<td>0.0018</td>
<td>0</td>
<td>0.0031</td>
<td>0</td>
<td>12.9</td>
<td>-0.9</td>
<td>52.3</td>
<td>0.0</td>
<td>NA</td>
</tr>
<tr>
<td>5FRI2SS15</td>
<td>13.11</td>
<td>15.16</td>
<td>FRI15</td>
<td>0.5</td>
<td>194.2</td>
<td>77.8</td>
<td>0.0034</td>
<td>0</td>
<td>0.0058</td>
<td>0</td>
<td>18.9</td>
<td>-0.2</td>
<td>74.5</td>
<td>-0.1</td>
<td>375</td>
</tr>
<tr>
<td>5FRI1SS15</td>
<td>13.11</td>
<td>15.16</td>
<td>FRI15</td>
<td>0.5</td>
<td>197.6</td>
<td>77.0</td>
<td>0.0034</td>
<td>0</td>
<td>0.0058</td>
<td>0</td>
<td>18.2</td>
<td>-0.2</td>
<td>72.5</td>
<td>0.2</td>
<td>595</td>
</tr>
<tr>
<td>10PI1SS</td>
<td>13.09</td>
<td>15.16</td>
<td>PI</td>
<td>0.5</td>
<td>192.4</td>
<td>76.2</td>
<td>0.0035</td>
<td>0.0035</td>
<td>0.0060</td>
<td>0.0060</td>
<td>19.0</td>
<td>2.0</td>
<td>75.8</td>
<td>6.0</td>
<td>22.0</td>
</tr>
<tr>
<td>10PI2SS</td>
<td>13.11</td>
<td>15.19</td>
<td>PI</td>
<td>0.5</td>
<td>198.2</td>
<td>76.5</td>
<td>0.0035</td>
<td>0.0035</td>
<td>0.0060</td>
<td>0.0060</td>
<td>18.6</td>
<td>1.3</td>
<td>76.9</td>
<td>6.0</td>
<td>27.3</td>
</tr>
</tbody>
</table>
Table 3.13  Axial-torsion variable amplitude stable block stress-strain analysis results for tubular specimens of 1050 QT steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>Load Path</th>
<th>$L_{stable}$ (Block)</th>
<th>$\nu$</th>
<th>$(\sigma_a)_{max}$ (MPa)</th>
<th>$(\sigma_m)_{max}$ (MPa)</th>
<th>$(\varepsilon_a)_{max}$</th>
<th>$(\Delta\varepsilon_p / 2)_{max}$ Calculated</th>
<th>$(\tau_a)_{max}$ (MPa)</th>
<th>$(\tau_m)_{max}$ (MPa)</th>
<th>$(\gamma_a)_{max}$</th>
<th>$(\Delta\gamma_p / 2)_{max}$ Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>10FRI2QT</td>
<td>FRI</td>
<td>3</td>
<td>0.40</td>
<td>724</td>
<td>-10</td>
<td>0.0069</td>
<td>0.0034</td>
<td>409</td>
<td>0</td>
<td>0.0112</td>
<td>0.0063</td>
</tr>
<tr>
<td>10FRI1QT</td>
<td>FRI</td>
<td>4</td>
<td>0.40</td>
<td>751</td>
<td>-12</td>
<td>0.0070</td>
<td>0.0034</td>
<td>428</td>
<td>-2</td>
<td>0.0114</td>
<td>0.0062</td>
</tr>
<tr>
<td>5FRI1QT</td>
<td>FRI</td>
<td>20</td>
<td>0.32</td>
<td>632</td>
<td>-10</td>
<td>0.0033</td>
<td>0.0003</td>
<td>371</td>
<td>-7</td>
<td>0.0054</td>
<td>0.0009</td>
</tr>
<tr>
<td>5FRI2QT</td>
<td>FRI</td>
<td>10</td>
<td>0.32</td>
<td>620</td>
<td>-65</td>
<td>0.0033</td>
<td>0.0004</td>
<td>361</td>
<td>-19</td>
<td>0.0054</td>
<td>0.0010</td>
</tr>
<tr>
<td>10FRR1QT</td>
<td>FRI</td>
<td>3</td>
<td>0.39</td>
<td>801</td>
<td>-17</td>
<td>0.0070</td>
<td>0.0031</td>
<td>454</td>
<td>7</td>
<td>0.0113</td>
<td>0.0059</td>
</tr>
<tr>
<td>10FRR2QT</td>
<td>FRI</td>
<td>4</td>
<td>0.39</td>
<td>800</td>
<td>-7</td>
<td>0.0070</td>
<td>0.0031</td>
<td>442</td>
<td>1</td>
<td>0.0112</td>
<td>0.0059</td>
</tr>
<tr>
<td>5FRR3QT</td>
<td>FRI</td>
<td>50</td>
<td>0.32</td>
<td>619</td>
<td>-10</td>
<td>0.0033</td>
<td>0.0004</td>
<td>369</td>
<td>-6</td>
<td>0.0055</td>
<td>0.0011</td>
</tr>
<tr>
<td>5FRR1QT</td>
<td>FRI</td>
<td>60</td>
<td>0.33</td>
<td>610</td>
<td>-3</td>
<td>0.0034</td>
<td>0.0004</td>
<td>350</td>
<td>-15</td>
<td>0.0055</td>
<td>0.0013</td>
</tr>
<tr>
<td>5FRI1QT90</td>
<td>FRI90</td>
<td>2,000</td>
<td>0.32</td>
<td>597</td>
<td>-9</td>
<td>0.0031</td>
<td>0.0002</td>
<td>362</td>
<td>-10</td>
<td>0.0054</td>
<td>0.0009</td>
</tr>
<tr>
<td>5FRI2QT90</td>
<td>FRI90</td>
<td>3,000</td>
<td>0.32</td>
<td>595</td>
<td>-22</td>
<td>0.0031</td>
<td>0.0002</td>
<td>359</td>
<td>-13</td>
<td>0.0054</td>
<td>0.0010</td>
</tr>
<tr>
<td>10PI1QT</td>
<td>PI</td>
<td>20</td>
<td>0.33</td>
<td>585</td>
<td>132</td>
<td>0.0034</td>
<td>0.0006</td>
<td>339</td>
<td>75</td>
<td>0.0057</td>
<td>0.0016</td>
</tr>
<tr>
<td>10PI2QT</td>
<td>PI</td>
<td>9</td>
<td>0.33</td>
<td>619</td>
<td>167</td>
<td>0.0035</td>
<td>0.0005</td>
<td>362</td>
<td>88</td>
<td>0.0057</td>
<td>0.0013</td>
</tr>
<tr>
<td>5PI1QT</td>
<td>PI</td>
<td>8</td>
<td>0.28</td>
<td>367</td>
<td>361</td>
<td>0.0017</td>
<td>0</td>
<td>229</td>
<td>210</td>
<td>0.0027</td>
<td>0</td>
</tr>
<tr>
<td>10PI1QT90</td>
<td>PI90</td>
<td>600</td>
<td>0.33</td>
<td>576</td>
<td>209</td>
<td>0.0034</td>
<td>0.0006</td>
<td>334</td>
<td>106</td>
<td>0.0057</td>
<td>0.0016</td>
</tr>
<tr>
<td>10PI2QT90</td>
<td>PI90</td>
<td>600</td>
<td>0.33</td>
<td>579</td>
<td>201</td>
<td>0.0034</td>
<td>0.0005</td>
<td>331</td>
<td>93</td>
<td>0.0057</td>
<td>0.0016</td>
</tr>
</tbody>
</table>
Table 3.14  Axial-torsion variable amplitude stable block stress-strain analysis results for tubular specimens of 304L stainless steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>$L_{\text{stable}}$ (Block)</th>
<th>Load Path</th>
<th>$\bar{V}$</th>
<th>$(\sigma_a)_{\text{max}}$</th>
<th>$(\sigma_m)_{\text{max}}$</th>
<th>$(\epsilon_a)_{\text{max}}$</th>
<th>$(\Delta \epsilon_p / \bar{V})_{\text{max}}$</th>
<th>$(\tau_a)_{\text{max}}$</th>
<th>$(\tau_m)_{\text{max}}$</th>
<th>$(\gamma_a)_{\text{max}}$</th>
<th>$(\Delta \gamma_p / \bar{V})_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10FRI2SS</td>
<td>3 FRI</td>
<td>0.43</td>
<td>459</td>
<td>-42</td>
<td>0.0070</td>
<td>0.0046</td>
<td>264</td>
<td>0</td>
<td>0.0112</td>
<td>0.0085</td>
<td></td>
</tr>
<tr>
<td>10FRI1SS</td>
<td>3 FRI</td>
<td>0.43</td>
<td>461</td>
<td>-4</td>
<td>0.0070</td>
<td>0.0046</td>
<td>254</td>
<td>0</td>
<td>0.0111</td>
<td>0.0086</td>
<td></td>
</tr>
<tr>
<td>5FRI1SS</td>
<td>15 FRI</td>
<td>0.38</td>
<td>391</td>
<td>-6</td>
<td>0.0034</td>
<td>0.0013</td>
<td>217</td>
<td>0</td>
<td>0.0053</td>
<td>0.0028</td>
<td></td>
</tr>
<tr>
<td>5FRI2SS</td>
<td>20 FRI</td>
<td>0.39</td>
<td>362</td>
<td>-4</td>
<td>0.0034</td>
<td>0.0015</td>
<td>204</td>
<td>0</td>
<td>0.0054</td>
<td>0.0031</td>
<td></td>
</tr>
<tr>
<td>2.5FRI1SS</td>
<td>1,600 FRI</td>
<td>0.33</td>
<td>283</td>
<td>-27</td>
<td>0.0018</td>
<td>0.0003</td>
<td>162</td>
<td>-1</td>
<td>0.0028</td>
<td>0.0009</td>
<td></td>
</tr>
<tr>
<td>10FRR1SS</td>
<td>3 FRR</td>
<td>0.41</td>
<td>604</td>
<td>-9</td>
<td>0.0070</td>
<td>0.0038</td>
<td>344</td>
<td>0</td>
<td>0.0112</td>
<td>0.0074</td>
<td></td>
</tr>
<tr>
<td>10FRR2SS</td>
<td>3 FRR</td>
<td>0.42</td>
<td>582</td>
<td>-9</td>
<td>0.0070</td>
<td>0.0039</td>
<td>333</td>
<td>0</td>
<td>0.0112</td>
<td>0.0075</td>
<td></td>
</tr>
<tr>
<td>5FRR1SS</td>
<td>20 FRR</td>
<td>0.37</td>
<td>434</td>
<td>-5</td>
<td>0.0034</td>
<td>0.0011</td>
<td>250</td>
<td>0</td>
<td>0.0054</td>
<td>0.0024</td>
<td></td>
</tr>
<tr>
<td>5FRR2SS</td>
<td>20 FRR</td>
<td>0.37</td>
<td>427</td>
<td>-3</td>
<td>0.0034</td>
<td>0.0012</td>
<td>246</td>
<td>0</td>
<td>0.0054</td>
<td>0.0025</td>
<td></td>
</tr>
<tr>
<td>2.5FRR1SS</td>
<td>150 FRR</td>
<td>0.34</td>
<td>284</td>
<td>-21</td>
<td>0.0018</td>
<td>0.0003</td>
<td>163</td>
<td>0</td>
<td>0.0028</td>
<td>0.0009</td>
<td></td>
</tr>
<tr>
<td>5FRI2SS15</td>
<td>200 FRI15</td>
<td>0.37</td>
<td>413</td>
<td>-5</td>
<td>0.0034</td>
<td>0.0012</td>
<td>231</td>
<td>0</td>
<td>0.0054</td>
<td>0.0027</td>
<td></td>
</tr>
<tr>
<td>5FRI1SS15</td>
<td>300 FRI15</td>
<td>0.38</td>
<td>398</td>
<td>-5</td>
<td>0.0034</td>
<td>0.0013</td>
<td>225</td>
<td>1</td>
<td>0.0054</td>
<td>0.0028</td>
<td></td>
</tr>
<tr>
<td>10PI1SS</td>
<td>10 PI</td>
<td>0.38</td>
<td>414</td>
<td>-43</td>
<td>0.0035</td>
<td>0.0013</td>
<td>234</td>
<td>19</td>
<td>0.0056</td>
<td>0.0029</td>
<td></td>
</tr>
<tr>
<td>10PI2SS</td>
<td>15 PI</td>
<td>0.38</td>
<td>402</td>
<td>28</td>
<td>0.0035</td>
<td>0.0014</td>
<td>235</td>
<td>19</td>
<td>0.0056</td>
<td>0.0029</td>
<td></td>
</tr>
</tbody>
</table>
Figure 3.1 The microstructure of 1050 steel in (a) normalized condition as pearlite and ferrite, (b) quenched and tempered condition as tempered martensite, and (c) induction-hardened condition as tempered martensite.
Figure 3.2 Tubular specimen configuration and dimensions in mm.

Figure 3.3 Instron closed-loop servo hydraulic axial-torsional fatigue testing frame.
Figure 3.4  Biaxial extensometer used to measure axial-torsion strains and control strain-controlled tests.

Figure 3.5  In-phase (proportional) and 90° out-of-phase (non-proportional) constant amplitude axial-torsional multiaxial loadings used in this study.
Figure 3.6 Axial-torsional star shape strain paths utilized in this study. Strain-time diagrams shown are for one complete strain block, except for the FRR path for which one quarter of the strain cycles in the block are shown for clarity.
Figure 3.7  Axial-torsional cruciform strain paths utilized in this study.
Figure 3.8  Axial monotonic test data points and curve for 1050 steel in normalized and quenched and tempered conditions.

Figure 3.9  Torsional monotonic test data points and curve for 1050 steel in normalized, quenched and tempered, and induction hardened conditions.
Figure 3.10 Stable cycle hysteresis loops for in-phase axial-torsion loading of 1050 N steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops.
Figure 3.11   Stable cycle hysteresis loops for 90° out-of-phase axial-torsion loading of 1050 N steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops. (c) Midsection shear strain versus axial strain. (d) Midsection shear stress versus axial stress.
Figure 3.12  Stable cycle hysteresis loops for in-phase axial-torsion loading of 1050 QT steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops.
Figure 3.13  Stable cycle hysteresis loops for 90° out-of-phase axial-torsion loading of 1050 QT steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops. (c) Midsection shear strain versus axial strain. (d) Midsection shear stress versus axial stress.
Figure 3.14 Stable cycle hysteresis loops for in-phase axial-torsion loading of 1050 IH steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops.
Figure 3.15 Stable cycle hysteresis loops for 90° out-of-phase axial-torsion loading of 1050 IH steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops. (c) Midsection shear strain versus axial strain. (d) Midsection shear stress versus axial stress.
Figure 3.16 Stable cycle hysteresis loops for in-phase axial-torsion loading of 304L stainless steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops.
Figure 3.17  Stable cycle hysteresis loops for 90° out-of-phase axial-torsion loading of 304L stainless steel. (a) Axial hysteresis loops. (b) Shear hysteresis loops. (c) Midsection shear strain versus axial strain. (d) Midsection shear stress versus axial stress.
Figure 3.18 Axial stress amplitudes for in-phase axial-torsion loading of 304L stainless steel versus (a) number of cycles, and (b) normalized number of cycles.
Figure 3.19  Midsection shear stress amplitudes for in-phase axial-torsion loading of 304L stainless steel versus (a) number of cycles, and (b) normalized number of cycles.
Figure 3.20  Axial stress amplitudes for 90° out-of-phase axial-torsion loading of 304L stainless steel versus (a) number of cycles, and (b) normalized number of cycles.
Figure 3.21 Midsection shear stress amplitudes for 90° out-of-phase axial-torsion loading of 304L stainless steel versus (a) number of cycles, and (b) normalized number of cycles.
Figure 3.22 Transient behaviors and mean stress relaxations for 1050 QT steel and 304L stainless steel (304L SS) under similar strain amplitude levels ($\varepsilon_a = 0.0034$ and $\gamma_a = 0.0058$) of FRI and PI strain paths in (a) axial, and (b) shear channels. The axial data plotted is for the pure axial loading cycle during the block load, while the shear data plotted is for the pure torsion cycle during the block load.
Figure 3.23 Maximum axial stress amplitudes for FRI and FRR axial-torsion strain paths of 1050 QT steel in versus (a) number of blocks and (b) normalized number of blocks. The axial and shear data plotted are respectively from the pure axial and pure torsion cycle during the block load.
Figure 3.24 Maximum midsection shear stress amplitudes for FRI and FRR axial-torsion strain paths of 1050 QT steel versus (a) number of blocks, and (b) normalized number of blocks. The axial and shear data plotted are respectively from the pure axial and pure torsion cycle during the block load. The axial and shear data plotted are respectively from the pure axial and pure torsion cycle during the block load.
Figure 3.25 Change in stress response over several load blocks for 1050 QT steel ($\epsilon_a = 0.007$ and $\gamma_a = 0.012$) under (a) FRI strain path (10FRI1QT), and (b) FRR strain path (10FRR1QT).
Figure 3.26 Maximum axial stress amplitudes for FRI, FRR, and FRI15 axial-torsion strain paths of 304L stainless steel versus (a) number of blocks, and (b) normalized number of blocks. The axial and shear data plotted are respectively from the pure axial and pure torsion cycle during the block load.
Figure 3.27  Maximum midsection shear stress amplitudes for FRI, FRR, and FRI15 axial-torsion strain paths of 304L stainless steel versus (a) number of blocks and (b) normalized number of blocks. The axial and shear data plotted are respectively from the pure axial and pure torsion cycle during the block load.
Figure 3.28 Change in stress response over several load blocks for 304L stainless steel ($\varepsilon_a = 0.007$ and $\gamma_a = 0.012$) under (a) FRI strain path (10FRI1SS), and (b) FRR strain path (10FRR1SS).
Chapter 4

Deformation Behavior and Predictions

4.1 Introduction

Constant and variable amplitude multiaxial deformation behaviors and predictions are discussed in this chapter. Constant amplitude multiaxial deformation behaviors of 1050 steel in normalized, quenched and tempered, and induction hardened conditions as well as 304L stainless steel are presented in Section 4.2. Effects of microstructure and hardness on non-proportional cyclic hardening of metallic materials are also investigated in this section. A new prediction model for non-proportional cyclic hardening coefficient based on simple uniaxial deformation constants is proposed and examined for generated multiaxial data in this study and a wide variety of available data in literature.

Variable amplitude multiaxial deformation behavior of 1050 QT steel and 304L stainless steel with low and high sensitivity to non-proportional loading, respectively, is presented in Section 4.3. Reviewing the stress response of these materials under FRI, FRR, FRI15, and PI strain paths, different issues such as effects of loading sequence on stress response, mean strain effects, and mean stress relaxation are discussed. Tanaka’s non-proportionality parameter coupled with a Fredrick-Armstrong incremental plasticity model as well as Kanazawa et al.’s empirical formulation as a representative of such
empirical models are also used to predict the stabilized stress response of the two materials under variable amplitude axial-torsion strain paths. Conclusions made from experimental observations and analysis results are presented in Section 4.4.

4.2 Constant amplitude deformation behavior and predictions

Effect of microstructure and hardness on multiaxial deformation behaviors of 1050 steel in normalized, quenched and tempered, and induction hardened conditions as well as 304L stainless steel are discussed in this section. An approximate equation for prediction of non-proportional cyclic hardening coefficient based on comparison of cyclic and monotonic uniaxial deformations is also proposed and examined for multiaxial data generated in this study as well as multiaxial deformation data available in the literature.

4.2.1 Constant amplitude deformation behavior

Axial and midthickness shear stresses and strains were calculated from the midlife stable cycles and von Mises equivalent stresses and strains were then obtained from:

\[\sigma_a = \frac{\Delta \sigma}{2} = \sqrt{\left(\frac{\Delta \sigma}{2}\right)^2 + 3\left(\frac{\Delta \tau}{2}\right)^2} \quad (4.1)\]

\[\bar{\varepsilon}_a = \frac{\Delta \bar{\varepsilon}}{2} = \frac{1}{\sqrt{2(1+\nu)}} \sqrt{2\left(\frac{\Delta \varepsilon}{2}\right)^2 (1+\nu)^2 + 3\left(\frac{\Delta \gamma}{2}\right)^2} \quad (4.2)\]

where \(\Delta \sigma/2\), \(\Delta \tau/2\), \(\Delta \varepsilon/2\), and \(\Delta \gamma/2\) are axial stress, shear stress, axial strain, and shear strain amplitudes, respectively. In the case of axial-torsion loading, transverse strain is also present and is a function of axial strain as \(\varepsilon_{tr} = -\nu \varepsilon\), with the equivalent Poisson’s ratio, \(\nu\), defined as:
where $\bar{e}_e$, $\bar{e}_p$, and $\bar{e}$ are equivalent elastic, plastic and total strains and $\nu_e$ and $\nu_p$ are elastic and plastic Poisson’s ratios. $\nu_e$ is about 0.27 and $\nu_p$ is equal to 0.5 for all materials and hardening conditions in this study.

Axial and shear hysteresis loops for in-phase loadings of 1050 N, 1050 QT, and 1050 IH steels as well as 304L stainless steel, presented in Figures 3.10, 3.12, 3.14, and 3.16, indicate the fact that extreme values of shear and axial stresses as well as shear and axial strains are simultaneous for in-phase loadings. Axial and shear stress and strain amplitudes are substituted into Equations (4.1) and (4.2) to evaluate equivalent stress and strain amplitudes. Extreme values of axial and shear channels are not simultaneous, however, in the case of 90° out-of-phase loading, as shown in Figures 3.11, 3.13, 3.15, and 3.17 for 1050 steel in three different hardening conditions and 304L stainless steel, respectively. In addition, extreme stress and strain values do not always occur at the same time for each separate channel and significant rounding at the tips of hysteresis loops is observed, especially for higher strain amplitudes (see Figures 3.11, 3.13, 3.15, and 3.17).

Therefore, for 90° out-of-phase loading, a time dependent analysis was followed and equivalent stress and strain values were calculated using Equations (4.1) and (4.2) for all the data points of the stable cycle, as shown in Figure 4.1 for 1050 N steel. Two points with 180° phase angle difference as the maximum and minimum values were selected in order to reflect both the maximum or near maximum equivalent stress and strain values in the cycle. Ramberg-Osgood curves fitted to the selected data, as well as to the maximum strain data as the lower bound and maximum stress data as the upper bound are
superimposed in Figure 4.1. Depending on the curve used, the non-proportional cyclic hardening coefficient value differs by about 7%. The difference might be more significant for other materials depending on the degree of plasticity and rounding at the tips of hysteresis loops.

Cyclic equivalent stress-strain relation can be represented by the Ramberg-Osgood type equation (Stephens et al., 2000):

\[
\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon}{2} + \frac{\Delta \varepsilon}{2} = \frac{\Delta \sigma}{2} + \left( \frac{\Delta \sigma}{2K} \right)^{\frac{1}{n}}
\]

The equivalent cyclic strength coefficient, \(K\), and the equivalent cyclic strain hardening exponent, \(n\), are the intercept and slope of the best line fit to equivalent stress amplitude versus equivalent plastic strain amplitude data in log-log scale, represented by:

\[
\frac{\Delta \sigma}{2} = K \left( \frac{\Delta \varepsilon}{2} \right)^{\frac{1}{n}}
\]

where the equivalent plastic strain amplitude, \(\Delta \varepsilon / 2\), can be calculated knowing the equivalent elastic strain amplitude, \(\Delta \varepsilon / 2\), from:

\[
\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon}{2} - \frac{\Delta \varepsilon}{2} = \frac{\Delta \sigma}{2E}
\]

Here \(\Delta \varepsilon / 2\) is total equivalent strain amplitude and \(E\) is modulus of elasticity. Midsection stable cycle equivalent stress and strain amplitudes are presented in Tables 4.1, 4.2, 4.3, and 4.4 for 1050 steel in normalized, quenched and tempered, and induction hardened conditions as well as 304L stainless steel, respectively. Equivalent stress amplitude versus equivalent plastic strain amplitude data and fit for 1050 steel in three
different hardening conditions and 304L stainless steel are presented in Figures 4.2 and 4.3 for in-phase and 90° out-of-phase loadings, respectively.

Not a notable scatter was found in cyclic deformation results and good fittings were obtained for all materials and hardening conditions examined in this study. For 1050 QT steel and 1050 IH steel as well as 304L stainless steel, test data with $\Delta\varepsilon_p / 2 \geq 0.03\%$ were considered in the curve fitting. This is due to the fact that for lower strain than this value, the plastic strain is too small causing much scatter. Nevertheless, lower values were considered for 1050 N steel and good fittings were obtained.

Cyclic in-phase and 90° out-of-phase deformation properties are presented in Table 4.5. The cyclic yield strength is calculated knowing the equivalent cyclic strength coefficient, $\bar{K}'$, and the equivalent cyclic strain hardening exponent, $\bar{\eta}'$, for in-phase and 90° out-of-phase loadings, and from the following relation:

$$\bar{\sigma}_y' = \bar{K}' (0.002)^{\bar{\eta}}$$  \hspace{1cm} (4.7)

Comparisons of the cyclic multiaxial deformation behavior for 1050 steel in the three hardness conditions as well as for 304L stainless steel are presented in Figures 4.4(a) and 4.4(b) for in-phase and 90° out-of-phase loadings, respectively. It can be observed that while stress levels of 1050 N steel and 304L stainless steel are similar under in-phase loading, 304L stainless steel exhibits higher stress levels and, therefore, more sensitivity under 90° out-of-phase loading as compared to 1050 N steel. It can also be observed that increase in hardness level of 1050 steel from 198 HB for normalized condition to 360 HB for quenched and tempered and 565 HB for induction hardened conditions significantly increases the stress response of this steel.
Comparisons of in-phase and 90° out-of-phase cyclic multiaxial test data and stress-strain curves for 1050 steel in normalized, quenched and tempered, and induction hardened conditions as well as for 304L stainless steel are presented in Figure 4.5. From this figure and Equation (2.17), $\alpha$ is estimated to be about 0.1 for 1050 N steel, -0.04 for 1050 QT steel, -0.05 for 1050 IH steel, and 0.55 for 304L stainless steel. These values are also reported in Table 4.2. Negative non-proportional cyclic hardening coefficients observed in this study for 1050 steel as a medium carbon steel is unexpected based on the literature (Fatemi and Stephens, 1989b; Socie and Marquis, 2000; Shang et al., 2000). Although small negative values of $\alpha$ for 1050 QT and 1050 IH steels might be related to the data scatter, all 90° out-of-phase stress data for these two materials are lower than the corresponding in-phase stress data.

### 4.2.2 Effects of microstructure on non-proportional cyclic behavior

Effects of microstructure and hardness level on the non-proportional cyclic hardening of material can be observed from Figure 4.5. Increase in hardness level and change in microstructure from pearlitic and ferritic in normalized condition of 1050 steel to tempered martensite in quenched and tempered and induction hardened conditions of the same steel reduces the non-proportional cyclic hardening from 0.1 to some negative values. Interaction of slip systems due to the rotation of maximum shear planes results in non-proportional cyclic hardening.

Non-proportional hardening phenomenon depends on the degree of strain path non-proportionality, as well as the level of plastic straining inducing slip systems in the material. The 90° out-of-phase loading generates the maximum degree of non-
proportionality due to the severe interaction of slip systems caused by continuous rotation of maximum shear plane in all directions.

The variation of non-proportional cyclic hardening observed may be qualitatively related to the decrease in stacking fault width between dislocations by increasing the hardness level of material and changing the microstructures to tempered martensite, and consequently, an increase in SFE. Indeed, due to the higher level of SFE in the quenched and tempered and induction hardened conditions with finer grains of 1050 steel, dislocations cannot split easily, therefore, they overcome the finer grains barriers without increase in stress level. As a result, no increase in stress response of material is observed in the case of 90° out-of-phase loading as compared to in-phase loading. Similar behavior was also reported by Batane et al. (2008) for nickel as the conventional grain size reduced to the ultrafine grain size. Although the load carrying capacity of the ultrafine nickel was found to increase more than twice of the conventional grain size nickel, the cyclic hardening behavior of the conventional grain size under non-proportional loading also changed to non-proportional cyclic softening behavior for ultrafine grain nickel.

