An assessment of surface ice sheet loads and their effects on an offshore wind turbine structure

Eric M. Wells
The University of Toledo

Follow this and additional works at: http://utdr.utoledo.edu/theses-dissertations

Recommended Citation
http://utdr.utoledo.edu/theses-dissertations/468
A Thesis

entitled

An Assessment of Surface Ice Sheet Loads and Their Effects on an Offshore Wind Turbine Structure.

by

Eric M. Wells

Submitted to the Graduate Faculty as partial fulfillment of the requirements for the Master of Science Degree in Mechanical Engineering

_________________________________________________
Dr. Abdollah Afjeh, Committee Co-Chair

_________________________________________________
Dr. Sorin Cioc, Committee Co-Chair

_________________________________________________
Dr. Efstratios Nikolaidis, Committee Member

_________________________________________________
Dr. Patricia R. Komuniecki, Dean
College of Graduate Studies

The University of Toledo

December 2012
This thesis examines the effects of surface ice sheets on an offshore wind turbine. First, the main ice load cases are presented, and methods used to calculate the loads from each of these cases are explained. These load cases consist of loads from moving ice sheets, loads from nonmoving ice sheets, and loads from agglomerated masses of ice, called ice ridges. Next, the data required to conduct the load calculations are presented from sources applicable to an offshore site in Lake Erie, which is the location of interest in this work. The load calculation methods were implemented into a wind turbine simulation software package, and simulations were run subjecting an offshore wind turbine to extreme ice loads combined with a large representative wind load. Results from these simulations are presented, which show the relative magnitude of the effects of the ice loads compared to the magnitude of the effects of the wind load. It was found that the effects on the foundation due to extreme ice loads can be much larger than the effects caused by a large representative wind load. Also presented in this work is an examination
of how the ice loads would influence the design of an offshore wind turbine foundation (i.e. how much bigger should the foundation be to support the ice loads). The simulation results presented in this study indicate that the surface ice sheet loads can be much larger than the wind loads and could be the driving parameter of the design of offshore wind turbine foundations in areas where ice can occur.
Acknowledgements

The author would like to thank Dr. Abdollah Afjeh, Dr. Sorin Cioc, and Dr. Efstratios Nikolaidis, for the time and effort they put in to helping complete this project. The author wishes to acknowledge his family, friends, and colleagues, for their support during the writing of this thesis. Finally the author wishes to gratefully acknowledge the Department of Energy, which supported this effort through the Grant DE-EE0003540.
Contents

Abstract iii
Acknowledgements v
Contents vi
List of Tables ix
List of Figures x
1 Introduction 1
2 Literature Survey 6
3 Ice Load Calculation Methods 8
  3.1 Loads from a Moving Flat Ice Sheet ......................... 8
  3.2 Loads from a Stationary Flat Ice Sheet ...................... 13
  3.3 Dynamic Load Profile ........................................ 15
  3.4 Loads from an Ice Ridge .................................... 17
4 Ice Environment Data 24
  4.1 Ice Thickness Data ........................................... 24
  4.2 Ice Concentration Data ...................................... 29
  4.3 Ice Floe Size Data ............................................ 30
  4.4 Ice Strength Data ............................................. 31
  4.5 Ice Movement Speed Data .................................... 33
  4.6 Ice Ridge Data ................................................. 34
  4.7 Ice Thickness Estimation from Temperature Data .......... 36
5 Lake Erie Specific Ice Conditions 38
10.3 Results ......................................................... 89

10.4 Future Work ................................................... 91

References ......................................................... 92

A Equations for coefficients A1, A2, A3, A4, B1 and B2, 94

used in Ice Load Calculations
# List of Tables

3.1 Summary of Moving Ice Sheet Load Equations. ................................. 11
4.1 Ice Thickness Data. ................................................................. 27
4.2 Ice Floe Size Data from the Cleveland Sample Region. .................... 31
4.3 Laboratory Measurements of Freshwater Ice Strength. .................... 32
5.1 Average and Standard Deviation of Cumulative FDD data. ................. 47
6.1 Ice Load Calculation Verification. ................................................. 49
6.2 Maximum Loads due to a 61.8cm thick Moving Ice Sheet. ................. 53
6.3 Maximum Loads due to a 92.7cm thick Moving Ice Sheet. ................. 53
6.4 Ice Keel Depths and Loads. ....................................................... 53
6.5 Loads from Non-Moving Ice Sheet. ............................................. 54
List of Figures

3-1 Ice Failure Mode against a Sloped Structure. ........................................ 9
3-2 Two Types of Conical Ice Breaking Structures. ............................... 10
3-3 Suggested Dynamic Horizontal Load Profile. ................................. 17
3-4 Ice Ridge Structure. ................................................................. 18
3-5 Local Failure of an Ice Keel. ..................................................... 19
3-6 Global Failure of the Ice Keel. .................................................. 20
3-7 Global and Local Failure Loads vs. Penetration Distance into Keel. .... 22
4-1 NOAA Ice Atlas Thickness Ranges. ........................................... 24
4-2 Great Lakes Ice Chart based on Estimated Thickness. .................. 26
4-3 Cleveland Ice Data Sample Region. ........................................... 27
4-4 Graph of NOAA Ice Data from Cleveland Sample Region. .......... 28
4-5 Median Great Lakes Ice Concentration Data. ............................. 29
4-6 Ice Floe Size Codes. ................................................................. 30
4-7 Lake Erie Ice Speed Histogram. ................................................. 34
4-8 Keel Draft Distribution from Northumberland Strait. .................... 35
5-1 Predicted Ice Thicknesses from Cumulative Freezing Degree Days. . 39
5-2 Maximum value of cumulative FDDs for 1984-2011 for Lake Erie. ... 41
5-3 Calculated Ice Thickness from FDD data for 1984-2011 for Lake Erie. 42
5-4 Weibull Distribution fit onto Calculated Ice Thickness Data. .......... 44
5-5 Logistic Distribution fit onto Calculated Ice Thickness Data. .......... 45
<table>
<thead>
<tr>
<th>Page</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-6</td>
<td>Weibull Distribution fit onto Cumulative FDD Data.</td>
</tr>
<tr>
<td>6-1</td>
<td>Waterline Structural Geometries including Ice Cones.</td>
</tr>
<tr>
<td>6-2</td>
<td>NASA satellite image of Lake Erie, Feb 22, 2011, showing Ice Floes.</td>
</tr>
<tr>
<td>7-1</td>
<td>Sample Ice Properties Input File.</td>
</tr>
<tr>
<td>7-2</td>
<td>Ice Keel Effective Height.</td>
</tr>
<tr>
<td>8-1</td>
<td>Foundation Base Shear Force for 61.8 cm Ice Sheet.</td>
</tr>
<tr>
<td>8-2</td>
<td>Foundation Base Shear Force for 61.8 cm Ice Sheet, excluding vertical structure.</td>
</tr>
<tr>
<td>8-3</td>
<td>Foundation Base Bending Moment for 61.8 cm Ice Sheet.</td>
</tr>
<tr>
<td>8-4</td>
<td>Foundation Base Bending Moment for 61.8 cm Ice Sheet, excluding vertical structure.</td>
</tr>
<tr>
<td>8-5</td>
<td>Location of Horizontal and Vertical Loads on Ice Cone Structures.</td>
</tr>
<tr>
<td>8-6</td>
<td>Tower Top Displacement for 61.8 cm Ice Sheet.</td>
</tr>
<tr>
<td>8-7</td>
<td>Blade Root Bending Moment for 61.8 cm Ice Sheet.</td>
</tr>
<tr>
<td>8-8</td>
<td>Foundation Base Shear Force for 92.7 cm thick Ice Sheet and 11.2 m deep Keel.</td>
</tr>
<tr>
<td>8-9</td>
<td>Foundation Base Bending Moment for 92.7 cm Ice Sheet and 11.2 m Keel.</td>
</tr>
<tr>
<td>8-10</td>
<td>Tower Top Displacement for 92.7 cm Ice Sheet and 11.2 m Keel.</td>
</tr>
<tr>
<td>8-11</td>
<td>Blade Root Bending Moment for 92.7 cm Ice Sheet and 11.2 m Keel.</td>
</tr>
<tr>
<td>8-12</td>
<td>Tower Top Displacement Comparison between FAST and ADAMS for original and hypothetical stiffened foundation.</td>
</tr>
<tr>
<td>9-1</td>
<td>Dimensions of Wind Turbine Foundation.</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

The depletion of fossil-fuel based energy and climate change from the burning of carbon-based fuels present significant challenges for meeting the energy needs of an increasing world population. Renewable sources of energy are actively being developed. With the exception of hydropower, wind energy is now the most successful renewable energy, producing the largest amount of global renewable energy to date. The vast majority of wind turbines today are land-based, contributing all of the installed wind energy generation in the U.S. and nearly all of the global installed generation. However, several important factors are driving the development of offshore wind energy. First, the average wind speeds tend to be significantly higher over water, yielding enhanced energy production. Second, limited availability of land due to visual obstruction and the lease cost of land for wind farms present a challenge in high wind areas. Finally, population centers, and thus energy needs, are predominantly near coastlines. To capture the vast resource of wind energy offshore, however, key technical and commercial challenges need to be addressed.

