Experimental high speed compressible subsonic flow pressure losses across gate, butterfly and ball valves and their use in natural gas transmission flow control simulation software

Kirk J. Gomes
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A Dissertation
entitled
Experimental High Speed Compressible Subsonic Flow Pressure Losses
Across Gate, Butterfly and Ball Valves and Their Use in
Natural Gas Transmission Flow Control Simulation Software

by
Kirk J. Gomes

Submitted to the Graduate Faculty as partial fulfillment of the requirements
for the Doctor of Philosophy Degree in Engineering

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The University of Toledo
May 2011
An Abstract of

Experimental High Speed Compressible Subsonic Flow Pressure Losses Across Gate, Butterfly and Ball Valves and Their Use in Natural Gas Transmission Flow Control Simulation Software

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The first part of the three parts that this work is divided into describes the simulation of a two-spool, two-stream turbofan engine modeled independently using Modelica (Dymola), MATLAB/Simulink and the Numerical Propulsion System Simulation (NPSS). The model contains steady-state performance maps for all of the components and has control volumes where continuity and energy balances are maintained. Rotor dynamics and duct momentum dynamics are also included. Steady state design and off-design points as well as transients can be simulated. The results of the simulation using the three modern tools are compared with the published results obtained using DIGTEM (Digital Computer Program for Generating Dynamic Turbofan Engine Models) and the pros and cons of each modern program are examined.

The second part describes the experimental results obtained by measuring pressure losses across gate, butterfly and ball valves subjected to compressible flow. Experimental data has been collected for all possible combinations of gate, butterfly and ball valves with nominal diameters of 0.5”, 0.75”, 1”, and 1.25” and various valve flow areas, i.e. fully open, three quarter open, half open, and quarter open. The results have been compared to a similar study conducted using a ball valve with a nominal diameter of 1.5”. 
The third and final part uses the results of the above two parts and describes the development and simulation of a natural gas transmission model. A gas turbine engine drives a compressor that is used to pump natural gas through a pipeline. Upstream of the compressor is a throttling valve used to control the mass flow rate. Downstream, the Weymouth equation is used to model the volumetric flow rate and pressure losses in the pipeline.
To my parents for their constant and never-ending support.
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First and foremost, my thanks go to Dr. Konstanty C. Masiulaniec. Not only has he been unbelievably helpful, resourceful and patient during this research, but he is the one person I have always been able to count on in times of trouble. It goes without saying that none of this would have been possible without him.

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List of Symbols

−1 ........ Inverse matrix when used as a superscript
( )* ........ Sonic flow condition when used as a superscript

β ........ Interpolation constant
γ ........ Ratio of specific heats
γ_g ........ Gas specific gravity (air = 1)
Δh ........ Enthalpy change, Btu/lbm
ΔT/T .... Temperature rise parameter
δ ........ Ratio of total pressure to sea-level pressure
η ........ Efficiency
θ ........ Ratio of total temperature to standard day temperature
ρ ........ Density

A ........ Cross-sectional area in Chapter 3, in^2
A ........ Air when used as a subscript in Chapter 3
A ........ Pipe cross-sectional area in Chapters 6, m^2
AB ........ Augmentor when used as a subscript
a ........ Altitude, ft
a ........ Actual value when used as a subscript
am ........ Ambient when used as a subscript
B ........ Main combustor when used as a subscript
BL ........ Bleed when used as a subscript
BLHT .... High pressure turbine cooling bleed when used as a subscript
BLLT .... Low pressure turbine cooling bleed when used as a subscript
BLOV .... Overboard bleed when used as a subscript
C ........ Compressor when used as a subscript
C_d ........ Nozzle flow coefficient
C_v ........ Nozzle velocity coefficient
CC ........ Correction coefficient
CVGP .... Compressor variable geometry parameter, deg
$c_p$        Specific heat at constant pressure, $Btu/lb_m-R$
$cr$        Critical flow when used as a subscript
$c_v$        Specific heat at constant volume, $Btu/lb_m-R$
$D$        Design input when used as a subscript in Chapter 3
$D$        Inside diameter of pipe in Chapter 8, in
$dt$        Differential time, s
$E$        Exit nozzle plane when used as a subscript
$es$        Expelled nozzle shock when used as a subscript
$F$        Fuel when used as a subscript
$F$        Thrust, $lb_f$
$F_f$      Pipe drag factor
$FVGP$      Fan variable geometry parameter, deg
$f$        Moody friction factor
$f_{an}$    Fan when used as a subscript
$f_i()$    Functional relationship, $i = 1..30$
$f/a$      Fuel-to-air ratio
$G_t$      Dimensionless mass flow rate
$g_c$      Gravitational constant, 386.3 $lb_m$-$in/lb_f$-$s^2$
$H$        Heat, $Btu$
$H$        High pressure spool when used as a subscript
$HT$      High pressure turbine when used as a subscript
$HVF$      Heating value of fuel, $Btu/lb_m$
$h$        Specific enthalpy, $Btu/lb_m$
$h_p$      Turbine enthalpy drop, $Btu/lb_m$-$R^{0.5-rpm}$
$I$        Inlet when used as a subscript
$I$        Polar moment of inertia, $lb_f$-$in$-$s^2$
$ID$      Fan hub region when used as a subscript
$i$        Initial condition when used as a subscript
$id$      Ideal when used as a subscript
$in$      Into volume when used as a subscript
$J$      Mechanical equivalent of heat, 9339.6 $lb_f$-$in/Btu$
$j$      Station when used as a subscript, $j = 0, 2, 1, 2.2, 3, 4, 4.1, 5, 6, 7, 8, 9, 13, 16$
$j'$      Entrance to volume at station $j$ when used as a subscript, $j = 3, 7, 13$
$K_{AB}$    Augmentor pressure loss coefficient, $lb_f^2$-$s^2/lb_m^2$-$in^4$-$R$
$K_B$      Main combustor pressure loss coefficient, $lb_f^2$-$s^2/lb_m^2$-$in^4$-$R$
$K_{BLWHT}$ Fraction of high pressure turbine doing work
$K_{BLWLT}$ Fraction of low pressure turbine doing work
$K_D$      Duct pressure loss coefficient, $lb_f^2$-$s^2/lb_m^2$-$in^4$-$R$
$K_{PR5}$ Low pressure turbine discharge pressure loss coefficient
$k$ .... Valve coefficient

$k_e$ .... Effective roughness of the internal pipe surface

$L$ .... Length of pipe in Chapter 8, miles

$L$ .... Low pressure spool when used as a subscript in Chapter 3

$LT$ .... Low pressure turbine when used as a subscript

$l$ .... Length, in

$load$ .... Load when used as a subscript

$M$ .... Mach number

$M$ .... Map when used as a subscript

$M_1$ .... Upstream Mach number

$M_2$ .... Downstream Mach number

$MV_1$ .... Mixing volume MV1 when used as a subscript

$MV_2$ .... Mixing volume MV2 when used as a subscript

$\dot{m}_t$ .... Mass flow rate of air, kg/s

$N$ .... Rotational speed, rpm

$NG$ .... Natural gas when used as a subscript

$NGHPC$ .... Natural gas compressor when used as a subscript

$n$ .... Net when used as a subscript

$new$ .... New when used as a subscript

$OD$ .... Fan tip region when used as a subscript

$old$ .... Old when used as a subscript

$out$ .... Out of volume when used as a subscript

$P$ .... Total pressure, psia

$P/P$ .... Pressure ratio

$P_{01}$ .... Upstream stagnation pressure, Pa

$P_{02}$ .... Downstream stagnation pressure, Pa

$P_1$ .... Upstream static pressure, Pa

$P_2$ .... Downstream static pressure, Pa

$p$ .... Static pressure, psia

$p_1$ .... Inlet pressure, psia

$p_2$ .... Outlet pressure, psia

$p_b$ .... Base pressure, psia

$Q$ .... Torque, in-lbf

$q_h$ .... Volumetric gas flow rate at $p_b$ and $T_b$, cfh

$R$ .... Gas constant, in-lbf/lb$_m$-R

$Re$ .... Reynolds number

$T$ .... Total temperature, R

$T/T$ .... Temperature ratio

$T_{01}$ .... Upstream stagnation temperature, K

$T_{02}$ .... Downstream stagnation temperature, K
\begin{align*}
T_1 & \quad \text{Upstream static temperature, } K \\
T_2 & \quad \text{Downstream static temperature, } K \\
T_b & \quad \text{Base temperature, } R \\
\bar{T} & \quad \text{Average flowing temperature, } R \\
t & \quad \text{Time, } s \\
u & \quad \text{Internal energy, } Btu/lb_m \\
V & \quad \text{Volume, } in^3 \\
V_1 & \quad \text{Upstream velocity} \\
V_2 & \quad \text{Downstream velocity} \\
v & \quad \text{Velocity, } in/s \\
\text{valve} & \quad \text{Valve when used as a subscript} \\
W & \quad \text{Stored mass, } lb_m \\
\dot{w} & \quad \text{Mass flow rate, } lb_m/s \\
\dot{w}_c & \quad \text{Corrected mass flow rate, } lb_m/s \\
\dot{w}_p & \quad \text{Turbine flow parameter, } lb_m-R-in/lb_f-rpm-s \\
x & \quad \text{Upstream side of shock when used as a subscript} \\
x_t & \quad \text{Ratio of static pressure loss to upstream static pressure} \\
x_{t,s} & \quad \text{Ratio of stagnation pressure loss to upstream stagnation pressure} \\
y & \quad \text{Downstream side of shock when used as a subscript} \\
\bar{Z} & \quad \text{Gas deviation factor at average flowing temperature and average pressure}
\end{align*}
Chapter 1

Introduction

Most natural gas reserves are not in the vicinity of consumers. One method to transport gas to the places where it is needed is by pumping it through pipelines.

Transmission lines cover the long distances and local distribution networks carry gas locally. To overcome the pressure losses in the pipeline, compression equipment is necessary at defined distances distributed over the length of the pipeline.

Compressor stations in long-distance pipelines are typically in remote locations without electrical infrastructure, so the compressors are usually driven by gas turbines [1]. A study conducted by Kansas State University’s National Gas Machinery Laboratory that was funded by the Department of Energy concluded that past simulation efforts “model the compressor station as a black box where the input pressure is increased by some percentage to determine the compressor station output pressure. Even when engines and compressors are included within the simulation, the models require the user to input an engine load line or the compressor load line. Few if any simulations offer the ability to incorporate a complete engine or compressor load map, and no references were found that focus on the fuel consumption and pollutant emissions of the compressor station” [2] and that is the motivation behind the work presented here. Since past efforts have focused on the simulation of the pipeline, the
aim of this work is to model the interaction of the components within that “black box” and the pipeline gas flow as well as incorporate the ability to model transients. MathWorks’ Simulink was chosen for the simulation due to its commercial availability, wide acceptance in industry and exceptional product support.

For a working simulation, in addition to the gas turbine engine model, control valve models were also needed that adequately described static and stagnation pressure losses for various mass flow rates, especially at high Mach numbers. Towards that end, experimental work was undertaken since a search of the literature revealed a distinct dearth of useful data.
Chapter 2

Previous Work

2.1 Gas Turbine Simulation

A major component of a natural gas transmission station is the gas turbine engine. Gas turbine simulation, which has been a research field for several decades provides a means of analyzing and predicting the behavior and performance of propulsion systems prior to their actual construction.

Early simulation programs such as GENENG I [3] and GENENG II [4] were written in FORTRAN and were developed for the analysis of a specific engine configuration. A change in the engine configuration involved the inconvenient and time-consuming task of rewriting the simulation code to match the new configuration.

Thus there was a need for a computer code that was flexible for modeling various engine configurations, could do both steady state and transient simulations and could be easily adapted to model real engine data. HYDES was one such program developed for the hybrid computer [5].

Hybrid simulations had a disadvantage in that they were generally not portable nor easy to modify. This led to the development of a digital computer model, DIGTEM, that was written in modular form in order to permit the simulation of various engine
configurations [6].

The introduction of graphical user interfaces and object-oriented programming led to newer simulation techniques [7] and simulation programs such as TESS [8]. Other simultaneous efforts involving object-oriented programming [9] eventually led to the development of a proprietary NASA simulation program called Numerical Propulsion System Simulation (NPSS), which until a few years ago, was restricted to use only within a select few organizations.

There have been recent efforts to simulate gas turbine engines using unrestricted and commercially available multidisciplinary packages such as Dymola [10]. Simulink is another similar simulation package that is commercially available. There was no comparison in terms of performance between these commercial packages and the aeronautical industry “standard”, NPSS. This is addressed in Chapters 3 and 4.

2.2 Valve Models

A search of the literature for data pertaining to the static and stagnation pressure losses across various types of valves when they were subjected to high speed compressible flow yielded no results. Subsequent searches yielded standard valve testing procedures for the compressible flow regime such as references [11] and [12]. The guidelines and procedures in these references were followed during the setup and collection of experimental data. A case study was also found involving the calculation of the valve flow coefficient of a ball valve in compressible flow [13]. The trends observed in the data collected in this work are compared the trends in the case study and this is discussed in Chapters 5 to 7. It must be reiterated however, that no actual data was found in the literature that could be used to develop the valve models.
2.3 Natural Gas Station and Pipeline Model

The most commonly used equations for steady state calculations of pipeline flows are the Weymouth equation and the Panhandle equations. Past as well as recent research in this area has focused on the steady state simulation of the gas pipeline and the development of computational algorithms for the transient case.