On the other hand, monotonic and cyclic strain hardening phenomenon of face-cube-center (fcc) metals can be related to the SFE level (Doong et al., 1990; Kida et al., 1997; Ohkubo et al., 1994; Ebrahimi et al., 2004; Borodii and Shukaev, 2007; Dumay et al, 2008; Li and Almazouzi, 2009). Lower value of SFE increases the possibility of splitting of dislocations, which results in decrease in dislocation mobility causing cyclic hardening behavior. For hardened materials with higher SFE, there is more dislocations mobility as a result of their rearrangement. This is facilitated by the cyclic plastic
deformation at high strain amplitude levels, which increases the ability of dislocations to circumnavigate around the finer grains of induction tempered microstructure.

Type 304L stainless steel with very low SFE exhibits planar slip microstructure under both proportional (uniaxial) and non-proportional loadings (Doong et al., 1990; Kida et al., 1997). Therefore, this material exhibits significant cyclic hardening as well as non-proportional cyclic hardening. However, multi-slip structures such as cells and labyrinths are also generated in stainless steels due to the rotation of maximum shear plane under non-proportional cyclic loadings resulting in more non-proportional cyclic hardening (Doong et al., 1990). Kida et al. (1997) related the significant non-proportional cyclic hardening of type 304 stainless steel to the reduction of the cell size as a result of severe interaction of slip systems under non-proportional loading condition. They also suggested the low SFE of stainless steels as the reason for splitting in dislocation bands and increasing the stacking faults between partial dislocations. These can explain both significant cyclic hardening as well as non-proportional cyclic hardening phenomena observed in stainless steels.

Figure 4.5 also includes the monotonic stress-strain curves for 1050 N steel, 1050 QT steel, and 304L stainless steel. As can be seen from this figure, 1050 N steel and 304L stainless steel are cyclic hardening materials, whereas 1050 QT steel is a cyclic softening material. Induction-hardened 1050 steel is also a material without cyclic hardening. These observations regarding the stable or steady-state (i.e. saturated) cyclic hardening behaviors for 1050 N steel and 304L stainless steel are different from the transient cyclic softening behavior observations, indicating the two phenomena are different.
Both 1050 N steel and 304L stainless steel exhibited initial cyclic softening as high as 30% as compared to midlife stable cycle for the lowest strain level of in-phase loading used ($\epsilon_a = 0.25\%$). The level of cyclic softening in in-phase loading was observed to decrease for both materials to less than 10% of the stable cycle for the highest strain level test at $\epsilon_a = 1\%$. However, 10% transient cyclic hardening at $\epsilon_a = 0.7\%$ in 90° out-of-phase loading was observed for both materials. The amount of initial cyclic hardening decreased for the lower strain amplitude level of 90° out-of-phase tests. Although, these two materials both exhibit the same degree of transient cyclic hardening under 90° out-of-phase loading, the overall non-proportional cyclic hardening observed for 304L stainless steel was much more than for 1050 N steel. Therefore, the transient cyclic softening or hardening behavior is not necessarily related to the stable or steady-state cyclic softening or hardening behavior.

Materials for which cyclic hardening was observed in this study (i.e. 1050 N steel and 304L stainless steel), exhibited some non-proportional cyclic hardening as well, whereas no non-proportional cyclic hardening was observed for a cyclic softening material (i.e. 1050 QT steel). Therefore, it appears that materials with cyclic hardening exhibit additional hardening due to the non-proportionality of loading. Considering the fact that both cyclic hardening and non-proportional cyclic hardening phenomena can be qualitatively related to the SFE, there should be also a qualitative relation between these two hardening phenomena.
4.2.3 Non-proportional cyclic hardening coefficient prediction

In order to predict the non-proportional cyclic hardening coefficient, $\alpha$, as a function of simple material properties, Borodii and Shukaev (2007) proposed a simple equation based on static hardening as a monotonic property of material, presented as:

$$\log|\alpha| = 0.705\beta - 1.22$$  \hspace{1cm} (4.8)

where the static hardening parameter, $\beta$, is a dimensionless parameter related to strain hardening exponent and defined as the ratio of the material ultimate tensile strength, $\sigma_U$, to the yield strength, $\sigma_y$, as:

$$\beta = \frac{\sigma_U}{\sigma_y} - 1$$  \hspace{1cm} (4.9)

These equations are based on experimental observations indicating the idea that as SFE decreases, both non-proportional cyclic hardening and static hardening coefficients have the tendency to increase.

Employing Equations (4.8) and (4.9), comparison of predicted and observed values of non-proportional cyclic hardening coefficient for the steels used in this study is presented in Figure 5.6. This figure also includes predictions for 1045 normalized (N) steel (Fatemi and Stephens, 1989b), pure titanium (Gladskyi, 2006), titanium alloy BT9 (Shukaev, 1996), 16MnR steel (Gao et al., 2009), 321 stainless steel (Shukaev, 1987), 800H stainless steel at room temperature (RT) (Portella and Osterle, 1999), and 316L stainless steel (He et al., 1999) from the literature. Significant scatter of data is observed, when relating the non-proportional cyclic hardening to only monotonic properties. This is due to the fact that non-proportional cyclic hardening also depends on the material slip
system induced under cyclic loading. Therefore, non-proportional cyclic hardening coefficient can not be predicted only based on monotonic properties.

Based on the earlier discussion on the effects of microstructure and SFE on monotonic, cyclic, and non-proportional cyclic hardening observed in fcc metallic materials, it is hypothesized that the uniaxial cyclic behavior also plays an important role in non-proportional cyclic hardening. Hence, comparing the cyclic and monotonic curves as shown in Figure 4.7, cyclic hardening coefficient, \( h \), is defined for a given strain level as follows:

\[
h = 1 - \frac{\sigma_M}{\sigma_c}
\]  

(4.10)

where \( \sigma_c \) is the uniaxial cyclic stress or in-phase equivalent stress, and \( \sigma_M \) is the monotonic uniaxial stress at the same strain level. The monotonic stress value is taken from the simple tension stress-strain curve.

Figure 4.8 represents the non-proportional hardening coefficient values for 1050 steel at the three hardness levels, and for 304L stainless steel versus cyclic hardening coefficient for \( \Delta\varepsilon_p/2 > 0.05\% \), and \( \Delta\varepsilon/2 < 1.2\% \). Effect of the non-proportional cyclic hardening is typically small for equivalent plastic strain amplitudes lower than 0.05%, and on the other hand, in applications components are usually subjected to cyclic equivalent strain amplitudes lower than 1.2%. The following empirical equation results in a good fit, therefore, the non-proportional cyclic hardening coefficient can be related to the cyclic hardening coefficient as (Shamsaei and Fatemi, 2010):

\[
\alpha = 1.6h^2 + 0.6h = 1.6 \left( \frac{\sigma_M}{\sigma_c} \right)^2 - 3.8 \left( \frac{\sigma_M}{\sigma_c} \right) + 2.2
\]  

(4.11)
Comparison of experimental and predicted non-proportional cyclic hardening coefficient based on the proposed relation (i.e. Equation (4.11)) for 1050 steel at different hardness levels, 304L stainless steel, and several other materials from the literature including 1045 normalized (N) steel (Fatemi and Stephens, 1989b), pure titanium (Gladskyi, 2006), titanium alloy BT9 (Shukaev, 1996), 16MnR steel (Gao et al., 2009), 321 stainless steel (Shukaev, 1987), 800H stainless steel at room temperature (RT) and 800ºC (Portella and Osterle, 1999), and 316L stainless steel (He et al., 1999) is presented in Figure 4.9. The range of data used in this figure is for $\frac{\Delta \varepsilon_p}{2} > 0.05\%$ and $\frac{\Delta \varepsilon}{2} < 1.2\%$. Interestingly, this equation seems to also work well for pure and alloyed titanium with hexagonal close pack (hcp) microstructure. Although very good correlations are observed and data scatter is relatively small, additional data for other materials are needed to further verify the predictive capability of the proposed relation.

As discussed earlier in the literature survey, fatigue life prediction of components and structures under variable amplitude non-proportional multiaxial loading often results in additional cyclic hardening and a decrease in fatigue life. It has been shown over the last two decades that multiaxial fatigue models taking into consideration constitutive behavior of material typically lead to more accurate life predictions. Therefore, the stress response of the material under variable amplitude non-proportional multiaxial straining is required in such models. Prediction of the non-proportional cyclic hardening coefficient based on the proposed relation facilitates calculation of the stress response needed for fatigue life predictions under such complex loading conditions.

Cyclic hardening coefficient and non-proportional cyclic hardening coefficient versus equivalent strain amplitudes (calculated from Equation (4.2)) are presented in
Figures 4.10(a) and 4.10(b), respectively. Similar behavior of these two coefficients with increasing strain amplitude is observed for each material presented in Figure 4.10. Cyclic hardening and non-proportional cyclic hardening coefficients of 1050 N steel have a tendency to decrease by an increase in strain level. These coefficients for 1045 N steel, titanium, and titanium BT9 are not sensitive to change in strain value. However, both cyclic hardening and non-proportional cyclic hardening coefficients increase by an increase in deformation for 304L SS, 316L SS, 800H SS RT, 800H SS 800ºC, 321 SS, and 16MnR steel, as shown in Figure 4.10. As can be seen from Figure 4.7, \( h \) increases by an increase in strain level if the slope of cyclic curve is greater than the slope of monotonic curve, such as for 304L stainless shown in Figure 4.5(d). For materials with approximately similar monotonic and cyclic curve slopes (i.e. parallel monotonic and cyclic deformation curves), such as 1050 N steel in Figure 4.5(a), \( h \) does not increase by an increase in deformation level. Variation of \( h \) and \( \alpha \) as a function of equivalent plastic strain amplitude were very similar to those as a function of equivalent total strain amplitude, presented in Figure 4.10.

Equation (4.11) can be rewritten in terms of uniaxial monotonic and cyclic deformation properties as well as the level of plastic strain, employing Equation (4.5), as follows (Shamsaei and Fatemi, 2010):

\[
\alpha = 1.6 \left( \frac{K}{K'} \left[ \varepsilon_p / 2 \right]_c^n \right) ^2 - 3.8 \left( \frac{K}{K'} \left[ \varepsilon_p / 2 \right]_M^n \right) + 2.2 \quad (4.12)
\]

where monotonic plastic strain, \((\varepsilon_p)_M\), and cyclic plastic strain amplitude, \((\Delta \varepsilon_p / 2)_C\), can be calculated, knowing monotonic stress and cyclic stress amplitude values at a given strain level.
In view of the fact that non-proportional cyclic hardening is most significant in presence of considerable plastic deformation, the elastic strain term in Equation (4.4) can be neglected, if small, compared to the plastic strain term. Then, Equation (4.5) can be written as:

\[
\frac{\Delta \sigma_c}{2} = K' \left( \frac{\Delta \varepsilon}{2} \right)^{n'}
\]  

(4.13)

where \( K' \) is the uniaxial or equivalent in-phase cyclic strength coefficient, and \( n' \) is the uniaxial or equivalent in-phase cyclic strain hardening exponent. Similar relation between the monotonic axial stress and axial strain also can be assumed, when the deformation is mostly plastic:

\[
\sigma_{st} = K(\varepsilon)^n
\]

(4.14)

where \( K \) and \( n \) are the strength coefficient and strain hardening exponent, respectively. Substituting Equations (4.13) and (4.14) into Equation (4.11), the following equation relates the non-proportional cyclic hardening coefficient to standard material properties, as a function of strain level:

\[
\alpha = 1.6 \left( \frac{K}{K'} \right)^2 \left( \frac{\Delta \varepsilon}{2} \right)^{2(n-n')} - 3.8 \left( \frac{K}{K'} \right) \left( \frac{\Delta \varepsilon}{2} \right)^{(n-n')} + 2.2
\]

(4.15)

For the case of 90° out-of-phase loading resulting in the maximum possible strain hardening, the factor of non-proportionality is assumed to be unity, \( F = 1 \). Therefore, the equivalent 90° out-of-phase stress amplitude, \( \Delta \bar{\sigma}_{op} / 2 \), can be simply related to the equivalent in-phase stress amplitude, \( \Delta \bar{\sigma}_{ip} / 2 \), as follows:

\[
\Delta \bar{\sigma}_{op} / 2 = (1 + \alpha) \Delta \bar{\sigma}_{ip} / 2
\]

(4.16)
Comparison of the experimental 90° out-of-phase equivalent stress amplitudes and the predicted values employing Equations (4.11) and (4.16) are presented in Figure 4.11. This comparison, including all materials presented in Figure 10, indicates an excellent correlation between experimental and predicted stress values based on Equation (4.11). It should be mentioned that the proposed equation (i.e. Equation (4.11)) was only based on the fit of data generated in this study (i.e. only 1050 steel in three hardness levels and 304L stainless steel). Therefore, excellent correlations obtained in Figures 4.9 and 4.11 for a variety of metallic materials from the literature is not resulting from any fitting of the same data and is an indication of broad applicability of the proposed model.

4.3 Variable amplitude deformation behavior and predictions

Variable amplitude multiaxial deformation behaviors of 1050 QT steel with no non-proportional cyclic hardening and 304L stainless steel with significant non-proportional cyclic hardening under discriminating strain paths are presented in this section. Kanazawa et al. (1979) empirical method as well as Tanaka’s non-proportionality parameter (1994) in conjunction with a Frederick-Armstrong plasticity model (Jiang and Sehitoglu, 1996) are used to predict the stabilized stress response of the two materials under these discriminating variable amplitude axial-torsion strain paths (Shamsaei et al., 2010a).

4.3.1 Variable amplitude deformation behavior

Midsection equivalent stress and strain amplitudes of the midlife blocks of each test of FRI, FRI15, FRR, and PI are calculated from Equations (4.1) and (4.2) and listed in Table 4.6 for 1050 QT steel and Table 4.7 for 304L stainless steel. Equivalent
Poisson’s ratio in these tables is calculated from Equation (4.3). Very similar equivalent stress levels for each of the cycles within a block were obtained for both materials.

Equivalent stress-strain data of FRI, FRR, and PI strain paths for 1050 QT steel are compared with in-phase (IP) and 90° out-of-phase (OP) curves and data in Figure 4.12(a). As can be seen from this figure, 1050 QT steel with no non-proportional hardening coefficient exhibits similar cyclic behaviors regardless of the strain path applied. Equivalent stress amplitude response of this material under 0.7% and 0.34% equivalent strain amplitudes for all strain paths used are presented in Figure 4.13(a). In-phase equivalent stress amplitude response for each strain level is also superimposed in this figure as the baseline for comparisons. Similar results are obtained for various strain paths utilized. Slightly higher stress response at equivalent strain of 0.7% for FRR strain path as compared to OP strain path can be related to a different mechanism of hardening (Tanaka et al., 1985b). FRR strain path results in the intersection of the stable dislocation structures due to sudden change in proportional strain direction, whereas the plastic strain rate vector and the corresponding plastic strain vector are not in the same direction for OP strain path.

Figure 4.12(b) represents equivalent stress-strain amplitude data of in-phase and 90° out-of-phase loadings as well as FRI, FRR, FRI15 and PI strain paths for 304L stainless steel. In addition, equivalent stress amplitude response of these strain paths under 0.7% and 0.34% equivalent strain amplitudes are presented in Figure 4.13(b), where the results are compared with the in-phase strain path. In contrast to 1050 QT steel, significant differences in equivalent stress responses are observed for different strain paths in these figures.
With about 55% non-proportional hardening coefficient for stainless steel, significant stress hardening is observed due to the interaction of the slip systems in the random sequence strain path (i.e. FRR) as compared to incremental sequence paths (i.e. FRI and PI). The FRR strain path, which consists of a combination of random sequence fully-reversed in-phase cycles within the constant equivalent strain circle, activates many of the slip systems within the material in many directions. This is similar to 90° out-of-phase loading, but with a different hardening mechanism. The sudden change in loading direction for FRR strain path causes cross hardening by activating intersecting slip systems within the material. Since FRR strain path is a combination of proportional strain cycles, the plastic strain rate vector and corresponding plastic strain vector are in the same direction. However, activated slip systems for the OP strain path are consequences of the local perpendicular plastic strain rate vector and the corresponding plastic strain vector (Tanaka et al., 1985b). Nevertheless, very comparable equivalent stress responses are observed for FRR and OP strain paths for both equivalent strain levels of 0.7% and 0.34% in Figure 4.13(b). Additional non-proportional hardening results for 304L stainless steel as the level of equivalent strain increases, which is due to more plastic deformation and activating more slip systems and, therefore, resulting in more effects of different mechanism of hardening.

The FRI strain path consists of 1° increments of in-phase straining. Therefore, the change in strain direction is very slow, activating the slip systems gradually but in all directions. This results in much less interaction of slip systems as compared to 90° out-of-phase or FRR strain paths, particularly at high equivalent strain amplitude levels.
Consequently, stress response for FRI loading is higher than for in-phase loading which only activates slip systems in one direction.

The FRI15 strain path is similar to the FRI path, except for 15° incremental change in the straining direction, rather than 1° increments. The stress response of 304L stainless steel for this path is slightly higher than for the FRI strain path, as shown in Figure 4.13(b), as the result of cross hardening. Nevertheless, the 304L stainless steel under the FRR strain path, at which the straining direction is changed in a random sequence, exhibits more hardening, as compared to FRI15 strain path (see Figure 4.13(b)).

Mean strain and consequently the resulting mean stress of PI strain path for 304L stainless steel slightly increases the level of non-proportional hardening, as compared to the FRI strain path. This may be explained by the higher maximum strain for the PI path, as compared to the FRI path. This higher strain activates more slip systems, causing more cyclic strain hardening, shown in Figure 4.13(b).

4.3.2 Predictions of the non-proportional cyclic deformation

- Deformation prediction by Kanazawa et al.’s empirical formulation

The method proposed by Kanazawa et al. (1979), relates the additional non-proportional hardening to the interaction of slip systems on different planes due to the change in the maximum shear plane. The factor of load non-proportionality, $F$, in this model is a factor of ellipticity of the circumscribed boundary around the strain path in the $\gamma/2-\epsilon$ plot. Observed equivalent stresses are compared to the predicted stresses employing this empirical model (i.e. Equation (2.17)) in Figure 4.14(a) for all the strain paths of
1050 QT steel. In view of the fact that 1050 QT steel exhibits no non-proportional hardening (i.e. $\alpha = 0$), and, therefore, its stress response is not sensitive to the strain path, all the predicted stress responses are similar to the in-phase stress. Therefore, all the predictions for 1050 QT steel fall within the error bands of 15% from the experimental observations.

The same comparisons for 304L stainless steel are also presented in Figure 4.14(a). Since all the load paths used are within the same circumstantial circle in $\gamma/\sqrt{3\varepsilon}$ plot, this method predicts the same value of the factor of non-proportionality for these strain paths as for the 90º out-of-phase straining (i.e. $F = 1$). It can be seen from this figure that predicted equivalent stress values for stainless steel data for different strain paths at 450 MPa and 650 MPa equivalent stress amplitude levels are the same, in spite of large observed equivalent stress amplitude variations among these strain paths. Significant over-predictions of stress as high as 40% at higher plastic strains can be observed in Figure 4.14(a) for stainless steel with FRI strain path. However, stress response predictions for 304L stainless steel are within 15% of experimental values for FRR, FRI15, and PI strain paths, which exhibit higher level of strain hardening. Although such empirical methods are simple to use, they can cause significant error in stress response predictions for materials with high sensitivity to the non-proportionality of loading. Another significant shortcoming of such empirical models is that they do not account for the progression of cross hardening with change in loading direction. Therefore, fatigue life analysis based on the stress responses obtained by such methods may result in very conservative predictions.
- Deformation prediction by Tanaka’s non-proportional parameter

As mentioned in the literature review, Jiang and Kurath (1996a, 1996b) reported the Armstrong-Frederick (1966) incremental plasticity model modified by Chaboche et al. (1979; Chaboche, 1987, 1991) as a proper basis to model the cyclic softening/hardening and non-proportional hardening behavior of materials. Armstrong-Frederick (1966) hardening models simulate the movement of the yield surface in the deviatoric stress space by a nonlinear kinematic hardening rule with the recovery term. These models consider the idea that the most recent part of the strain history dictates the mechanical behavior, and therefore, takes into account the strain memory effects.

The formulation of Jiang and Sehitoglu (1996) is employed in this investigation. A summary of the formulation is given here to show the various material constants needed for the model.

Yield Function

The von Mises yield function, \( Y \), is expressed as:

\[
Y = (\bar{S} - \alpha) : (\bar{S} - \alpha) - 2k^2 = 0
\]  
(4.17)

Here \( \bar{S} \) is the deviatoric stress tensor, \( \alpha \) is total backstress tensor and \( k \) is the yield stress in simple shear.

Flow Rule

The normality flow rule is given by:

\[
d\bar{e}^p = \frac{1}{h_p} \langle d\bar{S} : \bar{n} \rangle \bar{n}
\]  
(4.18)

where \( h_p \) is the plastic modulus function, \( \bar{e}^p \) is plastic strain tensor, and \( \bar{n} \) is the exterior unit normal to the yield surface at the loading point as described below:
\[
\eta = \frac{S - \alpha}{|S - \alpha|} 
\]  

(4.19)

**Hardening Rule**

Chaboche et al. (1979) and Chaboche (1987, 1991) considered that the total backstress \( \alpha \) is divided into \( M \) parts:

\[
\alpha = \sum_{i=1}^{M} \alpha^i 
\]  

(4.20)

The evolution of the backstress for each of the parts is given by:

\[
d\alpha^i = c^i r^i \left( \eta - \frac{\alpha^i}{r^i} \right) \mathbf{L}^i \ dp 
\]  

(4.21)

where \( dp \) is the equivalent plastic strain increment and \( c^i, r^i \) and \( \chi^i \) are three sets of non-negative single valued scalar functions and,

\[
\mathbf{L}^i = \frac{\alpha^i}{|\alpha^i|} \ (i = 1, 2, 3, \ldots, M) 
\]  

(4.22)

\[
|\alpha^i| = \sqrt{\alpha^i : \alpha^i} \ (i = 1, 2, 3, \ldots, M) 
\]  

(4.23)

\[
dp = \sqrt{d\varepsilon^p : d\varepsilon^p} 
\]  

(4.24)

Therefore, \( \mathbf{L}^i \) is unit vector of the backstress, \( \alpha^i \). Material memory is contained in the \( \alpha \) terms. The plastic modulus function \( h_p \) is derived from the consistency condition which requires that the stress state be on the yield surface when plastic deformation is occurring.
\[
\begin{align*}
 h_p &= \sum_{i=1}^{M} c^i r^i \left[ 1 - \left( \frac{\alpha^i}{r^i} \right)^{x^i+1} \right] L^i : n + \sqrt{2} \frac{dk}{dp} \\
 \text{(4.25)}
\end{align*}
\]

**Non-proportional Hardening**

Tanaka (1994) presented the internal structure by an evolution equation as a fourth rank tensor \( C \) in plastic strain space (Tanaka, 1984):

\[
\frac{dC_{ijkl}}{dp} = C_r \left[ n_{jl} n_{ki} - C_{ijkl} \right] dp
\]

The initial components of \( C \) are zero. The non-proportionality parameter is defined as:

\[
A = \sqrt{1 - \frac{n_{pq} C_{pq} C_{rs} n_{rs}}{C_{ijkl} C_{ijkl}}}
\]

Tanaka considered the history effects of non-proportional hardening by the evolution equation of an isotropic hardening variable as a function of the non-proportionality parameter, \( A \). Details and discussions of this model can be found in (Tanaka, 1994; Jiang and Kurath, 1997; Zhang and Jiang, 2008), where satisfactory predictions based on this model for various complex proportional and non-proportional loading paths for 304 and 316 stainless steels, polycrystalline copper, as well as 1045 normalized steel are reported.

The evolution equations for the yield stress and plastic modulus are given by:

\[
\begin{align*}
 dk &= \beta[k_a (1 + \alpha A) - k] dp \\
 dr^i &= \beta[r^i_a (1 + \alpha A) - r^i] dp
\end{align*}
\]

(4.28) \hspace{1cm} (4.29)

Here, the subscript \( \circ \) denotes the initial values.

The plasticity model employed here is a simplified version, implemented in *eFatigue* software, requiring experimentally determined cyclic stress-strain curves under
fully reversed axial (or in-phase) and 90° out-of-phase axial-torsion loading. In general, the model requires only five material constants for estimating the stabilized response. The modulus of elasticity, $E$, shear modulus, $G$, cyclic strength coefficient, $K'$, cyclic strain hardening exponent, $n'$, are needed to obtain the plastic modulus function constants, $r^i$ and $c^i$ and yield strength $k$. The non-proportional hardening coefficients, $\alpha$, is needed to describe the maximum degree of non-proportional hardening. $\alpha$ is obtained by comparing the uniaxial cyclic stress-strain curve with the cyclic stress-strain curve of 90° out-of-phase axial-torsion loading.

In-phase equivalent cyclic strength coefficient and exponent as well as the non-proportional hardening coefficient of the two materials from Table 4.5 were used as input for eFatigue. This leaves two additional rate constants, $\chi^i (i = 1, 2, 3, \ldots, M)$ which controls the ratcheting rate and $\beta$ which controls the non-proportional hardening rate. Since the steady state response is to be investigated here, the choice of rate constants is not very important and it has been found that $\chi^i (i = 1, 2, 3, 4, 5) = 10$ and $\beta = 5$ provide good estimates for a wide range of loading histories and materials.

Comparison of predicted and observed equivalent stress values of the strain paths utilized in this study for both materials using eFatigue is presented in Figure 4.14(b). Employing the aforementioned plasticity model, stress responses are satisfactorily predicted within the scatter bands of 7% and 12% correspondingly for 1050 QT and 304L stainless steels. Therefore, Tanaka’s non-proportionality parameter correctly predicts less cyclic hardening for paths with incremental change in strain direction such as FRI and PI, as compared to the paths with sudden change in strain direction such as FRR and FRI15.
Boundaries of the simulated stress paths from predictions using this plasticity model were also found to be very similar to those observed in experiments, as shown in Figure 4.15 for 304L stainless steel. One stress boundary from prediction and two experimental stress boundaries of duplicate tests for each strain path are presented in this figure. The predicted stress values are all within the scatter of 12% from the observed experimental stress values. Boundaries of all stress paths for 1050 QT steel also were predicted very well employing the above-mentioned plasticity model and Tanaka’s non-proportionality parameter. As mentioned earlier, empirical methods such as Kanazawa et al., cannot account for progression of cross hardening with change in loading direction. This can be seen from Figure 4.16, while the Tanaka’s non-proportionality parameter correctly predicts stress response for FRI strain path at \( \varepsilon_u = 0.7\% \), the Kanazawa et al.’s method considerably over-estimates the stress response of this strain path.

Mean stresses of the PI load path were found to significantly relax in presence of plastic deformations for both materials. Figure 4.17(a) represents the predicted and observed axial and shear stress values over a block at strain level of \( \bar{\varepsilon}_u = 0.35\% \) for 1050 QT steel. Predicted axial and shear mean stresses are about 10% of the corresponding stress amplitudes, whereas the observed mean stresses were about 20% of the corresponding stress amplitudes. Therefore, mean stress relaxations are over-estimated. The ratcheting rate parameter could be adjusted to obtain better estimates. However, the stress amplitudes for this strain path are still predicted fairly accurately, as shown in Figure 4.14(b). No stress relaxation was observed for the lower strain PI test of 1050 QT steel, \( \bar{\varepsilon}_u = 0.17\% \), without any plastic deformation. Exact results were obtained from the plasticity model in this case.
Considerable induced plastic deformations in PI test of 304L stainless steel resulted in significant mean stress relaxation to lower than 10% of the corresponding stress amplitudes. This significant mean stress relaxation is predicted by the plasticity model reasonably well, as shown in Figure 4.17(b). Since both stress amplitudes and mean stress relaxation are predicted well by the plasticity model, the predicted stress path is very similar to the experimental stress path for both axial and shear directions shown in this figure.