One of the key technology challenges for offshore wind turbines is operation in cold climates. While the icing of the turbine and tower are similar to the land-based wind turbine systems and can thus benefit from such developments, relatively little research and innovation have been focused on the development of the support structure of an offshore wind turbine in cold climates.
This project will address the design requirement of an offshore wind turbine support structure by conducting a study of the impact of ice loads on an offshore wind turbine. A simulation tool was developed to study the effects of ice on an offshore wind turbine. In cold climates where icing occurs, for example Lake Erie, it is important to determine to what extent the loads on an offshore wind turbine would be influenced by the freezing over of the water. In the United States, offshore sites in the Great Lakes are potentially very valuable, because of the excellent wind resources they possess. However, as parts of the lakes freeze over in the winter, an offshore turbine will be subjected to additional loads from the ice. Current wind turbine simulation tools, such as FAST [Jonkman, 2005] (Fatigue, Aerodynamics, Structures, and Turbulence) or GH-BLADED [GL Garrad Hassan, 2012] lack ice impact modeling capability. A simulation tool is needed to determine these loads and their effect on turbine operation.

The first step in studying the influence of ice on a wind turbine is to determine what loads a surface ice sheet could exert on the wind turbine structure. The ice loads come from three main cases: moving surface ice sheets, nonmoving surface ice sheets, and jumbled masses of moving ice called ice ridges.

The possible loads surface ice sheets could impose on an offshore wind turbine structure were determined using the standards set by the International Electrotechnical Commission (IEC). The IEC standard [IEC, 2009] describes two of the main loads: those from moving surface ice sheets and those from stationary surface ice sheets. When the ice sheet is moving, the structure must plow through the ice sheet, causing the ice sheet to break, which exerts a load on the structure. When the ice sheet is not moving relative to
the structure, loads appear due to the wind pushing on the ice sheet, changes in water level below the ice, and the fixed ice sheet exerting a load on other nearby structures.

The third main load case which should be examined is due to the ice ridge. When the ice initially forms, it is just a flat sheet. As the ice sheet is subjected to wind and thermal expansion loads and moves about the water, it may buckle and then re-freeze, forming a so-called “ice ridge”. The re-frozen area will be thicker than the original ice sheet, and can have a mass of broken ice rubble submerged beneath it, called an “ice keel”. As this ice ridge moves over the structure, it will impart two loads: one from the structure breaking through the thicker re-frozen surface ice, and a second from the submerged part of the foundation structure breaking through the ice rubble keel.

The main source of the ice sheet load calculation methods was the IEC standard. In order to calculate each of the ice loads, both the geometry of the wind turbine platform structure and the properties of the ice are required. The structural geometry is based on the NREL Offshore 5MW Baseline Wind Turbine; additional information about possible foundation shapes comes from the Center for Cold Ocean Resources Engineering (C-CORE). The shape of an offshore structure influences the magnitude of the ice loads to which it is subjected. In this work, two structural geometries were examined: cylinders and cones. These two geometries were selected because of their use in the IEC standard. As a vertical-walled cylindrical structure plows through a moving ice sheet, the ice sheet is crushed against the side of the structure. When a conical structure plows through a moving ice sheet, the ice sheet rides up the sloping sides of the structure; this causes the ice sheet to fail by bending. The calculation methods to determine the exact loads from both moving and nonmoving ice sheets as well as ice ridges are presented in Chapter 3.
In order to determine the loads imposed by ice on an offshore structure, ice conditions must be accurately defined. This work examines an offshore wind turbine situated in Lake Erie; therefore all ice properties will be specific to Lake Erie. The relevant ice properties which are presented in this work include: ice thickness, ice strength, ice movement speed, probability of impact with an ice ridge, and possible ice keel depths. The data for these properties comes from reports furnished by government agencies such as the National Oceanic and Atmospheric Administration, the Great Lakes Environmental Research Laboratory, National Research Council Canada, and others. Information about ice ridges is provided by C-CORE. The ice environment data is presented in Chapter 4. Some of the ice data, specifically the ice thickness data, has limited precision, and therefore must be inferred from other data (such as temperature data). The process of determining the ice thickness from temperature data is presented in Section 4.7, and the results are shown in Chapter 5.

Ice load calculation results are presented in Chapter 6. These results represent the peak loads caused by the ice (the maximum values of the time varying load profile). Also presented in Chapter 6 is a discussion about which of the three load cases (moving ice sheet, nonmoving ice sheet, and ice ridge) are most significant. It was found that the nonmoving ice sheet loads would always be lower than the moving ice sheet loads, so the nonmoving ice sheet loads did not need to be simulated.

To determine the response of an offshore turbine to ice loads, the software FAST was used, which performs dynamic simulation of a wind turbine system in the time domain. The FAST source code was modified to compute and apply the ice loads to the wind turbine foundation during simulation and determine the response. The approach of
finding the combined effect of wind and ice loads is more realistic than an approach of finding the effects of ice or wind loads in isolation. The simulation results for the moving ice sheet cases and for the ice ridge cases are plotted and discussed in Chapter 8. The implications of these results for offshore wind turbine foundation design are discussed in Chapter 9, and the main findings of this thesis are summarized in Chapter 10.
Chapter 2

Literature Survey

The ice related data for Lake Erie comes from a variety of sources. The ice thickness data are available from the NOAA Great Lakes Ice Atlas [Assel, 2004], which gives observational ice thickness data dating back to 1973. Other data sources consulted for ice thickness (such as the data used by the US Coast Guard, from [National Ice Center, 2012]) seemed to derive from the same observations as the NOAA Great Lakes Ice Atlas data, because they report thickness data in the same ranges (i.e. on gives 5-15cm, where the other gives 2-6 inches, these are the same). The NOAA Great Lakes Ice Atlas also provides data about ice coverage and ice floe size. Regarding the speed of the ice sheet on Lake Erie, only one study was found. This study [Campbell, 1987] used a satellite tracked buoy attached to the ice to measure its speed. To determine the bending and crushing strength of the freshwater ice in Lake Erie, one source was found [Timco, 1982] which measured the strength of lab-grown ice; the results from this report agree with the calculated bending strength from [Timco, 1994], which provides a bending strength equation for ice. Only one report [Lever, 2000] providing measured data for Lake Erie regarding the formation of ice ridges was found. This contains lakebed scour data, allowing the frequency of the deepest ice ridge keels to be determined. To determine the probability of ice ridge keels which do not reach the lakebed, sources referring to places other than Lake Erie had to be consulted. Data regarding the relative probabilities of ice ridge keels in the Northumberland Strait is given by [C-CORE, 2008]; it is assumed in this work that ice ridges in Lake Erie will behave similarly. Data for the
temperature over Lake Erie comes from [National Data Buoy Center SBIO1, 2012] and [National Data Buoy Center DBLN6, 2012].

Calculation methods used to determine the loads from moving ice sheets are presented in such sources as [Ralston, 1977] and [Croasdale, 1980]. Both of these reports present methods to find the loads from an ice sheet failing against a sloping sided structure. The methods from [Ralston, 1977] are recommended by the International Electrotechnical Commission (IEC) and so they will be used in this thesis. To determine the loads from ice ridges, [Timco, 1999] presents a number of different algorithms; the method used in this thesis is based on one of these algorithms and is used in [C-CORE 2008]. The equations to determine the loads from a nonmoving ice sheet come from [IEC, 2009].

Due to the limited precision of the ice thickness data, it became necessary to estimate the ice thickness from other measured data, namely temperature. Several sources ([C-CORE, 2008], [IEC, 2009], and [White, 2004]) provide similar methods for how to accomplish this; the method used in this thesis follows the method given by the IEC.
Chapter 3

Ice Load Calculation Methods

Chapter 3 will examine three main load cases which can occur for an offshore wind turbine foundation: loads from a moving ice sheet, loads from a non-moving ice sheet, and loads from an ice ridge. The ice loads acting on an offshore wind turbine can be determined using the IEC recommendations. The ice loads depend both on the foundation geometry as well as the conditions of the ice itself. Both static and dynamic loads should be studied. Section 3.1 shows how to calculate the loads from a moving flat ice sheet, and Section 3.2 discusses loads due to a non-moving flat ice sheet. Section 3.3 discusses how these loads are translated into a dynamic load profile. Section 3.4 addresses the loading case due to the ice ridge.