References [14], [15] and [16] describe the general steady state equations used in pipeline flow and these equations account for the pressure drops due to friction, elevation and kinetic energy. The general gas flow equation is shown below.

\[ q_h = 1.6156 \left( \frac{T_h}{p_h} \right) \left( \frac{p_1^2 - p_2^2}{LZ} \right)^{\frac{1}{2}} \left( \frac{D^5}{\gamma g T} \right)^{\frac{1}{2}} \left( \frac{1}{f} \right)^{\frac{1}{2}} \]  \hspace{1cm} (2.1)

Transmission Factor = \left( \frac{1}{f} \right)^{\frac{1}{2}} \hspace{1cm} (2.2)

The transmission factors for the various gas flow equations that are frequently used are shown in Table 2.1.

<table>
<thead>
<tr>
<th>Equations</th>
<th>Transmission Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weymouth</td>
<td>11.18 ( (D)^{\frac{1}{2}} )</td>
</tr>
<tr>
<td>Panhandle A</td>
<td>7.211 ( \left( \frac{q_h \gamma g}{D} \right)^{0.07/305} )</td>
</tr>
<tr>
<td>Panhandle B</td>
<td>16.70 ( \left( \frac{q_h \gamma g}{D} \right)^{0.001961} )</td>
</tr>
<tr>
<td>AGA Fully Turbulent</td>
<td>4 log ( \left( \frac{3.7D}{k_h} \right) )</td>
</tr>
<tr>
<td>AGA Smooth Pipe</td>
<td>4 log ( \left[ \frac{Re}{1.4126} \left( \frac{1}{f} \right)^{0.3} \right] )</td>
</tr>
<tr>
<td>AGA Partially Turbulent</td>
<td>( F_j \left[ 4 \log \left( \frac{Re}{1.4126} \left( \frac{1}{f} \right)^{0.3} \right) \right] )</td>
</tr>
</tbody>
</table>
Reference [17] presents the derivation of an analytical equation for compressible fluid flow in pipelines that is based on mass and momentum balance and relates flow rate, inlet pressure and outlet pressure. The equation can be used to estimate any of these variables if the others are known and can be used for any pipeline topology and configuration, including size and orientation.

Reference [18] performs a sensibility analysis to determine which flow equation such as Weymouth, Panhandle, AGA, etc. is best at predicting the natural gas flow behavior in the pipeline.

Reference [19] describes the development of a mathematical model and algorithm for the simulation of the steady state behavior of pipeline networks with compressible fluids. A polynomial expression is used to model the relationship between pressure and mass flow rate in the compressors. The model has the capability of determining pressures, flow rates, temperatures and gas compositions at any point in the network.

Reference [20] stresses the advantages of using transient simulations as a training tool for operation personnel as well as in the design phase of a gas pipeline. The model assumes isothermal flow and the inputs needed to initiate system calculations do not consider the elevation profile.

For many dynamic gas applications, the assumption that a process is isothermal or adiabatic is not valid. In these cases, the temperature of the gas is considered to be a function of distance and a mathematical model based upon the energy equation is used. Reference [21] compares these different isothermal and non-isothermal model and uses practical examples to emphasize the differences between the various models.

Reference [22] used the method of lines to solve the problem of transient simulation of gas pipeline flow.

Reference [23] characterizes the different transient models and numerical techniques used to solve the transient equations for pipeline gas flow. The reference also addresses the problems associated with choosing an appropriate numerical method.
for solving a particular mathematical model of a pipeline that balances accuracy and computational time.

Reference [24] presents an object-oriented methodology using a collection of C++ classes for modeling a natural gas transmission network. To simulate a gas pipeline network, a graph describing the structure of the network needs to be converted into a system of ordinary differential equations. To address this technically complicated task, solutions such as graph interpreters have been proposed. Reference [24] uses object-oriented technology to reduce these difficulties.

When simulating the transient flow of natural gas in pipelines, the inertia term in the momentum equation is usually neglected in an attempt to linearize the equations and this results in a loss of accuracy. Reference [25] presents a numerical algorithm and computer code that solves the non-linear conservative hyperbolic equations governing the behavior of transients in gas transmission pipelines without neglecting the kinetic energy term. This transient code is based on a new class of high resolution hybrid TVD schemes.

Reference [26] describes a fully implicit finite difference method for calculating the unsteady gas flow in pipeline networks and compares the results of the method to those produced by the method of characteristics and the two-step Lax-Wendroff method. When using a small time step, an excellent agreement between the methods is observed and this implicit method reduces the computation time.

Reference [27] analyzes the transients in gas flow and pressure in a horizontal straight pipe using numerical simulation and simulates eight representative cases of transient behavior. The numerical results show that transient effects in the pipeline may last for a long time and/or over significant length of pipe and are dependent upon the pipe dimensions and operating variables such as pressure and gas flow rate. The simulations predict that if the pressure drop across the pipe is increased, there is an initial surge in gas flow rate which is greater than the final steady-state value and
if the pressure drop is decreased, the initial flow rate may decrease below the final steady-state value. Oscillations in both pressure and mass flow rate are observed in the case of complete closure of a valve, but these gradually decay and the steady state conditions of no flow are ultimately achieved.

Reference [28] presents GNAP Version 1.0, a program capable of unsteady gas flow analysis in pipe networks. The method of characteristics and Total Variation Diminishing (TVD) method was used in the study. GNAP was created using Microsoft Visual C++ 6.0 and was designed for Windows NT/95/98.

Reference [29] describes and validates a special Runge-Kutta method that is used to simulate the transient phenomena in a two-dimensional natural gas flow. The upwind method of van Leer is chosen as the base solution algorithm and the Total Variation Diminishing (TVD) technique is employed for higher-order accuracy.

For the work presented here, the pipeline gas flow is modeled using the Weymouth equation. The mathematical model that for the compressor station is discussed in Chapter 8.
Chapter 3

Development of Gas Turbine Simulation Tools using Modern Computational Methods and Packages

3.1 Introduction to the Commercial Packages

The development of aircraft propulsion systems is dependent, to a great extent, on the ability of a simulation tool to predict the performance of the propulsion system and its associated controls prior to the building and testing of a prototype. Computer simulations provide the means for analyzing the behavior and interactions of these complex systems and can also serve as aids in understanding and solving problems that arise after the propulsion system is developed. In this chapter, three unique simulation tools, Modelica (Dymola), MATLAB/Simulink and the Numerical Propulsion System Simulation (NPSS) have been used to model a two-spool, two-stream augmented turbofan engine. The results from these tools are compared to the results
from DIGTEM [6] (Digital Computer Program for Generating Dynamic Turbofan Engine Models) which was written in Fortran in 1983. Additionally, a comparison between the three tools is provided in terms of usage and performance.

Modelica is primarily a modeling language that allows specification of mathematical models of complex natural or man-made systems, for the purpose of computer simulation of dynamic systems where behavior evolves as a function of time. Modelica is also an object-oriented equation-based programming language, oriented toward computational applications with high complexity requiring high performance [30].

Simulink is an environment for multi-domain simulation and model-based design for dynamic and embedded systems. It provides an interactive graphical environment and a customizable set of block libraries that facilitate design, simulation, implementation and testing of a variety of time-varying systems, including communications, controls, signal processing, video processing, and image processing. Add-on products extend Simulink software to multiple modeling domains, as well as provide tools for design, implementation, and verification and validation tasks. Simulink is integrated with MATLAB, providing immediate access to an extensive range of tools that allow the development of algorithms, analysis and visualization of simulations, creation of batch processing scripts, customization of the modeling environment, and the definition of signal, parameter, and test data.

The Numerical Propulsion System Simulation (NPSS) project is a technology developed by the NASA Glenn Research Center, in conjunction with the U.S. aero-propulsion industry and the Department of Defense and is capable of supporting detailed aerothermomechanical computer simulations of complete aircraft engines. NPSS can realistically model the physical interactions that take place throughout an engine, accelerating the concept-to-production development time and reducing the need for expensive full-scale tests and experiments. At its foundation, NPSS is a component-based object-oriented engine cycle simulator designed to perform cy-
cle design, steady state and transient off-design performance prediction, test data matching, and many other traditional tasks of engine cycle simulation codes. Like traditional codes, an NPSS engine model is assembled from a collection of interconnected components, and controlled through the implementation of an appropriate solution algorithm [31].

3.2 Engine Description and Mathematical Models of Components

The model being simulated is a two-spool, two-stream, low bypass ratio (bypass ratio of 0.808 at the design point) augmented turbofan engine. Figure 3-1 shows a schematic representation of that engine with the engine stations numbered. A single inlet is used to supply airflow to the fan. After leaving the fan, the air is separated into two streams. One stream passes through the engine core and the other stream passes through an annular bypass duct. The fan is driven by a low-pressure turbine. The core airflow passes through a compressor that is driven by a high-pressure turbine. Both the fan and the compressor are assumed to have variable geometry for better stability at low speeds. At the compressor exit, engine airflow bleeds are extracted and used for turbine cooling (flow returns to the cycle). Fuel flow is injected in the main combustor and burned to produce hot gas that drives the turbines. The engine core and bypass streams combine in an augmentor duct, where the flows are assumed to be thoroughly mixed and additional fuel is added to further increase the gas temperature (and thus thrust). Finally, the augmentor flow is discharged through a variable convergent-divergent nozzle. The nozzle throat area and exhaust area are varied to maintain airflow and to minimize drag during augmentor operation. The following section will briefly describe the mathematical models [32] of the engine components.
Figure 3-1: Schematic of the augmented turbofan engine [6].

3.2.1 Flight Conditions and Inlet

The following equations define the flight conditions and the inlet model.

\[ P_0 = f_1(a) \]  \hspace{1cm} (3.1)

\[ T_0 = f_2(a) + T_{am} \]  \hspace{1cm} (3.2)

\[ \eta_I = \begin{cases} 1.0 & \text{if } M_0 \leq 1.0 \\ 1.0 - 0.075 (M_0 - 1.0)^{1.35} & \text{if } M_0 > 1.0 \end{cases} \]  \hspace{1cm} (3.3)

\[ T_2 = T_0 \left[ 1.0 + \frac{(\gamma_I - 1) M_0^2}{2} \right] \]  \hspace{1cm} (3.4)

\[ P_2 = P_0 \eta_I \left( \frac{T_2}{T_0} \right)^{\gamma_I/(\gamma_I-1)} \]  \hspace{1cm} (3.5)

\[ \gamma_I = \gamma_0 = 1.4 \]  \hspace{1cm} (3.6)

Functions \( f_1 \) and \( f_2 \) are curve fits to atmospheric data [33].
3.2.2 Gas Properties

Functions $f_3$, $f_4$, and $f_5$ are curve fits of data [34] used to compute variable thermodynamic properties. The fuel is assumed to be JP-4 and the following equations are used.

$$c_p = f_3(T, f/a)$$ (3.7)

$$R = f_4(f/a) \approx R_A$$ (3.8)

$$c_v = c_p - \frac{R}{f}$$ (3.9)

$$\gamma = \frac{c_p}{c_v}$$ (3.10)

$$h = f_5(T, f/a)$$ (3.11)

3.2.3 Fan

Fan performance is represented by a set of overall performance maps and separate maps are used for the tip and hub sections [6]. The maps are assumed to represent fan performance with variable geometry at nominal, scheduled positions. Map-generated corrected fan airflow is adjusted to account for off-schedule geometry effects. The fan maps are shown in Figures 3-2 through 3-6. A linear interpolation algorithm is used to determine map values. The following equations are used to define the fan model.
Figure 3-2: Effect of variable inlet guide vane position on fan performance [32].

Figure 3-3: Fan performance maps with inlet guide vanes at their nominally scheduled position (mass flow) [32].
Figure 3-4: Fan performance maps with inlet guide vanes at their nominally scheduled position (tip efficiency) [32].