### 4.4 Conclusions

1) The non-proportional cyclic hardening coefficient, \( \alpha \) was found to be 0.1 for the normalized condition of 1050 steel with pearlitic-ferritic structure, and about - 0.05 for quenched and tempered and induction hardened conditions with tempered martensite. Therefore, the value of this coefficient for a given material depends on the material microstructure and hardness level. The value of this coefficient for 304L stainless steel was found to be 0.55.

2) The reduction in the non-proportional cyclic hardening of 1050 steel as it changes from the normalized condition to a finer grain tempered martensite condition can be qualitatively explained by the decrease in stacking fault width between dislocations by increasing the hardness level of the material, and consequently, an increase in stacking fault energy (SFE).

3) Significant cyclic hardening and non-proportional cyclic hardening observed for 304L stainless steel with low SFE can be explained by the planar slip mode of this material under both proportional and non-proportional loadings, as well as multi-slip structures such as cells and labyrinths due to the rotation of maximum shear
plane under non-proportional cyclic loadings. Formation of $\alpha'$ and $\varepsilon$ martensite in austenitic 304L stainless steel under plastic deformation also explains some of the observed irreversible hardenings.

4) Materials with cyclic hardening appear to also exhibit additional hardening due to the non-proportionality of loading. Both cyclic hardening and non-proportional cyclic hardening phenomena can be related to SFE.

5) Non-proportional cyclic hardening coefficient cannot be predicted only based on monotonic properties, as non-proportional cyclic hardening also depends on the material slip system induced under cyclic loading. The non-proportional cyclic hardening coefficient was found to strongly correlate with the cyclic hardening coefficient. Similar trends were observed for both coefficients with a change in total or plastic strain.

6) Based on experimental observations in this study, a predictive model for non-proportional cyclic hardening is proposed. This model only requires knowledge of the monotonic and uniaxial cyclic deformation behaviors of the material. Excellent predictions based on this model were obtained for available multiaxial data for a variety of materials from the literature.

7) While 1050 QT steel showed little or no sensitivity to non-proportionality of multiaxial straining or its direction, characterized by similar stress response, the 304L stainless steel showed significant sensitivity depending on both non-proportionality of straining as well as the direction or sequence of multiaxial straining.
8) Multiaxial in-phase straining of 304L stainless steel with very gradual change in strain direction (i.e. 1° increments) results in some non-proportional hardening, as compared with in-phase or proportional loading. Such loading activates the slip systems gradually but in all directions, while in-phase loading activates slip systems in only one direction. Increasing the straining direction angle (i.e. 15° increments) results in increased non-proportional hardening. This suggests the material has short term memory of prior deformation; therefore, the latest part of the loading block dictates most of the material deformation behavior.

9) Multiaxial in-phase (i.e. proportional) straining of 304L stainless steel with random sequence results in significant non-proportional hardening, equivalent to 90° out-of-phase loading. Such loading causes cross hardening due to the interaction of the slip systems by activating intersecting slip bands within the material.

10) Satisfactorily stress predictions were obtained for all the multiaxial strain paths investigated for both materials, employing the non-proportionality parameter proposed by Tanaka coupled with a simplified form of the Armstrong-Frederick incremental plasticity model. Predictions for 1050 QT steel and 304L stainless steel were respectively within scatter bands of 7% and 12% of the experimental observations.

11) Empirical methods, such as Kanazawa et al.’s non-proportionality formulation, can significantly over-estimate the stress response under in-phase straining with gradual change in strain direction. In addition, such methods cannot account for the progression of the cross hardening with change in loading direction.
Table 4.1  Midsection equivalent stress and strain components for tubular specimens of 1050 N steel under in-phase and 90° out-of-phase loading.

<table>
<thead>
<tr>
<th>ID</th>
<th>Load Type</th>
<th>$N_{\text{stable}}$ (Cycle)</th>
<th>$\bar{\nu}$</th>
<th>$\bar{\sigma}_a$ (MPa)</th>
<th>$\bar{\varepsilon}_a$</th>
<th>$\Delta \bar{\varepsilon}_e / 2$ Calculated</th>
<th>$\Delta \bar{\varepsilon}_p / 2$ Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>N 7</td>
<td>IP</td>
<td>2,048</td>
<td>0.44</td>
<td>496</td>
<td>0.0100</td>
<td>0.0024</td>
<td>0.0076</td>
</tr>
<tr>
<td>N 9</td>
<td>IP</td>
<td>1,200</td>
<td>0.44</td>
<td>498</td>
<td>0.0096</td>
<td>0.0024</td>
<td>0.0071</td>
</tr>
<tr>
<td>N 11</td>
<td>IP</td>
<td>8,304</td>
<td>0.41</td>
<td>401</td>
<td>0.0046</td>
<td>0.0019</td>
<td>0.0027</td>
</tr>
<tr>
<td>N 5</td>
<td>IP</td>
<td>9,982</td>
<td>0.41</td>
<td>391</td>
<td>0.0046</td>
<td>0.0019</td>
<td>0.0027</td>
</tr>
<tr>
<td>N 13</td>
<td>IP</td>
<td>189,626</td>
<td>0.35</td>
<td>301</td>
<td>0.0021</td>
<td>0.0015</td>
<td>0.0007</td>
</tr>
<tr>
<td>N 3</td>
<td>IP</td>
<td>103,984</td>
<td>0.35</td>
<td>283</td>
<td>0.0021</td>
<td>0.0014</td>
<td>0.0007</td>
</tr>
<tr>
<td>N 8</td>
<td>OP</td>
<td>528</td>
<td>0.45</td>
<td>477</td>
<td>0.0068</td>
<td>0.0023</td>
<td>0.0045</td>
</tr>
<tr>
<td>N 10</td>
<td>OP</td>
<td>400</td>
<td>0.45</td>
<td>474</td>
<td>0.0068</td>
<td>0.0023</td>
<td>0.0045</td>
</tr>
<tr>
<td>N 12</td>
<td>OP</td>
<td>3,678</td>
<td>0.39</td>
<td>402</td>
<td>0.0033</td>
<td>0.0020</td>
<td>0.0013</td>
</tr>
<tr>
<td>N 6</td>
<td>OP</td>
<td>2,248</td>
<td>0.39</td>
<td>390</td>
<td>0.0033</td>
<td>0.0019</td>
<td>0.0014</td>
</tr>
<tr>
<td>N 14</td>
<td>OP</td>
<td>100,000</td>
<td>0.30</td>
<td>279</td>
<td>0.0015</td>
<td>0.0014</td>
<td>0.0002</td>
</tr>
<tr>
<td>N 4</td>
<td>OP</td>
<td>169,300</td>
<td>0.30</td>
<td>282</td>
<td>0.0017</td>
<td>0.0014</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

Table 4.2  Midsection equivalent stress and strain components for tubular specimens of 1050 QT steel under in-phase and 90° out-of-phase loading.

<table>
<thead>
<tr>
<th>ID</th>
<th>Load Type</th>
<th>$N_{\text{stable}}$ (Cycle)</th>
<th>$\bar{\nu}$</th>
<th>$\sigma_a$ (MPa)</th>
<th>$\bar{\varepsilon}_a$</th>
<th>$\Delta \bar{\varepsilon}_e / 2$ Calculated</th>
<th>$\Delta \bar{\varepsilon}_p / 2$ Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q 7</td>
<td>IP</td>
<td>1,024</td>
<td>0.40</td>
<td>826</td>
<td>0.0098</td>
<td>0.0041</td>
<td>0.0057</td>
</tr>
<tr>
<td>Q 1</td>
<td>IP</td>
<td>928</td>
<td>0.40</td>
<td>823</td>
<td>0.0097</td>
<td>0.0041</td>
<td>0.0057</td>
</tr>
<tr>
<td>Q 9</td>
<td>IP</td>
<td>20,408</td>
<td>0.32</td>
<td>676</td>
<td>0.0047</td>
<td>0.0033</td>
<td>0.0013</td>
</tr>
<tr>
<td>Q 5</td>
<td>IP</td>
<td>6,478</td>
<td>0.32</td>
<td>704</td>
<td>0.0048</td>
<td>0.0035</td>
<td>0.0013</td>
</tr>
<tr>
<td>Q 11</td>
<td>IP</td>
<td>42,872</td>
<td>0.27</td>
<td>657</td>
<td>0.0035</td>
<td>0.0032</td>
<td>0.0003</td>
</tr>
<tr>
<td>Q 3</td>
<td>IP</td>
<td>173,122</td>
<td>0.26</td>
<td>602</td>
<td>0.0030</td>
<td>0.0030</td>
<td>0.0000</td>
</tr>
<tr>
<td>Q 12</td>
<td>OP</td>
<td>60</td>
<td>0.43</td>
<td>821</td>
<td>0.0105</td>
<td>0.0040</td>
<td>0.0064</td>
</tr>
<tr>
<td>Q 2</td>
<td>OP</td>
<td>400</td>
<td>0.40</td>
<td>726</td>
<td>0.0069</td>
<td>0.0036</td>
<td>0.0033</td>
</tr>
<tr>
<td>Q 6</td>
<td>OP</td>
<td>400</td>
<td>0.39</td>
<td>744</td>
<td>0.0069</td>
<td>0.0037</td>
<td>0.0032</td>
</tr>
<tr>
<td>Q 8</td>
<td>OP</td>
<td>4,514</td>
<td>0.30</td>
<td>592</td>
<td>0.0035</td>
<td>0.0029</td>
<td>0.0006</td>
</tr>
<tr>
<td>Q 15</td>
<td>OP</td>
<td>3,930</td>
<td>0.28</td>
<td>587</td>
<td>0.0033</td>
<td>0.0029</td>
<td>0.0005</td>
</tr>
<tr>
<td>Q 4</td>
<td>OP</td>
<td>3,998</td>
<td>0.27</td>
<td>610</td>
<td>0.0034</td>
<td>0.0030</td>
<td>0.0004</td>
</tr>
<tr>
<td>Q 10</td>
<td>OP</td>
<td>25,904</td>
<td>0.27</td>
<td>508</td>
<td>0.0027</td>
<td>0.0025</td>
<td>0.0002</td>
</tr>
<tr>
<td>Q 13</td>
<td>OP</td>
<td>47,334</td>
<td>0.26</td>
<td>404</td>
<td>0.0020</td>
<td>0.0020</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Table 4.3  Midsection equivalent stress and strain components for tubular specimens of 1050 IH steel under in-phase and 90° out-of-phase loading.

<table>
<thead>
<tr>
<th>ID</th>
<th>Load Type</th>
<th>$N_{stable}$ (Cycle)</th>
<th>$\nu$</th>
<th>$\sigma_a$ (MPa)</th>
<th>$\bar{\epsilon}_a$</th>
<th>$\Delta\bar{\epsilon}_a / 2$ Calculated</th>
<th>$\Delta\bar{\epsilon}_\sigma / 2$ Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>I 13</td>
<td>IP</td>
<td>40</td>
<td>0.32</td>
<td>2,154</td>
<td>0.0145</td>
<td>0.0106</td>
<td>0.0039</td>
</tr>
<tr>
<td>I 15</td>
<td>IP</td>
<td>40</td>
<td>0.32</td>
<td>2,100</td>
<td>0.0145</td>
<td>0.0107</td>
<td>0.0038</td>
</tr>
<tr>
<td>I 19</td>
<td>IP</td>
<td>40</td>
<td>0.30</td>
<td>2,001</td>
<td>0.0124</td>
<td>0.0103</td>
<td>0.0022</td>
</tr>
<tr>
<td>I 15</td>
<td>IP</td>
<td>40</td>
<td>0.30</td>
<td>1,943</td>
<td>0.0124</td>
<td>0.0100</td>
<td>0.0024</td>
</tr>
<tr>
<td>I 11</td>
<td>IP</td>
<td>140</td>
<td>0.28</td>
<td>1,874</td>
<td>0.0102</td>
<td>0.0092</td>
<td>0.0009</td>
</tr>
<tr>
<td>I 13</td>
<td>IP</td>
<td>100</td>
<td>0.28</td>
<td>1,870</td>
<td>0.0101</td>
<td>0.0093</td>
<td>0.0009</td>
</tr>
<tr>
<td>I 23</td>
<td>IP</td>
<td>1,652</td>
<td>0.26</td>
<td>961</td>
<td>0.0049</td>
<td>0.0049</td>
<td>0.0000</td>
</tr>
<tr>
<td>I 12</td>
<td>OP</td>
<td>40</td>
<td>0.26</td>
<td>1,824</td>
<td>0.0102</td>
<td>0.0091</td>
<td>0.0011</td>
</tr>
<tr>
<td>I 16</td>
<td>OP</td>
<td>40</td>
<td>0.26</td>
<td>1,753</td>
<td>0.0101</td>
<td>0.0090</td>
<td>0.0011</td>
</tr>
<tr>
<td>I 14</td>
<td>OP</td>
<td>60</td>
<td>0.27</td>
<td>1,601</td>
<td>0.0092</td>
<td>0.0084</td>
<td>0.0008</td>
</tr>
<tr>
<td>I 12</td>
<td>OP</td>
<td>70</td>
<td>0.26</td>
<td>1,625</td>
<td>0.0092</td>
<td>0.0085</td>
<td>0.0007</td>
</tr>
<tr>
<td>I 18</td>
<td>OP</td>
<td>40</td>
<td>0.26</td>
<td>1,385</td>
<td>0.0075</td>
<td>0.0072</td>
<td>0.0003</td>
</tr>
<tr>
<td>I 10</td>
<td>OP</td>
<td>512</td>
<td>0.26</td>
<td>753</td>
<td>0.0037</td>
<td>0.0037</td>
<td>0.0001</td>
</tr>
<tr>
<td>I 16</td>
<td>OP</td>
<td>2,000</td>
<td>0.26</td>
<td>753</td>
<td>0.0037</td>
<td>0.0037</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.4  Midsection equivalent stress and strain components for tubular specimens of 304L stainless steel under in-phase and 90° out-of-phase loading.

<table>
<thead>
<tr>
<th>ID</th>
<th>Load Type</th>
<th>$N_{stable}$ (Cycle)</th>
<th>$\nu$</th>
<th>$\sigma_a$ (MPa)</th>
<th>$\bar{\epsilon}_a$</th>
<th>$\Delta\bar{\epsilon}_a / 2$ Calculated</th>
<th>$\Delta\bar{\epsilon}_\sigma / 2$ Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSP 1</td>
<td>IP</td>
<td>512</td>
<td>0.44</td>
<td>453</td>
<td>0.0096</td>
<td>0.0023</td>
<td>0.0072</td>
</tr>
<tr>
<td>SSP 11</td>
<td>IP</td>
<td>2,048</td>
<td>0.43</td>
<td>379</td>
<td>0.0068</td>
<td>0.0020</td>
<td>0.0048</td>
</tr>
<tr>
<td>SSP 9</td>
<td>IP</td>
<td>4,096</td>
<td>0.40</td>
<td>348</td>
<td>0.0046</td>
<td>0.0018</td>
<td>0.0028</td>
</tr>
<tr>
<td>SSP 13</td>
<td>IP</td>
<td>16,928</td>
<td>0.40</td>
<td>269</td>
<td>0.0033</td>
<td>0.0014</td>
<td>0.0019</td>
</tr>
<tr>
<td>SSP 7</td>
<td>IP</td>
<td>101,980</td>
<td>0.39</td>
<td>234</td>
<td>0.0025</td>
<td>0.0012</td>
<td>0.0012</td>
</tr>
<tr>
<td>SSNP 1</td>
<td>OP</td>
<td>512</td>
<td>0.42</td>
<td>611</td>
<td>0.0068</td>
<td>0.0032</td>
<td>0.0037</td>
</tr>
<tr>
<td>SSNP 2</td>
<td>OP</td>
<td>398</td>
<td>0.42</td>
<td>621</td>
<td>0.0069</td>
<td>0.0032</td>
<td>0.0037</td>
</tr>
<tr>
<td>SSNP 4</td>
<td>OP</td>
<td>4,096</td>
<td>0.36</td>
<td>420</td>
<td>0.0033</td>
<td>0.0022</td>
<td>0.0011</td>
</tr>
<tr>
<td>SSNP 8</td>
<td>OP</td>
<td>4,096</td>
<td>0.36</td>
<td>427</td>
<td>0.0034</td>
<td>0.0022</td>
<td>0.0012</td>
</tr>
<tr>
<td>SSNP 6</td>
<td>OP</td>
<td>138,332</td>
<td>0.33</td>
<td>298</td>
<td>0.0021</td>
<td>0.0016</td>
<td>0.0006</td>
</tr>
</tbody>
</table>
Table 4.5  In-phase and 90° out-of-phase cyclic deformation properties and coefficients for 1050 steel and 304L stainless steel.

<table>
<thead>
<tr>
<th>In-phase Cyclic Properties</th>
<th>1050 N Steel</th>
<th>1050 QT Steel</th>
<th>1050 IH Steel</th>
<th>304L Stainless Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyclic strength coefficient, $K'$, MPa</td>
<td>1,480</td>
<td>1,558</td>
<td>3,328</td>
<td>2,939</td>
</tr>
<tr>
<td>Cyclic strain hardening exponent, $n'$</td>
<td>0.223</td>
<td>0.123</td>
<td>0.083</td>
<td>0.374</td>
</tr>
<tr>
<td>Cyclic yield strength, $\sigma'_y$, MPa</td>
<td>372</td>
<td>725</td>
<td>1,988</td>
<td>288</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>90º Out-of-phase Cyclic Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyclic strength coefficient, $K'$, MPa</td>
</tr>
<tr>
<td>Cyclic strain hardening exponent, $n'$</td>
</tr>
<tr>
<td>Cyclic yield strength, $\sigma'_y$, MPa</td>
</tr>
<tr>
<td>Non-proportional cyclic hardening coefficient, $\alpha$</td>
</tr>
</tbody>
</table>
Table 4.6  Midsection equivalent stress and strain components for tubular specimens of 1050 QT steel under variable amplitude star shape strain paths.

<table>
<thead>
<tr>
<th>ID</th>
<th>Load Type</th>
<th>( L_{stable} ) (Block)</th>
<th>( \bar{V} )</th>
<th>( \bar{\sigma}_a ) (MPa)</th>
<th>( \bar{\varepsilon}_a )</th>
<th>( \Delta \bar{\varepsilon}_e / 2 ) Calculated</th>
<th>( \Delta \bar{\varepsilon}_p / 2 ) Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>10FRI2Q</td>
<td>FRI</td>
<td>3</td>
<td>0.40</td>
<td>724</td>
<td>0.0069</td>
<td>0.0033</td>
<td>0.0036</td>
</tr>
<tr>
<td>10FRI1Q</td>
<td>FRI</td>
<td>4</td>
<td>0.40</td>
<td>751</td>
<td>0.0070</td>
<td>0.0036</td>
<td>0.0034</td>
</tr>
<tr>
<td>5FRI1QT</td>
<td>FRI</td>
<td>20</td>
<td>0.31</td>
<td>647</td>
<td>0.0033</td>
<td>0.0031</td>
<td>0.0002</td>
</tr>
<tr>
<td>5FRI2QT</td>
<td>FRI</td>
<td>10</td>
<td>0.32</td>
<td>630</td>
<td>0.0033</td>
<td>0.003</td>
<td>0.0003</td>
</tr>
<tr>
<td>10FRR1Q</td>
<td>FRR</td>
<td>3</td>
<td>0.39</td>
<td>801</td>
<td>0.0070</td>
<td>0.0038</td>
<td>0.0032</td>
</tr>
<tr>
<td>10FRR2Q</td>
<td>FRR</td>
<td>4</td>
<td>0.39</td>
<td>800</td>
<td>0.0069</td>
<td>0.0036</td>
<td>0.0033</td>
</tr>
<tr>
<td>5FRR3QT</td>
<td>FRR</td>
<td>50</td>
<td>0.32</td>
<td>645</td>
<td>0.0034</td>
<td>0.0031</td>
<td>0.0003</td>
</tr>
<tr>
<td>5FRR1QT</td>
<td>FRR</td>
<td>60</td>
<td>0.33</td>
<td>611</td>
<td>0.0034</td>
<td>0.0029</td>
<td>0.0005</td>
</tr>
<tr>
<td>10PI1QT</td>
<td>PI</td>
<td>20</td>
<td>0.33</td>
<td>591</td>
<td>0.0035</td>
<td>0.0029</td>
<td>0.0006</td>
</tr>
<tr>
<td>10PI2QT</td>
<td>PI</td>
<td>9</td>
<td>0.32</td>
<td>633</td>
<td>0.0035</td>
<td>0.0031</td>
<td>0.0004</td>
</tr>
<tr>
<td>5PI1QT</td>
<td>PI</td>
<td>8</td>
<td>0.27</td>
<td>400</td>
<td>0.0017</td>
<td>0.0017</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 4.7  Midsection equivalent stress and strain components for tubular specimens of 304L stainless steel under variable amplitude star shape strain paths.

<table>
<thead>
<tr>
<th>ID</th>
<th>Load Type</th>
<th>( L_{stable} ) (Block)</th>
<th>( \bar{V} )</th>
<th>( \bar{\sigma}_a ) (MPa)</th>
<th>( \bar{\varepsilon}_a )</th>
<th>( \Delta \bar{\varepsilon}_e / 2 ) Calculated</th>
<th>( \Delta \bar{\varepsilon}_p / 2 ) Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>10FRI2SS</td>
<td>FRI</td>
<td>3</td>
<td>0.43</td>
<td>469</td>
<td>0.0070</td>
<td>0.0025</td>
<td>0.0045</td>
</tr>
<tr>
<td>10FRI1SS</td>
<td>FRI</td>
<td>3</td>
<td>0.43</td>
<td>461</td>
<td>0.0069</td>
<td>0.0023</td>
<td>0.0046</td>
</tr>
<tr>
<td>5FRI1SS</td>
<td>FRI</td>
<td>15</td>
<td>0.38</td>
<td>391</td>
<td>0.0033</td>
<td>0.0020</td>
<td>0.0013</td>
</tr>
<tr>
<td>5FRI2SS</td>
<td>FRI</td>
<td>20</td>
<td>0.39</td>
<td>363</td>
<td>0.0034</td>
<td>0.0019</td>
<td>0.0015</td>
</tr>
<tr>
<td>2.5FRI1S</td>
<td>FRI</td>
<td>1600</td>
<td>0.33</td>
<td>288</td>
<td>0.0018</td>
<td>0.0016</td>
<td>0.0002</td>
</tr>
<tr>
<td>10FRR1S</td>
<td>FRR</td>
<td>3</td>
<td>0.41</td>
<td>613</td>
<td>0.0070</td>
<td>0.0032</td>
<td>0.0038</td>
</tr>
<tr>
<td>10FRR2S</td>
<td>FRR</td>
<td>3</td>
<td>0.41</td>
<td>593</td>
<td>0.0069</td>
<td>0.0030</td>
<td>0.0039</td>
</tr>
<tr>
<td>5FRR1SS</td>
<td>FRR</td>
<td>20</td>
<td>0.36</td>
<td>444</td>
<td>0.0033</td>
<td>0.0023</td>
<td>0.0010</td>
</tr>
<tr>
<td>5FRR2SS</td>
<td>FRR</td>
<td>20</td>
<td>0.36</td>
<td>437</td>
<td>0.0033</td>
<td>0.0023</td>
<td>0.0010</td>
</tr>
<tr>
<td>2.5FRR1S</td>
<td>FRR</td>
<td>150</td>
<td>0.33</td>
<td>290</td>
<td>0.0018</td>
<td>0.0015</td>
<td>0.0003</td>
</tr>
<tr>
<td>5FRI1SS1</td>
<td>FRI15</td>
<td>200</td>
<td>0.37</td>
<td>413</td>
<td>0.0033</td>
<td>0.0021</td>
<td>0.0012</td>
</tr>
<tr>
<td>5FRI1SSS1</td>
<td>FRI15</td>
<td>300</td>
<td>0.38</td>
<td>399</td>
<td>0.0034</td>
<td>0.0021</td>
<td>0.0013</td>
</tr>
<tr>
<td>10PI1SS</td>
<td>PI</td>
<td>10</td>
<td>0.38</td>
<td>415</td>
<td>0.0035</td>
<td>0.0022</td>
<td>0.0013</td>
</tr>
<tr>
<td>10PI2SS</td>
<td>PI</td>
<td>15</td>
<td>0.38</td>
<td>418</td>
<td>0.0035</td>
<td>0.0021</td>
<td>0.0014</td>
</tr>
</tbody>
</table>
Figure 4.1  Equivalent stress-strain data and generated stress-strain curves for 90° out-of-phase tests of 1050 N steel at three strain levels.
Figure 4.2 Midsection equivalent stress amplitude versus midsection equivalent plastic strain amplitude data and fit for in-phase loading of (a) 1050 N steel, (b) 1050 QT steel, (c) 1050 IH steel, and (d) 304L stainless steel.
Figure 4.3  Midsection equivalent stress amplitude versus midsection equivalent plastic strain amplitude data and fit for 90° out-of-phase loading of (a) 1050 N steel, (b) 1050 QT steel, (c) 1050 IH steel, and (d) 304L stainless steel.
Figure 4.4 Comparison of cyclic deformation behavior of 1050 steel at different hardness levels as well as 304L stainless steel under (a) in-phase, and (b) $90^\circ$ out-of-phase loadings.
Figure 4.5  Comparison of cyclic in-phase, cyclic 90° out-of-phase, and monotonic stress-strain curves for (a) 1050 N steel, (b) 1050 QT steel, (c) 1050 IH steel, and (d) 304L stainless steel.
Figure 4.6  Comparison of the non-proportional cyclic hardening coefficient, $\alpha$, observed from experiments with the predicted values based on Borodii-Shukaev equation.

Figure 4.7  Definition of cyclic hardening coefficient, $h$. 

\[ h = 1 - \frac{\sigma_M}{\sigma_C} \]
Figure 4.8 Correlation of non-proportional hardening coefficient, $\alpha$, and cyclic hardening coefficient, $h$, for steels used in this study.

\[ \alpha = 1.6h^2 + 0.6h \]

Figure 4.9 Comparison of experimental non-proportional cyclic hardening coefficient, $\alpha$, with predicted values based on Equation (4.11) proposed in this study.
Figure 4.10 (a) Experimental non-proportional cyclic hardening coefficient, $\alpha$, and (b) experimental cyclic hardening coefficient, $h$, versus equivalent strain amplitude.
Figure 4.11 Comparison of experimental 90° out-of-phase equivalent stress amplitudes with the predicted values based on Equations (4.11).
Figure 4.12  Cyclic deformation behavior under various strain paths for (a) 1050 QT steel, and (b) 304L stainless steel.
Figure 4.13  Equivalent stress amplitude responses for various strain paths under equivalent strain amplitudes of 0.7% and 0.34% for (a) 1050 QT steel, and (b) 304L stainless steel.
Figure 4.14 Comparison of observed and predicted equivalent stress amplitudes for 1050 QT steel and 304L stainless steel (304L SS) using (a) Kanazawa et al.’s empirical method, and (b) Tanaka’s non-proportionality parameter.
Figure 4.15  Comparison of the experimental midlife stress boundaries and predicted ones using the Tanaka’s non-proportionality parameter for 304L stainless steel. y-axis represents $\sqrt{3}\tau$ and x-axis represents $\sigma$, both in MPa. There are two experimental boundaries for each strain path.

(a) FRI load path at $\varepsilon_a = 0.7\%$
(b) FRR load path at $\varepsilon_a = 0.7\%$
(c) FRI load path at $\varepsilon_a = 0.33\%$
(d) FRR load path at $\varepsilon_a = 0.33\%$
(e) FRI15 load path at $\varepsilon_a = 0.33\%$
Figure 4.16  Experimental and predicted stress boundaries for 304L stainless steel for FRI strain path at $\bar{\varepsilon}_a = 0.7\%$ by Kanazawa et al.’s empirical method and Tanaka’s non-proportionality parameter.
Figure 4.17  Comparison of the experimental axial and shear stress boundaries and the predicted axial and shear stresses for PI strain path at strain level of $\varepsilon_a = 0.35\%$ using Tanaka’s non-proportionality parameter for (a) 1050 QT steel, and (b) 304L stainless steel.
Chapter 5

Fatigue Behavior and Life Predictions

5.1 Introduction

Effect of hardness and microstructure on multiaxial fatigue behavior is investigated by utilizing constant amplitude axial-torsion fatigue data of 1050 steel in three different hardness condition as well as 304L stainless steel. This chapter presents constant amplitude as well as variable amplitude multiaxial fatigue behavior and life predictions for 1050 QT steel with no non-proportional cyclic hardening and 304L stainless steel with a 55% additional non-proportional hardening coefficient.