3.1 Loads from a Moving Flat Ice Sheet

The IEC standard [IEC, 2009] describes load cases which would occur if the ice sheet were moving relative to the support structure(s). There are two main possible shapes for an offshore wind turbine support structure: a vertical cylinder and sloping conical structure. The cylindrical structure moves through the ice sheet by causing the ice sheet to fail by crushing against the vertical sides of the cylinder. The sloping structure moves through the ice sheet by causing the ice to ride up the side of the structure, deflecting the ice sheet out of the plane of the water surface. This causes the ice sheet to
crack at some distance away from the structure (caused by bending stress not crushing); as shown in Fig. 3-1.

![Figure 3-1: Ice Failure Mode against a Sloped Structure.](image)

For a structure located in open water (e.g. far from the shore in a lake), the ice sheet may approach from any direction. Thus, a convenient shape for the foundation is a radially symmetric conical structure. This is in contrast to bridge supports, for example, in which the ice approach direction is the same as the flow direction of the river, allowing other support structure shapes to be explored. In the case of an ice cone, the broken pieces of the ice sheet then move up or down (according to the cone type, see Fig. 3-2) and around the conical structure. The advantage of having a cone instead of simply a cylinder is that the cone causes the ice sheet to exert less force on the structure; the ice sheet is weaker in bending than crushing.

There are two possible types of ice cones: downward breaking and upward breaking, as shown in Fig. 3-2, taken from [C-CORE 2008]. The typical cone angle of structures of this type is 50 degrees from the horizontal.
The downward breaking cone pushes the ice sheet below the water surface, while the upward breaking cone lifts the ice sheet up out of the water to bend and break it. For the cylindrical structure, the ice is simply crushed against the sides of the structure. The IEC standard gives equations to determine the ice breaking loads on any of these structures. For the vertical walled cylindrical structure, [IEC, 2009] gives Eq. 3.1.

\[
H_d = k_1k_2k_3hD\sigma_c
\]  

(3.1)

where

- \(H_d\) = horizontal force
- \(D\) = the diameter of the structure at the waterline
- \(k_1\) = shape factor of the structure where the ice is hitting it
- \(k_2\) = contact factor between the ice and support structure
- \(k_3\) = aspect ratio factor, given as \(\sqrt{1+5h/D}\), where \(h\) is the ice thickness
- \(\sigma_c\) = crushing strength of the ice

Equation 3.2 is recommended by [IEC, 2009], taken from [Ralston, 1977], to calculate the horizontal load which will occur on a sloping structure when struck by a moving ice sheet. The magnitude of the vertical load is given by Eq. 3.3.
\[
H = A_4 [A_1 \sigma_b h^2 + A_2 \rho_w g h D^2 + A_3 \rho_w g h (D^2 - D_T^2)]
\] (3.2)

\[
V = B_1 H + B_2 \rho_w g h (D^2 - D_T^2)
\] (3.3)

where

\[A_1, A_2, A_3, A_4, B_1\] and \(B_2\) are dimensionless coefficients; these are functions of the cone angle and the ice to cone friction coefficient.

\(\sigma_b\) = ice bending strength

\(h\) = ice sheet thickness

\(\rho_w\) = water density

\(g\) = gravitational acceleration

\(D\) = cone diameter at the waterline

\(D_T\) = diameter at the narrow end of the cone (same as the tower diameter)

Table 3.1 summarizes which set of equations to use depending on the foundation geometry of the structure.

Table 3.1: Summary of Moving Ice Sheet Load Equations

<table>
<thead>
<tr>
<th></th>
<th>Vertical Walled Cylindrical Foundation</th>
<th>Upward Breaking Ice Cone</th>
<th>Downward Breaking Ice Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Load</td>
<td>Equation 3.1</td>
<td>Equation 3.2</td>
<td>Equation 3.2</td>
</tr>
<tr>
<td>Vertical Load</td>
<td>None</td>
<td>Equation 3.3</td>
<td>Equation 3.3</td>
</tr>
</tbody>
</table>

Equations 3.2 and 3.3 were derived by [Ralston, 1977] to apply to the case of an upward breaking ice cone subject to the forces from the advancing ice sheet; wherein the side of the cone being hit by the ice sheet is covered by a single layer of broken ice pieces
from the ice sheet. These ice pieces must be lifted out of the water and pushed up the side of the cone. In addition to pushing chunks of ice up or down the side of the ice cone, part of the flat ice sheet must also be deflected out of the plane of the water surface enough to bend it to the point of breaking. These effects exert both horizontal and vertical loads on the structure, with the vertical load pointing downwards for the upward breaking ice cone. [IEC, 2009] instructs that Eq. 3.2 can also be used to find the horizontal force on a downward breaking cone, by replacing the water density $\rho_w$ with $\rho_w/9$. For a downward breaking cone, the broken ice pieces on the surface of the ice cone must be forcibly submerged and pushed down the side of the cone underwater. Also, the ice sheet must be deflected downwards out of the plane of the water surface, submerging it. When lifting the ice, work must be done against the full weight density of the ice. But when submerging the ice, work only needs to be done against the buoyant density of the ice (the difference in density between the ice and the water). The IEC suggestion to simply replace $\rho_w$ with $\rho_w/9$ when using the downward breaking cone is clearly a rough rule of thumb used to account for the change from working against the full weight density of the ice to just the buoyant density. The numerical value of 9 used here is an approximation based on the differences in density between the ice and the water.

The failure criterion of the ice used to derive Eqs. 3.2 and 3.3 was a purely bending failure mode [Ralston, 1977]. The first two terms (associated with $A_1$ and $A_2$) in Eq. 3.2 correspond to the force required to break the ice sheet, while the third term (associated with $A_3$) comes from sliding the broken ice pieces along the surface of the cone. The vertical force is determined by Eq. 3.3, using the horizontal force and the coefficients $B_1$ and $B_2$, which are functions of the cone angle and ice-cone friction.
coefficient. The values for the coefficients $A_1, A_2, A_3, A_4, B_1$ and $B_2$, can be visually estimated from figures provided in both [Ralston, 1977] or in [IEC, 2009], requiring only the cone angle, ice-cone friction coefficient, ice density, thickness, ice strength, structural diameter, and water density. The values of these coefficients may also be directly calculated, this was the method used in this work. Appendix A shows the equations used to get the values for the coefficients $A_1, A_2, A_3, A_4, B_1$ and $B_2$.

### 3.2 Loads from a Stationary Flat Ice Sheet

For the case in which the ice sheet is stationary relative to the support structure, the IEC standard describes several load cases which would occur. For an isolated offshore wind turbine surrounded by nonmoving ice, vertical forces would appear due to changes in water level (the ice is frozen onto the support structure and cannot move with the water level), and horizontal forces would appear due to the wind and/or current applying loads to the ice sheet itself; these loads would then be transmitted to the foundation. The vertical forces due to changes in water level are limited by the adhesive shear strength of the ice to the structure, shown in Eq. 3.4, or by the bending strength of the ice, if the ice cracks in a ring around the structure, shown in Eq. 3.5

$$V_i = A \tau$$

(3.4)

where

$\tau = \text{adhesive shear strength}$

$A = \text{contact surface}$
\[ V_b = 0.6A\sqrt{\sigma_b \rho g \Delta z} \]  \hspace{1cm} (3.5)

where

\[ A = \text{contact surface} \]
\[ \sigma_b = \text{ice bending strength} \]
\[ \rho = \text{water density} \]
\[ g = \text{gravitational acceleration} \]
\[ \Delta z = \text{water level difference} \]

For a structure surrounded by an ice sheet fixed to the structure, wind drag on the ice sheet would induce a horizontal load on the structure, according to Eq. 3.6.

\[ H = C_d \rho \frac{U^2}{2} A \]  \hspace{1cm} (3.6)

Where \( C_d \) is the drag coefficient, [IEC, 2009] suggests a value of 0.004, \( \rho \) is the density of the air, and \( U \) is the free stream velocity of the air 10 meters above the ice.