Figure 3-5: Fan performance maps with inlet guide vanes at their nominally scheduled position (hub pressure ratio) [32].
Figure 3-6: Fan performance maps with inlet guide vanes at their nominally scheduled position (hub efficiency) [32].

\[
(\dot{w}_c)_{fan,M} = f_6 \left( \frac{P_{13}}{P_2}, \frac{N_L}{\theta_2^{1/2}} \right) 
\]

\[
P_{2.1} = P_{2.2} = P_2 f_7 \left( \frac{P_{13}}{P_2}, \frac{N_L}{\theta_2^{1/2}} \right) 
\]

\[
\dot{w}_2 = \frac{(\dot{w}_c)_{fan,M} \delta_2 \left[ 1 + f_8 \left( \frac{FVGP}{N_L}, \frac{N_L}{\theta_2^{1/2}} \right) \right]}{\theta_2^{1/2}} 
\]

\[
\eta_{fan,OD} = f_9 \left( \frac{P_{13}}{P_2}, \frac{N_L}{\theta_2^{1/2}} \right) 
\]

\[
\left( \frac{\Delta T}{T} \right)_{fan,OD, id} = \left( \frac{P_{13}}{P_2} \right)^{\frac{(\gamma_{fan}-1)}{\gamma_{fan}}} - 1.0 
\]

\[
T_{13}' = \left[ \frac{\left( \frac{\Delta T}{T} \right)_{fan,OD, id}}{\eta_{fan,OD}} + 1 \right] T_2 
\]
\[ \eta_{fan,ID} = f_{10} \left( \frac{P_{13}}{P_2}, \frac{N_L}{\theta_2^{1/2}} \right) \]  \hfill (3.18)

\[ \left( \frac{\Delta T}{T} \right)_{fan,ID, id} = \left( \frac{P_{2,1}}{P_2} \right) \frac{1}{\gamma_{fan} - 1} - 1.0 \]  \hfill (3.19)

\[ T_{2,1} = T_{2,2} = \left[ \frac{(\Delta T/T)_{fan,ID, id}}{\eta_{fan,ID}} + 1 \right] T_2 \]  \hfill (3.20)

\[ \gamma_{fan} = \gamma_2 \]  \hfill (3.21)

### 3.2.4 Compressor

A similar procedure is followed for the compressor with overall performance maps (shown in Figures 3-7 through 3-9) being used with a shift in the corrected airflow based on off-schedule values of variable geometry position [6]. The following equations are used to define the compressor model.

![Figure 3-7: Effect of variable stator vane position on compressor performance [32].](image)

Figure 3-7: Effect of variable stator vane position on compressor performance [32].
Figure 3-8: Compressor performance map with stator vanes at their nominally scheduled position (mass flow) [32].

Figure 3-9: Compressor performance map with stator vanes at their nominally scheduled position (efficiency) [32].

\[ (\dot{w}_c)_{C,M} = f_{11} \left( \frac{P_3}{P_{2,2}}, \frac{N_H}{\theta_{2,2}^{1/2}} \right) \]  

(3.22)
\[
\dot{w}_{2.2} = \frac{(\dot{w}_c)_{C,M} \delta_{2.2} [1 + f_{12} \left( CVGP, N_H / \theta_{2.2}^{1/2} \right)]}{\theta_{2.2}^{1/2}} 
\]

(3.23)

\[
\eta_C = f_{13} \left( \frac{P_3}{P_{2.2}}, \frac{N_H}{\theta_{2.2}^{1/2}} \right)
\]

(3.24)

\[
\left( \frac{\Delta T}{T} \right)_{C,id} = \left( \frac{P_3}{P_{2.2}} \right)^{(\gamma_C - 1)/\gamma_C} - 1.0
\]

(3.25)

\[
T_C = \beta_C T_{2.2} + (1 - \beta_C) T_3
\]

(3.26)

\[
T'_3 = \left[ \frac{(\Delta T/T)_{C,id}}{\eta_C} + 1 \right] T_{2.2}
\]

(3.27)

### 3.2.5 Bleeds

Flow through the bleed passages is assumed to be choked and both turbine cooling and overboard bleeds are modeled. The following equations are used.

\[
\left( \frac{\dot{w}}{A} \right)_{BL} = P_3 \left( \frac{g_c \gamma_3}{R_A T_3} \right)^{1/2} \left( \frac{2}{\gamma_3 + 1} \right)^{(\gamma_3 + 1)/(\gamma_3 - 1)}
\]

(3.28)

\[
\dot{w}_{BLHT} = A_{BLHT} \left( \frac{\dot{w}}{A} \right)_{BL}
\]

(3.29)

\[
\dot{w}_{BLLT} = A_{BLLT} \left( \frac{\dot{w}}{A} \right)_{BL}
\]

(3.30)

\[
\dot{w}_{BLOV} = A_{BLOV} \left( \frac{\dot{w}}{A} \right)_{BL}
\]

(3.31)
3.2.6 Turbines

Overall performance of the high and low pressure turbine is represented by bi-variate maps [6] shown in Figures 3-10 through 3-13. Cooling bleed for each turbine is assumed to reenter the cycle at the turbine discharge, although a portion of each bleed is assumed to do work. The following equations are used to define the model.

Figure 3-10: High pressure turbine performance map (flow parameter) [32].

Figure 3-11: High pressure turbine performance map (enthalpy parameter) [32].
Figure 3-12: Low pressure turbine performance map (flow parameter) [32].

Figure 3-13: Low pressure turbine performance map (enthalpy parameter) [32].

\[
(\dot{w}_p)_{HT} = f_{14} \left( \frac{P_{4,1}}{P_4}, \frac{N_H}{T_4^{1/2}} \right) \tag{3.32}
\]

\[
\dot{w}_4 = \frac{(\dot{w}_p)_{HT} P_4 N_H}{T_4} \tag{3.33}
\]
\[(h_p)_{HT} = f_{15} \left( \frac{P_{4,1}}{P_4}, \frac{N_H}{T_4^{1/2}} \right) \quad (3.34)\]

\[(\Delta h)_{HT} = (h_p)_{HT} N_H T_4^{1/2} \quad (3.35)\]

\[(\dot{w}_p)_{LT} = f_{16} \left( \frac{P_5}{P_{4,1}}, \frac{N_L}{T_{4,1}^{1/2}} \right) \quad (3.36)\]

\[\dot{w}_{4,1} = \frac{(\dot{w}_p)_{LT} P_{4,1} N_L}{T_{4,1}} \quad (3.37)\]

\[(h_p)_{LT} = f_{17} \left( \frac{P_5}{P_{4,1}}, \frac{N_L}{T_{4,1}^{1/2}} \right) \quad (3.38)\]

\[(\Delta h)_{LT} = (h_p)_{LT} N_L T_{4,1}^{1/2} \quad (3.39)\]

### 3.2.7 Combustors and Ducts

Total pressure losses are included in the models of the main combustor, bypass duct, mixer entrance, and augmentor. Heat addition associated with the burning of fuel in the main combustor and augmentor is assumed to take place in control volumes \(V_4\) and \(V_7\) respectively. The following equations are used to describe the combustor and duct models.

\[\dot{w}_3 = \left[ \frac{P_3 (P_3 - P_4)}{K_B T_3} \right]^{1/2} \quad (3.40)\]

\[T_B = \beta_B T_3 + (1 - \beta_B) T_4 \quad (3.41)\]
\( \Delta h_B = \eta_B HVF \)  

(3.42)

\[ \eta_B = f_{18} \left( \frac{f}{a} \right)_4 \]  

(3.43)

\( (f/a)_4 = \frac{\dot{w}_{FA}}{\dot{w}_3} \)  

(3.44)

\( P_6 = K_{PR5} P_6 \)  

(3.45)

\[ P'_7 = P_6 - \frac{K_{AB} \dot{w}_6^2 T_6}{P_6} \]  

(3.46)

\[ T_{AB} = \beta_{AB} T_6 + (1 - \beta_{AB}) T_7 \]  

(3.47)

\( \Delta h_{AB} = \eta_{AB} HVF \)  

(3.48)

\[ \eta_{AB} = f_{19} \left[ \frac{f}{a} \right]_7 \]  

(3.49)

\[ (f/a)_7 = \frac{\dot{w}_{F7} + \dot{w}_{FA}}{\dot{w}_6 - \dot{w}_{FA}} \]  

(3.50)

\[ P'_6 = P_{13} - \frac{K_D \dot{w}_{13}^2 T_{13}}{P_{13}} \]  

(3.51)

\[ T_6 = T_{13} \]  

(3.52)
3.2.8 Exhaust Nozzle

A convergent-divergent nozzle configuration is assumed and is defined by the following equations [35].

\[
\dot{w}_7 = P_7 A_E^* C_{d,N} \left( \frac{g_c \gamma_N}{R_A T_7} \right)^{1/2} \left( \frac{2}{\gamma_N} \right)^{(\gamma_N+1)/2(\gamma_N-1)}
\] (3.53)

\[
F_N = \frac{\dot{w}_7 v_E}{g_c} + A_E (P_E - P_0)
\] (3.54)

\[
C_{d,N} = f_{20} \left( \frac{P_0}{P_7} \right)
\] (3.55)

\[
\left( \frac{P_0}{P_7} \right)_{cr} = f_{21} \left( \frac{A_E}{A_8} \right)
\] (3.56)

If \( P_0/P_7 \geq (P_0/P_7)_{cr} \), the flow is subsonic in the nozzle and

\[
P_E = P_0
\] (3.57)

\[
\frac{A_E}{A_E^*} = f_{21}^{-1} \left( \frac{P_0}{P_7} \right)
\] (3.58)

\[
A_E^* = \frac{A_E}{A_E/A_E^*}
\] (3.59)

\[
M_E^* = f_{22} \left( \frac{P_0}{P_7} \right)
\] (3.60)

\[
v_E = M_E^* C_{v,N} \left( \frac{2g_c \gamma_N R_A T_7}{\gamma_N + 1} \right)^{1/2}
\] (3.61)
\[ C_{v,N} = f_{23} \left( \frac{P_0}{P_7} \right) \quad (3.62) \]

A shock may exist in the divergent portion of the nozzle. In that case, shock tables are used to compute the required parameters.

\[ M_x = f_{24} \left( \frac{A_E}{A_8} \right) \quad (3.63) \]

\[ \frac{P_y}{P_x} = f_{25} (M_x) \quad (3.64) \]

\[ \frac{P_y}{p_x} = f_{26} (M_x) \quad (3.65) \]

\[ \frac{P_y}{p_x} = f_{27} (M_x) \quad (3.66) \]

\[ \left( \frac{P_0}{P_7} \right)_{es} = \left( \frac{P_y}{P_x} \right) \left( \frac{p_y/p_x}{P_y/p_x} \right) \quad (3.67) \]

If \( P_0/P_7 = (P_0/P_7)_{es} \), the shock will be in the nozzle exit plane. Then,

\[ P_E = P_0 \quad (3.68) \]

\[ M_{x*} = f_{28} \left( \frac{A_E}{A_8} \right) \quad (3.69) \]

\[ v_x = M_{x*} \left( \frac{2 g_c \gamma_N R_A T_7}{\gamma_N + 1} \right)^{1/2} \quad (3.70) \]

\[ \frac{v_x}{v_y} = f_{29} (M_x) \quad (3.71) \]
\[ v_E = \frac{C_{v,N} v_x}{v_x v_y} \] (3.72)

If \( P_0/P_7 < (P_0/P_7)_{es} \), the shock is external to the nozzle. Then,

\[ M_E^* = f_{28} \left( \frac{A_E}{A_8} \right) \] (3.73)

\[ \frac{P_E}{P_7} = f_{30} \left( \frac{A_E}{A_8} \right) \] (3.74)

\[ P_E = P_7 \left( \frac{P_E}{P_7} \right) \] (3.75)

\[ v_E = M_E^* C_{v,N} \left( \frac{2 g_c \gamma_N R_A T_7}{\gamma_N + 1} \right)^{1/2} \] (3.76)

If \( (P_0/P_7)_{cr} > P_0/P_7 > (P_0/P_7)_{es} \), the shock is in the divergent section of the nozzle. Then,

\[ P_E = P_0 \] (3.77)

\[ \frac{A_x^*}{A_y^*} = \frac{P_y}{P_x} = f_{25} (M_x) \] (3.78)

\[ A_E = \left( \frac{A_E}{A_8} \right) \left( \frac{A_x^*}{A_y^*} \right) \] (3.79)

\[ \frac{P_E}{P_y} = f_{21} \left( \frac{A_E}{A_y^*} \right) \] (3.80)

\[ \frac{P_E}{P_x} = \left( \frac{P_E}{P_y} \right) \left( \frac{P_y}{P_x} \right) \] (3.81)
\[ P_E = P_t \left( \frac{P_E}{P_t} \right) \quad (3.82) \]

To solve these equations, Mx is varied until Equations (3.82) and (3.77) produce the same values for PE. Then,

\[ M_E^* = f_{28} \left( \frac{A_E}{A_y^*} \right) \quad (3.83) \]

The resulting value of \( M_E^* \) is used to compute the nozzle exit velocity.

\[ v_E = M_E^* C_e, N \left( \frac{2g_c \gamma_N R_A T_t}{\gamma_N + 1} \right)^{1/2} \quad (3.84) \]

The net thrust is computed by subtracting the inlet ram drag from the gross thrust.

\[ F_n = F_N - M_0 \dot{w}_2 \left( \frac{\gamma_0 R_A T_0}{g_c} \right)^{1/2} \quad (3.85) \]

### 3.2.9 Intercomponent Volumes

Intercomponent volumes are assumed at engine locations where gas dynamics are considered important and are required to avoid an iterative solution of the equations. Storage of mass and energy occurs in these volumes. The following equations define the dynamic models of the intercomponent volumes.