Constant amplitude multiaxial fatigue behavior of 1050 steel in normalized, quenched and tempered, and induction hardened conditions as well as 304L stainless steel are presented in Section 5.2. Effects of hardness on in-phase and 90° out-of-phase multiaxial fatigue behavior of 1050 steel are discussed in this section. Geometry effects on multiaxial fatigue behavior is also investigated by comparing in-phase and 90° out-of-phase multiaxial fatigue data of tubular and solid section 1050 N steel specimens. The capability of von Mises criterion as the most commonly used classical strength criterion to correlate in-phase and 90° out-of-phase multiaxial fatigue data is examined in Section 5.2. Fatemi-Socie (FS) critical plane approach is also used to correlate in-phase (IP) and
90° out-of-phase (OP) multiaxial fatigue data of 1050 steel in three different hardening conditions, as well as data of 304L stainless steel.

Furthermore, in order to predict multiaxial fatigue life of steels in the absence of any fatigue data, the Roessle-Fatemi hardness method for estimating the uniaxial fatigue behavior is extended to multiaxial fatigue. Then, IP and OP multiaxial fatigue lives are predicted using the FS multiaxial model based on only the hardness level of the material. The applicability of the prediction method based on hardness is also examined for Inconel 718 and 1Cr-18Ni-9Ti stainless steel under a wide range of biaxial loading conditions from literature and analysis results are presented at the end of Section 5.2.

Variable amplitude multiaxial fatigue behaviors of 1050 QT steel and 304L stainless steel with very different constitutive behaviors are presented in Section 5.3. Fatigue behavior of 1050 QT steel with no non-proportional cyclic hardening and 304L stainless steel with 55% non-proportional cyclic hardening coefficient are investigated under discriminating strain paths and compared with in-phase and 90° out-of-phase fatigue lives with the same equivalent strain amplitudes. This investigation also includes a discussion on the fatigue crack orientation angle and mechanism for materials and various loading conditions utilized in this study.

Bannantine-Socie (BS) and Wang-Brown (WB) variable amplitude multiaxial cycle counting methods are used to identify different cycles in variable amplitude multiaxial load histories. The FS critical plane approach is used to evaluate the fatigue damage caused by each counted cycle. Palmgeren-Miner linear damage rule is then employed to add damage at different levels to predict fatigue life under variable amplitude multiaxial strain blocks used in this study. Finally, some discussions regarding
BS and WB cycle counting methods are presented at the end of Section 5.3. Conclusions made from experimental observations and prediction results are presented in Section 5.4.

5.2 Constant amplitude fatigue behavior and predictions

Constant amplitude multiaxial fatigue behaviors of 1050 steel in three hardness conditions as well as 304L stainless steel are presented in this section. This section also includes discussion of effects of hardness on multiaxial fatigue behavior by comparing in-phase and 90° out-of-phase multiaxial fatigue data of 1050 steel at three hardness levels. Effects of out-of-phase loading and non-proportional hardening on multiaxial fatigue behavior are also presented in this section. von Mises classical strength theory as well as FS critical plane approach are used to correlate in-phase and 90° out-of-phase fatigue data. A multiaxial fatigue life prediction method only based on hardness level of steels is also proposed at the end of this section.

5.2.1 Constant amplitude fatigue behavior

Since shear stress is higher on the surface of specimens, crack initiation is expected to be on the surface; therefore, midsection shear stress needs to be extrapolated to the surface to be used for fatigue analysis. Knowing the midsection and the surface equivalent strain amplitudes and using equivalent stress-strain Ramberg-Osgood relation for proportional (in-phase) and non-proportional (90° out-of-phase) loading separately, the ratio of the surface equivalent stress to midsection equivalent stress was obtained by extrapolating effective strain amplitude to the surface for each fatigue test. Then, multiplying this ratio by the midsection equivalent stress, surface equivalent stress was calculated. Using this procedure, all midsection stress components were extrapolated to
surface and are presented in Tables 5.1, 5.2, 5.3, and 5.4 for 1050 N steel, 1050 QT steel, 1050 IH steel, and 304L stainless steel, respectively.

The following equation relates the surface equivalent stress amplitude to the fatigue life:

$$\sigma_s = \frac{\Delta \sigma}{2} = \sigma'_f (2N_f)^b$$  \hspace{1cm} (5.1)

where $\sigma'_f$ is the equivalent fatigue strength coefficient, $b$ is the equivalent fatigue strength exponent, and $2N_f$ is the number of reversals to failure (which was defined as the 5% load or torque drop, compared with the midlife cycle). The fatigue strength coefficient, $\sigma'_f$, and fatigue strength exponent, $b$, are the intercept and slope of the best line fit to surface equivalent stress amplitude, $\Delta \sigma / 2$, versus reversals to failure, $2N_f$, data in log-log scale. In accordance with ASTM Standard E739 (2007), when performing the least squares fit, the equivalent stress amplitude, $\Delta \sigma / 2$, was the independent variable and the reversals to failure, $2N_f$, was the dependent variable. All the test data were used for stress amplitude versus reversals to failure curve fitting, except for the run-out data and test data at the same level of run-out data.

The following equation relates the surface equivalent strain amplitude to fatigue life:

$$\bar{\varepsilon}_s = \frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_x}{2} + \frac{\Delta \varepsilon_p}{2} = \frac{\sigma'_f}{E} (2N_f)^b + \bar{\varepsilon}'_f (2N_f)^b$$  \hspace{1cm} (5.2)

where $\bar{\varepsilon}'_f$ is the equivalent fatigue ductility coefficient, $\bar{\varepsilon}$ is the equivalent fatigue ductility exponent, and $E$ is the average of the first cycle modulus of elasticity. Values of
\( \bar{\varepsilon}_f \) and \( \bar{\varepsilon} \) were obtained from the intercept and slope of the best line fit to equivalent plastic strain amplitude \( (\Delta \bar{\varepsilon}_p / 2) \), versus reversals to failure \( (2N_f) \), data in log-log scale:

\[
\frac{\Delta \bar{\varepsilon}_p}{2} = \varepsilon'_f (2N_f)^r
\]

(5.3)

In accordance with the ASTM Standard E739 (2007), when performing the least squares fit, the plastic strain amplitude, \( \Delta \bar{\varepsilon}_p / 2 \), was the independent variable and the reversals to failure, \( 2N_f \), was the dependent variable. The range of plastic strain data used in the curve fitting for all material conditions of steel 1050 was from tests with \( \Delta \bar{\varepsilon}_p / 2 \geq 0.02\% \). This is due to the fact that below this value the plastic strain is too small causing much scatter.

In-phase and 90° out-of-phase elastic, plastic, and total equivalent surface strain amplitude versus reversals to failure data and fittings are presented in Figures 5.1 through 5.4 for 1050 N steel, 1050 QT steel, 1050 IH steel, and 304L stainless steel, respectively. The scatter of data for IP and OP loadings are similar for a given material or hardening condition used in this study. Nevertheless, as hardness level increases, the scatter of data increases. More scatter of data for both IP and OP loadings of 1050 IH steel is because of the more sensitivity of higher hardness materials to defects and impurities.

Multiaxial fatigue equivalent coefficients and exponents for IP and OP loadings of normalized, quenched and tempered, and induction-hardened conditions of 1050 medium carbon steel as well as 304L stainless steel are listed in Table 5.5. Figure 5.5 represents proportional (in-phase) and non-proportional (90° out-of-phase) equivalent surface strain amplitudes versus reversals to failure comparison for 1050 N steel, 1050 QT steel, 1050 IH steel, and 304L stainless steel. Based on this figure, non-proportional
(OP) fatigue lives are found to be shorter compared to proportional (IP) fatigue lives for all materials and hardness levels in this study. Quenched and tempered 1050 steel with no non-proportional hardening and 304L stainless steel with 55% non-proportional hardening coefficient exhibit the maximum and the minimum differences between IP and OP fatigue data, respectively, amongst all materials and hardening conditions investigated. Therefore, additional non-proportional hardening is not the only reason of shorter fatigue lives under non-proportional loadings.

Specimen geometry effect can be seen in Figure 5.5(a) by comparing solid and tubular normalized specimen data. As can be seen from this figure, although fatigue lives of solid specimens are slightly longer, these differences in lives are small. This small difference can be explained by lower crack growth rate in solid specimens as compared to tubular specimens. In addition, cracks in solid specimens grow long but shallow; therefore, the specimen can carry the torque to a longer life than in the tubular specimens at the same applied strain amplitude. Since the definition of failure was based on torque drop, the solid specimen will result in a torque drop at a longer crack length. Short cracks in the solid specimens were observed much earlier than final failure, as compared to the tubular specimens at the same applied strain amplitudes in both in-phase and 90° out-of-phase fatigue tests. More differences between fatigue lives of normalized solid and tubular specimens in short life regime may be explained by the higher ratio of crack growth life to crack initiation life at shorter lives as compared to longer lives.

IP and OP strain-life multiaxial fatigue behaviors for 1050 steel in three different hardening conditions are compared separately in Figure 5.6. This figure shows similar behaviors as a function of hardness for both IP and OP loadings. Ductility of the
normalized steel is beneficial in short life regime, while strength of the quenched and tempered and induction-hardened steels is beneficial in long life regime. Comparing the fatigue lives at 1% equivalent strain amplitude (i.e. LCF region) indicates that the normalized specimens last about 3 times longer life than the induction-hardened specimens under both IP and OP loading conditions. Similar comparison at 0.3% equivalent strain amplitude (i.e. HCF region) shows that induction-hardened specimen lives are 8 and 25 times longer than the normalized specimens under IP and OP loadings, respectively. More details including the effect of hardness on stress-life behavior can be found in (Shamsaei and Fatemi, 2009).

5.2.2 Constant amplitude fatigue life predictions

- Life prediction by von Mises criterion

von Mises criterion as the most commonly used classical strength theory considers an average of distortion energy as the equivalent strain. The following equation relates the surface equivalent strain amplitude to the fatigue life:

\[
\varepsilon' = \frac{1}{\sqrt{2(1+\nu)}} \sqrt{2 \left( \frac{\Delta\varepsilon}{2} \right)^2 (1+\nu)^2 + \frac{3}{2} \left( \frac{\Delta\gamma}{2} \right)^2} = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (5.4)
\]

where \( \sigma'_f \) is the fatigue strength coefficient, \( b \) is the fatigue strength exponent, \( \varepsilon'_f \) is the fatigue ductility coefficient, \( c \) is the fatigue ductility exponent, \( E \) is the average of the first cycle modulus of elasticity and \( 2N_f \) is the number of reversals to failure (which was defined as 5% load or torque drop, whichever occurred first, compared with the near midlife stable cycle). In-phase equivalent fatigue properties from Table 5.5 are used.
instead of uniaxial properties for the right side of this equation, since uniaxial properties were not available.

Employing Equation (5.4), the correlation of IP and OP test data for 1050 steel in three different hardening condition as well as 304L stainless steel are presented in Figure 5.7. As can be seen from this figure, out-of-phase (OP) data fall within scatter bands of 15 for 1050 N steel, 60 for 1050 QT steel, 20 for 1050 IH steel, and 5 for 304L stainless steel. The difference between OP and IP data is much greater for 1050 QT steel with no non-proportional hardening, as compared to 304L stainless steel with 55% non-proportional hardening. Significant differences in fatigue lives observed for IP and OP loadings at the same equivalent strain amplitudes suggest the von Mises criterion underestimates the fatigue damage under non-proportional loadings. In addition, crack orientation angle cannot be predicted by von Mises criterion, which represents distortion energy as a scalar quantity. This is not consistent with the experimental observations indicating the fact that all cracks are around a preferred direction and not in random directions. Figure 5.8 presents cracks observed for IP and OP loading of 1050 N steel, 1050 QT steel, and 304L stainless steel. As can be seen from this figure, cracks orientations are not random and they are all around a specific direction.

- **Life prediction by FS critical plane approach**

  Overall crack orientations for IP and OP loadings were around the maximum shear plane, as shown in Figure 5.8. Therefore, a shear based critical plane approach should be used for life predictions. Although it is typically expected for high hardness materials, such as the induction-hardened specimens, to fail on the maximum principal plane, this was not the case in this study. Shear failure mode in high hardness steel under
torsion was also observed by Kurath and Jiang (1995) and McClaflin and Fatemi (2004). McClaflin and Fatemi reported that even though the final fracture occurred on the maximum principal stress plane, crack initiation appeared to be on the maximum shear plane. Therefore, a shear-based multiaxial fatigue model should be used for fatigue data correlation in such cases.

The FS critical plane approach is a function of maximum shear strain amplitude \( \Delta \gamma_{\text{max}} / 2 \) and maximum normal stress acting on the maximum shear strain plane, \( \sigma_{n,\text{max}} \) as follows:

\[
\frac{\Delta \gamma_{\text{max}}}{2} \left( 1 + k \frac{\sigma_{n,\text{max}}}{\sigma_y} \right) = \frac{\tau'_f}{G} (2N_f)^{b_o} + \gamma'_f (2N_f)^{c_o}
\]

(5.5)

where \( \sigma_y \) is the material monotonic yield strength, \( k \) is a material constant and can be found by fitting fatigue data from uniaxial tests to fatigue data from pure torsion tests, \( \tau'_f \) is the shear fatigue strength coefficient, \( b_o \) is the shear fatigue strength exponent, \( \gamma'_f \) is the shear fatigue ductility coefficient, \( c_o \) is the shear fatigue ductility exponent, and \( G \) is the cyclic shear modulus. The maximum normal stress on the maximum shear strain plane, \( \sigma_{n,\text{max}} \), takes into account constitutive behavior including additional hardening due to the non-proportionality of loading.

Figure 5.9 presents correlation of IP and OP multiaxial fatigue data by FS parameter for 1050 N steel \( (k = 0.4) \), 1050 QT steel \( (k = 0.6) \), 1050 IH steel \( (k = 0.7) \), and 304L stainless steel \( (k = 0.15) \). These figures also include scatter bands of 3 and 5 in life with respect to fatigue life. Since shear strain-life fatigue properties (i.e. \( \tau'_f, b_o, \gamma'_f, c_o \)) were not available, they were found from the equivalent fatigue properties obtained from
in-phase tests (i.e. $\sigma_f', \bar{b}, \bar{\varepsilon}_f', \bar{c}$) and converted based on the von Mises criterion as:

$$
\tau_f' = \frac{\sigma_f'}{\sqrt{3}}, \quad b_o = \bar{b}, \quad \gamma_f' = \sqrt{3\varepsilon_f'}, \quad \text{and} \quad c_o = \bar{c}.
$$

IP and OP fatigue data correlated based on the FS parameter (see Figure 5.9) fall within scatter bands of 4, 6, 8, and 3 for 1050 N steel, 1050 QT steel, 1050 IH steel, and 304L stainless steel, respectively. Considering typically higher scatter of experimental data at high hardness levels and inherent data scatter in duplicate tests of 1050 QT steel and 1050 IH steel, presented in Tables 5.2 and 5.3, the predictions can be considered fairly reasonable. Therefore, much better data correlation can be obtained by using the FS critical plane approach as compared to von Mises criterion, especially for 1050 steel in normalized and quenched and tempered conditions.

Figure 5.10 presents the comparison of the FS damage parameter versus plane orientation angle for IP and OP loading of 1050 QT steel with no non-proportional cyclic hardening and 304L stainless steel with significant non-proportional cyclic hardening. As can be seen from this figure, due to the higher stress on the critical plane, out-of-phase loading results in higher damage value as compared to in-phase loading. This is also the case for 1050 QT steel, as a material with no non-proportional cyclic hardening. The higher damage value can explain shorter fatigue lives observed for both materials under OP loading, as compared to IP loading. In addition, due to the non-proportionality of loading, more planes around the critical plane experience higher value of FS damage parameter; thus, higher probability of the presence of a weak plane and defect orientation around the critical plane increases the possibility of early crack initiation for out-of-phase loading.
Not a considerable difference was observed by using the other definition of the FS damage parameter which is the maximum damage value, instead of damage on the maximum shear plane, as follows:

\[
FS = \left[ \frac{\Delta \gamma}{2} \left( 1 + k \frac{\sigma_{n,\text{max}}}{\sigma_y} \right) \right]_{\text{max}} \tag{5.6}
\]

Since the maximum shear plane and the plane experiencing the maximum FS damage parameter are exactly the same for OP loading condition and they are very close to each other for IP loading, Equations (5.5) and (5.6) result in very similar fatigue life predictions for the materials and loading conditions used in this section. However, this is not necessarily true for general multiaxial loading condition.

Similar results to the shear form were obtained by using the uniaxial form of the FS parameter, using \( \sigma_f' = \bar{\sigma}_f \), \( b = \bar{b} \), \( \varepsilon_f' = \bar{\varepsilon}_f \), and \( c = \bar{c} \), as listed in Table 5.5, since uniaxial properties were not available:

\[
\frac{\Delta \gamma_{\text{max}}}{2} \left( 1 + k \frac{\sigma_{n,\text{max}}}{\sigma_y} \right) = \left[ \left( 1 + \nu_p \right) \frac{\sigma_f'}{E} (2N_f)^b + \left( 1 + \nu_p \right) \varepsilon_f' (2N_f)^c \right] \left[ 1 + k \frac{\sigma_f'}{2\sigma_y} (2N_f)^b \right] \tag{5.7}
\]

As an approximation, if test data are not available, one may consider \( k = 1 \) in Equation (5.7). It should also be mentioned that by using \( k = 0.5 \) in this equation, very similar results were obtained, indicating not a very high sensitivity of this equation to the exact value of \( k \). This is because the parameter \( k \) appears on both sides of the uniaxial form of this parameter.
5.2.3 Fatigue life predictions based only on hardness

To enable fatigue life predictions of steels under multiaxial loadings based on simple and easy to obtain properties such as hardness, the Roessle-Fatemi hardness method (2000) is used in combination with the FS parameter. The uniaxial form of the FS parameter (i.e. Equation (5.7)) can be expressed based on Brinell hardness as follows:

\[
\frac{\Delta \gamma_{\text{max}}}{2} \left( 1 + k \frac{\sigma_{n,\text{max}}}{\sigma_y} \right) = \left[ A(2N_f)^{-0.09} + B(2N_f)^{-0.56} \right] \left[ 1 + kC(2N_f)^{-0.09} \right]
\]

(5.8)

where \( k = 1 \), \( E = 200,000 \) MPa for steels, and \( \sigma_f', \epsilon_f' \) in this equation given in terms of hardness, resulting in:

\[
A = \frac{5.53(HB) + 293}{200,000}
\]

(5.9)

\[
B = \frac{0.48(HB)^2 - 731(HB) + 286,500}{200,000}
\]

(5.10)

\[
C = \frac{1}{0.0022(HB) + 0.382}
\]

(5.11)

The yield strength of material, \( \sigma_y \), can also be estimated as a function of Brinell hardness as follows (Roessle, 1988):

\[
\sigma_y = 0.0044(HB)^2 + 1.33(HB)
\]

(5.12)

More details regarding this hardness method can be found in (Shamsaei and Fatemi, 2009). In-phase and 90° out-of-phase fatigue data of 1050 steel in three different hardening conditions as well as 304L stainless steel are compared with predicted fatigue lives only based on hardness in Figure 5.11. All 304L stainless steel data are within scatter bands of 5 and 90% of the data are within scatter bands of 3. For the normalized and quenched and tempered conditions, all the data are within factors of 3 and 5 from...
predictions, respectively. For the induction hardened condition, 85% of test data are within a factor of 5. Considering typically higher scatter of experimental data at high hardness levels, the predictions can be considered acceptable.

In addition to data generated in this study, to examine the applicability of the proposed method to other steels under various multiaxial loadings, Inconel 718 (Morrow, 1988) and 1Cr-18Ni-9Ti stainless steel data (Chen et al., 2004) from the literature were also used. The Inconel 718 data include in-phase axial-torsional data with or without mean stress, 90° out-of-phase axial-torsional data, and tension-tension multiaxial data. Comparison of the Inconel 718 data with predicted lives based on only hardness shown in Figure 5.11 indicates that 75% and 93% of the data are within the scatter bands of 3 and 5 in life from the predictions by the proposed method, respectively. The ability of the proposed method for predicting 1Cr-18Ni-9Ti stainless steel data is also presented in Figure 5.11. All the 1Cr-18Ni-9Ti stainless steel data are within a factor of 5 and 86% of the data are within a factor of 3 from the predicted fatigue life curve based on hardness.

Considering all the data in Figure 5.11, 94% and 81% of the data fall within scatter bands of 5 and 3, respectively. These data include four very different steels, three very different hardness levels, a broad range of loading conditions, and the predictions were based on only hardness as the sole material property. Therefore, the proposed method based on the FS and Roessle-Fatemi models for multiaxial fatigue life prediction of steels appears to be simple to use, relatively accurate, and a promising technique, although additional data are needed to further verify its robustness. However, this method is limited to the shear failure mode materials and for $150 < HB < 700$. 


The shear form of FS parameter (i.e. Equation (5.5)), can also be used, with the shear fatigue properties calculated based on von Mises criterion as \( \tau' = \sigma' / \sqrt{3} \), \( b_s = b \), \( \gamma' = \sqrt{3} \varepsilon' \), and \( c_s = c \). Then, estimating \( \sigma' \) and \( \varepsilon' \) as a function of hardness based on Roessle-Fatemi equation (2000) results in:

\[
\frac{\Delta \gamma'_{\text{max}}}{2} \left(1 + k \frac{\sigma_{n,\text{max}}}{\sigma_y}\right) = \frac{6.37(HB) + 338}{200,000} (2N_f)^{-0.09} + \frac{0.55(HB)^2 - 842(HB) + 331000}{200,000} (2N_f)^{-0.56}
\] (5.13)

where yield strength, \( \sigma_y \), can be calculated from Equation (5.12). Since \( k \) only appears on the left side of Equation (5.13), predictions are more sensitive to the value of \( k \), as compared to predictions based on Equation (5.8). Therefore, rather than using \( k = 1 \), a more accurate value can be obtained from the following equation, which relates \( k \) to the uniaxial and shear fatigue properties as well as fatigue life (Socie and Marquis, 2000):

\[
k = \left[\frac{\tau_f (2N_f)^b + \gamma_f (2N_f)^c}{1.3 \frac{\sigma_f}{E} (2N_f)^b + 1.5 \varepsilon_f (2N_f)^c}\right]^{-1} \frac{2\sigma_y}{\sigma'_f (2N_f)^b}
\] (5.14)

Using von Mises criterion, \( G = E / 2(1 + \nu_c) \), \( \nu_c = 0.3 \), and Roessle-Fatemi hardness model, this equation simplifies to the following expression as a function of Brinell hardness and fatigue life:

\[
k = [0.0003(HB) + 0.0585] (2N_f)^{0.09}
\] (5.15)

More details including derivation of these equations can be found in (Shamsaei and Fatemi, 2009). Variation of \( k \) versus fatigue life for several Brinell hardness of steels is shown in Figure 5.12. As can be seen from this figure, \( k \) increases as fatigue life...
increases, but not a significant change can be noticed for $k$ in short and midlife regimes for a specific hardness level. Furthermore, $k$ increases as hardness increases for a given fatigue life. Higher value of $k$ with increasing fatigue life and hardness level indicates the fact that in the long life regime and for higher hardness materials the effect of normal stress on fatigue life is more significant. Fatemi and Kurath (1988) related the increase of $k$ with fatigue life to the crack closure concept. In the short life regime due to more nominal plastic deformation, multiple cracks initiate and then link to cause failure, while in the long life regime with less nominal plasticity cracks extend with increasing number of cycles. The same concept explains the lower value of $k$ for lower hardness materials (i.e. more plastic deformation) and higher value of $k$ for higher hardness materials (i.e. less plastic deformation).

Life predictions for 1050 steel in different hardness levels and 304L stainless steel using the shear form of the Fatemi-Socie parameter based on hardness and employing Equations (5.13) and (5.15) were similar to those using the uniaxial form (i.e. Equations (5.8) through (5.12)).

### 5.3 Variable amplitude fatigue behavior and predictions

Variable amplitude multiaxial fatigue behaviors of 1050 QT steel and 304L stainless steel with very different additional cyclic hardening sensitivity from non-proportional loading are investigated in this section. Some discriminating star shape and cruciform strain paths with incremental and random load sequences are applied to specimens of 1050 QT steel with about zero non-proportional cyclic hardening and 304L
stainless steel with 55% non-proportional cyclic hardening coefficient and fatigue lives are compared with IP and OP fatigue lives with the same equivalent strain amplitudes.

To identify cycles within a variable amplitude multiaxial loading block, Bannantine-Socie (BS) and Wang-Brown (WB) cycle counting methods are used. FS critical plane approach is then employed to evaluate fatigue damage caused by each counted cycle. Palmgeren-Miner linear damage rule is used to add damage from each cycle to predict fatigue life under variable amplitude multiaxial strain blocks. Predicted fatigue lives are then compared with experimental fatigue lives. Some details and challenges regarding variable amplitude multiaxial fatigue prediction techniques and cycle counting methods are also presented.

5.3.1 Variable amplitude fatigue behavior

Similar to constant amplitude fatigue tests, shear stress is higher on the surface of variable amplitude test specimens; therefore, crack initiation was observed to be on the surface. The same procedure as constant amplitude fatigue test was used to extrapolate midsection shear stress to the surface to be used for fatigue analysis. Using this procedure all midsection stress components were extrapolated to surface for FRI, FRR, FRI15, FRI90, and PI90 strain paths and presented in Tables 5.6 and 5.7 for 1050 QT steel and 304L stainless steel, respectively.

Fatigue lives and stress responses of 1050 QT steel specimens under various strain paths normalized based on the in-phase (IP) data at the same equivalent strain level are presented in Figure 5.13. Average values of stress and fatigue life obtained from duplicate tests at each strain level are presented in this figure. Since 1050 QT steel is a material with no non-proportional hardening, all strain paths result in the same stabilized
equivalent stress response, as shown in Figure 5.13(b). However, Figure 5.13(a) indicates fatigue lives of this material under FRI and FRR strain paths are about half of the fatigue lives under IP strain path at $\bar{\varepsilon}_a = 0.7\%$. The OP strain path results in more than an order of magnitude shorter fatigue life as compared to IP strain path at $\bar{\varepsilon}_a = 0.7\%$. The ratios for fatigue lives of non-proportional strain paths to in-phase strain path decreases to even smaller values at $\bar{\varepsilon}_a = 0.34\%$ (0.15 for FRI, 0.25 for FRR, and 0.03 for OP strain paths). Therefore, shorter fatigue lives observed for non-proportional strain paths cannot be only related to the higher equivalent stress response under non-proportional cyclic loading.

Comparison of the observed fatigue lives for FRI90 strain path with fatigue lives observed for PI90 strain path shown in Figure 5.13(a) indicates the pulsating strain path (i.e. PI90) results in shorter fatigue lives as compared to the fully-reversed strain path (i.e. FRI90). Although both strain paths have the same strain amplitudes, PI90 path reaches to twice the maximum strain, compared to the FRI90 strain path. Coupling of cycles from both sides (i.e. top and bottom and left and right of the circle in Figure 3.7(b)) of the PI90 strain path results in a larger cycle with twice the amplitude, as compared to the fully-reversed strain path, FRI90. As a result of the higher amplitude cycle, fatigue lives under the PI90 strain path are shorter than fatigue lives under the FRI90 strain path. Coupling of the applied PI90 strain path cycles reduces the effective number of cycles by a factor of two, as compared to the FRI90 strain path. However, the coupled higher amplitude cycle has much more significant effect on fatigue life and results in a considerably shorter life of the PI90 path compared to the FRI90 path (by a factor of 5).