For a group of multiple wind turbines surrounded by nonmoving ice, additional forces would come into play: horizontal loads caused by thermal ice pressure and horizontal loads due to changes in water level causing an arch effect. The horizontal loads from thermal ice pressure would be directed outwards radially away from the center of a cluster of wind turbines; this is shown in Eq. 3.7.

\[ H_i = f_i D \]  \hspace{1cm} (3.7)

where

\[ D = \text{diameter of support structure}; \text{ use a value of 4m if support structure diameter is less than 4m.} \]
\[ f_i = \text{force per unit width of the support structure}; 300\text{kN/m at outer edge of wind farm, 100 \text{kN/m for structures in the interior of a wind farm} \]

The load due to the arch effect caused by changes in water level would happen as ice is held in place between support elements as the water level changes, this would form an arch. Ice trapped between two structures forms an arch as the water level lowers, creating horizontal loads.

As the water level changes, the so called “arch effect” occurs as ice trapped between two structures forms an arch as the water level lowers, forming an arch, which creates a horizontal load. The horizontal load from this arch effect is shown in Eq. 3.8.

\[ H_v = f_v D \quad (3.8) \]

where

\[ D = \text{diameter of support structure; use a value of 4m if support structure diameter is less than 4m.} \]

\[ f_v = \text{force per unit width of the support structure; 200kN/m} \]

### 3.3 Dynamic Load Profile

[IEC, 2009] gives two methods for determining the dynamic loads that would be present on an offshore wind turbine support platform due to moving ice. One method is to use an upwards shifted sinusoidal wave. The other method to determine the dynamic load is to use a triangular wave. The previously calculated force values for moving ice will be the maxima of the dynamic loading. For a vertical support structure, the dynamic loading is given by Eq. 3.9, from [IEC, 2009] (this is the upwards shifted sine wave).
\[ H_{dyn} = H_d \left( \frac{3}{4} + \frac{1}{4} \sin(f_N t / 2\pi) \right) \]  

(3.9)

where

- \( H_d \) = horizontal load from the ice resulting from Eq. 3.2
- \( t \) = time
- \( f_N \) = the natural frequency of the structure.

Equation 3.9 shows that the amplitude of the sine wave is ±25% of the maximum value, while the maximum load is still \( H_d \).

For the dynamic load on a conical support structure, [IEC, 2009] gives Eq. 3.10, wherein the frequency of the loading is related to the movement speed of the ice rather than the structure natural frequency.

\[ H_{dyn} = H_d \left( \frac{3}{4} + \frac{1}{4} \sin(f_b t / 2\pi) \right) \]  

(3.10)

where

- \( H_d \) = horizontal load from the ice resulting from Eq. 3.2
- \( t \) = time
- \( f_b = U/Kh \), where \( U \) is the speed of the moving ice sheet, \( h \) is the ice sheet thickness, and \( K \) is an arbitrary constant between 4 and 7, chosen to maximize the load on the wind turbine.

It is assumed in this work that the vertical load on the ice cone will vary with time in the same way as the horizontal load; simply replace the horizontal static load in Eq. 3.10 with the vertical static load to calculate the vertical dynamic load profile.

Alternately, a saw tooth or triangular waveform can be used instead of the sinusoidal wave. The period of the triangular wave should be either \( 1/f_N \) or \( 1/f_b \).
depending on whether the structure is cylindrical or conical. The mean value of the triangular load profile is specified to be 55% of the maximum value [IEC, 2009], however, no rationale is given for this choice. An example of such a triangular load profile is shown in Fig. 3-3, from IEC standard.

![Figure 3-3: Suggested Dynamic Horizontal Load Profile from [IEC, 2009]](image)

**3.4 Loads from an Ice Ridge**

An ice ridge occurs when the flat surface ice sheet buckles and cracks, forming a pile of ice rubble at the crack location. Ice rubble accumulates both on top of the ice and below the ice surface. In the middle of the ice ridge (at the plane of the water surface) a new frozen layer of solid ice forms, with a thickness of about 1.5 times the thickness of the original ice sheet. This is called the consolidated layer. The structure of an ice ridge is shown in Fig. 3-4, taken from [Heinonen, 2004].
In addition to the thicker consolidated layer of ice in the middle of the ice ridge, the ice rubble can pile up in very large amounts. It is possible for the rubble below the water surface (the “ice keel”) to reach all the way down to the lakebed. This is known from observations of lakebed scour, where the grooves cut in the lakebed by the ice keel were observed.

As shown in Fig. 3-4, the ice ridge is made of two portions: a layer of solid ice at the water surface, and a mass of ice rubble below the water surface. The consolidated layer and the ice rubble keel will each put their own load on the structure when the ice ridge comes into contact with a structure. The consolidated ice layer in the ice ridge will create loads on the structure according to Eqs. 3.2 and 3.3. The rubble keel, however, will exert an additional load, which will arise via a different mechanism than the breaking of an ice sheet. For the rubble keel, the structure shape (cylindrical or conical) does not affect the loads, because the rubble keel is not susceptible to the bending failure mode induced by a conical structure. Also, on a structure with an ice cone, the cone exists only near the water line, as shown in Fig. 3-2, while the ice keel is below the waterline, where the foundation is assumed here to be cylindrical. Therefore it will be assumed in this work that structures with or without ice cones will behave the same with respect to ice
rubble keels. When the ice rubble keel meets a structure, there are two modes of failure which could occur; these are global failure and local failure.

Figure 3-5: Local Failure of an Ice Keel [Timco, 1999]

Figure 3-5 shows the local failure mode of a rubble keel. The horizontal load required to fail the keel is given by Eqs. 3.11 and 3.12, given in both [C-CORE 2008] and [Timco, 1999]. The local failure mode occurs when the portion of the ice rubble keel in contact with the foundation fails through crushing.

\[
F_{\text{local}}(x) = W_e \left(1 + \frac{2h_e}{3W_e} \left(\frac{h_e^2 \gamma_{\text{eff}} K_p}{2} + 2ch_e K_p\right)\right) \tag{3.11}
\]

and

\[
h_e = x \tan \theta_k \left(1 + \frac{S W_e}{h_k - h_c}\right) \tag{3.12}
\]

where

\[
W_e = \text{effective structure width}
\]
\( x \) = penetration distance of the foundation into the keel

\( c \) = cohesion within the rubble keel

\( S \) = effect of any surcharge, C-CORE Report does not specify what this is

\( \theta_k \) = angle that the keel rubble makes with the horizontal

\( h_k \) = total draft of the keel as measured from the waterline

\( h_c \) = consolidated layer thickness

\( \gamma_{eff} \) = effective buoyant density of the keel rubble, including effects of porosity

\( K_p = tan \left( 45^\circ + \frac{\phi}{2} \right) \) = passive pressure coefficient of the rubble

\( \phi \) = internal friction angle of the rubble

Figure 3-6: Global Failure of the Ice Keel [Timco, 1999]

The global failure mode of the ice rubble keel is described by Eqs. 3.13, 3.14, and 3.15, from [C-CORE 2008]. The global load (given by Eq. 3.13) is a combination of the
loads required to fail the rubble along the “global failure planes” (shown in Fig. 3-6), and the loads required to separate the rubble from the underside of the ice sheet.

\[ F_{global}(x) = F_{side}(x) + F_{top}(x) \]  
\[ F_{side}(x) = K \gamma_{eff} \tan(\phi) \left[ (h_k - h_c)^2 \left( \frac{w_k - w_b}{3} + w_b \right) - \frac{x \tan(\theta_k)^2}{3} \right] + c \left[ (w_k - w_b)(h_k - h_c) - x^2 \tan(\theta_k) \right] \]  
\[ F_{top}(x) = W \left[ \gamma_{eff} \tan(\phi) \left[ (w_k - w_b)(h_k - h_c) - x^2 \tan(\theta_k) + c(w_k - x) \right] \right] \]

Equation 3.14 is the force required to cause failure of the rubble keel along the “global failure planes” shown in Fig. 3-6. Equation 3.15 is the load required to begin pushing the rubble keel along the underside of the ice sheet. Both of these combine to give the total load of the “Global” failure mode.
Figure 3-7 shows the behavior of both loads (global and local) as the penetration depth of the structure into the keel increases. As long as the local failure mode requires a lower load, it will occur and the global failure mode will not occur. When the penetration distance is reached where the global mode requires lower load, the global failure will occur. Therefore the maximum keel load (from either global or local failure modes) occurs when the two failure modes occur for the same load.
For the purposes of this project, a time history of the force caused by the ice keel is required. Note that the load from the ice keel depends on the distance which the foundation has crushed into the ice rubble keel. Because the ice keel moves with the ice, and the size of the keel is known (or assumed), then the time that the foundation will reach each position within the ice keel is known. From this, a time history of the load generated by the ice keel can be calculated and applied to the simulated model. The failure mode with the lower force will occur and apply its load to the foundation. It will be assumed in this work that once the global failure occurs, the ice keel will no longer exert any force on the foundation structure, because once global failure occurs, no more material remains to be broken or sheared by the foundation.
Chapter 4

Ice Environment Data

In order to properly calculate the ice loads which will act upon the foundation, certain ice properties are required. These include ice thickness, ice strength, ice movement speed, and dimensions of ice ridges.