\[ W_{13} = \int_0^t (\dot{w}_2 - \dot{w}_{2.2} - \dot{w}_{13}) \, dt + W_{13,i} \quad (3.86) \]

\[ T_{13} = \int_0^t \left( \frac{\left( \dot{w}_2 - \dot{w}_{2.2} \right) \left( h'_{13} - h_{13} \right)}{c_{v,13}} + T_{13} \left( \dot{w}_2 - \dot{w}_{2.2} - \dot{w}_{13} \right) \left( \gamma_{13} - 1 \right) } \right) \frac{W_{13}}{W_{13}} \, dt + T_{13,i} \quad (3.87) \]
\[ P_{13} = \frac{RAW_{13}T_{13}}{V_{13}} \quad (3.88) \]

\[ W_3 = \int_0^t (\dot{w}_{2.2} - \dot{w}_{BLHT} - \dot{w}_{BLLT} - \dot{w}_{BLOV} - \dot{w}_3) \, dt + W_{3,i} \quad (3.89) \]

\[ T_3 = \int_0^t \left( \frac{\dot{w}_{2.2}(h'_3 - h_3)}{c_{v,3}} + T_3(\dot{w}_{2.2} - \dot{w}_{BLHT} - \dot{w}_{BLLT} - \dot{w}_{BLOV} - \dot{w}_3)(\gamma_3 - 1) \right) \frac{W_3}{W_3} \, dt + T_{3,i} \quad (3.90) \]

\[ P_3 = \frac{R_A W_3 T_3}{V_3} \quad (3.91) \]

\[ W_4 = \int_0^t (\dot{w}_3 + \dot{w}_{F,4} - \dot{w}_4) \, dt + W_{4,i} \quad (3.92) \]

\[ T_4 = \int_0^t \left( \frac{\dot{w}_3 h_B + \dot{w}_{F,4} \Delta h_B - h_4 (\dot{w}_3 + \dot{w}_{F,4})}{c_{v,4}} + T_4(\dot{w}_3 + \dot{w}_{F,4} - \dot{w}_4)(\gamma_4 - 1) \right) \frac{W_4}{W_4} \, dt + T_{4,i} \quad (3.93) \]

\[ P_4 = \frac{R_A W_4 T_4}{V_4} \quad (3.94) \]

\[ W_{4,1} = \int_0^t (\dot{w}_4 + \dot{w}_{BLHT} - \dot{w}_{4,1}) \, dt + W_{4,1,i} \quad (3.95) \]
\[ T_{4.1} = \int_0^t \left( \frac{[\dot{w}_4(h_4 - \Delta h_{HT}) + \dot{w}_{BLHT}(h_3 - K_{BLWHT}\Delta h_{HT}) - h_{4.1}(\dot{w}_4 + \dot{w}_{BLHT})]/c_{v,4.1} + T_{4.1}(\dot{w}_4 + \dot{w}_{BLHT} - \dot{w}_{4.1})(\gamma_{4.1} - 1)}{W_{4.1}} \right) dt + T_{4.1,i} \]  

(3.96)

\[ P_{4.1} = \frac{R_A W_{4.1} T_{4.1}}{V_{4.1}} \]  

(3.97)

\[ W_6 = \int_0^t (\dot{w}_{4.1} + \dot{w}_{BLLT} + \dot{w}_{13} - \dot{w}_6) \, dt + W_{6,i} \]  

(3.98)

\[ T_6 = \int_0^t \left( \frac{[\dot{w}_{4.1}(h_{4.1} - \Delta h_{LT}) + \dot{w}_{BLLT}(h_3 - K_{BLWLT}\Delta h_{LT}) + \dot{w}_{13}h_{16} - h_6(\dot{w}_{4.1} + \dot{w}_{BLLT} + \dot{w}_{13})]/c_{v,6} + T_6(\dot{w}_{4.1} + \dot{w}_{BLLT} + \dot{w}_{13} - \dot{w}_6)(\gamma_{6} - 1)}{W_6} \right) dt + T_{6,i} \]  

(3.99)

\[ P_6 = \frac{R_A W_6 T_6}{V_6} \]  

(3.100)

\[ W_7 = \int_0^t (\dot{w}_6 + \dot{w}_{F,7} - \dot{w}_7) \, dt + W_{7,i} \]  

(3.101)

\[ T_7 = \int_0^t \left( \frac{\dot{w}_6 h_{AB} + \dot{w}_{F,7} \Delta h_{AB} - h_7(\dot{w}_6 + \dot{w}_{F,7})}{c_{v,7}} + T_7(\dot{w}_6 + \dot{w}_{F,7} - \dot{w}_7)(\gamma_7 - 1)}{W_7} \right) dt + T_{7,i} \]  

(3.102)

\[ P_7 = \frac{R_A W_7 T_7}{V_7} \]  

(3.103)
3.2.10 Fluid Momentum

The effects of fluid momentum are considered in the bypass duct and augmentor duct models.

\[ \dot{w}_{13} = g_c \left( \frac{A}{l} \right)_D \int_0^t \left( P'_6 - P_6 \right) dt + \dot{w}_{13,i} \quad (3.104) \]

\[ \dot{w}_6 = g_c \left( \frac{A}{l} \right)_{AB} \int_0^t \left( P'_7 - P_7 \right) dt + \dot{w}_{6,i} \quad (3.105) \]

3.2.11 Rotor Inertias

Rotor speeds are computed from dynamic forms of the angular momentum equations.

\[ N_L = \left( \frac{30}{\pi} \right)^2 \frac{J}{I_L} \int_0^t \left( \frac{\Delta h_{LT} (\dot{w}_{4,1} + K_{BWLT} \dot{w}_{BLLT})}{N_L} \right) - (\dot{w}_2 - \dot{w}_{2,2}) (h'_{13} - h_2) - \dot{w}_{2,2} (h_{2,2} - h_2) dt + N_{L,i} \quad (3.106) \]

\[ N_H = \left( \frac{30}{\pi} \right)^2 \frac{J}{I_H} \int_0^t \left( \frac{\Delta h_{HT} (\dot{w}_4 + K_{BLWHT} \dot{w}_{BLHT})}{N_H} \right) - \dot{w}_{2,2} (h'_{3} - h_{2,2}) dt + N_{H,i} \quad (3.107) \]

3.2.12 Correction Coefficients

To balance the engine at the design point, correction coefficients are introduced. If the design point data (which are specified as input) are not exact, incompatibilities between the data and the engine model will result in non-zero derivatives or
mismatches between the specified and calculated outputs of the component maps. To compensate for these differences, a “self-trimming” feature is built in through the use of correction coefficients. These coefficients are calculated using the design point values and balance the engine by making the derivative terms become zero and compensating for numerical inconsistencies created when values are interpolated from performance map data. The correction coefficients are then part of the model and are used at both design and off-design points. Ideally,

\[ P_{2,a} = P_{2,D} \]  
\[ T_{2,a} = T_{2,D} \]  
\[ P_{0,a} = P_{0,D} \]

However, if they are not equal, the equations that use these values will be scaled by correction coefficients. The scaling coefficients for the inlet conditions are,

\[ CC_1 = \frac{P_{2,D}}{P_{2,a}} \]  
\[ CC_2 = \frac{T_{2,D}}{T_{2,a}} \]  
\[ CC_3 = \frac{P_{0,D}}{P_{0,a}} \]

Equations (3.1), (3.4), and (3.5) are then modified as shown below.

\[ P_0 = f_1(a) \times CC_3 \]
\[ T_2 = T_0 \left[ 1.0 + \frac{(\gamma_I - 1) M_0^2}{2} \right] \times CC_2 \]  

(3.115)

\[ P_2 = P_0 \eta_I \left( \frac{T_2}{T_0} \right)^{\gamma_I/(\gamma_I-1)} \times CC_1 \]  

(3.116)

The equations used to calculate the correction coefficients for the fan are shown below.

\[ CC_4 = \frac{\dot{\omega}_{2,D}}{\dot{\omega}_{2,a}} \]  

(3.117)

\[ CC_5 = \frac{(\Delta T/T)_{\text{fan,OD,}id,D}}{\eta_{\text{fan,OD,}D} \left( T_{13,\text{D}}/T_{2,\text{D}} - 1.0 \right)} \]  

(3.118)

\[ CC_6 = \frac{P_{2,2,D}}{P_{2,2,a}} \]  

(3.119)

\[ CC_7 = \frac{(\Delta T/T)_{\text{fan,ID,}id,D}}{\eta_{\text{fan,ID,}D} \left( T_{2,2,\text{D}}/T_{2,\text{D}} - 1.0 \right)} \]  

(3.120)

Equations (3.13), (3.14), (3.17), and (3.20) are then modified as shown below.

\[ P_{2,1} = P_{2,2} = P_2 f_7 \left( \frac{P_{13}}{P_2}, \frac{N_L}{\theta_{2,1/2}} \right) \times CC_6 \]  

(3.121)

\[ \dot{\omega}_2 = \frac{(\dot{\omega}_{c})_{\text{fan,M}} \delta_2 \left[ 1 + f_8 \left( \frac{FVGP, N_L}{\theta_{2,1/2}} \right) \right]}{\theta_{2,1/2}} \times CC_4 \]  

(3.122)

\[ T_{13}' = \left[ \frac{(\Delta T/T)_{\text{fan,OD,}id}}{\eta_{\text{fan,OD}} \times CC_5} + 1 \right] \times T_2 \]  

(3.123)
\[ T_{2,1} = T_{2,2} = \left[ \frac{(\Delta T/T)_{\text{fan,ID},\text{id}}}{\eta_{\text{fan,ID}} \times CC_7} + 1 \right] T_2 \] (3.124)

The procedure is repeated for the compressor.

\[ CC_8 = \frac{\dot{w}_{2,2,D}}{\dot{w}_{2,2,a}} \] (3.125)

\[ CC_9 = \frac{(\Delta T/T)_{C,\text{id},D}}{\eta_{C,D} \left( \frac{T_{2,2,D}}{T_{2,D}} - 1.0 \right)} \] (3.126)

Equations (3.23) and (3.27) are then modified as shown below.

\[ \dot{w}_{2,2} = (\dot{w}_c)_{C,M} \delta_{2,2} \left[ 1 + f_{12} \left( \frac{CVGP,N_H}{\theta_{2,2}^{1/2}} \right) \right] \times CC_8 \] (3.127)

\[ T'_3 = \left[ \frac{(\Delta T/T)_{C,\text{id}}}{\eta_C \times CC_9} + 1 \right] T_{2,2} \] (3.128)

The correction coefficients for the turbines are shown below.

\[ CC_{11} = \frac{\dot{w}_{4,D}}{\dot{w}_{4,a}} \] (3.129)

\[ CC_{13} = \frac{\dot{w}_{4,1,D}}{\dot{w}_{4,1,a}} \] (3.130)

Equations (3.33) and (3.37) are then modified as shown below.

\[ \dot{w}_4 = \frac{(\dot{w}_p)_{HT} P_4 N_H}{T_4} \times CC_{11} \] (3.131)

\[ \dot{w}_{4,1} = \frac{(\dot{w}_p)_{LT} P_{4,1} N_L}{T_{4,1}} \times CC_{13} \] (3.132)

The next set of correction coefficients zeros out the state variable derivatives.
associated with energy balances in the intercomponent volumes.

\[ CC_{10} = \frac{h_{4,D} (\dot{w}_{3,D} + \dot{w}_{F,4,D}) - \dot{w}_{3,D} h_{B,D}}{\dot{w}_{F,4,D} \Delta h_{B,D}} \]  \hspace{1cm} (3.133)

\[ CC_{12} = \frac{\dot{w}_{4,D} h_{4,D} + \dot{w}_{BLHT,D} h_{3,D} - h_{4,1,D} (\dot{w}_{4,D} + \dot{w}_{BLHT,D})}{\Delta h_{HT,D} (\dot{w}_{4,D} + \dot{w}_{BLHT,D} K_{BLWHT,D})} \]  \hspace{1cm} (3.134)

\[ CC_{14} = \frac{\left( \frac{\dot{w}_{4,1,D} h_{4,1,D} + \dot{w}_{BLLT,D} h_{3,D} + \dot{w}_{13,D} h_{16,D}}{-h_{6,D} (\dot{w}_{4,1,D} + \dot{w}_{BLLT,D} + \dot{w}_{13,D})} \right)}{\Delta h_{LT,D} (\dot{w}_{4,1,D} + \dot{w}_{BLLT,D} K_{BLWLT,D})} \]  \hspace{1cm} (3.135)

The steady state versions of Equations (3.92), (3.95), and (3.98) are shown below as well as the modified Equations (3.93), (3.96), and (3.99).