Since only two cycles in a block of 360 PI cycles couple and make a cycle with twice the amplitude as compared to the FRI strain path cycles, not a considerable
difference in the fatigue life can be observed from Figure 5.13(a) for these two paths. The more difference in fatigue lives observed for FRI90 and PI90 as compared to FRI and PI, shown in Figure 5.13(a), can be explained by the fact that FRI90 and PI90 strain paths only include four extreme cycles (pure axial and pure torsion cycles) of the FRI and PI strain paths. Coupling of these extreme cycles causes more effect on fatigue life of PI90 strain path, as compared to PI strain path, which has many other damaging cycles on the critical plane. This explains much shorter fatigue lives observed for PI90 strain path as compared to the PI strain path (by a factor of 3.5). Very similar fatigue lives are observed for FRI and FRI90 strain paths. Although more damage accumulates on the critical plane for the FRI90 strain path, as compared to FRI strain path, there is a wider distribution of damage around the critical plane for the FRI path. This is discussed later in more detail.

It should be noted that although the PI and PI90 path cycles have a mean stress, as compared to the fully-reversed FRI and FRI90 paths, this effect is small. This is due to the fact that the mean stresses are relatively small due to stress relaxation. More importantly, the mean stresses have opposite signs in the two halves of each strain block, therefore, positive and negative mean stresses mostly cancel each other’s effects in a complete strain block.

Fatigue lives and stress responses of 304L stainless steel specimens for various strain paths are also normalized based on the in-phase (IP) data at the same equivalent strain level and presented in Figure 5.14. As can be seen from Figure 5.14(a), IP, FRR, and FRI strain paths result in comparable fatigue lives, although the stress response of FRR path is much higher than IP and FRI strain paths (see Figure 5.14(b)). This is
observed at both strain amplitude levels ($\bar{\varepsilon}_a = 0.7\%$ and $0.34\%$). Therefore, higher stress is not the only factor affecting fatigue life under non-proportional loading.

In view of the fact that each strain path consists of individual proportional cycles, they can be directly decomposed into several proportional cycles with the same equivalent strain amplitudes. Based on this direct cycle counting, FRI and FRR strain paths are decomposed into 360 fully-reversed proportional cycles, FRI15 strain path is decomposed into 24 fully-reversed proportional cycles, PI strain path with 360 proportional cycles and FRI90 and PI90 paths decomposed into 4 proportional cycles.

Decomposing strain blocks directly into proportional cycles, von Mises criterion predicts nearly the same fatigue damage for all cycles. Comparison of observed and predicted fatigue lives based on the von Mises criterion for both materials is presented in Figure 5.15 for all strain paths used in this study. Scatter bands of 3, 5, and 10 are also superimposed in this figure. Fatigue data for 1050 QT steel fall within scatter band of 60 from predictions based on the von Mises criterion and within scatter band of 5 for 304L stainless steel.

As crack orientations were all around the maximum shear plane for both materials, as shown in Figure 5.8, the shear-based FS critical plane parameter, presented by Equation (5.5), was used. Value of $k$ was found to be 0.6 for 1050 QT steel and 0.15 for 304L stainless steel. The FS parameter considers much higher damage value for out-of-phase loading than von Mises criterion, due to the higher normal stress on the critical plane, as presented in Figure 5.16 for 1050 QT steel and 304L stainless steel at $\bar{\varepsilon}_a = 0.7\%$. This is true even for materials with no non-proportional cyclic hardening, such as 1050 QT steel, as shown in Figure 5.16(a).
Figure 5.16 also includes the distribution of the cumulative fatigue damage based on FS parameter versus plane orientation angle for FRI strain path. For each plane, stress and strain components of each cycle are mapped and fatigue damage based on the FS parameter is evaluated. Then, the accumulation of the fatigue damage on each plane is calculated by linearly adding the fatigue damage associated with each cycle. The cumulative fatigue damage is normalized for one cycle of FRI strain path in these figures.

As can be seen from Figure 5.16, more planes around the critical plane experience a considerable amount of damage for the FRI strain path, as compared to the IP strain path. Cracks can initiate and grow on any of these planes around the critical plane. The higher value of the damage parameter and the higher probability of crack initiation and growth on a wider range of planes result in shorter fatigue lives. A wider range of planes subjected to a high percentage of damage results in activating more slip systems and/or more defects oriented around the critical plane orientations, leading to shorter fatigue life. This is analogous to size effects, where a larger volume of the material typically results in shorter fatigue life due to higher probability of defects which affect fatigue behavior.

By direct cycle counting of FRI, FRR, or PI strain path, very similar cumulative fatigue damage distribution versus plane orientation angle as that for FRI strain path, presented in Figure 5.16, is obtained. This is due to the fact that all the above-mentioned paths consist of the same cycles and the only differences are either the sequence of straining or the presence of mean strains. Therefore, similar to FRI strain path, shorter fatigue lives observed for all variable amplitude strain paths may be partially explained by the wider range of planes experiencing a considerable amount of damage compared to
IP strain path. Figure 5.17 indicates that while cracks are observed to be oriented close to the critical plane orientation, they are not exactly on this plane for all the strain paths.

5.3.2 Cycle counting and fatigue life predictions

A cycle counting method is required to identify loading cycles in variable amplitude multiaxial loading. As previously mentioned in Chapter 2, BS and WB variable amplitude multiaxial methods are the only known proposals in the literature which reflect the concept of critical plane approach. After cycle counting and evaluating the fatigue damage associated with each counted cycle, one can use linear or one of the nonlinear cumulative damage rules (Fatemi and Yang, 1998) to account for the accumulation of damage on candidate planes within the material. As the linear damage rule is universally used and does not require any additional material constant, it is used in this study in the form of Equation (2.20) for life predictions under variable amplitude loading. Number of cycles to failure, $N_f$, is found from the in-phase curve and fatigue properties listed in Table 5.5. As mentioned earlier, the in-phase curve is used because uniaxial or pure torsion fatigue data were not available.

To verify the capability of the linear damage rule for analysis of a load history similar to the FRI strain path, uniaxial tests with the same strain history, as presented in Figure 5.18(a), and in the same life regime as the multiaxial experiments were conducted. These uniaxial tests were performed with small solid specimens (with 5 mm diameter) of 1050 QT steel. The uniaxial loading block consists of 90 incrementally increasing cycles in tension and 90 incrementally increasing cycles in compression with the same amplitude levels, $(\varepsilon_a)_{\text{max}}$, forming a sinusoidal strain block. Three different maximum
strain levels of 0.75%, 1%, and 1.5% were employed. The FS parameter and linear cumulative damage rule were used to predict fatigue lives. Comparison of the predicted and experimentally observed fatigue lives is presented in Figure 5.18(b). As can be seen from this figure, all data fall between 50% error bands from the prediction line. Therefore, linear cumulative damage rule works well for such loading history.

As mentioned earlier, BS cycle counting method (Bannantine and Socie, 1991a, 1991b) is based on the rainflow cycle counting of the main damage component and on various planes within the material. Depending on the damage parameter used, different components of strain and stress are mapped on different planes. For the FS parameter, shear strain as the main component and normal stress as the auxiliary component are mapped on different planes. Then, rainflow cycle counting is performed on the main component (i.e. shear strain channel) to identify cycles. The associated auxiliary component (i.e. normal stress channel) is also determined for each counted cycle. As stresses and strains usually get their extreme values on the free surface, critical plane should be searched on the free surface. After cycle counting on candidate planes within the material, the FS parameter in the following form is used to evaluate fatigue damage associated with every counted cycle on each plane:

$$FS = \left[ \frac{\Delta \gamma}{2} \left( 1 + k \frac{\sigma_{n,\text{max}}}{\sigma_y} \right) \right]$$ (5.16)

Linear cumulative fatigue damage rule is then employed to evaluate total damage on each plane. The plane with the greatest value of the cumulative damage is considered as the critical plane and fatigue life is evaluated based on the damage on this critical plane.
Predicted fatigue lives employing BS cycle counting method implemented in eFatigue software, FS damage parameter, and linear cumulative damage rule are compared with experimental fatigue lives for 1050 QT steel and 304L stainless steel in Figure 5.19. As can be seen from Figure 5.19, 81% of the data are within scatter bands of 3, while 96% of data fall within scatter bands of 5 from predictions. Figure 5.19 also indicates that 82% and 97% of variable amplitude strain path data are within scatter bands of 3 and 5 from predictions based on the aforementioned multiaxial fatigue life prediction technique. Therefore, by employing BS cycle counting method and based on FS critical plane approach and linear cumulative damage rule, reasonable fatigue life predictions for two materials with different constitutive behaviors and under various strain paths are obtained. Reviewing Tables 5.6 and 5.7, some inherent data scatter in duplicate tests as high as a factor of 3 is noticeable for OP tests of 1050 QT steel. Thus, not all the scatters observed in Figure 5.19 can be related to the described fatigue life estimation technique. The ratio of predicted to experimental fatigue lives for all variable amplitude strain paths and for both materials is also presented in Table 5.8.

Another important factor in life estimation is definition of failure. When using crack initiation approaches, failure life is typically defined as the life associated with micro-crack nucleation as well as micro-crack growth on the order of several hundred microns. In experiments, however, fatigue life is typically defined as a crack length corresponding to a certain load drop or decreased stiffness. Such cracks are typically longer than several hundred microns in length. Depending on the material fracture toughness, the discrepancy between the two definitions may be small or large. The 1050 QT steel used in this study has much lower fracture toughness than the 304L stainless
As a result, many micro-cracks were observed on the surface of the 304L stainless steel specimens for all strain paths investigated. In addition, while many such micro-cracks were present during a significant portion of the fatigue life for 304L stainless steel, they were much smaller in number and appeared much later in life for the 1050 QT steel. This can explain some of the differences between the observed fatigue lives of the two materials. This can also explain the higher ratios of predicted to experimental fatigue lives of the 1050 QT steel, as compared with the 304L stainless steel, listed in Table 5.8.

Distribution of the linear cumulative fatigue damage for 304L stainless steel as the result of one strain block of FRI path at $\varepsilon_a = 0.7\%$ on different planes within the material and based on rainflow cycle counting method and FS damage parameter is presented in Figure 5.20(a). As can be seen from this figure, the plane perpendicular to the specimen longitudinal axis (0° plane) experiences the maximum damage, which is consistent with the experimentally observed critical plane, shown in Figure 5.17(c). Very similar results were obtained for all other strain paths indicating that the plane perpendicular to the specimen axis is the critical plane based on rainflow cycle counting method, FS parameter, and linear cumulative damage rule. It was also experimentally observed that cracks initiated and grew around this plane (i.e. 0° plane) for all variable amplitude strain paths and for both materials (see Figures 5.17(c) through 5.17(f) for 304L stainless steel).

Damage contribution of each cycle of the FRI strain path at $\varepsilon_a = 0.7\%$ on the critical plane (plane perpendicular to the specimen longitudinal axis) and based on FS parameter for 304L stainless steel is presented in Figure 5.20(b), considering FRI stain path as 360 proportional cycles. The first cycle in this figure is the pure axial cycle of the
FRI strain path. This figure indicates the fact that contributions of different cycles to fatigue damage on the critical plane are not similar and cycles with the higher portion of shear loading are more damaging. This is due to the fact that maximum shear strain is much larger for shear cycles as compared to axial cycles within a circle in von Mises shear strain-axial strain plot.

As described in Chapter 2, Wang and Brown (1996a) proposed a cycle counting method based on reversal counting of the modified equivalent strain history. They used this cycle counting and the Kandil et al. strain-based parameter (Kandil et al., 1982) for fatigue life predictions under variable amplitude multiaxial loading. To identify the components of the damage parameter in a multiaxial strain block, Wang and Brown performed reversal counting on the modified von Mises equivalent strain history and identified axial and shear strain reversals associated with every counted equivalent strain reversal. This method modifies the strain history by assigning a reference point with the greatest value of the equivalent strain to avoid the sign problem in the von Mises equivalent strain criterion and to identify out-of-phase cycles.

The WB cycle counting method implemented in eFatigue software is also employed to identify reversals of the utilized strain blocks with the FS parameter and the critical plane associated with that to evaluate fatigue damage, and linear damage rule is then used to predict fatigue life. Predicted fatigue lives based on this procedure and using eFatigue software are compared with experimental fatigue lives in Figure 5.21. As can be seen from this figure, the overall predictions are similar to those based on BS cycle counting method, shown in Figure 5.19. Based on WB cycle counting method, FS damage parameter and linear damage rule, 83% and 94% of the data are within scatter
bands of 3 and 5 from predictions, as presented in Figure 5.21. However, the critical plane cannot be identified by WB cycle counting method, as this method adds damage caused by each counted reversal, no matter what plane represents the critical plane for these reversals.

The ratios of predicted to experimental fatigue lives using WB cycle counting method, FS critical plane approach and linear damage rule are also presented in Table 5.8. Comparison of the prediction results employing BS and WB cycle counting methods presented in Table 5.8 indicates better predictions with WB for PI and FRI15 strain paths, while BS cycle counting method resulted in better predictions for FRI90 strain path. These two cycle counting methods result in very similar fatigue damage predictions for other strain paths in this study. Comparison of Figures 5.15, 5.19, and 5.21 indicates that most strain paths fatigue life data fall between IP and OP fatigue data. It can, therefore, be concluded that by using a proper damage parameter to correlate IP and OP fatigue data, fatigue life predictions under general multiaxial loading will be improved. Although, a proper cycle counting method to correctly identify cycles within a multiaxial load block can greatly affect the results, having a suitable damage parameter taking into account various factors involved in such an analysis may be even more essential.

5.3.3 Some discussions on variable amplitude fatigue analysis

- Identification of the different components of the damage parameter

Although it was shown in the previous section that both BS and WB cycle counting methods along with FS damage parameter and linear cumulative damage rule result in acceptable variable amplitude multiaxial fatigue life predictions, still such analysis method does not consider the interaction of different components of the damage
parameter. Figure 5.22(a) presents shear strain and normal stress time histories as the components of FS damage parameter on the loading plane for a fully-reversed star-shaped strain path in $\varepsilon-\gamma$ axes with 45° increments. These time histories are rearranged in Figure 5.22(b) to start from the maximum shear strain point, and rainflow as the representative of cycle counting methods is performed. As previously mentioned, the cycle counting should be performed on the main channel (shear strain for FS damage parameter), then the associated auxiliary parameter (maximum normal stress for FS damage parameter) needs to be identified. Counted shear strain cycles are shown by arrows representing rain flow on the shear strain history and associated normal stress is shown by a circle with the same color. As can be seen from Figure 5.22(b), two of the rain flows experience the same maximum normal stress, where following the regular rainflow cycle counting or any other cycle counting for multiaxial loadings, the maximum normal stress point will be discarded by counting the smaller loop (green rain flow) within the largest loop in the history. Thus, this discarded maximum normal stress is not considered for the largest loop in the history (blue rain flow).

The challenge of variable amplitude multiaxial fatigue prediction methods can be explained better by rainflow cycle counting on PI90 and FRI90 strain paths. Shear strain and normal stress time histories for these strain paths are presented in Figures 5.23(a) and 5.24(a) for PI90 and FRI90 strain paths, respectively. These histories are rearranged to start from the maximum point in the shear strain history in Figures 5.23(b) and 5.24(b) for PI90 and FRI90, respectively, to perform rainflow cycle counting. Based on the BS cycle counting method, rain flows are drawn to identify shear strain cycles and associated normal stresses are presented by a circle with the same color as the counted shear strain
cycle in these figures. Counted shear strain ranges and associated maximum normal stresses are listed in a table for PI90 strain path in Figure 5.23 and for FRI90 in Figure 5.24.

Adding some very small shear cycles with no effect on fatigue life to PI90 and FRI90 as presented in Figures 5.25 and 5.26, and performing rainflow cycle counting, the maximum normal stresses may be discarded by counting these small shear cycles for both strain paths. Thus, a new set of counted shear strain cycles and associated maximum normal stresses can be obtained, as presented in the tables in Figures 5.25 and 5.26 for PI90 and FRI90 strain paths, respectively. Following this procedure, for the largest counted shear cycle, the maximum normal stress decreases from 800 MPa to 600 MPa for PI90 and from 600 MPa to zero for FRI90. Assuming no effect of the small added shear cycle to fatigue damage, very different fatigue lives will be predicted for these new sets of data, presented in Figures 5.25 and 5.26, as compared to the data presented in Figure 5.23 and 5.24. It should be noted that if the stress response for PI90 was fully-reversed, the associated maximum normal stress would be zero instead of 600 MPa according to this procedure. An alternate procedure may be to use the maximum normal stress associated with every counted shear cycle, even if it is used more than once. This, however, may result in conservative predictions. Therefore, there is no unique method of cycle counting for general applicability to multiaxial variable amplitude service load histories. Interaction of the stress or strain components of the damage parameter and a robust methodology for general purpose multiaxial variable amplitude fatigue life predictions remain as challenging problems.
- **Deficiency of WB cycle counting method**

As previously described, WB method is based on reversal counting on von Mises strain history modified based on a reference point with the greatest von Mises strain value. Then, critical plain is searched within the material for each counted reversal. Finally, total damage is estimated by summation of the critical damage of each counted reversal, no matter if they are all on the same plane or not. Figure 5.27 presents WB reversal counting method for FRI90 strain path, assuming point A on the axial strain as the first reference point. It should be mentioned here that points A, B, C, and D have the same von Mises equivalent value; thus, any of them can be selected as the first reference point. Since point B is the highest point in the modified von Mises strain history and strain is always increasing from point A to point B, the first counted reversal on the modified von Mises history is AOB, which is on the axial strain cycle, as presented in Figure 5.27(a). Then, the next reference point is point B, and BOC is counted as the second reversal on the modified von Mises strain history. As can be seen from Figure 5.27(b), this counted reversal is associated with reversals BO and OC on axial and shear strain histories, respectively. The same process is continued until all eight reversals are counted, as shown in Figure 5.27.

Now, by considering point C on the shear cycle as the first reference point, as presented in Figure 5.28(a) for FRI90 strain path, and modifying the von Mises history based on this point, the highest point on this modified history will be point D and, therefore, reversal COD on the shear strain history will be the first counted reversal. Then, point D is selected as the next reference point; thus, point B is the next turning point and reversal DOB is counted on the modified von Mises history. Reversals DO and
OB on shear and axial strain histories are associated with reversal DOB on the modified von Mises history, presented in Figure 5.28(b). This process is continued, as presented in Figure 5.28, until all reversals are counted. Comparing Figures 5.27 and 5.28 which represent WB reversal counting method on FRI90 strain path starting with different reference points, similar reversals are counted but in different orders. This different order in counting reversals based on WB method for this strain path has no effect on fatigue life calculations and similar predictions are obtained based on the linear cumulative damage rule.

The WB reversal counting method on PI90 strain path, assuming point A on the axial cycle as the first reference point, is presented in Figure 5.29. Similar to FRI90 strain path and due to the fact that these strain paths are within \( e - \gamma / \sqrt{3} \) strain plot, points A, B, C, and D have the same von Mises equivalent strain values; thus, any of these points can be selected as the first reference point. Starting from point A on the axial cycle, modified von Mises is always increasing to point C, then continues to point B' and finally point B as the turning point. Therefore, the first counted reversal, presented in Figure 5.29(a), is a mix of axial and shear reversals, AB and OC, respectively. Then, the remainder of this process, COB' should be counted by assuming point C as the new reference point. This counted reversal is also a mixture of axial and shear reversals, CO on the shear strain history and OB' on the axial strain history, presented in Figure 5.29(b). This process continues until all four reversals are counted as shown in Figure 5.29.

As previously mentioned, there are four possible starting points for WB reversal counting on PI90. Selecting point D on the shear cycle as the first reference point,
presented in Figure 5.30(a), the first counted reversal will be DOAC’C as a mix of axial and shear reversals. Continuing the WB reversal counting, four other reversals are identified as shown in Figure 5.30(b) through 5.30(d). Comparing Figures 5.29 and 5.30 which represent the WB reversal counting method by two different reference points for PI90, very different reversals are identified. Therefore, the WB reversal counting method does not lead to a unique answer and different reversals are obtained depending on the selected reference point. This is a fundamental weakness of this method which results in different fatigue life predictions for the same loading path.

5.4 Conclusions

1) Similar variations in strain-life behavior as a function of hardness were observed for in-phase and the 90º out-of-phase loadings. Ductility of the normalized steel is beneficial in short life regime, while strength of the quenched and tempered and induction-hardened steels is beneficial in long life regime.

2) At 1% equivalent strain amplitude (LCF region) the normalized specimens had about 3 times longer life than the induction-hardened specimens under both in-phase and 90º out-of-phase loading conditions. At 0.3% equivalent strain amplitude (HCF region), the induction-hardened specimen lives were 8 and 25 times longer than the normalized specimen lives under in-phase and 90º out-of-phase loadings, respectively.

3) The scatter of data for in-phase and 90º out-of-phase loadings were similar for a given material or hardening condition. As hardness level increases, the scatter of data increases, with more scatter for the induction-hardened condition related to higher sensitivity to inclusions.
4) Overall crack orientations for in-phase and 90° out-of-phase loadings for 304L stainless steel and 1050 steel at all hardness levels were found to be around the maximum shear plane. Therefore, a shear-based multiaxial fatigue model is appropriate for fatigue data correlations for all materials in this study.

5) No significant specimen geometry effect on fatigue lives was observed between solid and tubular specimens for either in-phase or 90° out-of-phase loadings. Slightly longer lives of solid specimens can be explained by shallow cracks, enabling carrying a torque to a longer life, as compared to tubular specimens.

6) von Mises criterion did not correlate the in-phase and 90° out-of-phase fatigue data. Based on the Fatemi-Socie multiaxial fatigue parameter, 92% of the 1050 N steel test data and 80% of 1050 IH steel test data were within a factor of 3 in life, while 86% of the 1050 QT steel test data were within a factor of 5 in life. All the in-phase and 90° out-of-phase fatigue data of 304L stainless steel were predicted within a factor of 3 in life.

7) A simple method for multiaxial fatigue life prediction of steels is proposed based on the Fatemi-Socie parameter in combination with the Roessle-Fatemi model. This method uses hardness as the only needed material property. Considering data from four very different steels, three very different hardness levels for one of the steels, and a broad range of loading conditions, life predictions were found to be relatively accurate, with 94% and 81% of the data within scatter bands of 5 and 3 in life with respect to the predicted curve, respectively.

8) Fatigue life of 1050 QT steel with no non-proportional cyclic hardening was found to be more sensitive to non-proportional loading than for 304L stainless
steel with significant non-proportional cyclic hardening. Therefore, additional cyclic hardening due to the non-proportionality of loading is not the only factor causing shorter fatigue lives under non-proportional loading.

9) FRR strain path with random sequences of axial-torsion cycles and sudden changes in straining direction resulted in a considerably higher stress response as compared to FRI strain path with gradual change in straining direction for 304L stainless steel. However, not a significant effect of this higher stress was observed for fatigue life of this material. Similarly, no effect of loading sequence was observed for 1050 QT steel with no non-proportional cyclic hardening.

10) Shorter fatigue lives were observed for the pulsating PI90 strain path as compared to the fully-reversed FRI90 strain path for 1050 QT steel with no non-proportional hardening. These observed shorter fatigue lives can be explained by coupling of cycles from the two sides of the PI90 strain path resulting in a cycle with twice the amplitude, as compared to the fully-reversed FRI90 strain path.

11) FRI90 and PI90 strain paths only include four extreme cycles (pure axial and pure torsion cycles) of the FRI and PI strain paths; therefore, coupling of these extreme cycles causes more effect on fatigue life of PI90 strain path, as compared to PI strain path, which has many other damaging cycles on the critical plane. This explains more difference in fatigue lives observed for FRI90 and PI90 as compared to FRI and PI strain paths. FRI90 and FRI strain paths resulted in similar fatigue lives for 1050 QT steel.

12) von Mises criterion cannot be used for fatigue life predictions of variable amplitude strain blocks, even when the load block consists of only proportional
cycles. This criterion is independent of fatigue damage plane orientation which is in contrast with the experimental observations indicating all cracks are around a preferred plane (i.e. critical plane) within the material.

13) Satisfactory multiaxial fatigue life predictions were obtained by either Bannantine-Socie or Wang-Brown cycle counting method, Fatemi-Socie damage parameter, and linear cumulative damage rule for 1050 QT steel and 304L stainless steel under various strain paths. Crack orientation plane predicted by using Bannantine-Socie rainflow cycle counting method and Fatemi-Socie critical plane parameter corresponded to the observed crack orientations.

14) Robust cycle counting method and damage parameter are needed for general applicability to variable amplitude multiaxial loading as the realistic condition for many industrial components and structures. Available life prediction methods do not consider interaction of different components of damage parameter. This may have a significant effect on fatigue life prediction.

15) Depending on the selected reference point to start Wang-Brown cycle counting, different results may be obtained. This is a fundamental weakness which may result in very different fatigue life predictions for the same load path. Although this lack of uniqueness was not observed for FRI90 strain path, different sets of counted cycles were resulted by Wang-Brown cycle counting for PI90 strain path.
Table 5.1  Surface stress-strain components for stable cycle in axial-torsion constant amplitude fatigue experiments of 1050 N steel under in-phase and 90° out-of-phase loading.

<table>
<thead>
<tr>
<th>ID</th>
<th>Load Type</th>
<th>Maximum $\sigma_a$ (MPa)</th>
<th>$\sigma_a$ @ $\varepsilon_{\text{Max}}$ (MPa)</th>
<th>$\varepsilon_a$</th>
<th>Maximum $\tau_a$ (MPa)</th>
<th>$\tau_a$ @ $\varepsilon_{\text{Max}}$ (MPa)</th>
<th>$\gamma_a$</th>
<th>$\sigma_a$ (MPa)</th>
<th>$\bar{\varepsilon}_a$</th>
<th>$\Delta \varepsilon_{\text{p}} / 2$</th>
<th>$2N_f$ (Reversals)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N 7</td>
<td>IP</td>
<td>350</td>
<td>350</td>
<td>0.0069</td>
<td>208</td>
<td>208</td>
<td>0.0130</td>
<td>502</td>
<td>0.0104</td>
<td>0.0080</td>
<td>3,122</td>
</tr>
<tr>
<td>N 9</td>
<td>IP</td>
<td>364</td>
<td>364</td>
<td>0.0069</td>
<td>201</td>
<td>201</td>
<td>0.0119</td>
<td>498</td>
<td>0.0099</td>
<td>0.0075</td>
<td>3,334</td>
</tr>
<tr>
<td>N 11</td>
<td>IP</td>
<td>287</td>
<td>287</td>
<td>0.0033</td>
<td>166</td>
<td>166</td>
<td>0.0057</td>
<td>406</td>
<td>0.0048</td>
<td>0.0028</td>
<td>18,530</td>
</tr>
<tr>
<td>N 5</td>
<td>IP</td>
<td>282</td>
<td>282</td>
<td>0.0033</td>
<td>161</td>
<td>161</td>
<td>0.0057</td>
<td>396</td>
<td>0.0048</td>
<td>0.0029</td>
<td>22,706</td>
</tr>
<tr>
<td>N 13</td>
<td>IP</td>
<td>210</td>
<td>210</td>
<td>0.0015</td>
<td>131</td>
<td>131</td>
<td>0.0026</td>
<td>307</td>
<td>0.0022</td>
<td>0.0007</td>
<td>378,378</td>
</tr>
<tr>
<td>N 3</td>
<td>IP</td>
<td>201</td>
<td>201</td>
<td>0.0014</td>
<td>121</td>
<td>121</td>
<td>0.0025</td>
<td>289</td>
<td>0.0021</td>
<td>0.0007</td>
<td>774,746</td>
</tr>
<tr>
<td>N 8</td>
<td>OP</td>
<td>478</td>
<td>315</td>
<td>0.0069</td>
<td>264</td>
<td>155</td>
<td>0.0119</td>
<td>479</td>
<td>0.0069</td>
<td>0.0046</td>
<td>1,046</td>
</tr>
<tr>
<td>N 10</td>
<td>OP</td>
<td>475</td>
<td>320</td>
<td>0.0069</td>
<td>267</td>
<td>157</td>
<td>0.0119</td>
<td>476</td>
<td>0.0069</td>
<td>0.0046</td>
<td>1,146</td>
</tr>
<tr>
<td>N 12</td>
<td>OP</td>
<td>401</td>
<td>344</td>
<td>0.0033</td>
<td>223</td>
<td>177</td>
<td>0.0057</td>
<td>403</td>
<td>0.0033</td>
<td>0.0014</td>
<td>7,654</td>
</tr>
<tr>
<td>N 6</td>
<td>OP</td>
<td>390</td>
<td>337</td>
<td>0.0033</td>
<td>220</td>
<td>179</td>
<td>0.0057</td>
<td>390</td>
<td>0.0033</td>
<td>0.0014</td>
<td>12,046</td>
</tr>
<tr>
<td>N 14</td>
<td>OP</td>
<td>276</td>
<td>272</td>
<td>0.0015</td>
<td>165</td>
<td>163</td>
<td>0.0026</td>
<td>283</td>
<td>0.0016</td>
<td>0.0002</td>
<td>205,140</td>
</tr>
<tr>
<td>N 4</td>
<td>OP</td>
<td>279</td>
<td>275</td>
<td>0.0015</td>
<td>164</td>
<td>159</td>
<td>0.0026</td>
<td>294</td>
<td>0.0018</td>
<td>0.0004</td>
<td>266,838</td>
</tr>
</tbody>
</table>
Table 5.2  Surface stress-strain components for stable cycle in axial-torsion constant amplitude fatigue experiments of 1050 QT steel under in-phase and 90° out-of-phase loading.