4.1 Ice Thickness Data

Ice thickness is a key variable in ice load calculations. The main source of ice thickness data is the NOAA Great Lakes Ice Atlas [Assel, 2002]. The observations in this dataset come from the winters (December through April) of the years 1973-2000. The NOAA Ice Atlas data on ice thickness is reported in ranges, as shown in Fig. 4-1.

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Ice Thickness Range (in cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>no stage of development</td>
<td>00 (no ice)</td>
</tr>
<tr>
<td>1</td>
<td>new ice</td>
<td>00 to 5</td>
</tr>
<tr>
<td>4</td>
<td>thin lake ice</td>
<td>5 to 15</td>
</tr>
<tr>
<td>5</td>
<td>medium lake ice</td>
<td>15 to 30</td>
</tr>
<tr>
<td>7</td>
<td>thick lake ice</td>
<td>30 to 70</td>
</tr>
<tr>
<td>8</td>
<td>1st stage thick ice</td>
<td>30 to 50</td>
</tr>
<tr>
<td>9</td>
<td>2nd stage thick ice</td>
<td>50 to 70</td>
</tr>
<tr>
<td>10</td>
<td>very thick lake ice</td>
<td>70 to 120</td>
</tr>
<tr>
<td>-8</td>
<td>place holder</td>
<td></td>
</tr>
<tr>
<td>-9</td>
<td>missing data</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4-1: NOAA Ice Atlas thickness ranges, from [Assel, 2002]
In Fig. 4.1, it can be seen that the ice thickness measurements are grouped in thickness bins, e.g. thick lake ice ranging from 30 to 70 cm in thickness. The 30 to 50 cm and 50 to 70 cm ice thickness bins contain no data (all the data is in the 30 to 70 cm bin). These measurements come from a variety of sources (such as satellites, shore stations, aircraft, and ships) [Assel, June 2004] which were limited in their measurement precision. It is likely that instead of directly measuring the ice thickness, it was only estimated during observation, which would explain why the thickness bins have limited precision in the NOAA ice atlas dataset.

For the calculations, however, a precise value is needed to plug into the equations, rather than the large range provided by this data. Other sources have been consulted in an effort to find more precise ice thickness data. One source is the data provided by the National Ice Center; this is the data used by the US Coast Guard. A sample of this data can be seen in Fig. 4-2, taken from http://www.natrice.noaa.gov/products/great_lakes.html [National Ice Center, 2012].
Figure 4-2: Great Lakes Ice Chart based on Estimated Thickness in Inches [National Ice Center, 2012]

Note the thickness scale in the lower right of Fig. 4-2, given here in inches: 0-2 in, 2-6 in, 6-12 in, 12-28 in, 28 in and up. Converting to metric, these thickness bins are identical to the thickness bins from the NOAA Ice Atlas [Assel, 2002]. In our research, no other datasets of ice thickness in the open water of the Great Lakes were found which did not have this limited precision (i.e. all open-water ice thickness datasets found had this limitation). Nevertheless, we extracted the data relevant to the Lake Erie site from the NOAA data.

The NOAA Ice Atlas gives ice thickness data on a grid covering all of the Great Lakes with a nominal resolution of 2.5 km per grid cell [Assel, May 2004]. To get data
for the Cleveland region of Lake Erie, a subset of the grid data from the NOAA atlas was examined, shown in Fig. 4-3. The thickness data from the ice data sample region are shown in Table 4.1. Figure 4-4 shows the NOAA ice data from Table 4.1 in graph form.

Figure 4-3: Cleveland Ice Data Sample Region.

Table 4.1: Ice thickness data from [Assel, 2002]

<table>
<thead>
<tr>
<th>Ice Thickness Range (cm)</th>
<th>Samples in Range</th>
<th>Percent of Total Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 5</td>
<td>6138</td>
<td>5.48%</td>
</tr>
<tr>
<td>5 to 15</td>
<td>20260</td>
<td>18.10%</td>
</tr>
<tr>
<td>15 to 30</td>
<td>32265</td>
<td>28.82%</td>
</tr>
<tr>
<td>30 to 70</td>
<td>52998</td>
<td>47.34%</td>
</tr>
<tr>
<td>70 to 120</td>
<td>286</td>
<td>0.26%</td>
</tr>
</tbody>
</table>
As can be seen in this NOAA ice atlas data, ice of a thickness between 30 and 70 cm occurs very frequently (nearly 50% of ice observations are in this range). Also, ice between 70 and 120 cm thickness can occur, although very rarely. However, for this study, the exact ice thickness is needed which occurs on average once during a given period (the choice of the number of years in the period is arbitrary). The NOAA ice atlas data simply are inadequate to find this thickness. As discussed, other sources of more precise data were not found in the literature; therefore the required ice thickness data must be obtained some other way. As will be discussed in Sections 4.7 and 5.1, the method used in this study estimates ice thickness based on air temperature data. The NOAA ice thickness data will still be useful, however, as a sanity check on the estimated ice thicknesses.
4.2 Ice Concentration Data

Ice concentration data describes how frequently a given area of the lake surface will be covered with ice. This is important because it indicates how frequently the ice loads will be present on the structure. As can be seen in Fig. 4-5, the median ice concentration in mid-February is above 90% in Lake Erie, showing that ice could be encountered very frequently, making it a significant concern. (February is the time when the most ice is present on the Great Lakes).

![Median Ice Concentration by Cell](image)

Figure 4-5: Median Ice Concentration data for February 8-14, years 1973-2002, from [Assel, 2002]

Using the NOAA Ice Atlas data on ice concentration, it would be possible to find the probability of encountering ice at a given offshore site in a given period of time, and combine this with the ice thickness data to determine the chance of an offshore site
seeing ice thicknesses within a given thickness range during a given time period. However, the NOAA ice concentration data does not play a major role in our calculations; it merely serves to illustrate how frequently ice can occur on Lake Erie.

### 4.3 Ice Floe Size Data

The NOAA Great Lakes Ice Atlas [Assel, 2002] gives observational data about the size of the ice floes. Ice floe size observations are divided into multiple categories, as shown in Fig. 4-6, from [Assel, 2002].

![Figure 4-6: Ice Floe size codes from [Assel, 2002]](image)

The following codes are used to denote forms of lake ice:

<table>
<thead>
<tr>
<th>Code</th>
<th>Description of Ice Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>pancake ice (30 cm – 3 m)</td>
</tr>
<tr>
<td>1</td>
<td>small ice cake (&lt;3m) or brash ice (&lt;2m)</td>
</tr>
<tr>
<td>2</td>
<td>ice cake (3m – 20m)</td>
</tr>
<tr>
<td>3</td>
<td>small floe (20m – 100m)</td>
</tr>
<tr>
<td>4</td>
<td>medium floe (100m – 500m)</td>
</tr>
<tr>
<td>5</td>
<td>big floe (500m – 2km)</td>
</tr>
<tr>
<td>6</td>
<td>vast floe (2km – 10 km)</td>
</tr>
<tr>
<td>8</td>
<td>fast ice</td>
</tr>
<tr>
<td>-8</td>
<td>place holder</td>
</tr>
<tr>
<td>-9</td>
<td>missing data</td>
</tr>
</tbody>
</table>

The NOAA ice atlas data from the Cleveland sample area reveals that four different ice floe sizes were observed: small floe, medium floe, big floe, and vast floe (none were observed in other categories). Table 4.2 gives the number of times each floe size was observed during the entire period of record (1973-2000).
### Table 4.2: Ice Floe Size Data from the Cleveland sample region

<table>
<thead>
<tr>
<th>Ice Floe Size</th>
<th>Number of Observations</th>
<th>Percentage of Total Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Floe (20m – 100m)</td>
<td>7964</td>
<td>18.06%</td>
</tr>
<tr>
<td>Medium Floe (100m – 500m)</td>
<td>11928</td>
<td>27.05%</td>
</tr>
<tr>
<td>Big Floe (500m -2km)</td>
<td>23768</td>
<td>53.89%</td>
</tr>
<tr>
<td>Vast Floe (2km – 10km)</td>
<td>442</td>
<td>1.00%</td>
</tr>
</tbody>
</table>

From the data in Table 4.2 one can roughly conclude that “big” ice floes (between 500m to 2km across) are a common occurrence; also it can be stated that “vast” ice floes (2km – 10km across) are possible, and should be considered in the design. Unfortunately, the ice floe size data are limited in precision, just like the ice thickness data. For example, the relative probability of a 5km wide ice floe compared to a 10km wide ice floe cannot be determined. All that can be inferred from these data is that very large ice floes do occur and must be taken into design considerations.