\[ \dot{w}_3 + \dot{w}_{F,4} - \dot{w}_4 = 0 \]  \hspace{1cm} (3.136)

\[ T_4 = \int_0^t \left( \frac{\dot{w}_3 h_B + \dot{w}_{F,4} \Delta h_B \times CC_{10} - h_4 (\dot{w}_3 + \dot{w}_{F,4})}{c_{v,4}} + T_4 (\dot{w}_3 + \dot{w}_{F,4} - \dot{w}_4) (\gamma_4 - 1) \right) \frac{dt}{W_4} + T_{4,i} \]  \hspace{1cm} (3.137)

\[ \dot{w}_4 + \dot{w}_{BLHT} - \dot{w}_{4,1} = 0 \]  \hspace{1cm} (3.138)

\[ T_{4,1} = \int_0^t \left( \frac{[\dot{w}_4 h_4 + \dot{w}_{BLHT} h_3 - h_{4,1} (\dot{w}_4 + \dot{w}_{BLHT}) - CC_{12} \Delta h_{HT,D} (\dot{w}_4 + K_{BLWHT} \dot{w}_{BLHT})] / c_{v,4,1} + T_{4,1} (\dot{w}_4 + \dot{w}_{BLHT} - \dot{w}_{4,1}) (\gamma_{4,1} - 1)}{W_{4,1}} \right) dt + T_{4,1,i} \]  \hspace{1cm} (3.139)
\[ \dot{w}_{4.1} + \dot{w}_{BLLT} + \dot{w}_{13} - \dot{w}_6 = 0 \]  

\[ T_6 = \int_0^t \left( \frac{[\dot{w}_{4.1}h_{4.1} + \dot{w}_{BLLT}h_3 + \dot{w}_{13}h_{16} - CC_{14}\Delta h_{LT} (\dot{w}_{4.1} + K_{BLWLT}\dot{w}_{BLLT}) - h_6 (\dot{w}_{4.1} + \dot{w}_{BLLT} + \dot{w}_{13})/c_{v, 6} + T_6 (\dot{w}_{4.1} + \dot{w}_{BLLT} + \dot{w}_{13} - \dot{w}_6) (\gamma_6 - 1)]}{W_6} \right) dt + T_{6,i} \]  

The following two correction coefficients shown below are used to zero the speed derivatives at the design point.

\[ CC_{15} = \frac{\dot{w}_{2.2,D} (h_{3,D} - h_{2.2,D})}{\Delta h_{HT,D} (\dot{w}_{4,D} + \dot{w}_{BLHT,D} K_{BLWHT,D})} \]  

\[ CC_{16} = \frac{(\dot{w}_{2,D} - \dot{w}_{2.2,D}) (h_{13,D} - h_{2,D}) + \dot{w}_{2.2,D} (h_{2.2,D} - h_{2,D})}{\Delta h_{LT,D} (\dot{w}_{4.1,D} + \dot{w}_{BLLT,D} K_{BLWLT,D})} \]  

Equations (3.106) and (3.107) are then modified as shown below.

\[ N_L = (\frac{30}{\pi})^2 \frac{J}{T_L} \int_0^t \left( \frac{\Delta h_{LT} (\dot{w}_{4.1} + K_{BLWLT}\dot{w}_{BLLT}) \times CC_{16} - (\dot{w}_2 - \dot{w}_{2.2}) (h_{13} - h_2) - \dot{w}_{2.2} (h_{2.2} - h_2)}{N_L} \right) dt + N_{L,i} \]  

\[ N_H = (\frac{30}{\pi})^2 \frac{J}{T_H} \int_0^t \left( \frac{\Delta h_{HT} (\dot{w}_4 + K_{BLWHT}\dot{w}_{BLHT}) \times CC_{15} - \dot{w}_{2.2} (h_{3} - h_{2.2})}{N_H} \right) dt + N_{H,i} \]
The following correction coefficients are used to compensate for the imbalances in the augmentor and nozzle models.

\[ CC_{17} = \frac{\dot{w}_{7,D}}{\dot{w}_{7,a}} \quad (3.146) \]

\[ CC_{18} = \frac{h_{7,D} (\dot{w}_{6,D} + \dot{w}_{F,7,D}) - \dot{w}_{6,D} h_{AB,D}}{\dot{w}_{F,7,D} \Delta h_{AB,D}} \quad (3.147) \]

\[ CC_{19} = \frac{F_{N,D} - A_E (P_E - P_0)}{F_{N,a} - A_E (P_E - P_0)} \quad (3.148) \]

The steady state version of Equation (3.101) and the modified Equations (3.53), (3.54), and (3.102) are shown below.

\[ \dot{w}_6 + \dot{w}_{F,7} - \dot{w}_7 = 0 \quad (3.149) \]

\[ \dot{w}_7 = P_7 A_E^* C_{d,N} \left( \frac{g_c \gamma_N}{R_A T_7} \right)^{1/2} \left( \frac{2}{\gamma_N + 1} \right)^{(\gamma_N+1)/2(\gamma_N-1)} \times CC_{17} \quad (3.150) \]

\[ F_N = \frac{\dot{w}_7 V_E}{g_c} \times CC_{19} + A_E (P_E - P_0) \quad (3.151) \]

\[ T_7 = \int_0^t \left( \frac{\dot{w}_6 h_{AB} + \dot{w}_{F,7} \Delta h_{AB} \times CC_{18} - h_7 (\dot{w}_6 + \dot{w}_{F,7})}{c_{v,7}} \right) \frac{+ T_7 (\dot{w}_6 + \dot{w}_{F,7} - \dot{w}_7) (\gamma_7 - 1)}{W_7} \ dt + T_{7,i} \quad (3.152) \]

The correction coefficients calculated for the dry design point are shown in Table 3.1.
Table 3.1: Correction coefficients for the dry design point.

<table>
<thead>
<tr>
<th>CC</th>
<th>Value</th>
<th>CC</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>11</td>
<td>1.00109</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>12</td>
<td>1.02671</td>
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<td>1</td>
<td>13</td>
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<td>7</td>
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</tr>
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<td>9</td>
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<td>1.00893</td>
</tr>
<tr>
<td>10</td>
<td>1.00243</td>
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</tr>
</tbody>
</table>

3.2.13 Transient Control Inputs

During the transient simulation, the values of six inputs are varied over a period of 20 seconds. These inputs are the combustor and augmentor fuel flow rates, nozzle throat and exit areas, and the compressor and fan variable geometry parameters. A plot of these inputs is shown in Figure 3-14.

The results obtained from the simulations are discussed in Chapter 4.
Figure 3-14: Transient control inputs.
Chapter 4

Verification and Comparison of Simulation Tools for Gas Turbines

4.1 Simulation Results

The variation of rotational speed, pressure and temperature in response to the transient control inputs over a period of 20 seconds is shown in Figures 4-2 to 4-15. The individual results from Dymola, Simulink and NPSS are superimposed over the results from DIGTEM. A fixed time step of 0.0005 s and the default solver was used in the three modern tools. Dymola uses the DASSL solver by default, Simulink uses the Euler solver and NPSS uses the Gear first order implicit solver. From the plots, one can see that the results from the three modern tools are fairly close to those from DIGTEM. A few of the plots show differences such as the low pressure spool rotational speed (Figure 4-15) and the augmentor inlet pressure and temperature (Figures 4-6 and 4-12). However, these differences are fairly small and range from about 1 to 5%.

For performance comparison purposes, the 20-second transient was repeated using identical solver parameters. Once again, a fixed time step of 0.0005 s was used. The Euler solver was chosen since it was the only one common to all three programs.
Simulink and NPSS were run on a 3.8 GHz Pentium 4 PC with 1024MB RAM while Dymola was run on 2.4 GHz Pentium 4 PC with 512MB RAM. Both machines were running the latest updated version of the Windows XP Professional operating system. Table 4.1 shows the time in seconds taken by each program to simulate the 20-second transient using the Euler solver and the default solver of each individual program.

<table>
<thead>
<tr>
<th>Solver</th>
<th>Dymola</th>
<th>Simulink</th>
<th>NPSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler</td>
<td>25.6 s</td>
<td>47 s</td>
<td>150 s</td>
</tr>
<tr>
<td>Default</td>
<td>7.5 s (DASSL)</td>
<td>47 s (Euler)</td>
<td>148.3 s (Gear)</td>
</tr>
</tbody>
</table>

A time step sensitivity study was also performed using the Euler solver. In the case of Dymola and Simulink, it was found that the largest time step that could be used was 0.00058 s, whereas with NPSS, it was 0.002 s. Values larger than these resulted in the inputs to the performance maps going out of range and subsequent convergence problems. The time steps used and the simulation run times are summarized in Table 4.2 and depicted graphically in Figure 4-1. In all cases, the differences observed in the results of the transient simulation were smaller than 0.01.

<table>
<thead>
<tr>
<th>Time step in seconds</th>
<th>Dymola</th>
<th>Simulink</th>
<th>NPSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005 s</td>
<td>25.6 s</td>
<td>47 s</td>
<td>150 s</td>
</tr>
<tr>
<td>0.00058 s</td>
<td>22.7 s</td>
<td>41.4 s</td>
<td>129.3 s</td>
</tr>
<tr>
<td>0.00075 s</td>
<td>-</td>
<td>-</td>
<td>102.8 s</td>
</tr>
<tr>
<td>0.001 s</td>
<td>-</td>
<td>-</td>
<td>79.8 s</td>
</tr>
<tr>
<td>0.0015 s</td>
<td>-</td>
<td>-</td>
<td>56.4 s</td>
</tr>
<tr>
<td>0.002 s</td>
<td>-</td>
<td>-</td>
<td>43.2 s</td>
</tr>
</tbody>
</table>
Figure 4-1: Time sensitivity results.

Figure 4-2: Variation of bypass duct inlet pressure during the transient.
Figure 4-3: Variation of combustor inlet pressure during the transient.

Figure 4-4: Variation of high pressure turbine inlet pressure during the transient.
Figure 4-5: Variation of low pressure turbine inlet pressure during the transient.

Figure 4-6: Variation of augmentor inlet pressure during the transient.
Figure 4-7: Variation of nozzle inlet pressure during the transient.

Figure 4-8: Variation of bypass duct inlet temperature during the transient.
Figure 4-9: Variation of combustor inlet temperature during the transient.

Figure 4-10: Variation of high pressure turbine inlet temperature during the transient.
Figure 4-11: Variation of low pressure turbine inlet temperature during the transient.

Figure 4-12: Variation of augmentor inlet temperature during the transient.
Figure 4-13: Variation of nozzle inlet temperature during the transient.

Figure 4-14: Variation of high pressure spool speed during the transient.
4.2 Comments/Observations

From the results, one can see that Dymola, Simulink and NPSS produce nearly identical results. One can also take advantage of the object-oriented nature of these tools when writing code and hence they are well suited for use in the simulation of propulsion systems. Among the three programs, Dymola was found to be the fastest and the easiest to use. One of the advantages of Dymola is that the equations being solved do not need rearranging and they do not have to be entered strictly in the order in which they are to be solved. Another time-saving advantage is that plots of every possible variable used in the simulation as a function of time can be viewed as soon as the simulation is completed, eliminating the need for post-processing software. A disadvantage of Dymola, which uses the Modelica programming language, is a lack of extensive documentation.

On the other hand, Simulink, aside from being extremely well documented by Mathworks, is also the subject of numerous books containing plenty of examples that highlight the program’s features. Like Dymola, Simulink also includes a post-
processor that enables the viewing of results during and after the simulation. Simulink also permits the embedding of programs and functions written using MATLAB.

NPSS was found to be the slowest in simulating the mathematical model in this work. Despite being more flexible than Dymola and Simulink in selecting the time step, NPSS still took longer to simulate the model. NPSS is available in a GUI version and a command line version. An attempt to use the GUI version of the program was rendered useless due to the CPU usage spiking to 100% when performing even mundane tasks like opening a file. It was also felt that the inclusion of post-processing functions would be beneficial and the documentation could use some improvement.
Chapter 5

Experimental Setup for Measuring Pressure Drops Across Valves

5.1 Experimental Setup and Procedure

Figure 5-1: Equipment used in the experiment.

Figure 5-1 shows the equipment that forms a part of the experimental setup. Ambient air is drawn in by the two compressors, each of which has an inlet capacity of 1000 acfm and can develop outlet pressures up to 225 psi. Following compression, the
air passes through the dryers and is finally stored in the two tanks. Each compressed air storage tank has a diameter of 66 in. and a height of 228 in. and a rated capacity of 3000 gallons. These tanks serve as reservoirs feeding air at a constant stagnation pressure to the pipe/valve setup described next.

As seen in Figure 5-2, the valve being tested is located between two flanged data-measuring sections. Each of these sections consists of a spacer sandwiched between two flanges and houses a pitot static tube. A schematic is shown in Figure 5-3. The pressure transducers and thermocouples are wired to a LabVIEW data acquisition system. The pressure transducers have a range of 0 to 300 psi (absolute), response time of 1 ms and an accuracy of 0.25% BFSL. A gate valve is provided at the end of a fixed length of pipe after the downstream flange section to throttle the flow. Generic gate, butterfly and ball valves were selected for testing and sectional views of these valves are shown in Figures 6-1, 6-18 and 6-35.

5.2 Testing Procedure

The following procedure is used to collect the data:
Figure 5-3: Schematic of the data-measuring section.

1. The compressor control panel is set to maintain a constant pressure of 110 psi in the storage tanks.

2. The test valve (for eg. 1.25” nominal diameter) is opened so that the flow area is a quarter of the total area.

3. Starting at fully shut, the throttling gate valve at the end is opened in small increments and held at each displacement position for short intervals of time (around 6 to 8 seconds) until it is fully open. Data recording begins shortly before the throttling gate valve is opened for the first time and is stopped shortly after the throttling gate valve is fully open.

4. Adjust the test valve so that flow area is half of the total area and repeat step 3.

5. Adjust the test valve so that flow area is three quarters of the total area and repeat step 3.
6. Adjust the test valve so that it is fully open and repeat step 3.