<table>
<thead>
<tr>
<th>ID</th>
<th>Load Type</th>
<th>Maximum $\sigma_a$ (MPa)</th>
<th>$\sigma_a$ @ $\varepsilon_{Max}$ (MPa)</th>
<th>$\varepsilon_a$</th>
<th>Maximum $\tau_a$ (MPa)</th>
<th>$\tau_a$ @ $\varepsilon_{Max}$ (MPa)</th>
<th>$\gamma_a$</th>
<th>$\sigma_a$ (MPa)</th>
<th>$\bar{\varepsilon}_a$</th>
<th>$\Delta\varepsilon_f/2$ Calculated</th>
<th>$2N_f$ (Reversals)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q 7</td>
<td>IP</td>
<td>594</td>
<td>594</td>
<td>0.0069</td>
<td>336</td>
<td>336</td>
<td>0.0120</td>
<td>831</td>
<td>0.0101</td>
<td>0.0060</td>
<td>2,450</td>
</tr>
<tr>
<td>Q 1</td>
<td>IP</td>
<td>596</td>
<td>596</td>
<td>0.0068</td>
<td>333</td>
<td>333</td>
<td>0.0119</td>
<td>830</td>
<td>0.0101</td>
<td>0.0060</td>
<td>3,064</td>
</tr>
<tr>
<td>Q 9</td>
<td>IP</td>
<td>477</td>
<td>477</td>
<td>0.0032</td>
<td>284</td>
<td>284</td>
<td>0.0055</td>
<td>683</td>
<td>0.0048</td>
<td>0.0015</td>
<td>24,096</td>
</tr>
<tr>
<td>Q 5</td>
<td>IP</td>
<td>495</td>
<td>495</td>
<td>0.0033</td>
<td>296</td>
<td>296</td>
<td>0.0057</td>
<td>711</td>
<td>0.0050</td>
<td>0.0015</td>
<td>26,876</td>
</tr>
<tr>
<td>Q 11</td>
<td>IP</td>
<td>457</td>
<td>457</td>
<td>0.0024</td>
<td>291</td>
<td>291</td>
<td>0.0040</td>
<td>665</td>
<td>0.0036</td>
<td>0.0004</td>
<td>122,068</td>
</tr>
<tr>
<td>Q 3</td>
<td>IP</td>
<td>403</td>
<td>403</td>
<td>0.0020</td>
<td>273</td>
<td>273</td>
<td>0.0035</td>
<td>618</td>
<td>0.0031</td>
<td>0.0001</td>
<td>417,144</td>
</tr>
<tr>
<td>Q 12</td>
<td>OP</td>
<td>814</td>
<td>592</td>
<td>0.0105</td>
<td>463</td>
<td>316</td>
<td>0.0182</td>
<td>821</td>
<td>0.0105</td>
<td>0.0065</td>
<td>230</td>
</tr>
<tr>
<td>Q 2</td>
<td>OP</td>
<td>722</td>
<td>577</td>
<td>0.0069</td>
<td>412</td>
<td>333</td>
<td>0.0119</td>
<td>726</td>
<td>0.0069</td>
<td>0.0033</td>
<td>708</td>
</tr>
<tr>
<td>Q 6</td>
<td>OP</td>
<td>740</td>
<td>609</td>
<td>0.0069</td>
<td>425</td>
<td>347</td>
<td>0.0120</td>
<td>744</td>
<td>0.0069</td>
<td>0.0032</td>
<td>812</td>
</tr>
<tr>
<td>Q 8</td>
<td>OP</td>
<td>581</td>
<td>568</td>
<td>0.0033</td>
<td>341</td>
<td>335</td>
<td>0.0056</td>
<td>602</td>
<td>0.0036</td>
<td>0.0007</td>
<td>4,702</td>
</tr>
<tr>
<td>Q 15</td>
<td>OP</td>
<td>582</td>
<td>580</td>
<td>0.0031</td>
<td>346</td>
<td>344</td>
<td>0.0053</td>
<td>604</td>
<td>0.0035</td>
<td>0.0006</td>
<td>7,980</td>
</tr>
<tr>
<td>Q 4</td>
<td>OP</td>
<td>593</td>
<td>589</td>
<td>0.0031</td>
<td>358</td>
<td>355</td>
<td>0.0054</td>
<td>627</td>
<td>0.0036</td>
<td>0.0005</td>
<td>13,394</td>
</tr>
<tr>
<td>Q 10</td>
<td>OP</td>
<td>488</td>
<td>487</td>
<td>0.0025</td>
<td>302</td>
<td>301</td>
<td>0.0043</td>
<td>527</td>
<td>0.0029</td>
<td>0.0003</td>
<td>44,586</td>
</tr>
<tr>
<td>Q 13</td>
<td>OP</td>
<td>371</td>
<td>371</td>
<td>0.0018</td>
<td>247</td>
<td>247</td>
<td>0.0031</td>
<td>430</td>
<td>0.0021</td>
<td>0</td>
<td>2,058,680</td>
</tr>
</tbody>
</table>
Table 5.3  Surface stress-strain components for stable cycle in axial-torsion constant amplitude fatigue experiments of 1050 IH steel under in-phase and 90° out-of-phase loading.

| ID | Load Type | Maximum $\sigma_a$ (MPa) | $\sigma_a$ @ $\varepsilon_{Max}$ (MPa) | Maximum $\varepsilon_a$ | $\varepsilon_a$ @ $\varepsilon_{Max}$ (MPa) | Maximum $\tau_a$ (MPa) | $\tau_a$ @ $\varepsilon_{Max}$ (MPa) | $\gamma_a$ | $\sigma_a$ (MPa) | $\varepsilon_a$ | $\Delta \varepsilon_p / 2$ | $2N_f$ Calculated | $2N_f$ (Reversals) |
|----|-----------|--------------------------|--------------------------------------|------------------------|-------------------------------------|------------------------|-------------------------------------|---------|----------------|----------------|-----------------|-------------------|-----------------|----------------|
| I 13 | IP        | 1.493                    | 1.493                                | 0.0100                 | 930                                 | 930                    | 0.0174                             | 2.246   | 0.0152         | 0.0041         | 84              |                   |                 |
| I 15 | IP        | 1.443                    | 1.443                                | 0.0100                 | 913                                 | 913                    | 0.0174                             | 2.190   | 0.0152         | 0.0041         | 234             |                   |                 |
| I 19 | IP        | 1.369                    | 1.369                                | 0.0085                 | 888                                 | 888                    | 0.0147                             | 2.026   | 0.0130         | 0.0026         | 576             |                   |                 |
| I 15 | IP        | 1.328                    | 1.328                                | 0.0085                 | 861                                 | 861                    | 0.0148                             | 1.966   | 0.0130         | 0.0028         | 608             |                   |                 |
| I 11 | IP        | 1.279                    | 1.279                                | 0.0069                 | 844                                 | 844                    | 0.0119                             | 1.920   | 0.0106         | 0.0012         | 674             |                   |                 |
| I 13 | IP        | 1.274                    | 1.274                                | 0.0069                 | 848                                 | 848                    | 0.0120                             | 1.918   | 0.0106         | 0.0011         | 716             |                   |                 |
| I 23 | IP        | 656                      | 656                                  | 0.0033                 | 440                                 | 440                    | 0.0057                             | 1.007   | 0.0051         | 0              | 13,982          |                   |                 |
| I 17 | IP        | 643                      | 643                                  | 0.0033                 | 446                                 | 446                    | 0.0057                             | 1.004   | 0.0051         | 0.0001         | 29,696          |                   |                 |
| I 19 | IP        | 512                      | 512                                  | 0.0025                 | 348                                 | 348                    | 0.0043                             | 792     | 0.0039         | 0.0001         | 43,908          |                   |                 |
| I 21 | IP        | 509                      | 509                                  | 0.0025                 | 348                                 | 348                    | 0.0043                             | 789     | 0.0039         | 0              | 1,780,108       |                   |                 |
| I 10 | OP        | 1.778                    | 1.758                                | 0.0095                 | 1,106                               | 1,101                  | 0.0166                             | 1,882   | 0.0108         | 0.0014         | 34              |                   |                 |
| I 16 | OP        | 1.702                    | 1.669                                | 0.0095                 | 1,066                               | 1,054                  | 0.0165                             | 1,829   | 0.0107         | 0.0013         | 120             |                   |                 |
| I 14 | OP        | 1.517                    | 1.517                                | 0.0085                 | 977                                 | 977                    | 0.0147                             | 1,697   | 0.0099         | 0.0010         | 486             |                   |                 |
| I 12 | OP        | 1.585                    | 1.585                                | 0.0085                 | 988                                 | 988                    | 0.0148                             | 1,713   | 0.0099         | 0.0010         | 634             |                   |                 |
| I 16 | OP        | 1.301                    | 1.301                                | 0.0069                 | 854                                 | 854                    | 0.0120                             | 1,478   | 0.0082         | 0.0005         | 1,738           |                   |                 |
| I 18 | OP        | 670                      | 670                                  | 0.0033                 | 449                                 | 449                    | 0.0057                             | 775     | 0.0039         | 0              | 29,120          |                   |                 |
| I 12 | OP        | 520                      | 520                                  | 0.0025                 | 346                                 | 346                    | 0.0043                             | 611     | 0.0030         | 0              | 51,696          |                   |                 |
| I 18 | OP        | 501                      | 501                                  | 0.0024                 | 335                                 | 335                    | 0.0042                             | 590     | 0.0029         | 0              | 274,656         |                   |                 |
| I 22 | OP        | 509                      | 509                                  | 0.0025                 | 340                                 | 340                    | 0.0043                             | 590     | 0.0030         | 0              | 1,119,844       |                   |                 |
| 14  | OP        | 416                      | 416                                  | 0.0020                 | 274                                 | 274                    | 0.0035                             | 478     | 0.0023         | 0              | > 3,706,120     |                   |                 |
Table 5.4 Surface stress-strain components for stable cycle in axial-torsion constant amplitude fatigue experiments of 304L stainless steel under in-phase and 90° out-of-phase loading.

<table>
<thead>
<tr>
<th>ID</th>
<th>Load Type</th>
<th>Maximum σ_a @ ε_{Max} (MPa)</th>
<th>ε_a (MPa)</th>
<th>Maximum τ_a @ ε_{Max} (MPa)</th>
<th>γ_a (MPa)</th>
<th>σ_a (MPa)</th>
<th>ε_a</th>
<th>Δε_p / 2</th>
<th>2N_f (Reversals)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSP 1</td>
<td>IP</td>
<td>333</td>
<td>0.0069</td>
<td>184</td>
<td>0.0120</td>
<td>461</td>
<td>0.0100</td>
<td>0.0020</td>
<td>1,668</td>
</tr>
<tr>
<td>SSP 11</td>
<td>IP</td>
<td>269</td>
<td>0.0049</td>
<td>159</td>
<td>0.0084</td>
<td>385</td>
<td>0.0070</td>
<td>0.0050</td>
<td>4,502</td>
</tr>
<tr>
<td>SSP 9</td>
<td>IP</td>
<td>250</td>
<td>0.0030</td>
<td>145</td>
<td>0.0057</td>
<td>355</td>
<td>0.0048</td>
<td>0.0030</td>
<td>6,500</td>
</tr>
<tr>
<td>SSP 13</td>
<td>IP</td>
<td>191</td>
<td>0.0023</td>
<td>114</td>
<td>0.0040</td>
<td>269</td>
<td>0.0034</td>
<td>0.0020</td>
<td>31,456</td>
</tr>
<tr>
<td>SSP 7</td>
<td>IP</td>
<td>167</td>
<td>0.0018</td>
<td>103</td>
<td>0.0030</td>
<td>239</td>
<td>0.0026</td>
<td>0.0013</td>
<td>194,062</td>
</tr>
<tr>
<td>SSNP 10</td>
<td>OP</td>
<td>608</td>
<td>0.0069</td>
<td>365</td>
<td>0.0120</td>
<td>629</td>
<td>0.0072</td>
<td>0.0037</td>
<td>760</td>
</tr>
<tr>
<td>SSNP 2</td>
<td>OP</td>
<td>625</td>
<td>0.0069</td>
<td>374</td>
<td>0.0119</td>
<td>642</td>
<td>0.0073</td>
<td>0.0037</td>
<td>800</td>
</tr>
<tr>
<td>SSNP 4</td>
<td>OP</td>
<td>421</td>
<td>0.0033</td>
<td>255</td>
<td>0.0051</td>
<td>442</td>
<td>0.0035</td>
<td>0.0011</td>
<td>8,338</td>
</tr>
<tr>
<td>SSNP 8</td>
<td>OP</td>
<td>431</td>
<td>0.0033</td>
<td>257</td>
<td>0.0057</td>
<td>444</td>
<td>0.0036</td>
<td>0.0012</td>
<td>11,700</td>
</tr>
<tr>
<td>SSNP 6</td>
<td>OP</td>
<td>269</td>
<td>0.0017</td>
<td>160</td>
<td>0.0030</td>
<td>308</td>
<td>0.0022</td>
<td>0.0006</td>
<td>317,544</td>
</tr>
</tbody>
</table>
Table 5.5  In-phase and 90° out-of-phase fatigue properties for 1050 steel and 304L stainless steel.

<table>
<thead>
<tr>
<th>In-phase Fatigue Properties</th>
<th>1050 N Steel</th>
<th>1050 QT Steel</th>
<th>1050 IH Steel</th>
<th>304L Stainless Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatigue strength coefficient, $\sigma'_f$, MPa</td>
<td>1,109</td>
<td>1,346</td>
<td>4,974</td>
<td>1,287</td>
</tr>
<tr>
<td>Fatigue strength exponent, $b$</td>
<td>-0.100</td>
<td>-0.062</td>
<td>-0.152</td>
<td>-0.145</td>
</tr>
<tr>
<td>Fatigue ductility coefficient, $\varepsilon'_f$</td>
<td>0.292</td>
<td>2.01</td>
<td>0.529</td>
<td>0.122</td>
</tr>
<tr>
<td>Fatigue ductility exponent, $\bar{c}$</td>
<td>-0.456</td>
<td>-0.725</td>
<td>-0.910</td>
<td>-0.394</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>90° Out-of-phase Fatigue Properties</th>
<th>1050 N Steel</th>
<th>1050 QT Steel</th>
<th>1050 IH Steel</th>
<th>304L Stainless Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatigue strength coefficient, $\sigma'_f$, MPa</td>
<td>936</td>
<td>1,193</td>
<td>4,062</td>
<td>1,392</td>
</tr>
<tr>
<td>Fatigue strength exponent, $b$</td>
<td>-0.095</td>
<td>-0.073</td>
<td>-0.154</td>
<td>0.125</td>
</tr>
<tr>
<td>Fatigue ductility coefficient, $\varepsilon'_f$</td>
<td>0.204</td>
<td>0.228</td>
<td>0.006</td>
<td>0.034</td>
</tr>
<tr>
<td>Fatigue ductility exponent, $\bar{c}$</td>
<td>-0.540</td>
<td>-0.652</td>
<td>-0.317</td>
<td>-0.335</td>
</tr>
</tbody>
</table>
Table 5.6  Surface stress-strain components for stable cycle of axial-torsion variable amplitude fatigue experiments of 1050 QT steel.

<table>
<thead>
<tr>
<th>ID</th>
<th>Load Type</th>
<th>( L_{\text{stable}} ) (Block)</th>
<th>( \sigma_a ) max (MPa)</th>
<th>( \varepsilon_a ) max</th>
<th>( \tau_a ) max (MPa)</th>
<th>( \gamma_a ) max (MPa)</th>
<th>( \sigma ) (MPa)</th>
<th>( \varepsilon_a )</th>
<th>( \Delta \varepsilon_p / 2 )</th>
<th>( L_f ) Calculated (Blocks)</th>
<th>( 2N_f ) (Reversals)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10FRI2QT</td>
<td>FRI</td>
<td>3</td>
<td>724</td>
<td>0.0069</td>
<td>413</td>
<td>0.0120</td>
<td>724</td>
<td>0.0069</td>
<td>0.0034</td>
<td>4.5</td>
<td>3,240</td>
</tr>
<tr>
<td>10FRI1QT</td>
<td>FRI</td>
<td>4</td>
<td>751</td>
<td>0.0070</td>
<td>432</td>
<td>0.0122</td>
<td>751</td>
<td>0.0070</td>
<td>0.0033</td>
<td>6.3</td>
<td>4,536</td>
</tr>
<tr>
<td>5FRI1QT</td>
<td>FRI</td>
<td>20</td>
<td>632</td>
<td>0.0033</td>
<td>374</td>
<td>0.0057</td>
<td>648</td>
<td>0.0033</td>
<td>0.0001</td>
<td>41.0</td>
<td>29,520</td>
</tr>
<tr>
<td>5FRI2QT</td>
<td>FRI</td>
<td>10</td>
<td>620</td>
<td>0.0033</td>
<td>364</td>
<td>0.0058</td>
<td>630</td>
<td>0.0033</td>
<td>0.0002</td>
<td>90.0</td>
<td>64,800</td>
</tr>
<tr>
<td>10FRR1QT</td>
<td>FRR</td>
<td>3</td>
<td>801</td>
<td>0.0070</td>
<td>458</td>
<td>0.0121</td>
<td>801</td>
<td>0.0070</td>
<td>0.0031</td>
<td>4.2</td>
<td>3,024</td>
</tr>
<tr>
<td>10FRR2QT</td>
<td>FRR</td>
<td>4</td>
<td>800</td>
<td>0.0070</td>
<td>446</td>
<td>0.0120</td>
<td>800</td>
<td>0.0070</td>
<td>0.0031</td>
<td>6.5</td>
<td>4,680</td>
</tr>
<tr>
<td>5FRR3QT</td>
<td>FRR</td>
<td>50</td>
<td>619</td>
<td>0.0033</td>
<td>372</td>
<td>0.0059</td>
<td>644</td>
<td>0.0034</td>
<td>0.0002</td>
<td>94.0</td>
<td>67,680</td>
</tr>
<tr>
<td>5FRR1QT</td>
<td>FRR</td>
<td>60</td>
<td>610</td>
<td>0.0034</td>
<td>353</td>
<td>0.0059</td>
<td>611</td>
<td>0.0034</td>
<td>0.0004</td>
<td>118.0</td>
<td>84,960</td>
</tr>
<tr>
<td>5FRI1QT90</td>
<td>FRI90</td>
<td>2,000</td>
<td>597</td>
<td>0.0031</td>
<td>365</td>
<td>0.0058</td>
<td>632</td>
<td>0.0033</td>
<td>0.0002</td>
<td>4,975</td>
<td>39,800</td>
</tr>
<tr>
<td>5FRI2QT90</td>
<td>FRI90</td>
<td>3,000</td>
<td>595</td>
<td>0.0031</td>
<td>362</td>
<td>0.0058</td>
<td>627</td>
<td>0.0033</td>
<td>0.0002</td>
<td>7,325</td>
<td>58,600</td>
</tr>
<tr>
<td>10PI1QT</td>
<td>PI</td>
<td>20</td>
<td>585</td>
<td>0.0034</td>
<td>341</td>
<td>0.0061</td>
<td>591</td>
<td>0.0035</td>
<td>0.0006</td>
<td>42.0</td>
<td>30,240</td>
</tr>
<tr>
<td>10PI2QT</td>
<td>PI</td>
<td>9</td>
<td>619</td>
<td>0.0035</td>
<td>365</td>
<td>0.0060</td>
<td>632</td>
<td>0.0035</td>
<td>0.0004</td>
<td>54.0</td>
<td>38,880</td>
</tr>
<tr>
<td>10PI1QT90</td>
<td>PI90</td>
<td>600</td>
<td>576</td>
<td>0.0034</td>
<td>336</td>
<td>0.0061</td>
<td>582</td>
<td>0.0035</td>
<td>0.0006</td>
<td>1,125</td>
<td>9,000</td>
</tr>
<tr>
<td>10PI2QT90</td>
<td>PI90</td>
<td>600</td>
<td>579</td>
<td>0.0034</td>
<td>333</td>
<td>0.0060</td>
<td>579</td>
<td>0.0035</td>
<td>0.0006</td>
<td>1,275</td>
<td>10,200</td>
</tr>
<tr>
<td>ID</td>
<td>$L_{\text{stable}} \text{ (Block)}$</td>
<td>Load Type</td>
<td>$(\sigma_a)_{\text{max}}$ (MPa)</td>
<td>$(\varepsilon_a)_{\text{max}}$</td>
<td>$(\tau_a)_{\text{max}}$ (MPa)</td>
<td>$(\gamma_a)_{\text{max}}$</td>
<td>$\sigma_a$ (MPa)</td>
<td>$\varepsilon_a$</td>
<td>$\Delta\varepsilon_p / 2$ Calculated</td>
<td>$L_f$ (Blocks)</td>
<td>$2N_f$ (Reversals)</td>
</tr>
<tr>
<td>-------------</td>
<td>-------------------------------------</td>
<td>-----------</td>
<td>---------------------------------</td>
<td>-------------------------------</td>
<td>----------------------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>-----------------------------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>10FRI2SS</td>
<td>3</td>
<td>FRI</td>
<td>459</td>
<td>0.0070</td>
<td>271</td>
<td>0.0120</td>
<td>469</td>
<td>0.0070</td>
<td>0.0046</td>
<td>5.5</td>
<td>3.960</td>
</tr>
<tr>
<td>10FRI1SS</td>
<td>3</td>
<td>FRI</td>
<td>461</td>
<td>0.0070</td>
<td>262</td>
<td>0.0120</td>
<td>461</td>
<td>0.0070</td>
<td>0.0046</td>
<td>5.8</td>
<td>4.176</td>
</tr>
<tr>
<td>5FRI1SS</td>
<td>15</td>
<td>FRI</td>
<td>391</td>
<td>0.0034</td>
<td>224</td>
<td>0.0057</td>
<td>391</td>
<td>0.0034</td>
<td>0.0014</td>
<td>28.0</td>
<td>20.160</td>
</tr>
<tr>
<td>5FRI2SS</td>
<td>20</td>
<td>FRI</td>
<td>362</td>
<td>0.0034</td>
<td>210</td>
<td>0.0058</td>
<td>364</td>
<td>0.0034</td>
<td>0.0015</td>
<td>53.0</td>
<td>38.160</td>
</tr>
<tr>
<td>10FRR1SS</td>
<td>3</td>
<td>FRR</td>
<td>604</td>
<td>0.0070</td>
<td>354</td>
<td>0.0120</td>
<td>613</td>
<td>0.0070</td>
<td>0.0038</td>
<td>4.0</td>
<td>2.880</td>
</tr>
<tr>
<td>10FRR2SS</td>
<td>3</td>
<td>FRR</td>
<td>582</td>
<td>0.0070</td>
<td>342</td>
<td>0.0120</td>
<td>592</td>
<td>0.0070</td>
<td>0.0039</td>
<td>4.0</td>
<td>2.880</td>
</tr>
<tr>
<td>5FRR1SS</td>
<td>20</td>
<td>FRR</td>
<td>434</td>
<td>0.0034</td>
<td>256</td>
<td>0.0058</td>
<td>443</td>
<td>0.0034</td>
<td>0.0011</td>
<td>39.0</td>
<td>28.080</td>
</tr>
<tr>
<td>5FRR2SS</td>
<td>20</td>
<td>FRR</td>
<td>427</td>
<td>0.0034</td>
<td>252</td>
<td>0.0058</td>
<td>436</td>
<td>0.0034</td>
<td>0.0011</td>
<td>45.0</td>
<td>32.400</td>
</tr>
<tr>
<td>5FRI2SS15</td>
<td>200</td>
<td>FRI15</td>
<td>413</td>
<td>0.0034</td>
<td>237</td>
<td>0.0058</td>
<td>413</td>
<td>0.0034</td>
<td>0.0012</td>
<td>375</td>
<td>18.000</td>
</tr>
<tr>
<td>5FRI1SS15</td>
<td>300</td>
<td>FRI15</td>
<td>398</td>
<td>0.0034</td>
<td>231</td>
<td>0.0058</td>
<td>400</td>
<td>0.0034</td>
<td>0.0013</td>
<td>595</td>
<td>28.560</td>
</tr>
<tr>
<td>10PI1SS</td>
<td>10</td>
<td>PI</td>
<td>414</td>
<td>0.0035</td>
<td>240</td>
<td>0.0060</td>
<td>416</td>
<td>0.0035</td>
<td>0.0013</td>
<td>22.0</td>
<td>15.840</td>
</tr>
<tr>
<td>10PI2SS</td>
<td>15</td>
<td>PI</td>
<td>402</td>
<td>0.0035</td>
<td>241</td>
<td>0.0060</td>
<td>417</td>
<td>0.0035</td>
<td>0.0013</td>
<td>27.3</td>
<td>19.656</td>
</tr>
</tbody>
</table>
Table 5.8  Comparison of fatigue life predictions employing BS and WB cycle counting methods using eFatigue software.