### 4.4 Ice Strength Data

In order to determine the loads caused by an ice sheet impinging on an offshore structure, the ice strength must be known; in the case of a moving ice sheet, the ice must break or it will subject the structure to an ever increasing force. In this work, placing an offshore wind turbine in Lake Erie is considered. Lake Erie is a freshwater lake, and therefore freshwater ice strength properties, not sea ice strength properties, must be considered. From the IEC standards, crushing and bending strength of the ice are both required parameters. For the case of an offshore structure plowing through a moving ice
sheet (likely to produce the highest loads on any offshore structure), the loads on a vertical cylindrical structure are governed by the ice crushing strength, and the loads on an ice cone are governed by the ice bending strength. Because the ice loads on an ice cone are lower than the loads on a vertical-walled cylindrical structure, the ice cone is the main object of study in this work, and thus the ice bending strength is the primary parameter of interest.

To determine the bending strength properties of freshwater ice, the equations for the flexural strength of sea ice may be used [Timco, 1994], given in Eq. 4.1. This equation is made applicable to freshwater ice by setting the salinity parameter to zero.

\[
\sigma_f = 1.76e^{-5.88/\sqrt{v_b}} \tag{4.1}
\]

where

- \(\sigma_f\) is the flexural strength in MPa
- \(v_b\) is the brine volume fraction of the ice

In addition to results from Eq. 4.1, measured data from laboratory grown freshwater ice may be used. [Timco, 1982] gives ice strengths measured from laboratory grown ice; this is shown in Table 4.3.

**Table 4.3: Laboratory Measurements of Freshwater Ice Strength [Timco, 1982]**

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Strength (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantilever Beam Bending Strength</td>
<td>770 ± 150 kPa</td>
</tr>
<tr>
<td>Simple Beam Bending Strength (top tension)</td>
<td>2200 ± 320 kPa</td>
</tr>
<tr>
<td>Simple Beam Bending Strength (bottom tension)</td>
<td>1770 ± 190 kPa</td>
</tr>
<tr>
<td>Compressive Strength (horizontal loading)</td>
<td>4400 ± 700 kPa</td>
</tr>
<tr>
<td>Shear Strength</td>
<td>500 ± 220 kPa</td>
</tr>
</tbody>
</table>
By setting the brine volume fraction to zero in Eq. 4.1, we find that the flexural strength for freshwater ice should be 1.76 MPa. The value in Table 4.3, for the simple beam bending strength (bottom tension) agrees well with this value.

### 4.5 Ice Movement Speed Data

The movement speed of the ice sheet is important for calculating the dynamic loads which the ice sheet induces on an offshore structure equipped with an ice cone. Specifically, the ice sheet movement speed controls the ice breaking frequency, which controls the period of the dynamic ice load (recall that the dynamic load is modeled as a shifted sine wave or as a periodic triangular waveform). Also, the ice speed controls the movement speed of ice ridges, influencing how quickly the ice ridge dynamic loads are applied. For Lake Erie, only limited data on the ice sheet movement speeds is available. In a study [Campbell, 1987] conducted during the winter of 1984, a buoy was placed on the ice of Lake Erie, and tracked via satellite to record its speed. The buoy was periodically inspected to ensure that it remained fixed to the ice sheet; therefore the buoy speeds were equivalent to the ice speeds. The significant observations made by the study were that the ice moved with an average speed of 8 centimeters per second, the maximum observed speed was 46 centimeters per second, and that 6.5% of the measurements showed speeds greater than 20 centimeters per second. A speed histogram from this report is shown in Fig. 4-7. Unfortunately, this study does not provide a tabular record of all the data collected, making further analysis of the data impossible.
4.6 Ice Ridge Data

In addition to flat ice sheets, an offshore structure may encounter an ice ridge. An ice ridge is a formation which results from the flat ice sheet cracking and then crushing back together. At a crack site, the ice sheet buckles causing a mass of broken ice rubble to pile up both above and below the water surface. The surface layer then may refreeze, possibly thicker than before the crack occurred (the consolidated layer). This thick layer of ice accompanied by the masses of ice rubble is called an ice ridge. The mass of ice rubble below the surface is called the ice keel. [C-CORE 2008] suggests that the consolidated layer may refreeze to a thickness of 1.5 to 2 times the original ice thickness.
This report also gives some data sources which may be used to estimate the depth of the ice rubble keel below the surface of the ice ridge.

The data cited by [C-CORE 2008] shows that the probability of occurrence of an ice keel decreases by one order of magnitude for each 4.5 meter increment of increased keel depth. (For example a 14.5 m deep keel will be only one tenth as likely as a 10 m deep keel). This data is shown in Fig. 4-8, taken from [C-CORE 2008].

![Figure 4-8: Keel Draft Distribution from Northumberland Strait [C-CORE 2008]](image)

While the measurements shown in Fig. 4-8 do not apply directly to Lake Erie (they come from the Northumberland Strait in the Gulf of Saint Lawrence) the two areas are similar because they can both freeze over in the winter and thaw in the warmer
seasons, thus these measurements can be used as a good starting point to estimate the ice keel depth probability in Lake Erie.

The C-CORE report lists [Lever, 2000] as a source for scour frequency data in Lake Erie. Specifically, [Lever, 2000] found that along a proposed natural gas pipeline route near the southern shore of the lake in north-east Ohio, there were approximately 0.1 scours per kilometer per year. This value is specific to a water depth of 17 meters. These measurements imply that a 17 meter deep ice keel will cross a 10 kilometer long line once every year, or alternately that such a keel will cross a 10 meter line (the same order of magnitude as the size of a wind turbine support structure) once every 1000 years. Combined with the previous information from [C-CORE 2008] this means that a 12.5 meter deep keel will occur once every 100 years in a 10 m wide area, while an 8 meter keel will occur every 10 years. (Recall that for each decrement of 4.5 meters from the keel depth, the probability is increased by ten times). By using this data, the keel depth which will occur on average once every 50 years (or any desired period) can be found. In Section 5.3, Eq. 5.5 is created based on the data in this section to predict the keel depth which occurs once in a given period. The once per 50 year keel depth is 11.2 meters.

4.7 Ice Thickness Estimation from Temperature Data

Air temperature data can be used to estimate the ice thickness. To do this, a concept called Freezing Degree-Days is used. A Freezing Degree-Day (FDD) is the difference between the average daily temperature and the freezing point of water (here measured in degrees Centigrade) [Assel, 1980]. A temperature below freezing gives a positive FDD, and a temperature above freezing gives a negative FDD. To estimate the
ice thickness on a body of water at the end of a given time period, the cumulative amount of FDDs which have occurred during that time period must be found [White, 2004]. Also, we can find the maximum ice thickness which occurs during that period from the maximum value of the cumulative FDDs during that period.

Following the method of [Assel, 1980], for a given winter season, the accumulation time period is taken to be October 1st through April 30th; also, a winter season is named according to the year in which it ends (For example, the 1984 winter season ranges from Oct 1st 1983 through April 30th 1984). If the cumulative FDDs ever become negative (i.e. if on any day there have been more cumulative above freezing degree-days than below freezing) the cumulative sum is reset to zero, and accumulation of FDDs begins again from zero starting on that date.

Equations to find ice thickness from cumulative FDDs will be shown in Section 5.1.
5.1 Ice Thickness from Lake Erie Temperature Data

To determine an appropriate design ice thickness for an offshore site in Lake Erie, the cumulative freezing degree days (as previously discussed in Section 4.7) are used.

Several sources give equations are available to change cumulative FDDs into ice thickness, shown below: [IEC, 2009] gives Eq. 5.1, [C-CORE 2008] gives Eq. 5.2, and the Cold Regions Research and Engineering Laboratory (CRREL) [White, 2004] gives Eq. 5.3.

\[ h_{\text{ice}} = 0.032\sqrt{0.9FDD - 50} \]  \hspace{1cm} (5.1)

\[ h_{\text{ice}} = 1.308(FDD)^{0.6} \]  \hspace{1cm} (5.2)

\[ h_{\text{ice}} = 0.8\sqrt{FDD} \]  \hspace{1cm} (5.3)

\( h_{\text{ice}} \) = ice thickness in meters

FDD = accumulated freezing degree-days (in degrees C)

\( h_{\text{ice}} \) = ice thickness in centimeters

FDD = accumulated freezing degree-days (in degrees C)

\( h_{\text{ice}} \) = ice thickness in inches

FDD = accumulated freezing degree-days (in degrees F)
Converting the output of Eqs. 5.1, 5.2, and 5.3 to meters, the predicted ice thickness can be plotted for any given amount of cumulative FDDs, shown in Fig. 5-1.