7. Repeat steps 1 through 6 for storage tank pressures of 135 psi, 160 psi, 185 psi and 210 psi.

8. Repeat steps 1 through 7 for a different sized valve, i.e. 1”, 0.75” and 0.5” nominal diameter.

9. Repeat steps 1 through 8 for a different valve type, i.e. gate, butterfly and ball.

## 5.3 Determination of the Valve Flow Area

Valve flow areas were calculated using a projected view as if one were looking through the valve from end to end. Accordingly, the shape of the flow area would be a perfect circle when the valve was fully open. Figure 5-4 shows what the flow areas look like in the case of the three different valves. The hatched area in the figure is the valve body.

![Valve flow areas](image-url)
Chapter 6

Valve Results

The pressure loss ratio \( x_t \) and dimensionless mass flow rate \( G_t \) are defined as shown below [13].

\[
x_t = \frac{P_1 - P_2}{P_1} \quad (6.1)
\]

\[
G_t = \frac{\dot{m}_t \sqrt{RT_1}}{AP_1} \quad (6.2)
\]

Similarly, the stagnation pressure loss ratio is also defined as shown in Equation 6.3.

\[
x_{t,s} = \frac{P_{01} - P_{02}}{P_{01}} \quad (6.3)
\]

The pressure loss is related to the valve coefficient by Equation 6.4.

\[
P_1 - P_2 = k \rho \frac{V_1^2}{2} \quad (6.4)
\]
6.1 Gate Valve Results

6.1.1 Sectional View

Figure 6-1 shows a sectional view of the gate valves used in this experiment.

![Sectional view of the gate valve.](image)

6.1.2 Static Pressure Losses

Figures 6-2 - 6-5 show plots of $G_t$ versus $x_t$ for the different valve sizes. A curve fit of the data from [13] using Equation 6.5 for the 1.5” ball valve has been superimposed on to each plot. It should be noted that [13] neglects to mention the setting at which the ball valve was tested, i.e. fully open, three quarter open, half open or quarter open.

$$G_t = 0.0625 \ln x_t + 0.361 \quad (6.5)$$

It is evident from the plots that the same linear relationship (although with the
A larger valve opening, which implies a lower resistance to the flow, results in a lower pressure loss for the same mass flow rate. This trend is common to all four sizes.

Figure 6-2: Static pressure loss ratio versus mass flow rate for the 0.5” gate valve.

Figure 6-3: Static pressure loss ratio versus mass flow rate for the 0.75” gate valve.
6.1.3 Stagnation Pressure Losses

Figures 6-6 - 6-9 show plots of the stagnation pressure loss ratio $x_{t,s}$ versus the dimensionless mass flow rate $G_t$. A linear trend, almost identical to that observed in
the case of the static pressure loss ratio, is evident in this case also.

Figure 6-6: Stagnation pressure loss ratio versus mass flow rate for the 0.5” gate valve.

Figure 6-7: Stagnation pressure loss ratio versus mass flow rate for the 0.75” gate valve.
Figure 6-8: Stagnation pressure loss ratio versus mass flow rate for the 1” gate valve.

Figure 6-9: Stagnation pressure loss ratio versus mass flow rate for the 1.25” gate valve.

6.1.4 Stagnation Pressure Losses as a Function of $P_2/P_{01}$

Figures 6-10 - 6-13 shows the variation of stagnation pressure loss ratio with the ratio $P_2/P_{01}$. The ratio, $P_2/P_{01}$, generally used in the context of converging-diverging
nozzles, is a measure of the mass flow rate and the compressibility of the flow, with smaller values indicating a larger mass flow rate passing through the valve. A larger mass flow rate would imply a higher flow speed through the valve and one would expect higher pressure losses. Pressure losses would also depend on the valve opening, with a smaller opening producing larger pressure losses and vice-versa.

Figure 6-10: Stagnation pressure loss ratio versus $P_2/P_{01}$ for the 0.5” gate valve.
Figure 6-11: Stagnation pressure loss ratio versus $P_2/P_{01}$ for the 0.75" gate valve.

Figure 6-12: Stagnation pressure loss ratio versus $P_2/P_{01}$ for the 1" gate valve.
Figure 6-13:  Stagnation pressure loss ratio versus $P_2/P_{01}$ for the 1.25” gate valve.

### 6.1.5 Valve Coefficient

Plots of $k$ versus the upstream Mach number $M_1$ are shown in Figures 6-14 - 6-17. For the fully open and three quarter open setting, $k$ appears to be constant for all the valve sizes. For the half open setting, $k$ is constant at slower flow rates and there appears to be some variation at the higher Mach numbers. There appears to be no discernible common trend for the quarter open setting. This could be due to the fact that at the quarter open setting, the valve chokes at a fairly low upstream Mach number of approximately 0.10. The valve coefficients are summarized in Table 6.1. For better scaling, plots of $k$ versus $M_1$ excluding the quarter open setting are included in Appendix A.
Table 6.1: Gate valve coefficient $k$.

<table>
<thead>
<tr>
<th>Size</th>
<th>Setting</th>
<th>Valve coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5”</td>
<td>Fully open</td>
<td>1.8541</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>3.0367</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>8.8980</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>0.75”</td>
<td>Fully open</td>
<td>1.3197</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>2.0414</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>5.085</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>1”</td>
<td>Fully open</td>
<td>0.8911</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>1.4733</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>6.1250</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>1.25”</td>
<td>Fully open</td>
<td>1.2351</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>1.7633</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>7.9997</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>Inconclusive</td>
</tr>
</tbody>
</table>

Figure 6-14: Valve coefficient versus the upstream Mach number for the 0.5” gate valve.
Figure 6-15: Valve coefficient versus the upstream Mach number for the 0.75" gate valve.

Figure 6-16: Valve coefficient versus the upstream Mach number for the 1" gate valve.
6.2 Butterfly Valve Results

6.2.1 Sectional View

Figure 6-18 shows a sectional view of the butterfly valves used in this experiment.

6.2.2 Static Pressure Losses

Plots of $G_t$ versus $x_t$ are shown in Figures 6-19 - 6-22. Just as in the case of the gate valves, higher pressure losses are observed at larger mass flow rates and with a smaller valve opening.
Figure 6-18: Sectional view of the butterfly valve.

Figure 6-19: Static pressure loss ratio versus mass flow rate for the 0.5” butterfly valve.
Figure 6-20: Static pressure loss ratio versus mass flow rate for the 0.75” butterfly valve.

Figure 6-21: Static pressure loss ratio versus mass flow rate for the 1” butterfly valve.
6.2.3 Stagnation Pressure Losses

Plots of $x_{ts}$ versus $G_t$ are shown in Figures 6-23 - 6-26. These plots exhibit trends similar to those seen in the case of static pressure losses in section 6.2.2.
Figure 6-24: Stagnation pressure loss ratio versus mass flow rate for the 0.75” butterfly valve.

Figure 6-25: Stagnation pressure loss ratio versus mass flow rate for the 1” butterfly valve.
Figure 6-26: Stagnation pressure loss ratio versus mass flow rate for the 1.25” butterfly valve.

6.2.4 Stagnation Pressure Losses as a Function of $P_2/P_{01}$

Plots of $x_{ls}$ versus $P_2/P_{01}$ are shown in Figures 6-27 - 6-30. At higher flow speeds and smaller valve openings, pressure losses are higher. These trends are common to all four valve sizes.
Figure 6-27: Stagnation pressure loss ratio versus $P_2/P_{01}$ for the 0.5” butterfly valve.

Figure 6-28: Stagnation pressure loss ratio versus $P_2/P_{01}$ for the 0.75” butterfly valve.
Figure 6-29: Stagnation pressure loss ratio versus $P_2/P_{01}$ for the 1” butterfly valve.

Figure 6-30: Stagnation pressure loss ratio versus $P_2/P_{01}$ for the 1.25” butterfly valve.

6.2.5 Valve Coefficient

Plots of $k$ versus the upstream Mach number $M_1$ are shown in Figures 6-31 - 6-34. For the butterfly valves also, $k$ appears to be constant for all the valve sizes at the
fully open and three quarter open setting. Just as in the case of the gate valves, for the half open setting, $k$ is constant at slower flow rates, but there appears to be slightly more variation at the higher Mach numbers. Once again, there appears to be no discernible common trend for the quarter open setting. The valve coefficients are summarized in Table 6.2. Appendix A shows plots of $k$ versus $M_1$ excluding the quarter open setting.

Table 6.2: Butterfly valve coefficient $k$.

<table>
<thead>
<tr>
<th>Size</th>
<th>Setting</th>
<th>Valve coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5”</td>
<td>Fully open</td>
<td>1.6561</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>1.7666</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>3.6797</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>0.75”</td>
<td>Fully open</td>
<td>1.4172</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>1.9847</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>4.2390</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>1”</td>
<td>Fully open</td>
<td>1.3861</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>2.2332</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>3.7020</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>1.25”</td>
<td>Fully open</td>
<td>1.5471</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>1.6294</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>3.7213</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>Inconclusive</td>
</tr>
</tbody>
</table>
Figure 6-31: Valve coefficient versus the upstream Mach number for the 0.5” butterfly valve.

Figure 6-32: Valve coefficient versus the upstream Mach number for the 0.75” butterfly valve.
Figure 6-33: Valve coefficient versus the upstream Mach number for the 1” butterfly valve.

Figure 6-34: Valve coefficient versus the upstream Mach number for the 1.25” butterfly valve.
6.3 Ball Valve Results

6.3.1 Sectional View

Figure 6-35 shows a sectional view of the ball valves used in this experiment.

![Figure 6-35: Sectional view of the ball valve.](image)

6.3.2 Static Pressure Losses

Linear trends, similar to those in the case of gate and butterfly valves are observed in the graphs for the ball valves as well. These are shown in Figures 6-36 - 6-39 for $G_t$ versus $x_t$. 

76
Figure 6-36: Static pressure loss ratio versus mass flow rate for the 0.5” ball valve.

Figure 6-37: Static pressure loss ratio versus mass flow rate for the 0.75” ball valve.
Figure 6-38: Static pressure loss ratio versus mass flow rate for the 1” ball valve.

Figure 6-39: Static pressure loss ratio versus mass flow rate for the 1.25” ball valve.

6.3.3 Stagnation Pressure Losses

Figures 6-40 - 6-43 show plots of $x_{t,s}$ versus $G_t$. The results are consistent with those obtained for the previous two types of valves and linear trends are observed in
this case as well.

Figure 6-40: Stagnation pressure loss ratio versus mass flow rate for the 0.5” ball valve.

Figure 6-41: Stagnation pressure loss ratio versus mass flow rate for the 0.75” ball valve.
Figure 6-42: Stagnation pressure loss ratio versus mass flow rate for the 1” ball valve.

Figure 6-43: Stagnation pressure loss ratio versus mass flow rate for the 1.25” ball valve.

6.3.4 Stagnation Pressure Losses as a Function of $P_2/P_{01}$

Figures 6-44 - 6-47 show plots of $x_{t,s}$ versus $P_2/P_{01}$. At the risk of being redundant, pressure losses in butterfly valves increase with mass flow rate, and decrease as the
valve opening increases.

Figure 6-44: Stagnation pressure loss ratio versus $P_2/P_{01}$ for the 0.5” ball valve.

Figure 6-45: Stagnation pressure loss ratio versus $P_2/P_{01}$ for the 0.75” ball valve.
6.3.5 Valve Coefficient

Plots of $k$ versus the upstream Mach number $M_1$ are shown in Figures 6-48 - 6-51. As seen in the case of gate and butterfly valves previously, ball valves also appear to
have a constant $k$ value for all sizes at the fully open and three quarter open setting. However, for the half open setting, when compared to the other two valve types, ball valves appear to have the largest variation in $k$ at the higher Mach numbers, while staying more or less constant at slower flow rates. In this case as well, there appears to be no discernible common trend for the quarter open setting. The valve coefficients are summarized in Table 6.3 and Appendix A shows plots of $k$ versus $M_1$ excluding the quarter open setting.

Table 6.3: Ball valve coefficient $k$.

<table>
<thead>
<tr>
<th>Size</th>
<th>Setting</th>
<th>Valve coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5&quot;</td>
<td>Fully open</td>
<td>1.2334</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>1.9689</td>
</tr>
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<td></td>
<td>Half open</td>
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<td></td>
<td>Quarter open</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>0.75&quot;</td>
<td>Fully open</td>
<td>0.8852</td>
</tr>
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<td></td>
<td>Three quarter open</td>
<td>2.9644</td>
</tr>
<tr>
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<td>Half open</td>
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<td>Quarter open</td>
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</tr>
<tr>
<td>1&quot;</td>
<td>Fully open</td>
<td>0.9386</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>4.0412</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>13.920</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>1.25&quot;</td>
<td>Fully open</td>
<td>1.3412</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>3.3026</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>10.150</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>Inconclusive</td>
</tr>
</tbody>
</table>
Figure 6-48: Valve coefficient versus the upstream Mach number for the 0.5” ball valve.