<table>
<thead>
<tr>
<th>Strain Path</th>
<th>$\bar{\varepsilon}$</th>
<th>$\sigma$ (MPa)</th>
<th>$L_f$</th>
<th>Exp. Blocks</th>
<th>Pred. BS Blocks</th>
<th>Ratio Pred. BS</th>
<th>Pred. WB Blocks</th>
<th>Ratio Pred. WB</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRI 0.0069</td>
<td>724</td>
<td>4.5</td>
<td>15</td>
<td>3.3</td>
<td>12</td>
<td>2.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRI 0.0070</td>
<td>751</td>
<td>6.25</td>
<td>15</td>
<td>2.4</td>
<td>12</td>
<td>1.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRI 0.0033</td>
<td>647</td>
<td>41</td>
<td>305</td>
<td>3.4</td>
<td>198</td>
<td>2.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRI 0.0033</td>
<td>630</td>
<td>90</td>
<td>305</td>
<td>3.4</td>
<td>198</td>
<td>2.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR 0.0070</td>
<td>801</td>
<td>4.2</td>
<td>13</td>
<td>3.1</td>
<td>11</td>
<td>2.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR 0.0069</td>
<td>800</td>
<td>4.65</td>
<td>13</td>
<td>2.8</td>
<td>11</td>
<td>2.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR 0.0034</td>
<td>645</td>
<td>94</td>
<td>163</td>
<td>1.7</td>
<td>192</td>
<td>2.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR 0.0034</td>
<td>611</td>
<td>118</td>
<td>163</td>
<td>1.4</td>
<td>192</td>
<td>1.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRI90 0.0031</td>
<td>597</td>
<td>4,975</td>
<td>14,785</td>
<td>3.0</td>
<td>46,959</td>
<td>9.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRI90 0.0031</td>
<td>595</td>
<td>7,325</td>
<td>14,785</td>
<td>2.0</td>
<td>46,959</td>
<td>6.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI 0.0035</td>
<td>591</td>
<td>42</td>
<td>216</td>
<td>5.1</td>
<td>139</td>
<td>3.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI 0.0035</td>
<td>633</td>
<td>54</td>
<td>216</td>
<td>4.0</td>
<td>139</td>
<td>2.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI90 0.0034</td>
<td>576</td>
<td>1,125</td>
<td>1,727</td>
<td>1.5</td>
<td>1,728</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI90 0.0034</td>
<td>579</td>
<td>1,275</td>
<td>1,727</td>
<td>1.4</td>
<td>1,728</td>
<td>1.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1050 QT Steel

304L Stainless Steel

<table>
<thead>
<tr>
<th>Strain Path</th>
<th>$\bar{\varepsilon}$</th>
<th>$\sigma$ (MPa)</th>
<th>$L_f$</th>
<th>Exp. Blocks</th>
<th>Pred. BS Blocks</th>
<th>Ratio Pred. BS</th>
<th>Pred. WB Blocks</th>
<th>Ratio Pred. WB</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRI 0.0070</td>
<td>469</td>
<td>5.5</td>
<td>6</td>
<td>1.1</td>
<td>4</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRI 0.0069</td>
<td>461</td>
<td>5.75</td>
<td>6</td>
<td>1.0</td>
<td>4</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRI 0.0033</td>
<td>391</td>
<td>28</td>
<td>63</td>
<td>2.3</td>
<td>39</td>
<td>1.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRI 0.0034</td>
<td>363</td>
<td>53</td>
<td>63</td>
<td>1.2</td>
<td>39</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR 0.0070</td>
<td>613</td>
<td>4</td>
<td>5</td>
<td>1.3</td>
<td>4</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR 0.0069</td>
<td>593</td>
<td>4</td>
<td>5</td>
<td>1.3</td>
<td>4</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR 0.0033</td>
<td>444</td>
<td>39</td>
<td>61</td>
<td>1.6</td>
<td>47</td>
<td>1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRR 0.0033</td>
<td>437</td>
<td>45</td>
<td>61</td>
<td>1.4</td>
<td>47</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRI15 0.0033</td>
<td>413</td>
<td>375</td>
<td>910</td>
<td>2.4</td>
<td>544</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRI15 0.0034</td>
<td>399</td>
<td>595</td>
<td>910</td>
<td>1.5</td>
<td>544</td>
<td>0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI 0.0035</td>
<td>415</td>
<td>22</td>
<td>53</td>
<td>2.4</td>
<td>34</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI 0.0035</td>
<td>418</td>
<td>27.3</td>
<td>53</td>
<td>1.9</td>
<td>34</td>
<td>1.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.1  Equivalent surface strain amplitude versus reversals to failure for (a) in-phase and (b) 90° out-of-phase loadings of 1050 N steel.
Figure 5.2  Equivalent surface strain amplitude versus reversals to failure for (a) in-phase and (b) 90° out-of-phase loadings of 1050 QT steel.
Figure 5.3 Equivalent surface strain amplitude versus reversals to failure for (a) in-phase and (b) 90° out-of-phase loadings of 1050 IH steel.
Figure 5.4  Equivalent surface strain amplitude versus reversals to failure for (a) in-phase and (b) 90° out-of-phase loadings of 304L stainless steel.
Figure 5.5 Comparison of in-phase and 90° out-of-phase equivalent strain amplitudes versus fatigue life for (a) 1050 N steel, (b) 1050 QT steel, (c) 1050 IH steel, and (d) 304L stainless steel.
Figure 5.6  Equivalent surface strain amplitude versus reversals to failure for (a) in-phase and (b) 90° out-of-phase axial-torsion loading of 1050 steel.
Figure 5.7 Correlation of in-phase and 90° out-of-phase fatigue data by von Mises criterion for (a) 1050 N steel, (b) 1050 QT steel, (c) 1050 IH steel, and (d) 304L stainless steel.
Figure 5.8 Cracks observed under in-phase (IP) and 90° out-of-phase (OP) loading for (a) 1050 N steel, (b) 1050 QT steel, and (c) 304L stainless steel. Maximum critical planes for IP (20° and 110°) and OP (0°) loading are also presented.
Figure 5.9  Correlation of in-phase and 90° out-of-phase fatigue data by the Fatemi-Socie critical plane parameter for (a) 1050 N steel, (b) 1050 QT steel, (c) 1050 IH steel, and (d) 304L stainless steel.
Figure 5.10  Variation of the FS damage parameter with plane orientation for in-phase (IP) and 90° out-of-phase (OP) loadings at $\varepsilon_a = 0.7\%$ for (a) 1050 QT steel, and (b) 304L stainless steel.
Figure 5.11 Predicted fatigue lives based on only hardness versus observed fatigue lives for different materials and hardesses under various multiaxial loading conditions.

Figure 5.12 Variation of $k$ in the Fatemi-Socie parameter with fatigue life and Brinell hardness for steels.
Figure 5.13  Fatigue lives (a) and equivalent stress amplitude (b) for various strain paths under equivalent strain amplitudes of 0.7% and 0.34% for 1050 QT steel. Values are normalized by in-phase (IP) data at each strain level and represent average values for duplicate tests.
Figure 5.14  Fatigue lives (a) and equivalent stress amplitude (b) for various strain paths under equivalent strain amplitudes of 0.7% and 0.34% for 304L stainless steel. Values are normalized by in-phase (IP) data at each strain level and represent average values for duplicate tests.
Figure 5.15  Experimental and predicted fatigue lives based on von Mises criterion for 1050 QT steel and 304L stainless steel.
Figure 5.16  Variation of the cumulative damage based on FS parameter with plane orientation for in-phase (IP), 90° out-of-phase (OP), and FRI strain paths at $\bar{\varepsilon}_a = 0.7\%$ for (a) 1050 QT steel, and (b) 304L stainless steel. Values are normalized for one cycle for FRI strain path.
Figure 5.17 Examples of cracks observed on the surface of 304L stainless steel specimens at $\varepsilon_a = 0.34\%$ under (a) IP, (b) OP, (c) FRI, (d) FRR, (e) FRI15, and (d) PI strain paths. Maximum damage plane is also presented for all strain paths.
Figure 5.18  (a) Uniaxial variable amplitude strain block used to verify the capability of the linear cumulative damage rule for similar loadings used in the multiaxial tests. Every 5th cycle is presented for clarity. (b) Comparison of the predicted and experimental fatigue lives using FS parameter and linear cumulative damage rule.
Figure 5.19 Experimental and predicted fatigue lives based on BS cycle counting method, FS critical plane approach, and linear cumulative damage rule using eFatigue for 1050 QT steel and 304L stainless steel.
Figure 5.20  (a) Linear cumulative fatigue damage, and (b) fatigue damage caused by each cycle of one block of FRI strain path based on the FS critical plane approach for 304L stainless steel at $\varepsilon_a = 0.7\%$ on $0^\circ$ plane.
Figure 5.21 Experimental and predicted fatigue lives based on WB reversal counting method, FS critical plane approach, and linear cumulative damage rule using eFatigue for 1050 QT steel and 304L stainless steel.
Figure 5.22  (a) Shear strain and normal stress time histories on the loading plane, and (b) rainflow cycle counting for a fully-reversed incremental strain path with 45° increments.
Figure 5.23 (a) Shear strain and normal stress time histories on the loading plane, and (b) BS cycle counting and results for PI90 strain path.
Figure 5.24 (a) Shear strain and normal stress time histories on the loading plane, and (b) BS cycle counting and results for FRI90 strain path.
Table 5.25 BS cycle counting results on the modified PI90 strain path.

<table>
<thead>
<tr>
<th>Number of Cycles</th>
<th>Shear Strain Range</th>
<th>Maximum Normal Stress, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0240</td>
<td>600</td>
</tr>
<tr>
<td>1</td>
<td>0.0040</td>
<td>800</td>
</tr>
<tr>
<td>1</td>
<td>0.0040</td>
<td>-200</td>
</tr>
</tbody>
</table>

Figure 5.25 BS cycle counting results on the modified PI90 strain path.
Figure 5.26  BS cycle counting results on the modified FRI90 strain path.

<table>
<thead>
<tr>
<th>Number of Cycles</th>
<th>Shear Strain Range</th>
<th>Maximum Normal Stress, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0116</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.0010</td>
<td>600</td>
</tr>
<tr>
<td>1</td>
<td>0.0058</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.0058</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.0010</td>
<td>600</td>
</tr>
</tbody>
</table>
Figure 5.27  WB reversal counting on FRI90 strain path. Point A on the axial cycle is considered as the first reference point.
Figure 5.28  WB reversal counting on FRI90 strain path. Point C on the shear cycle is considered as the first reference point.
Figure 5.29  WB reversal counting on PI90 strain path. Point A on the axial cycle is considered as the first reference point.
Figure 5.30  WB reversal counting on PI90 strain path. Point D on the shear cycle is considered as the first reference point.
Chapter 6

Cracking Behavior

6.1 Introduction

Fatigue process includes initiation and growth of micro-cracks. The portion of crack initiation or growth may be negligible or may occupy the entire fatigue life. The crack initiation life consists of crack nucleation and micro-crack growth up to a length of typically about several hundred micrometer (i.e. microscopic growth), as presented in Figure 6.1. The crack growth stage is usually divided into a period spent in propagation of small cracks and a period of long crack growth (i.e. macroscopic growth). Small crack growth life consists of the life spent for crack propagation from crack nucleation to about 1 mm, as shown in Figure 6.1. Up to this length, crack is affected by the local plasticity and does not generate its own plastic zone. Cracks longer than 1 mm (i.e. macroscopic level in Figure 6.1) are not usually affected by the local plasticity and they generate their own plastic zone at the tip. The modeling of such cracks requires the use of fracture mechanics.

Crack growth from about 1 mm to the crack length which causes 5% load or torque drop (i.e. cracks with the length of about 10 mm) was observed to be fast for tubular and solid specimens. Therefore, the total life observed is considered the life to a
crack length of about 1 mm. This chapter discusses small crack growth from about 100 µm to about 1 mm.

Cracking behaviors and related analyses are presented in Section 6.2 for crack initiation and growth up to about 1 mm. This includes the effect of strain amplitude level, load non-proportionality, material, and specimen geometry. Section 6.3 presents the correlation of crack growth rate data by Reddy-Fatemi strain-based effective intensity factor. Finally, conclusions made based on cracking observations and analyses in this chapter are presented in Section 6.4.

6.2 Parameter definitions and cracking analysis

Parameters defining a crack in this study are presented in Figure 6.2(a). Crack length, \( l_c \), is defined as the length of a straight line connecting two tips of a given crack. Crack angle, \( \theta_c \), is the angle between the crack length direction (straight line connecting two tips of the crack) and the line perpendicular to the specimen longitudinal axis, presented in Figure 6.2(b). Crack segment angle, \( \theta_r \), is defined as the angle between a crack segment and the line perpendicular to the specimen longitudinal axis; however, only crack segments with at least 50 µm length are considered in this study.

Final crack length and direction were recorded by direct examination of the surface of the specimens. The specimens examined included those from constant amplitude tests of 1050 N steel, 1050 QT steel, and 304L stainless steel. Due to the very high hardness level of 1050 IH steel, fracture occurred as soon as crack initiated; thus, no crack analysis was conducted for this material. Final crack length and orientation for all tubular specimens of 1050 N steel, 1050 QT steel, and 304L stainless steel are presented.
in Tables 6.1, 6.2, and 6.3, respectively. It should be noted that for some specimens more than one crack was observed. Due to relatively high hardness of 1050 QT steel, fewer crack data could be obtained for this material, as can be seen from Table 6.2.

One specimen for each strain level of in-phase and 90° out-of-phase strain paths was carefully examined under a Nikon optical microscope, equipped with a Nikon digital camera to detect cracks with lengths ranging from 50 µm to 1 mm. Results of these direct observations are presented in Tables 6.1 through 6.3 for tubular specimens of 1050 N steel, 1050 QT steel, and 304L stainless steel. Cracks observed under the microscope are categorized based on their length into cracks between 50 µm and 200 µm (crack type A), 200 µm and 500 µm (crack type B), and 500 µm and 1 mm (crack type C). Number of cracks, range of crack angles, and range of crack segment angle in each category are reported in the aforementioned tables. A square area of 2 mm × 2 mm around the center of the gage section was considered for 1050 N steel and 304L stainless steel, while the whole gage section surface was examined for 1050 QT steel. This is because fewer cracks were observed for 1050 QT steel with the higher hardness level, as compared to 1050 N steel and 304L stainless steel.

Cellulose acetate sheet replication method was also employed to monitor specimen surface crack growth for some in-phase (IP) and 90° out-of-phase (OP) fatigue experiments. One of the duplicate constant amplitude tests of normalized and quenched and tempered tubular specimens at each strain level was used for crack replication. The description of cellulose acetate sheet replication method can be found in Chapter 3. After midlife, replication of the surface of the specimen was performed at each 10% of the expected fatigue life. This technique gives the opportunity to go backward from the last
replica to the first one and detect the length and angle of initial cracks causing the final failure. It also permits to compare the crack growth rates for different loading or material conditions.

Crack analysis in this section includes the effects of strain amplitude, in-phase (proportional) versus 90° out-of-phase (non-proportional) strain path, solid versus tube specimen geometry, and type of material. These comparisons are made based on both the presence of different cracks observed on the surface after failure, as well as cellulose acetate sheet replicas showing the crack growth behavior under each test condition. Overall observations of cracking behaviors under variable amplitude strain paths are also presented.

6.2.1 Effects of strain amplitude level on cracking behavior

As can be seen from Tables 6.1 through 6.3, more cracks were observed for higher amplitude level tests, as compared to lower amplitude level tests, for all materials and all loading conditions. While more than 100 and 50 cracks of type A were observed for high and intermediate amplitude level in-phase tests of 1050 N steel specimens, respectively, only 2 cracks were observed for the low amplitude level test (see Table 6.1).

Figure 6.3 presents examples of cracks observed for 1050 N steel under IP loading with high, intermediate, and low amplitude levels. Many more cracks can be seen in this figure for high amplitude level test, as compared to intermediate and low amplitude level tests. Similar trend was observed for other crack types, other materials, or OP strain path; as amplitude level increased, more cracks were observed. As can be noticed from Tables 6.1 trough 6.3, cracks were observed on wider plane orientations for higher amplitude strain level tests of both IP and OP loading. This can be explained by
the fact that damage value on planes neighbored to the critical plane is a high percentage of the damage value on the critical plane. As an example, for OP loading, planes within the range of about ±16° from the critical plane experience more than 95% of the damage value on the critical plane. Higher strain amplitude level results in wider range of planes experiencing the majority of the maximum damage value. Cracks in shear failure mode materials are consequence of active slip systems. Shear stress and number of applied cycles (i.e. cumulative damage) should reach a critical level to activate slip systems on a specific plane. Higher amplitude loading results in exceeding of the shear stress on a wider range of planes around the maximum shear plane and, therefore, slip systems are activated on more planes. As a result of this activated slip systems, cracks are nucleated on the wider range of plane for higher amplitude level tests.

Evolutions of micro-cracks for solid specimens of 1050 N steel under IP loading with high, intermediate, and low amplitudes are presented separately in Figures 6.4 through 6.6. Mechanism of micro-crack evolution for high and intermediate amplitude strain level tests of 1050 N steel, presented in Figures 6.4 and 6.5, was a combination of crack growth and coalescence, whereas the evolution mechanism for low amplitude test was found to be only crack growth, as shown in Figure 6.6. As can be seen from Figure 6.4, many micro-cracks nucleated around the maximum shear plane (i.e. plane of 110° for IP loading in this study) and developed by the combination of growth and coalescence mechanisms. Presence of other shorter cracks around the critical plane was also noticeable (see Figure 6.4). Similarly, for intermediate amplitude level test, shown in Figure 6.5, some micro-cracks nucleated and grew around the maximum shear plane. Coalescence of cracks was also observed for this amplitude level, as can be seen in
Figure 6.5 at 12000, 17000, and 18000 cycles. Again, presence of some shorter cracks around the critical plane is noticeable. Nevertheless, micro-crack evolution for low amplitude level of 1050 N steel under IP loading, shown in Figure 6.6, indicates very few cracks nucleated around the critical plane and then grew fast around the same plane till final failure.

Crack length versus normalized number of cycles for high, intermediate, and low strain amplitude level tests of IP loading for 1050 N steel solid specimens is presented in Figure 6.7(a) for crack evolution from about 60 µm to 10 mm. It can be observed from this figure that crack initiation occurred at much earlier stage of fatigue life for high strain amplitude tests, as compared to lower amplitude fatigue tests. A crack with 60 µm length was observed at 40%, 50%, and 75% of the fatigue life for high, intermediate, and low strain amplitude in-phase fatigue tests of 1050 N steel solid specimens, respectively. This figure also explains more cracks observed for higher amplitude level tests as compared to lower amplitude level tests, presented in Tables 6.1 through 6.3 and Figures 6.4 through 6.6. Shorter crack growth stage as compared to crack initiation stage of fatigue life for low amplitude level tests caused fatigue failure before other cracks could nucleate and grow. In the case of high amplitude level test, crack growth stage was longer and gave the opportunity to other cracks to nucleate and grow before fatigue failure occurred.

Crack growth rate was found to be the highest for the highest amplitude test and the lowest for the lowest amplitude level test. Crack evolution from 60 µm to 1,000 µm took 1350, 7200, and 47600 cycles for high, intermediate and low amplitude in-phase tests of 1050 N steel solid specimens, respectively. Similarly, cracks were observed at
earlier stage of fatigue life for higher amplitude OP tests of 1050 N steel solid specimens, presented in Figure 6.7(b) as crack length versus normalized number of cycles. Crack growth rate was also found to be higher for the higher amplitude level tests of 1050 N steel under OP loading.

### 6.2.2 Effects of load non-proportionality on cracking behavior

More cracks were observed for IP loading as compared to OP loading for all materials, when the comparison was made for the same crack type and at the same strain level in Tables 6.1 through 6.3. The only exception was for cracks type A ($50 \, \mu m < l_c < 200 \, \mu m$) for the highest level test of 304L stainless steel, where more micro-cracks were observed for OP loading as compared to IP loading. However, only a few of these micro-cracks could grow to about 500 $\mu m$ and fatigue failure occurred due to the nucleation of many micro-cracks. While 22 cracks of type B and 3 cracks of type C were observed for the highest strain level test of IP loading for 304L stainless steel, only 5 cracks of type B and no crack of type C were observed under OP loading test of this material. Similarly, while some cracks of type B were observed for all three strain amplitude level tests of 1050 N steel tubular specimens under IP loading, as listed in Table 6.1, no crack of type B was detected for OP loading of these specimens. In the case of 1050 QT steel under OP loading, only the main crack causing fatigue failure was observed and no other crack nucleated, as listed in Table 6.2.

Evolutions of micro-cracks from 60 $\mu m$ to 10 mm for solid specimens of 1050 N steel under IP and OP loadings are compared for high and intermediate strain amplitude levels in Figure 6.8, as crack length versus normalized number of cycles. As can be seen
from this figure, crack initiation occurred at earlier stage of fatigue life for IP tests as compared to OP tests with the same strain amplitude level. A crack with 60 µm length for high strain level tests of 1050 N steel solid specimens was observed at 40% and 60% of the fatigue life for IP and OP loadings, respectively. Crack length versus number of cycles is presented for IP and OP tests of 1050 N steel solid specimens for high and low strain level tests in Figure 6.9. As can be seen from this figure, for both strain amplitude levels, crack evolution rate was higher for OP loading as compared to IP loading with the same strain level. As a result of this higher crack evolution rate and shorter stage of crack growth relative to crack initiation for the OP loading as compared to IP loading, as soon as a crack initiated, it grew and caused fatigue failure; therefore, other cracks could not nucleate and grow. Lower crack evolution rate and longer stage of crack growth relative to crack initiation for IP loading gave the opportunity to other cracks to nucleate and grow. This explains more cracks observed under IP loading as compared to OP loading. It can be also found from the comparison of Figures 6.9(a) and 6.9(b) that the difference of crack growth rate between IP and OP loadings decreased by a decrease in strain amplitude level.

Micro-cracks were observed over a wider range of plane orientations for OP loading as compared to IP loading, as observed from Table 6.3. Presence of cracks on the wider range of planes around the critical plane for OP loading as compared to IP loading can be explained by a wider range of planes experiencing a high percentage of the damage value on the critical plane for OP loading. Figure 6.10 presents some cracks observed for the highest strain amplitude level tests of 304L stainless steel under IP and OP loadings. As can be seen from this figure, micro-cracks were observed on planes with
wider orientation range for OP loading in Figure 6.10(b), as compared to IP loading in Figure 6.10(a). However, the difference of crack orientation ranges observed for IP and OP loadings decreased by a decrease in strain amplitude level.

6.2.3 Effects of material on cracking behavior

As listed in Tables 6.1 through 6.3, more cracks were observed for more ductile behaving materials (i.e. 304L stainless steel and 1050 N steel) as compared to more brittle behaving material (i.e. 1050 QT steel). As example, for the highest strain amplitude level test of tubular specimens under IP loading, 22, 16, and 10 cracks of type B were observed for 304L stainless steel, 1050 N steel, and 1050 QT steel, respectively. It should be also taken into consideration that crack observations were made on an area of $4 \text{ mm}^2$ for 1050 N steel and 304L stainless steel, while the whole gage section was examined for 1050 QT steel; thus, fewer cracks were observed for 1050 QT steel as a harder material. These observations can be explained by the higher crack growth rate for harder materials (i.e. more brittle behaving materials), as compared to more ductile behaving materials.

Evolutions of micro-cracks from 100 $\mu$m to 10 mm for high strain amplitude IP test of tubular specimens for 1050 N steel and 1050 QT steel at the highest strain level are compared in Figure 6.11(a), as crack length versus normalized number of cycles. A short crack was detected at the earlier stage of fatigue life for 1050 N steel, as compared to 1050 QT steel. By the slight extension of the curves in Figure 6.11(a), a crack with 60 $\mu$m length could be observed at 55% and 65% of the fatigue life for 1050 N steel and 1050 QT steel, respectively. Crack length versus number of cycles for the highest strain level IP loading of 1050 N steel and 1050 QT steel is compared in Figure 6.11(b). As can
be seen from this figure, crack growth rate was higher for 1050 QT steel as a more brittle material as compared to 1050 N steel as a more ductile material. This explains more cracks observed for more ductile materials such as 1050 N steel and 304L stainless steel, as listed in Tables 6.1 and 6.3.

All cracks for both IP and OP loadings appeared to be around the maximum shear plane (i.e. plane of 110° for IP loading and plane of 0° for OP loading) for all materials investigated. It can be noted from Tables 6.1 through 6.3 that longer cracks were observed on the planes oriented on a narrower range, whereas shorter cracks appeared to be on a wider range of planes around the critical plane. For instance, cracks type A, B, and C for the highest IP strain amplitude test of 304L stainless steel, presented in Table 6.3, were observed to be on the planes oriented between, 130° and 180°, 125° and 160°, and 130° and 145°, respectively. This may be explained by the fact that the grain shape, size, and orientation affect the micro-cracks behavior, whereas longer cracks grow under control of the shear driving force and independent of the grain properties.

6.2.4 Effects of specimen geometry on cracking behavior

Many more cracks in various sizes were observed for solid specimens as compared to tubular specimens of 1050 N steel. Evolutions of micro-cracks from 60 µm to 10 mm for high strain amplitude IP test of solid and tubular specimens of 1050 N steel are compared in Figure 6.12(a), as crack length versus normalized number of cycles. It can be observed form this figure that cracks nucleated at smaller percentage of fatigue life for the solid specimen as compared to the tubular specimen. A crack with 60 µm length was detected at 40% and 55% of the fatigue life for solid and tubular specimens of 1050 N steel, respectively, under IP loading at the highest strain amplitude level. This is
due to the fact that cracks can grow longer on solid specimens before 5% load or torque drop criterion, as the definition of the failure in this study.

Figure 6.12(b) presents crack size versus number of cycles for solid and tubular specimens of 1050 N steel under IP loading at the highest strain amplitude level. As can be seen from this figure, nucleation of a crack with 60 µm length for 1050 N steel at the aforementioned level was observed at about N = 1,000 cycles for both geometries under investigation. As mentioned earlier, this life is associated with 40% and 55% of the fatigue life for solid and tubular specimens, respectively. It can be also observed from Figure 6.12(b) that crack growth rate was higher for tubular specimens as compared to solid specimens, which explains the presence of more cracks for solid specimens. Lower crack growth rate observed for solid specimens can result from the gradient of shear stress in the solid section, while shear stress is nearly uniform in thin-walled tubular section.

6.2.5 Cracking behaviors under variable amplitude strain paths

Cracking behaviors observed under star shape strain paths (FRI, FRR, FRI15, and PI) were very similar to the cracking behaviors under OP loading at the same equivalent strain amplitude level, when the comparison was made in terms of number of cracks in each category and the range of crack plane orientations. Similar to 1050 QT steel specimens under OP loading, only one crack nucleated, grew, and caused failure for this material subjected to the star shape strain paths, with no other crack detected. In the case of 304L stainless steel, many more cracks were observed in higher strain amplitude level tests and very similar results to the OP loading of the material at the same strain amplitude level, as reported in Table 6.3. Not a specific difference in cracking behaviors
was observed for FRI, FRR, and FRI15 strain path tests for both 1050 QT steel and 304L stainless steel. The only difference was found for the PI strain path, for which cracking behavior at 0.34% and equivalent strain amplitude (with 0.69% maximum equivalent strain value) appeared to be more similar to the FRI loading with 0.69% strain amplitude level. For this specific strain path, more cracks, and on the wider range of plane orientations were observed as compared to other star shape strain paths in this study. This observation leads to the point that not only strain amplitude as the driving force for shear failure mode materials is an important parameter affecting the cracking behavior, maximum shear strain value may also play an important role. As described in Chapter 5, cycles from both sides (top and bottom, left and right) couple and generate cycles with twice of amplitudes for PI strain path.

Figure 5.17 presents some examples of cracks observed for 304L stainless steel under FRI, FRR, FRI15, and PI strain paths at 0.34% equivalent strain amplitude. This figure indicates that all observed cracks were around the 0° plane, which is the critical plane based on FS parameter, as discussed in section 5.3. Although cracks nucleate and grow around the critical plane with the highest value of damage, these cracks are not necessarily exactly on the critical plane. As an example, cracks were observed to be on planes oriented on about ± 15° around the critical plane for 304L stainless steel under star shape strain paths. As can be seen from Table 6.3, very similar plane orientation range was observed for this material under OP loading.

6.3 Correlation of crack growth rate data

The strain-based effective intensity factor proposed by Reddy and Fatemi (1992) is used here to correlate crack growth data generated in this study. Reddy-Fatemi
The effective strain-based intensity factor is based on the FS critical plane approach (1988) proposed for shear failure mode materials. This strain intensity factor parameter considers the maximum shear strain amplitude as the main parameter driving the crack, and the normal stress on the maximum shear strain amplitude plane as the secondary parameter accelerating the crack growth rate if tensile, and retarding crack growth rate if compressive. The Reddy-Fatemi effective strain-based intensity factor range, $\Delta K_{\text{CPA}}$, is expressed as follows:

$$\Delta K_{\text{CPA}} = G \Delta \gamma_{\text{max}} \left( 1 + k \frac{\sigma_n}{\sigma_y} \right) (\pi c)^{1/2} \quad (6.1)$$

where $G$ is the shear modulus and $c$ is half the surface crack length. The other variables are from the FS critical plane approach described in Chapter 5. Reddy and Fatemi hypothesized that the effective strain intensity factor they proposed can be used for shear failure mode materials since it represents the physical damage mechanism of such materials. This parameter was shown to correlate small crack growth rate data for a wide variety of multiaxial loading conditions of tubular specimens of 1045 N steel and Inconel 718 (Reddy and Fatemi, 1992), as well as hourglass solid specimens of A533B steel (Park et al., 2001).