**Figure 5-1: Predicted Ice Thicknesses from Cumulative Freezing Degree Days**

Figure 5-1 shows that the equation given by the IEC standard returns the highest ice thickness values for a given amount of cumulative FDDs greater than about 100 centigrade degree-days. It will be shown from our data that the yearly maxima for the cumulative FDDs have an average of slightly more than 200 deg C days; the IEC equation is most conservative in this range. Also, because the IEC standard is being used as the standard for this work, the IEC equation is more authoritative in this context.
For Lake Erie, the available temperature data comes from the National Data Buoy Center. Two locations in this database have a sufficient period of record (1983-2011) to give data suitable for a statistical analysis. These locations are South Bass Island, Ohio [National Data Buoy Center SBIO1, 2012], and Dunkirk New York [National Data Buoy Center DBLN6, 2012]. (Other sites on Lake Erie had periods of record too short to be useful). These two sites come from the extreme east and west ends of Lake Erie, but give similar data, as shown in Fig. 5-2. (The data for each of these sites is given as hourly temperature measurements, which were processed to find average daily temperatures and then cumulative FDDs for each winter season). The similarity between these two sites indicates that the conditions across the entire lake do not vary greatly, so data from either of these sites should be applicable to an offshore site near Cleveland.
Figure 5-2: Maximum value of cumulative FDDs for winter seasons 1984-2011 for two Lake Erie sites.

Applying the IEC ice thickness equation to the FDD data, the yearly maximum ice thicknesses can be obtained, as shown in Fig. 5-3.
Figure 5-3: Calculated Ice Thickness from FDD data for winter seasons 1984-2011 for two Lake Erie sites.

For the years 1998 and 2002, the IEC equation could not predict any ice thickness, because the cumulative FDDs were too low (below 50 degree C days). In these cases, the ice thickness was assumed to be zero.

The ice thickness values calculated from Eq. 5.1 and shown in Fig. 5-3 represent the ice thickness after the ice has been allowed to grow into a level ice sheet. If the ice were to buckle and form ice ridges, a solid layer would refreeze in the middle of the ice ridges (the consolidated layer). The thickness of the consolidated layer is (according to [C-CORE 2008]) about 1.5 times the level ice thickness.
5.2 Once per 50 year Ice Thickness

From this data, it is needed to determine the ice thickness which will occur on average once in a 50 year period (the 50 year period was selected according to [IEC, 2009]). The once per 50 year event is an ice thickness which has 98% chance of not being exceeded in any single given year. This once per 50 year ice thickness event was used in our simulations.

There are two determine the once per 50 year event: find the once per 50 year FDD event from the measured FDD data, and calculate the ice thickness from that value; or calculate the once per 50 year thickness event from our calculated thickness data. Calculating the once per 50 year event from the calculated thickness data is more desirable because it is closer to the procedure which would be used if we had actual measured thickness data.

To get the “best result” a statistical distribution which best fits either the FDD data, or the computed ice thickness data must be selected. If a distribution (Gumbel, Weibull, or others) is found that fits the FDD data better than any distribution fits the computed ice thickness data, then that distribution and the FDD data should be used. To fit distributions to the FDD data and the calculated thickness data, the software Minitab was used.

After inputting the calculated ice thickness data into Minitab, no good distributions were found for the computed thickness data, because the probability of zero thickness ice is nonzero (for the Dunkirk NY site, there were two out of 28 winters which had zero calculated ice thickness from the IEC equation).
The distributions examined in this study all tended to either predict zero probability of zero ice thickness, or give positive nonzero probability for negative ice thicknesses. Neither of these properties can give a “good” result.

Figure 5-4: Weibull Distribution fit onto Calculated Ice Thickness Data

Figure 5-4 shows a Weibull distribution fit for the calculated ice thickness data. By visual inspection, we can see that it does not fit the data very well. In addition, this distribution predicts zero probability for zero ice thickness. This is an undesirable property as previously mentioned.
Figure 5-5: Logistic Distribution fit onto Calculated Ice Thickness Data

Figure 5-5 shows a logistic distribution fit for the calculated thickness data. Visually, it fits better than the Weibull distribution (of all the distributions examined in Minitab, this distribution “looked” the best). However, the logistic distribution gives a nonzero probability for negative ice thicknesses. This is unacceptable.

Next distribution fits were attempted for the cumulative Freezing Degree Day data. For the FDD data, having a zero probability for zero maximum cumulative FDDs in a year is acceptable. (Zero maximum cumulative FDDs for a given year means that the average daily temperature was never below freezing for the entire winter; intuitively this should be much rarer than the lake not freezing over during a given year). The Weibull distribution fits the FDD data well, as shown in Fig. 5-6.
Because a good distribution was not found to fit the calculated ice thickness data, the FDD data was used, in conjunction with the Weibull distribution fit, to determine the once per 50 year maximum cumulative FDD value, and compute the once per 50 year ice thickness from this FDD value.

Using the Minitab software, a Weibull distribution was fitted to the FDD data from the Dunkirk NY site (this site is more severe than the South Bass Island site and thus gives a more conservative result); this Weibull distribution can be seen in Fig. 5-6. Minitab reports that the Weibull distribution fits this data with a p-value greater than 0.25; any p-value larger than 0.05 indicates that the distribution fits the data adequately. From the Weibull distribution parameters provided by Minitab, the event which has exactly 98% chance of not being exceeded in any given year was found: this once per 50 year event worked out to be 469.3 FDDs. Inputting this value into Eq. 5.1 value of about 61.8 cm ice thickness can be calculated. That is, the ice in Lake Erie will be thicker than
61.8 cm only once out of every 50 years on average. This thickness value will be used for our simulations.

Another distribution which was tried to fit to the FDD data was the Gumbel distribution (The Minitab software did not include this distribution). By taking the mean value and the standard deviation of the FDD data, one may calculate the value which will be exceeded on average once per 50 years. This is done using Eq. 5.4, taken from [C-CORE 2008]. This is called Gumbel extrapolation.

Table 5.1: Average and Standard Deviation of Cumulative FDD data

<table>
<thead>
<tr>
<th></th>
<th>Dunkirk, NY</th>
<th>South Bass Island, OH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average of FDD data</td>
<td>220.66</td>
<td>231.31</td>
</tr>
<tr>
<td>Standard Deviation of FDD data</td>
<td>108.84</td>
<td>102.17</td>
</tr>
</tbody>
</table>

\[
FDD_T = \mu - \left[ \frac{\sqrt{\pi} \left( 0.5772 + \ln\left( \frac{T}{T-1} \right) \right)}{\sigma} \right] \sigma
\] (5.4)

In Eq. 5.4, \(T\) is the desired return period in years, \(\sigma\) is the standard deviation of the data, and \(\mu\) is the mean of the data.

Using the Gumbel extrapolation, it was found that the once per 50 year FDD value for the Dunkirk site to be 502.8 degree C days, yielding a value of 64 cm ice thickness. This is consistent with the previously obtained value of 61.8 cm, indicating that this value is in the correct range.
5.3 Once per 50 year Ice Ridge Keel Depth

From the ice scour data [C-CORE 2008] and [Lever, 2000] the once per 50 year ice ridge event was determined, according to Eq. 5.5, which was created to agree with the ice ridge data from Section 4.6.

\[ h_{kee} = 1.954 \ln(T) + 3.501 \]  \hspace{1cm} (5.5)

In Eq. 5.5, \( h_{kee} \) is the depth of the ice keel in meters, and \( T \) is the desired return period in years. The once per 50 year ice keel depth is 11.2 meters.
Chapter 6

Ice Load Calculation Results

6.1 Calculation Verification

To verify the ice load calculations conducted in this work, the calculation results from the [Ralston, 1977] source are compared with those of this study in Table 6.1, using the input values given in that source. These calculations apply to the case of an ice cone being struck by a 3 foot thick ice sheet with a flexural strength of 100 psi; the cone has a waterline diameter of 60 feet, a diameter of 20 feet at the narrowest part of the cone, a cone angle of 45°, and an ice-cone frictional coefficient of 0.15.