Figure 6-49: Valve coefficient versus the upstream Mach number for the 0.75” ball valve.
Figure 6-50: Valve coefficient versus the upstream Mach number for the 1” ball valve.

Figure 6-51: Valve coefficient versus the upstream Mach number for the 1.25” ball valve.

6.4 Valve Comparison

When fully open, the flow passage of a ball valve is essentially a continuation of the pipe with no changes in cross-sectional area. Therefore, in the case of a fully open
valve, one would expect ball valves to have the lowest pressure losses among the three
types of valves in this study simply by virtue of their construction. Butterfly valves
on the other hand, due to the presence of the “butterfly”, would be expected to have
the highest pressure losses. Gate valves would be expected to have losses which fall
somewhere in between. The results obtained (Figures 6-52 - 6-59) attest to this for all
valve sizes except the 0.5” size which appears to have almost a two-way tie between
gate and butterfly valves for highest pressure losses (Figures 6-52 and 6-53).

For the case where the valves were three quarter open and half open, butterfly
valves were found to have the lowest pressure losses followed by gate valves and finally
ball valves. Exceptions were observed in the case of the 0.5” valves where gate valves
(instead of ball valves) were found to have the highest pressure losses. For any valve
setting aside from fully open, the flow path through the butterfly valves appears to
be somewhat less tortuous than the path through the gate and ball valves, which
explains the lower pressure losses. Results for the three quarter open case are shown
in Figures 6-60 - 6-67 and those for the half open case are shown in Figures 6-68 - 6-75.

Finally, for the quarter open case, butterfly valves were once again found to have
the lowest pressure losses followed by gate valves and then ball valves, except in the
case of the 0.5” and 0.75” sizes where gate valves instead of ball valves were found
to have the highest pressure losses. Results for the quarter open case are shown in
Figures 6-76 - 6-83.
Figure 6-52: Comparison of stagnation pressure loss ratio versus mass flow rate for the fully open 0.5” valves.

Figure 6-53: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the fully open 0.5” valves.
Figure 6-54: Comparison of stagnation pressure loss ratio versus mass flow rate for the fully open 0.75” valves.

Figure 6-55: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the fully open 0.75” valves.
Figure 6-56: Comparison of stagnation pressure loss ratio versus mass flow rate for the fully open 1” valves.

Figure 6-57: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the fully open 1” valves.
Figure 6-58: Comparison of stagnation pressure loss ratio versus mass flow rate for the fully open 1.25" valves.

Figure 6-59: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the fully open 1.25" valves.
Figure 6-60: Comparison of stagnation pressure loss ratio versus mass flow rate for the three quarter open 0.5” valves.

Figure 6-61: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the three quarter open 0.5” valves.
Figure 6-62: Comparison of stagnation pressure loss ratio versus mass flow rate for the three quarter open 0.75” valves.

Figure 6-63: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the three quarter open 0.75” valves.
Figure 6-64: Comparison of stagnation pressure loss ratio versus mass flow rate for the three quarter open 1” valves.

Figure 6-65: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the three quarter open 1” valves.
Figure 6-66: Comparison of stagnation pressure loss ratio versus mass flow rate for the three quarter open 1.25” valves.

Figure 6-67: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the three quarter open 1.25” valves.
Figure 6-68: Comparison of stagnation pressure loss ratio versus mass flow rate for the half open 0.5” valves.

Figure 6-69: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the half open 0.5” valves.
Figure 6-70: Comparison of stagnation pressure loss ratio versus mass flow rate for the half open 0.75” valves.

Figure 6-71: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the half open 0.75” valves.
Figure 6-72: Comparison of stagnation pressure loss ratio versus mass flow rate for the half open 1” valves.

Figure 6-73: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the half open 1” valves.
Figure 6-74: Comparison of stagnation pressure loss ratio versus mass flow rate for the half open 1.25” valves.

Figure 6-75: Comparison of stagnation pressure loss ratio versus $\frac{P_2}{P_{01}}$ for the half open 1.25” valves.
Figure 6-76: Comparison of stagnation pressure loss ratio versus mass flow rate for the quarter open 0.5” valves.

Figure 6-77: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the quarter open 0.5” valves.
Figure 6-78: Comparison of stagnation pressure loss ratio versus mass flow rate for the quarter open 0.75” valves.

Figure 6-79: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the quarter open 0.75” valves.
Figure 6-80: Comparison of stagnation pressure loss ratio versus mass flow rate for the quarter open 1" valves.

Figure 6-81: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the quarter open 1" valves.
Figure 6-82: Comparison of stagnation pressure loss ratio versus mass flow rate for the quarter open 1.25” valves.

Figure 6-83: Comparison of stagnation pressure loss ratio versus $P_2/P_{01}$ for the quarter open 1.25” valves.
Chapter 7

Valve Curve Fits and Error Analysis

7.1 Valve Curve Fits

Reference [13] presents data for the static pressure loss with mass flow rate in the form of Equation 6.5 on p. 55. The results presented in Chapter 6 have been curve-fitted as well and the coefficients of the general equation $G_t = a \ln x_t + b$ are shown in Table 7.1.

7.2 Error Analysis

Measurement errors at the upstream and downstream measuring locations and the DAQ system are depicted in Figure 7-1.

The accuracy of the pressure (both static and stagnation) and temperature data acquisition system (DAQ) is a function of the signal being measured and decreases as the magnitude of the signal decreases. Consequently, the error due to DAQ system accuracy is at its highest when the magnitude of the pressure or temperature being measured is at its lowest, thereby producing the smallest input signals. The
Table 7.1: Curve fit coefficients of \( G_t = a \ln x_t + b \).

<table>
<thead>
<tr>
<th>Size</th>
<th>Setting</th>
<th>Gate</th>
<th></th>
<th>Butterfly</th>
<th></th>
<th>Ball</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( a )</td>
<td>( b )</td>
<td>( a )</td>
<td>( b )</td>
<td>( a )</td>
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</tr>
<tr>
<td>0.5&quot;</td>
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<td>0.6201</td>
<td>0.1159</td>
<td>0.6924</td>
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</tr>
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<td>0.0901</td>
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</tr>
<tr>
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<td>0.0285</td>
<td>0.1944</td>
<td>0.0372</td>
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</tr>
<tr>
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</tr>
<tr>
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<tr>
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</tr>
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<td>1.1409</td>
</tr>
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<td>0.5781</td>
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<td>0.6186</td>
<td>0.0727</td>
<td>0.4750</td>
</tr>
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<td>0.3983</td>
<td>0.0376</td>
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<td>0.1705</td>
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<td>0.0252</td>
<td>0.1906</td>
<td>0.0169</td>
<td>0.1138</td>
</tr>
</tbody>
</table>

The accuracy of the system was obtained from a handy online calculator provided by the manufacturer of the DAQ system, National Instruments. Due to the sheer amount of data collected, it was not practically feasible to determine the DAQ system accuracy for every pressure and temperature data point. Instead, for each pipe size and for each valve setting, i.e. fully open, three quarter open, half open and quarter open, the data was examined to determine the maximum and minimum values of pressure and temperature that were measured and these values were then used to determine the corresponding DAQ system accuracy. These values are summarized in Tables 7.2 and 7.3.

Manufacturer’s (Omega Engineering, Inc.) specifications rate the pressure transducers as having an accuracy of ±0.25%, which includes linearity, hysteresis and repeatability.

The pitot tubes introduce an error in the measurement of static pressure due to
blockage. For subsonic flows, this error is a function of the Mach number, area of the pipe and area of the measuring instrument inside the flow and can be calculated using Equation 7.1 [36]. At higher Mach numbers, the error due to blockage is larger.

\[
Error_{\text{static pressure}} = \frac{-\gamma M^2}{1 - M^2} \left[ \frac{1.15 + 0.75(M - 0.2)}{2} \right] \left( \frac{\text{Area}_{\text{pitot}}}{\text{Area}_{\text{pipe}}} \right)
\]  

(7.1)

In order to accurately determine these errors using the above equations, the Mach number would have to be measured independently from the static and stagnation pressures. With the existing equipment and resources, this was not practical in the case of this experiment. Hence, minimum and maximum Mach numbers were calculated using the measured static and stagnation pressure data for each pipe size and for each valve setting. At these minimum and maximum Mach numbers, errors due to blockage would respectively be, lowest and highest in the measurement of static
pressure. These minimum and maximum calculated error values are summarized in Table 7.4.

Pitot tubes also introduce errors in the measurement of stagnation pressures due to misalignment. However, using [37] in conjunction with the manufacturer’s (Dwyer Instruments, Inc.) specifications, which permit misalignment of up to 15°, it can be deduced that these errors are negligible and can be neglected.

The thermocouples (manufactured by Omega Engineering, Inc.) also introduce an error due to accuracy of ±0.5% according to the specifications sheet.
Table 7.2: DAQ system error limits for pressure.

<table>
<thead>
<tr>
<th>Size</th>
<th>Setting</th>
<th>Upstream pressure</th>
<th>Downstream pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Range (psi)</td>
<td>DAQ error (±%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Min.</td>
<td>Max.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Min.</td>
<td>Max.</td>
</tr>
<tr>
<td>0.5&quot;</td>
<td>Fully open</td>
<td>91</td>
<td>211</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>93</td>
<td>211</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>101</td>
<td>214</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>106</td>
<td>214</td>
</tr>
<tr>
<td>0.75&quot;</td>
<td>Fully open</td>
<td>84</td>
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<tr>
<td></td>
<td>Half open</td>
<td>99</td>
<td>210</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>106</td>
<td>212</td>
</tr>
<tr>
<td>1&quot;</td>
<td>Fully open</td>
<td>77</td>
<td>209</td>
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<tr>
<td></td>
<td>Three quarter open</td>
<td>84</td>
<td>213</td>
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<tr>
<td></td>
<td>Half open</td>
<td>96</td>
<td>214</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>106</td>
<td>214</td>
</tr>
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<td>214</td>
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<td>82</td>
<td>213</td>
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<tr>
<td></td>
<td>Half open</td>
<td>94</td>
<td>212</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>104</td>
<td>213</td>
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</table>

<table>
<thead>
<tr>
<th>Size</th>
<th>Setting</th>
<th>Range (psi)</th>
<th>DAQ error (±%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min.</td>
<td>Max.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Min.</td>
<td>Max.</td>
</tr>
<tr>
<td>0.5&quot;</td>
<td>Fully open</td>
<td>72</td>
<td>211</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>68</td>
<td>211</td>
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<tr>
<td></td>
<td>Half open</td>
<td>45</td>
<td>214</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>20</td>
<td>213</td>
</tr>
<tr>
<td>0.75&quot;</td>
<td>Fully open</td>
<td>63</td>
<td>214</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>59</td>
<td>210</td>
</tr>
<tr>
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<td>Half open</td>
<td>40</td>
<td>209</td>
</tr>
<tr>
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<td>Quarter open</td>
<td>16</td>
<td>209</td>
</tr>
<tr>
<td>1&quot;</td>
<td>Fully open</td>
<td>62</td>
<td>209</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>56</td>
<td>213</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>40</td>
<td>214</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>15</td>
<td>214</td>
</tr>
<tr>
<td>1.25&quot;</td>
<td>Fully open</td>
<td>68</td>
<td>214</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>63</td>
<td>213</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>46</td>
<td>211</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>21</td>
<td>209</td>
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</table>
Table 7.3: DAQ system error limits for temperature.

<table>
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<th>Upstream temperature</th>
<th>Downstream temperature</th>
</tr>
</thead>
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<td></td>
<td>Range (°C)</td>
<td>DAQ error (±%)</td>
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<td>9.5</td>
<td>16.5</td>
</tr>
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<td></td>
<td>Three quarter open</td>
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<td>16.4</td>
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<td></td>
<td>Half open</td>
<td>12.9</td>
<td>16.7</td>
</tr>
<tr>
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<td>Quarter open</td>
<td>12.9</td>
<td>17.4</td>
</tr>
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<td>0.75&quot;</td>
<td>Fully open</td>
<td>6.0</td>
<td>15.9</td>
</tr>
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<td>Three quarter open</td>
<td>7.6</td>
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<td></td>
<td>Half open</td>
<td>10.3</td>
<td>16.1</td>
</tr>
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<td>Quarter open</td>
<td>11.8</td>
<td>16.2</td>
</tr>
<tr>
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<td>Fully open</td>
<td>8.5</td>
<td>21.0</td>
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<tr>
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<td>Three quarter open</td>
<td>10.8</td>
<td>20.2</td>
</tr>
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<td>Half open</td>
<td>13.2</td>
<td>20.8</td>
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<td>15.4</td>
<td>21.6</td>
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<td>9.1</td>
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<td>20.7</td>
</tr>
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<td>11.5</td>
<td>19.6</td>
</tr>
<tr>
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<td>Quarter open</td>
<td>13.9</td>
<td>20.3</td>
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</table>
Table 7.4: Error limits for static pressure due to blockage.