Crack growth rate data for IP and OP loading of 1050 N steel solid specimens based on the Reddy-Fatemi effective strain intensity factor is presented in Figure 6.13(a). Although it was found that non-proportionality of loading and change in strain amplitude affect the crack growth rate, these data are correlated well by the Reddy-Fatemi effective strain intensity factor, as can be seen from this figure. Similarly, the correlation of crack growth rate for IP loading of 1050 N steel and 1050 QT steel tubular specimens at
the Reddy-Fatemi strain intensity factor is presented in Figure 6.13(b). Although crack growth rate was found to be faster for 1050 QT steel as a more brittle behaving material, as compared to 1050 N steel as a more ductile behaving material, the Reddy-Fatemi effective intensity factor can correlate the crack growth data well for these two materials, as presented in Figure 6.13(b). This is due to the fact that this parameter considers the secondary effect of the tensile normal stress on crack growth rate. This secondary effect is presented in this model by the combination of the FS material constant, $k$, normal stress on the critical plane, $\sigma_n^{\text{max}}$, and yield strength, $\sigma_y$. This combination was found to be higher for 1050 QT steel as compared to 1050 N steel, and therefore, higher crack growth rate observed for 1050 QT steel could be predicted well by the Reddy-Fatemi strain-based intensity factor, as presented in Figure 13(b).

6.4 Conclusions

1) More cracks and also on a wider range of planes were observed for higher amplitude level tests for all strain paths employed in this study. Crack growth rate was found to decrease by a decrease in strain amplitude level. Short cracks were observed at much earlier stage of fatigue life for higher strain amplitude fatigue tests as compared to lower strain amplitude fatigue tests.

2) While more cracks were detected for in-phase loading as compared to 90° out-of-phase loading for all materials, cracks were observed on a wider range of plane orientations for 90° out-of-phase loading. Short cracks were observed at earlier stage of fatigue life for in-phase loading and with lower crack growth rate as compared to 90° out-of-phase loading. Due to the higher crack growth rate for 90°
out-of-phase loading, as soon as a crack initiated, it grew and caused fatigue failure; therefore, other cracks could not nucleate and grow.

3) More cracks and lower crack growth rates were observed for more ductile materials in this study. Not a specific effect of the type of materials, utilized in this study, was observed on the range of planes with cracks. All cracks were found to be around the maximum shear plane for both in-phase and 90° out-of-phase loadings and for all materials in this study.

4) Many more cracks were observed for solid specimens as compared to tubular specimens for 1050 N steel for both in-phase and 90° out-of-phase loadings. Short cracks were also detected at an earlier stage of fatigue life for solid specimens as compared to tubular specimens. These observations can be explained by the lower crack growth rate of solid specimens, which can result from the gradient of shear stress in the solid section, while shear stress is nearly uniform in thin-walled tubular section.

5) Cracking behaviors under FRI, FRR, PI, and FRI15 star shape strain paths were found to be very similar to the 90° out-of-phase (OP) loading. This observation indicates the fact that cracking behavior of star shape loading paths consisting of in-phase cycles within a circular boundary in $\varepsilon-\gamma/\sqrt{3}$ axes is more similar to cracking behavior for out-of-phase loading, than in-phase loading. Cracks for these strain paths were observed to be around the critical plane based on the Fatemi-Socie critical plane approach for both 1050 QT steel and 304L stainless steel.
6) Reddy-Fatemi effective strain-based intensity factor, which is based on the Fatemi-Socie critical plane approach and applicable for shear failure mode materials, was found to satisfactorily correlate in-phase and $90^\circ$ out-of-phase crack growth data of 1050 N steel solid specimens. Multiaxial crack growth data of 1050 N steel with high ductility and 1050 QT steel with low ductility were also correlated well by the Reddy-Fatemi effective strain intensity factor.
Table 6.1  Summary of cracking observations for tubular specimens of 1050 N steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>Load Type</th>
<th>$\varepsilon_a$</th>
<th>$\gamma_a$</th>
<th>$2N_f$ (Rev.)</th>
<th>Overall Failure Crack</th>
<th>$50 \mu m &lt; l_c &lt; 200 \mu m$</th>
<th>$100 \mu m &lt; l_c &lt; 200 \mu m$</th>
<th>$500 \mu m &lt; l_c &lt; 1,000 \mu m$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$n$</td>
<td>$l_c$ (mm)</td>
<td>$\theta_c$ (°)</td>
<td>$\theta_c$ (°)</td>
</tr>
<tr>
<td>N 7</td>
<td>IP</td>
<td>0.0069</td>
<td>0.0130</td>
<td>3,122</td>
<td>1</td>
<td>5</td>
<td>145</td>
<td>&gt;100</td>
</tr>
<tr>
<td>N 9</td>
<td>IP</td>
<td>0.0069</td>
<td>0.0119</td>
<td>3,334</td>
<td>1</td>
<td>15</td>
<td>135</td>
<td>&gt;50</td>
</tr>
<tr>
<td>N 11</td>
<td>IP</td>
<td>0.0033</td>
<td>0.0057</td>
<td>18,530</td>
<td>1</td>
<td>5</td>
<td>140</td>
<td>&gt;50</td>
</tr>
<tr>
<td>N 13</td>
<td>IP</td>
<td>0.0015</td>
<td>0.0026</td>
<td>378,378</td>
<td>1</td>
<td>10</td>
<td>140</td>
<td>3</td>
</tr>
<tr>
<td>N 3</td>
<td>IP</td>
<td>0.0014</td>
<td>0.0025</td>
<td>774,746</td>
<td>1</td>
<td>10</td>
<td>145</td>
<td>&gt;100</td>
</tr>
<tr>
<td>N 8</td>
<td>OP</td>
<td>0.0069</td>
<td>0.0119</td>
<td>1,046</td>
<td>1</td>
<td>15</td>
<td>175</td>
<td>&gt;100</td>
</tr>
<tr>
<td>N 10</td>
<td>OP</td>
<td>0.0069</td>
<td>0.0119</td>
<td>1,146</td>
<td>1</td>
<td>20</td>
<td>180</td>
<td>&gt;50</td>
</tr>
<tr>
<td>N 12</td>
<td>OP</td>
<td>0.0033</td>
<td>0.0057</td>
<td>7,654</td>
<td>1</td>
<td>10</td>
<td>170</td>
<td>&gt;50</td>
</tr>
<tr>
<td>N 14</td>
<td>OP</td>
<td>0.0015</td>
<td>0.0026</td>
<td>205,140</td>
<td>1</td>
<td>10</td>
<td>185</td>
<td>0</td>
</tr>
<tr>
<td>N 4</td>
<td>OP</td>
<td>0.0015</td>
<td>0.0026</td>
<td>266,838</td>
<td>1</td>
<td>10</td>
<td>175</td>
<td></td>
</tr>
</tbody>
</table>
Table 6.2  Summary of cracking observations for tubular specimens of 1050 QT steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>Load Type</th>
<th>$\varepsilon_a$</th>
<th>$\gamma_a$</th>
<th>$2N_j$ (Rev.)</th>
<th>$n$</th>
<th>$l_c$ (mm)</th>
<th>$\theta_c$ (°)</th>
<th>$\theta_r$ (°)</th>
<th>Overall Failure Crack</th>
<th>$50 \mu m&lt;l_c&lt;200 \mu m$</th>
<th>$200 \mu m&lt;l_c&lt;500 \mu m$</th>
<th>$500 \mu m&lt;l_c&lt;1,000 \mu m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q 7</td>
<td>IP</td>
<td>0.0069</td>
<td>0.0120</td>
<td>2,450</td>
<td>1</td>
<td>10</td>
<td>150</td>
<td></td>
<td></td>
<td>20</td>
<td>130-190</td>
<td>10</td>
</tr>
<tr>
<td>Q 1</td>
<td>IP</td>
<td>0.0068</td>
<td>0.0119</td>
<td>3,064</td>
<td>1</td>
<td>10</td>
<td>130</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q 9</td>
<td>IP</td>
<td>0.0032</td>
<td>0.0055</td>
<td>24,096</td>
<td>1</td>
<td>6</td>
<td>130</td>
<td></td>
<td></td>
<td>4</td>
<td>130-150</td>
<td>0</td>
</tr>
<tr>
<td>Q 11</td>
<td>IP</td>
<td>0.0024</td>
<td>0.0040</td>
<td>122,068</td>
<td>1</td>
<td>5</td>
<td>140</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q 3</td>
<td>IP</td>
<td>0.0020</td>
<td>0.0035</td>
<td>417,144</td>
<td>1</td>
<td>5</td>
<td>135</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q 12</td>
<td>OP</td>
<td>0.0105</td>
<td>0.0182</td>
<td>230</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q 2</td>
<td>OP</td>
<td>0.0069</td>
<td>0.0119</td>
<td>708</td>
<td>1</td>
<td>20</td>
<td>185</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q 6</td>
<td>OP</td>
<td>0.0069</td>
<td>0.0120</td>
<td>812</td>
<td>1</td>
<td>7</td>
<td>182</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q 8</td>
<td>OP</td>
<td>0.0033</td>
<td>0.0056</td>
<td>4,702</td>
<td>1</td>
<td>7</td>
<td>185</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q 15</td>
<td>OP</td>
<td>0.0031</td>
<td>0.0053</td>
<td>7,980</td>
<td>1</td>
<td>20</td>
<td>170</td>
<td></td>
<td></td>
<td>1</td>
<td>170</td>
<td>0</td>
</tr>
<tr>
<td>Q 4</td>
<td>OP</td>
<td>0.0031</td>
<td>0.0054</td>
<td>13,394</td>
<td>1</td>
<td>10</td>
<td>175</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q 10</td>
<td>OP</td>
<td>0.0025</td>
<td>0.0043</td>
<td>44,586</td>
<td>1</td>
<td>5</td>
<td>185</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Q 13</td>
<td>OP</td>
<td>0.0018</td>
<td>0.0031</td>
<td>2,058,686</td>
<td>1</td>
<td>12</td>
<td>190</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 6.3 Summary of cracking observations for tubular specimens of 304L stainless steel.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>Load Type</th>
<th>$\varepsilon_a$</th>
<th>$\gamma_a$</th>
<th>$\theta_f$ (Rev.)</th>
<th>Overall Failure Crack</th>
<th>50 µm &lt; $l_c$ &lt; 200 µm</th>
<th>200 µm &lt; $l_c$ &lt; 500 µm</th>
<th>500 µm &lt; $l_c$ &lt; 1,000 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSP 1</td>
<td>IP</td>
<td>0.0069</td>
<td>0.0120</td>
<td>1,668</td>
<td>1 n 10 135</td>
<td>42 n 130-180</td>
<td>22 n 120-160 110-180</td>
<td>3 n 130-145 115-180</td>
</tr>
<tr>
<td>SSP 11</td>
<td>IP</td>
<td>0.0049</td>
<td>0.0084</td>
<td>4,502</td>
<td>1 n 15 10 150</td>
<td>19 n 130-180</td>
<td>10 n 125-155 115-185</td>
<td>3 n 130-140 115-165</td>
</tr>
<tr>
<td>SSP 9</td>
<td>IP</td>
<td>0.0030</td>
<td>0.0057</td>
<td>6,500</td>
<td>1 n 5 140</td>
<td>8 135-180</td>
<td>4 120-145 110-180</td>
<td>0</td>
</tr>
<tr>
<td>SSP 9</td>
<td>IP</td>
<td>0.0030</td>
<td>0.0057</td>
<td>6,500</td>
<td>1 n 5 140</td>
<td>8 135-180</td>
<td>4 120-145 110-180</td>
<td>0</td>
</tr>
<tr>
<td>SSP 13</td>
<td>IP</td>
<td>0.0023</td>
<td>0.0040</td>
<td>31,456</td>
<td>1 n 15 130</td>
<td>3 150-180</td>
<td>1 150 150 1 150</td>
<td>1 155 155</td>
</tr>
<tr>
<td>SSP 7</td>
<td>IP</td>
<td>0.0018</td>
<td>0.0030</td>
<td>194,062</td>
<td>1 n 8 130</td>
<td>3 150-180</td>
<td>1 150 150 1 150</td>
<td>1 155 155</td>
</tr>
<tr>
<td>SSNP 10</td>
<td>OP</td>
<td>0.0069</td>
<td>0.0120</td>
<td>760</td>
<td>&gt;100 Micro NA</td>
<td>&gt;100 150-210</td>
<td>5 165-185 145-200</td>
<td>0</td>
</tr>
<tr>
<td>SSNP 2</td>
<td>OP</td>
<td>0.0069</td>
<td>0.0119</td>
<td>800</td>
<td>&gt;100 3 165 175</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSNP 4</td>
<td>OP</td>
<td>0.0033</td>
<td>0.0057</td>
<td>8,338</td>
<td>1 n 5 170-180</td>
<td>15 155-210</td>
<td>1 165 160-180</td>
<td>0</td>
</tr>
<tr>
<td>SSNP 8</td>
<td>OP</td>
<td>0.0033</td>
<td>0.0057</td>
<td>11,700</td>
<td>1 n 10 180</td>
<td>2 180-185</td>
<td>1 175 175</td>
<td>0</td>
</tr>
<tr>
<td>SSNP 6</td>
<td>OP</td>
<td>0.0017</td>
<td>0.0030</td>
<td>317,544</td>
<td>1 n 20 175</td>
<td>2 180-185</td>
<td>1 175 175</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 6.1 Different stages of crack initiation and growth during the fatigue process and the approach used for analysis.

Figure 6.2 (a) Definition of crack parameters. (b) Definition of the plane angle.
Figure 6.3  Examples of cracks observed for tubular specimens of 1050 N steel under in-phase tests at (a) high amplitude ($\varepsilon_a = 0.0100$), (b) intermediate amplitude ($\varepsilon_a = 0.0048$), and (c) low amplitude ($\varepsilon_a = 0.0022$) levels. The orientation of the critical planes on the specimen is also presented.
Figure 6.4 Crack length and orientation observed at different stages of fatigue life for the high strain level test ($\bar{\varepsilon}_a = 0.0100$) of 1050 N steel solid specimen under in-phase loading (i.e. NS 3). The orientation of the critical planes on the specimen is also presented.
Figure 6.5 Crack length and orientation observed in different stages of fatigue life for the intermediate strain level test ($\bar{\varepsilon}_a = 0.0049$) of 1050 N steel solid specimen under in-phase loading (i.e. NS 7). The orientation of the critical planes on the specimen is also presented.
Figure 6.6 Crack length and orientation observed at different stages of fatigue life for the low strain level test ($\bar{e}_a = 0.0023$) of 1050 N steel solid specimen under in-phase loading (i.e. NS 5). The orientation of the critical planes on the specimen is also presented.
Figure 6.7 Effect of strain amplitude level on cracking behavior for solid specimens of 1050 N steel under (a) in-phase, and (b) 90° out-of-phase loadings.
Figure 6.8 Effects of load non-proportionality on cracking behavior for solid specimens of 1050 N steel under high and intermediate strain amplitude levels.
Figure 6.9 Effects of load non-proportionality on crack growth rate of 1050 N steel solid specimens under (a) high strain amplitude ($\varepsilon_a = 0.0069$ and $\gamma_a = 0.0120$), and (b) intermediate strain amplitude level tests ($\varepsilon_a = 0.0033$ and $\gamma_a = 0.0057$).
Figure 6.10  Some cracks observed for the highest strain amplitude level tests ($\varepsilon_a = 0.0069$ and $\gamma_a = 0.0120$) of 304L stainless steel under (a) in-phase, and (b) 90º out-of-phase loadings.
Figure 6.11  Effects of material ductility on the cracking behavior at $\varepsilon_a = 0.0100$. Crack length versus (a) normalized number of cycles, and (b) number of cycles.
Figure 6.12  Effects of solid and tubular geometries on cracking behavior for 1050 N steel at $\varepsilon_a = 0.0100$. Crack length versus (a) normalized number of cycles, and (b) number of cycles.
Figure 6.13 Crack growth rate versus Reddy-Fatemi effective strain-stress-based intensity factor for (a) in-phase (IP) and 90° out-of-phase (OP) loading of 1050 N steel solid specimens, and (b) in-phase loading of 1050 N steel and 1050 QT steel tubular specimens at $\varepsilon_a = 0.0100$. 
Chapter 7

Summary and Possible Future Research

There are a variety of issues involved in fatigue life predictions under variable amplitude multiaxial loading including load cycle counting technique, cyclic plastic behavior, and fatigue damage evolution and accumulation process. The main goal of this research was to investigate fatigue and deformation behaviors and to evaluate the capabilities of life prediction methodologies under some discriminating variable amplitude axial-torsional strain paths. As fatigue damage is a local phenomenon affected by microstructure of material, the effect of microstructure and hardness on deformation and fatigue behavior was also investigated using medium carbon 1050 steel at three hardness levels. Stainless steel 304L with significant non-proportional hardening was also utilized in this study, to contrast the behavior with 1050 QT steel without any non-proportional hardening.

Cyclic plasticity and hardening rules, multiaxial fatigue models, cycle counting methods, and cumulative damage rules were briefly reviewed and discussed in Chapter 2. It has been suggested that the Armstrong-Frederick incremental plasticity model is a proper foundation for modeling material behaviors, including additional non-proportional hardening. However, cyclic plasticity models often require a large number of material constants to be identified. Empirical stress-strain models estimate the non-proportional...
cyclic hardening directly from the strain path amplitude using a phenomenological approach. With regards to fatigue damage parameters, the classical multiaxial fatigue models, such as von Mises equivalent stress or strain, do not reflect damage mechanism and under-estimate fatigue damage under non-proportional loading. Critical plane approaches have been shown to result in more accurate predictions. Critical plane approaches are categorized into stress-based, strain-based, and strain-stress-based models. Strain-stress-based multiaxial fatigue models have the advantage of reflecting the material constitutive behavior and non-proportional hardening.

7.1 Summary

Effect of strain path non-proportionality as well as hardness level on cyclic deformation behavior and a predictive method for non-proportional cyclic hardening coefficient based on simple material properties were presented. The non-proportional cyclic hardening coefficient, $\alpha$, was found to be 0.1 for the normalized condition of 1050 steel with pearlitic-ferritic structure, and about $-0.05$ for quenched and tempered and induction hardened conditions with tempered martensite. Therefore, the value of this coefficient for a given material depends on the material microstructure and hardness level. The reduction in the non-proportional cyclic hardening of 1050 steel as it changes from the normalized condition to a finer grain tempered martensite condition can be qualitatively explained by the decrease in stacking fault width between dislocations by increasing the hardness level of the material, and consequently, an increase in stacking fault energy (SFE). In addition, it was found that materials with cyclic hardening also exhibit additional hardening due to the non-proportionality of loading. Both cyclic hardening and non-proportional cyclic hardening phenomena can be related to SFE. The
non-proportional cyclic hardening coefficient was found to strongly correlate with the cyclic hardening coefficient. A predictive model for non-proportional cyclic hardening is proposed which only requires knowledge of the monotonic and uniaxial cyclic deformation behaviors of the material. Excellent predictions based on this model were obtained for the materials in this study as well as for a variety of materials from the literature.

Multiaxial in-phase straining of 304L stainless steel with very gradual change in strain direction (i.e. 1° increments in axial-torsion strain space) results in some non-proportional hardening, as compared with in-phase or proportional loading. Such loading activates the slip systems gradually but in all directions, while in-phase loading activates slip systems in only one direction. However, as the material has short term memory of prior deformation, the latest part of the loading block dictates most of the material deformation behavior. In-phase (i.e. proportional) straining of 304L stainless steel with random sequence resulted in significant non-proportional hardening, similar to 90° out-of-phase loading. Such loading causes cross hardening due to the interaction of the slip systems by activating intersecting slip bands within the material. Satisfactorily stress predictions were obtained for all the multiaxial strain paths investigated for 1050 QT steel and 304L stainless steel, employing the non-proportionality parameter proposed by Tanaka coupled with a simplified form of the Armstrong-Frederick incremental plasticity model. Empirical methods, such as Kanazawa et al.’s non-proportionality formulation, can significantly over-estimate the stress response under in-phase straining with gradual change in strain direction. In addition, such methods cannot account for the progression of the cross hardening with change in loading direction.
The effect of hardness on fatigue behavior was evaluated using in-phase and 90° out-of-phase axial-torsional fatigue data of 1050 steel in normalized, quenched and tempered, and induction hardened conditions. At 1% equivalent strain amplitude (LCF region) the normalized specimens had about 3 times longer life than the induction-hardened specimens under both in-phase and 90° out-of-phase loading conditions. At 0.3% equivalent strain amplitude (HCF region), the induction-hardened specimen lives were 8 and 25 times longer than the normalized specimen lives under in-phase and 90° out-of-phase loadings, respectively. No significant specimen geometry effect on fatigue lives was observed between solid and tubular specimens for either in-phase or 90° out-of-phase loadings. von Mises criterion did not correlate the in-phase and 90° out-of-phase fatigue data. Overall crack orientations for in-phase and 90° out-of-phase loadings for 304L stainless steel and 1050 steel at all hardness levels were found to be around the maximum shear plane. Based on the shear-based Fatemi-Socie multiaxial fatigue parameter, 92% of the 1050 N steel test data and 80% of 1050 IH steel test data were within a factor of 3 in life, while 86% of the 1050 QT steel test data were within a factor of 5 in life. All the in-phase and 90° out-of-phase fatigue data of 304L stainless steel were predicted within a factor of 3 in life. In addition, a simple method for multiaxial fatigue life prediction of steels was proposed based on the Fatemi-Socie parameter in combination with the Roessle-Fatemi model. This method uses hardness as the only needed material property. Considering data from four very different steels, three very different hardness levels of one of the steels, and a broad range of loading conditions, life predictions were found to be relatively accurate, with 94% and 81% of the data within scatter bands of 5 and 3 in life with respect to the predicted curve, respectively.
Fatigue behavior of 1050 steel in quenched and tempered condition with no non-proportional hardening and 304L stainless steel with 55% non-proportional cyclic hardening coefficient was also investigated under discriminating strain paths. Fatigue life of 1050 QT steel with no non-proportional cyclic hardening was found to be more sensitive to non-proportional loading than for 304L stainless steel with significant non-proportional cyclic hardening. Therefore, additional cyclic hardening due to the non-proportionality of loading is not the only factor causing shorter fatigue lives under non-proportional loading. This is contrary to common expectations, where shorter fatigue life in non-proportional loading is often attributed to additional non-proportional hardening. The Fatemi-Socie parameter can account for the higher damage in non-proportional loading by the higher stress value in the parameter, regardless of non-proportional hardening.

Although FRR strain path with random sequences of axial-torsion cycles and sudden changes in straining direction resulted in a considerably higher stress response as compared to FRI strain path with gradual change in straining direction for 304L stainless steel, not a significant effect of this higher stress was observed in fatigue life. Similarly, no effect of loading sequence was observed for 1050 QT steel with no non-proportional cyclic hardening. Shorter fatigue lives were observed for the pulsating PI90 strain path as compared to the fully-reversed FRI90 strain path. This was explained by coupling of cycles from the two sides of the PI90 strain path resulting in a cycle with twice the amplitude, as compared to the fully-reversed FRI90 strain path.

The commonly used von Mises criterion was found not to be suitable for fatigue life predictions of variable amplitude strain blocks, even when the load block consists of
only proportional cycles. This criterion is independent of fatigue damage plane orientation which is in contrast with the experimental observations indicating all cracks are around a preferred plane (i.e. critical plane) within the material. Satisfactory multiaxial fatigue life predictions were obtained by either Bannantine-Socie or Wang-Brown cycle counting method, Fatemi-Socie damage parameter, and linear cumulative damage rule for 1050 QT steel and 304L stainless steel under various strain paths. Crack orientation plane predicted by using this procedure corresponded to the observed crack orientations. However, this may not be a robust procedure for general applicability to variable amplitude multiaxial loading since a unique cycle count may not result and the interaction of different components of damage parameter is not considered.

More cracks and also on a wider range of planes around the critical plane were observed for higher amplitude level tests for all strain paths investigated. Short cracks were observed at much earlier stage of fatigue life for higher strain amplitude fatigue tests as compared to lower strain amplitude fatigue tests. While more cracks were detected for in-phase loading as compared to 90° out-of-phase loading for all materials, cracks were observed on a wider range of plane orientations for 90° out-of-phase loading. Although more cracks and lower crack growth rates were observed for more ductile materials in this study, all cracks were found to be around the maximum shear plane for both in-phase and 90° out-of-phase loadings and for all materials used. Cracking behaviors under variable amplitude strain paths were found to be very similar to the 90° out-of-phase (OP) loading. Cracks for these strain paths were observed to be around the critical plane based on the Fatemi-Socie critical plane approach for both 1050 QT steel and 304L stainless steel. Reddy-Fatemi effective strain-based intensity factor, which is
based on the Fatemi-Socie critical plane parameter and applicable for shear failure mode materials, was found to satisfactorily correlate in-phase and 90° out-of-phase crack growth data of 1050 N steel solid specimens. Multiaxial crack growth data from tubular specimens of 1050 N steel with high ductility and 1050 QT steel with low ductility were also correlated well by the Reddy-Fatemi effective strain intensity factor.

7.2 Possible future research

Robust cycle counting method and damage parameter are needed for general applicability to variable amplitude multiaxial loading which is the realistic condition for many industrial components and structures. Improving the available multiaxial damage assessment methods or developing a new more robust method to correctly identify cycles and the corresponding damage in a variable amplitude multiaxial history can be of a great interest in many applications. Data generated in this study under a variety of strain paths can be later used to verify the accuracy of such a method.

The interaction of different stress and strain parameters on the critical plane and their effect on fatigue damage is not yet well understood. This can be better explained by the load paths presented in Figure 7.1. All these load paths have the same shear strain amplitude of 0.005 and a maximum normal stress of 700 MPa on the loading plane (which is also the critical plane for these load paths). However, shear strain and normal stress components for each load path interact differently. During the shear strain cycle of load path “I”, tensile normal stress is constantly applied, opening the crack and resulting in less friction. However, normal stress is pulsating only once at the zero shear strain for load path “P” and, therefore, it may have minimal effect on fatigue damage. The normal stress component gradually increases as the shear strain component gradually decreases,
and vise versa, for load path “D”. Thus, the effect of normal stress on fatigue life may be different for any of these load paths.

Figure 7.2 presents shear strain amplitude and maximum normal stress on different plane orientation angles for 1050 QT steel and the three above-mentioned load paths. Since 1050 QT steel is a material with no sensitivity to non-proportional hardening, all the load paths result in the same stress response and, therefore, the same FS damage parameter value on the critical plane (which is the loading plane for all three load paths), as can be seen from Figure 7.2. These strain paths result in FS damage value of 0.0068 on the loading plane (critical plane) for 1050 QT specimens. This FS parameter value is associated with $N_f = 17000$ cycles based on the fatigue curve fit to the in-phase data. Interestingly, fatigue lives of $N_f = 28000$ and $N_f = 52500$ cycles were experimentally obtained for 1050 QT steel specimens under load path “P” in some preliminary tests which were conducted. Therefore, the average of fatigue lives for “P” load path is 2.5 times longer than the predicted value based on the in-phase fatigue curve. Fatigue life observed for the load path “I”, however, was $N_f = 8000$ cycles which is half of the predicted value based on the FS parameter. However, significant ratcheting in this test may have contributed some damage for strain path “I”, as a tensile strain of 4% at $10^3$ cycles was measured, which linearly increased to about 10% at failure. Better understanding of the interaction of different stress and strain components on the critical plane is an interesting topic for further research in multiaxial fatigue analysis. The loading paths presented in Figure 7.1 can be a starting point for this research topic.
Figure 7.1 Some specific load paths designed to challenge FS critical plane approach; (a) load path “I”, (b) load path “P”, and (c) load path “D”.
Figure 7.2 Variation of shear strain amplitude, maximum normal stress, and FS parameter for 1050 QT steel under (a) load path “I”, (b) load path “P”, and (c) load path “D”.
References

A


“ASME boiler and pressure vessel code,” 1986, Section III, Div. 1, Subsection NB, Class 1 Components, American Society of Mechanical Engineers, New York.


B


Colin, J. A., 2009, Deformation history and load sequence effects on cumulative fatigue damage and life predictions, Ph.D. Dissertation, Mechanical, Industrial, and Manufacturing Department, The University of Toledo, Toledo, Ohio.


**D**


**E**


eFatigue Software: [https://efatigue.com/](https://efatigue.com/)

**F**


**G**


**H**


**I**


**J**


K


L


N


O


P


Rankine, W. J. M., 1842, “On the causes of the unexpected breakage of the journals of railway axles, and on the means of preventing such accidents by observing the law of continuity in their construction”, *Institution of Civil Engineers, Minutes of Proceedings*, pp. 105-108.


T


V


W


Z