<table>
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<th>Size</th>
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<th>Upstream Mach number</th>
<th>Downstream Mach number</th>
</tr>
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<td></td>
</tr>
<tr>
<td>0.5&quot;</td>
<td>Fully open</td>
<td>0.04</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>0.03</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>0.03</td>
<td>0.22</td>
</tr>
<tr>
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<td>Quarter open</td>
<td>0.03</td>
<td>0.10</td>
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<tr>
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<td>Fully open</td>
<td>0.03</td>
<td>0.49</td>
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<td>0.03</td>
<td>0.44</td>
</tr>
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<td>0.03</td>
<td>0.27</td>
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<tr>
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<td>Quarter open</td>
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<td>0.10</td>
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<td>Fully open</td>
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<td>0.53</td>
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<tr>
<td></td>
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<td></td>
<td>Half open</td>
<td>0.05</td>
<td>0.28</td>
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<tr>
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<td>0.44</td>
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<td>0.38</td>
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<td>Half open</td>
<td>0.04</td>
<td>0.25</td>
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<td>0.12</td>
</tr>
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<td></td>
<td>Half open</td>
<td>0.02</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>0.02</td>
<td>0.45</td>
</tr>
<tr>
<td>1.25&quot;</td>
<td>Fully open</td>
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<td>0.51</td>
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<td>0.56</td>
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<td>0.04</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>0.04</td>
<td>0.51</td>
</tr>
</tbody>
</table>
Chapter 8

Natural Gas Transmission Control

Simulation with Sample Results

A typical natural gas pipeline compressor station is shown in Figure 8-1 [38] and can be represented by the schematic in Figure 8-2.

Figure 8-1: Gas turbines driving centrifugal compressors in a pipeline compressor station.
8.1 Model Description

The engine model described in Chapter 3 is a fairly complicated one and convergence is dependent on a delicate balance between approximately 50 design data values. Modifying the engine in order to match the schematic in Figure 8-2 would essentially invalidate all the design data, thereby eliminating the only available source of reference values needed for mathematical convergence. For this reason, coupled with the reluctance of engine manufacturers (for obvious reasons) to publish design data in the open literature, the engine model from Chapter 3 was subjected to only one modification. Instead of the high pressure turbine driving just the compressor as described in section 3.2, it will now drive the natural gas compressor as well. Although natural gas is composed of a mixture of gases, it will be assumed that methane only is flowing through the pipeline. Even though the results of Chapter 4 suggest
Dymola as the winner among the three simulation tools, it will not be used in this section. Instead, Simulink will be used due to its largely graphical interface, availability of documentation and user friendliness. The following sections describe the development of any new models or the modification of any existing models that will be used in the formulation of the natural gas transmission station model.

### 8.1.1 Rotor Inertias

The only modification to the engine model detailed in section 3.2 is the inclusion of work needed to run the natural gas compressor. Since it is powered by the high pressure turbine, Equation 3.107 is modified accordingly and the resulting equation is shown below.

\[
N_H = \left( \frac{30}{\pi} \right)^2 \frac{J}{I_H} \int_0^t \left( \frac{\Delta h_{HT}(\dot{w}_4 + K_{BLWHT}\dot{w}_{BLHT}) - \dot{w}_{22}(h'_3 - h_{22})}{\dot{w}_{NGHPC}(h_{NGHPC,\text{out}} - h_{NGHPC,\text{in}})} \right) \frac{dt}{N_H} + N_{H,i}
\]

(8.1)

### 8.1.2 Valve

Mathematical models for three types of valves relating pressure losses to mass flow rates were developed in Chapter 7 and these results are summarized in Table 7.1. These results were obtained through experimentation with air as the fluid as detailed in Chapter 5. However, in this section, the valves are being used in an application involving natural gas (assumed to contain methane only, as mentioned earlier). Assuming the valve coefficient \( k \) is the same for air and methane, Equation 6.4 is used to derive valve models for use with methane. These curve fit coefficients are summarized in Table 8.1. Stagnation temperature is assumed to be constant across the valve.
Table 8.1: Curve fit coefficients of $G_t = a \ln x_t + b$ (methane).

<table>
<thead>
<tr>
<th>Size</th>
<th>Setting</th>
<th>Gate</th>
<th></th>
<th>Butterfly</th>
<th></th>
<th>Ball</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>a</td>
<td></td>
<td>b</td>
<td></td>
<td>a</td>
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<td>Inconclusive</td>
<td>Inconclusive</td>
<td>Inconclusive</td>
<td>Inconclusive</td>
<td></td>
</tr>
<tr>
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<td>0.2477</td>
<td>0.9912</td>
<td>0.2776</td>
<td>1.0192</td>
<td>0.3033</td>
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<td>Half open</td>
<td>0.1276</td>
<td>0.4708</td>
<td>0.1443</td>
<td>0.5727</td>
<td>0.1122</td>
<td>0.3465</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
<td>Inconclusive</td>
<td>Inconclusive</td>
<td>Inconclusive</td>
<td>Inconclusive</td>
<td>Inconclusive</td>
<td></td>
</tr>
<tr>
<td>1.25&quot;</td>
<td>Fully open</td>
<td>0.2261</td>
<td>0.9462</td>
<td>0.2085</td>
<td>0.8608</td>
<td>0.2164</td>
<td>0.9086</td>
</tr>
<tr>
<td></td>
<td>Three quarter open</td>
<td>0.1907</td>
<td>0.7921</td>
<td>0.2035</td>
<td>0.8378</td>
<td>0.1580</td>
<td>0.6160</td>
</tr>
<tr>
<td></td>
<td>Half open</td>
<td>0.1152</td>
<td>0.4179</td>
<td>0.1432</td>
<td>0.5722</td>
<td>0.0895</td>
<td>0.3493</td>
</tr>
<tr>
<td></td>
<td>Quarter open</td>
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<td>Inconclusive</td>
<td>Inconclusive</td>
<td>Inconclusive</td>
<td>Inconclusive</td>
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</tr>
</tbody>
</table>

8.1.3 Natural Gas Compressor

The natural gas compressor model is assumed to be identical to the compressor model described in section 3.2.4.

8.1.4 Pipeline Flow

The Weymouth equation for horizontal pipes is used to model the gas volumetric flow rate inside the pipeline [39].

$$q_h = 3.23 \left( \frac{T_b}{p_b} \right) \sqrt{\frac{(p_1^2 - p_2^2) D^5}{\gamma_g Z TL f}} \quad \text{where} \quad f = \frac{0.032}{D^{1/3}} \quad (8.2)$$
8.1.5 Intercomponent Volumes

Similar to the engine, mixing volumes are placed between the valve and the natural gas compressor (denoted as MV1) as well as between the natural gas compressor and the pipeline (denoted as MV2). The following equations define the models.

\[ W_{MV1} = \int_{0}^{t} (\dot{w}_{valve} - \dot{w}_{NGHPC}) \, dt + W_{MV1,i} \]  

\[ T_{MV1} = \int_{0}^{t} \left( \frac{\dot{w}_{valve} (h_{valve} - h_{MV1})}{c_{v,MV1}} + T_{MV1} \left( \frac{\dot{w}_{valve} - \dot{w}_{NGHPC}}{W_{MV1}} \right) (\gamma_{MV1} - 1) \right) \, dt + T_{MV1,i} \]  

\[ P_{MV1} = \frac{R_{NG} W_{MV1} T_{MV1}}{V_{MV1}} \]  

\[ W_{MV2} = \int_{0}^{t} (\dot{w}_{NGHPC} - \dot{w}_{pipe}) \, dt + W_{MV2,i} \]  

\[ T_{MV2} = \int_{0}^{t} \left( \frac{\dot{w}_{NGHPC} (h'_{NGHPC} - h_{MV2})}{c_{v,MV2}} + T_{MV2} \left( \frac{\dot{w}_{NGHPC} - \dot{w}_{pipe}}{W_{MV2}} \right) (\gamma_{MV2} - 1) \right) \, dt + T_{MV2,i} \]  

\[ P_{MV2} = \frac{R_{NG} W_{MV2} T_{MV2}}{V_{MV2}} \]
8.2 Simulation Results

Due to the absence of any numerical results that could be used for comparisons, the natural gas transmission station model was qualitatively tested to ensure that the model obeyed the physics of the problem. The following observations were made.

The mass flow rate of methane exiting the pipeline at a fixed pressure can be controlled by the valve setting as well as the amount of fuel being supplied to the engine. As more fuel is supplied to the engine, its compressor-turbine bundle spins faster which in turn causes the natural gas compressor to spin faster and increase the output methane mass flow rate. Similarly, reducing the fuel flow results in a drop in methane mass flow rate. Figure 8-3 shows the variation of methane mass flow rate during a 10 second transient and Figure 8-4 shows the variation of the methane compressor spool speed. Fuel flow rate stays constant at 4.5 lb/s during the first 3 seconds, then linearly increases to 5.5 lb/s over a period of 4 seconds and stays at that value for the remainder of the transient.

![Figure 8-3: Variation of pipeline exit mass flow rate with fuel flow rate.](image)
Figure 8-4: Variation of methane compressor RPM with fuel flow rate.

An alternative means of control is closing the valve upstream of the natural gas compressor which reduces the mass flow rate while opening has the opposite effect. Variation of mass flow rate and spool speed as the valve opening is reduced from fully open to three quarter open is shown in Figures 8-5 and 8-6 respectively. A step function is used to model the closing of the valve.

Figure 8-5: Variation of pipeline exit mass flow rate with valve setting.
The mass flow rate of methane exiting the pipeline is also dependent on the density of methane entering the system from the reservoir. An increase in the temperature of methane in the reservoir lowers the density, thereby reducing the mass flow rate. This variation is depicted in Figure 8-7 where the transient input consists of the methane inlet temperature staying constant at 520$^\circ$R for the first 3 seconds, then increasing linearly to 550$^\circ$R during the next 4 seconds and finally staying there for the remaining 3 seconds. It was also observed that if all other variables stayed constant and inlet temperature gradually dropped, eventually the methane compressor would be unable to handle the increasing mass flow rate and maintain pipeline outlet pressure, unless more fuel was supplied to the engine.
Figure 8-7: Variation of pipeline exit mass flow rate with methane reservoir temperature.

If a higher delivery pressure is desired at the pipeline exit while maintaining a constant mass flow rate, the fuel flow rate to the engine has to be increased. Simulation of this situation exposes a drawback of the current model. Even though this situation requires a solution by trial and error due to the fact that pipeline exit pressure and fuel flow rate are inputs to the model, it does demonstrate the validity of the model.
Chapter 9

Conclusions and Future Work

This is the first known complete simulation of a natural gas transmission line that includes the natural gas compressor and driver, a control valve and a simple model of the flow dynamics within a long pipeline. A study of computational robustness and speed of various commercial packages was undertaken to determine the best software platform in which to execute the simulation. The choice for the simulation tool was Simulink because it had the best trade-off between speed, user friendliness and documentation.

A valve study was conducted for low to high speed subsonic flows. This is the first known study for gate and butterfly valves and generic valves were used in order not to limit the usability of the experimental results which were curve-fitted for future use. Thus, with a knowledge of the mass flow rate and the upstream static pressure, the resulting downstream static pressure can be determined. The results obtained exhibit the same trends as those seen in the case of the ball valve in [13] and the errors in the measurands are reasonably low.

The natural gas compressor station model can be used to simulate transients as well as the interactions between the engine and the pipeline flow, but it is in no way complete. However, it does provide a means for extension. Since the model
is developed in Simulink, it is relatively easy to replace components such as, for example, the maps governing the performance of the methane compressor. A more robust model of the dynamics of compressible flow in a long pipeline such as the true lag in pressure and mass flow rate with a change in the upstream compressor pressure needs to replace the existing model based on the Weymouth equation. There are many references cited that can accomplish this with different degrees of complexity.
References


[21] Osiadacz, A. J. and Chaczykowski, M., “Comparison of Isothermal and Non-


Appendix A

Valve Coefficient Plots

A.1 Gate Valves

Figure A-1: Valve coefficient versus the upstream Mach number for the 0.5” gate valve (excluding the quarter open setting).
Figure A-2: Valve coefficient versus the upstream Mach number for the 0.75” gate valve (excluding the quarter open setting).

Figure A-3: Valve coefficient versus the upstream Mach number for the 1” gate valve (excluding the quarter open setting).
Figure A-4: Valve coefficient versus the upstream Mach number for the 1.25” gate valve (excluding the quarter open setting).

A.2 Butterfly Valves

Figure A-5: Valve coefficient versus the upstream Mach number for the 0.5” butterfly valve (excluding the quarter open setting).
Figure A-6: Valve coefficient versus the upstream Mach number for the 0.75” butterfly valve (excluding the quarter open setting).

Figure A-7: Valve coefficient versus the upstream Mach number for the 1” butterfly valve (excluding the quarter open setting).
Figure A-8: Valve coefficient versus the upstream Mach number for the 1.25” butterfly valve (excluding the quarter open setting).

A.3 Ball Valves

Figure A-9: Valve coefficient versus the upstream Mach number for the 0.5” ball valve (excluding the quarter open setting).
Figure A-10: Valve coefficient versus the upstream Mach number for the 0.75” ball valve (excluding the quarter open setting).

Figure A-11: Valve coefficient versus the upstream Mach number for the 1” ball valve (excluding the quarter open setting).
Figure A-12: Valve coefficient versus the upstream Mach number for the 1.25” ball valve (excluding the quarter open setting).