Table 6.1: Ice Load Calculation Verification

<table>
<thead>
<tr>
<th></th>
<th>Present Study</th>
<th>Ralston 1977</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Load</td>
<td>3.17 MN</td>
<td>3.16 MN</td>
</tr>
<tr>
<td>Vertical load</td>
<td>3.01 MN</td>
<td>3.01 MN</td>
</tr>
</tbody>
</table>

6.2 Foundation Geometry

The simulations will use the NREL Offshore 5MW Baseline Wind Turbine system, described in [Jonkman, 2006], configured for shallow water operation. The wind turbine tower is mounted atop a monopile support structure. The monopile extends from the mudline (the seabed) to 10m above the water surface, where it connects to the tower. (The base of the monopole is assumed to be rigidly connected to the seabed; other foundation models are outside the scope of this work). The tower extends up to give a
hub-height of 87.6m above the water level. The NREL Offshore 5MW Baseline system assumes a water depth of 20m, giving a total length of the structure (tower and monopile together) of 107.6m from the seabed to the rotor hub. This choice of depth was made by NREL when they designed this turbine model, but it is appropriate for Lake Erie, which has an average depth of about 20m.

Three different foundation geometries at the waterline will be examined: cylindrical, upward breaking cone, and downward breaking cone. The monopole will be a 6m diameter vertical-walled cylinder (which is the configuration of the NREL 5MW offshore turbine provided by NREL). To test the effects of an ice cone, such a cone was added in this study onto the pre-existing monopole structure. Based on the ice cone geometry presented in [C-CORE 2008] shown in Fig. 6-1, typical ice cone geometry has a cone angle of 50° from the horizontal, and a waterline diameter of 1.84 times the tower diameter.

![Figure 6-1: Waterline Structural Geometries including Ice Cones [C-CORE 2008]](image)

This means that to add an ice cone to the 6m diameter tower, an 11m diameter cone should be used; for both the upward breaking cone and downward breaking cone simulations. In this work, an 11m waterline diameter cone with a 50 degree angle from
the horizontal was be used. Below the waterline, where the ice ridge keel acts on the structure, the structure is assumed to be a vertical walled cylinder with a diameter of 6m. This follows the given design of the NREL Offshore 5MW Baseline tower, which uses a 6m diameter monopile extending to the seabed. This work follows that design, except for the addition of the ice cone at the waterline.

6.3 Ice Properties used for All Calculations

Three loading cases were examined: wind only, wind plus ice sheet, and wind plus ice sheet plus ice ridge rubble keel. The ice conditions for the simulations were based on the 50 year event calculated in Section 5.2. For the ice sheet only case, a value of 61.8 cm was used for the ice sheet thickness. For the ice sheet plus ice keel, an ice sheet thickness of 92.7 cm was used (this is the 1.5 times thicker consolidated layer which refreezes when the ice sheet buckles and forms a ridge [C-CORE 2008]), and an ice keel depth of 11.2m was used. The angle which the submerged ice rubble makes with the horizontal is assumed to be 30 degrees according to [C-CORE 2008], and the ice keel is assumed to come to a point at the depth of 11.2m, giving an ice keel width of approximately 39m.

For these simulations, the force from the ice sheet is assumed in this work to act at a point 20m up from the seabed, exactly at the waterline. The force of the ice keel is assumed to have a triangular load distribution (according to [C-CORE, 2008]), so it will be applied at a point one third of the effective keel height down from the waterline.
The ice bending strength used is 1770 kPa [Timco, 1982] (this value agrees with the value of 1760 kPa from [Timco, 1994]) and the crushing strength of the ice is 4400 kPa [Timco, 1982].

The ice motion direction is the same direction as the wind direction, corresponding to an extreme case, although any direction can be used. The ice motion speed is 20 centimeters per second (a plausible value picked from the data in [Campbell, 1987] discussed in Section 4.5). Given the 20 cm/sec ice speed, a breaking period of about 25 seconds is calculated for the ice cone cases, according to the breaking frequency used in Eq. 3.10. For the vertical cylinder case, the ice breaking frequency is set to the first tower natural frequency, which for the NREL Offshore 5MW Baseline system gives a breaking period of close to 2.5 seconds.

The ice-on-steel friction coefficient used is 0.15. The water density used is 999.8 kg per cubic meter for the upward breaking cone. For the downward breaking cone, [IEC, 2009] states that one should use one ninth of the water density in the calculations; this accounts for the fact that it is easier to submerge the ice pieces rather than lift them out of the water. The cone geometry is as described above. For the ice keel calculations, an ice density of 916 kilograms per cubic meter was used. Also, for the ice keel calculations, an effective structure width of 6 meters was used (matching the tower diameter).

### 6.4 Calculated Lake Erie Ice Loads

Using the stated foundation geometry and stated ice properties, the horizontal and vertical loads from each possible tower shape (vertical cylinder, upward breaking cone, downward breaking cone) and for both the case of just the once per 50 year ice sheet, as
well as for the once per 50 year ice keel (including the thicker consolidated layer ice sheet) were calculated. The horizontal loads caused by a moving ice sheet are shown in Table 6.2; these loads depend on both the ice properties, and on the structural geometry; all of which has been described previously.

Table 6.2: Maximum Loads due to a 61.8cm thick moving ice sheet

<table>
<thead>
<tr>
<th></th>
<th>Vertical Cylindrical Structure</th>
<th>Upward Breaking Ice Cone</th>
<th>Downward Breaking Ice Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Load</td>
<td>9.04 MN</td>
<td>2.42 MN</td>
<td>1.88 MN</td>
</tr>
<tr>
<td>Vertical Load</td>
<td>0 MN</td>
<td>1.87 MN</td>
<td>1.44 MN</td>
</tr>
</tbody>
</table>

Note that the vertical cylindrical structure receives much greater loads than either the up or downward breaking ice cone, with the downward breaking ice cone receives the smallest loads.

Table 6.3: Maximum Loads due to a 92.7cm thick Moving Ice Sheet

<table>
<thead>
<tr>
<th></th>
<th>Vertical Cylindrical Structure</th>
<th>Upward Breaking Ice Cone</th>
<th>Downward Breaking Ice Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Load</td>
<td>14.66 MN</td>
<td>5.02 MN</td>
<td>4.18 MN</td>
</tr>
<tr>
<td>Vertical Load</td>
<td>0 MN</td>
<td>3.86 MN</td>
<td>3.19 MN</td>
</tr>
</tbody>
</table>

Table 6.4: Ice Keel Depths and Loads

<table>
<thead>
<tr>
<th></th>
<th>Ice Keel Depth</th>
<th>Maximum Keel Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Once Per 50 year event</td>
<td>11.2 m</td>
<td>3.43 MN</td>
</tr>
</tbody>
</table>
As the ice keel breaks against the foundation, the force it exerts on the structure increases, until global failure of the keel occurs. Table 6.4 shows the maximum force which is placed on the structure; this occurs just before global failure of the keel.

The horizontal loads from a nonmoving ice sheet are shown in Table 6.5; these loads do not depend on ice thickness. The loads from thermal ice pressure and arch effect depend on the waterline diameter of the structure. The horizontal load due to wind drag on the ice sheet depends on the ice sheet surface area and the wind speed, as shown in Eq. 3.6.

Table 6.5: Loads from non-moving ice sheet

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Load due to Wind Drag</td>
<td>Less than moving ice load</td>
</tr>
<tr>
<td>Horizontal Load due to Thermal Ice Pressure</td>
<td>3.312 MN</td>
</tr>
<tr>
<td>Horizontal Load due to Arch Effect</td>
<td>2.208 MN</td>
</tr>
</tbody>
</table>

### 6.5 Significance of Moving vs. Stationary Ice

The reader should realize that the ice sheet would break and begin to move if any combination of the loads from Table 6.5 were to exceed the force of a moving ice sheet (keep in mind that these loads do not necessarily act in the same direction). In order for the wind drag on the ice sheet to exceed the ice sheet breaking force (e.g. 2.42 MN for an upward breaking ice cone, from Table 6.2), given a wind speed of 11.4 m/s (the turbine rated wind speed), an ice sheet of about 7 square kilometers (3 km diameter disk) would
be required (this value was calculated using Eq. 3.6). Figure 6-2 from [NASA Earth Data, 2012] shows some actual ice floes on Lake Erie. Figure 6-2 was taken on Feb 22, 2011.

![Image of Lake Erie with ice floes](image)

Figure 6-2: NASA satellite image of Lake Erie, Feb 22, 2011, showing ice floes [NASA Earth Data, 2012]

The scale of this image is 250 m per pixel in the original image. Ice floe A is about 142 pixels wide, or about 35 km. Ice floe B is about 60 pixels across, or 15 km. Ice floe C is about 28 pixels across, or 7 km. Using Eq. 3.6, we find that a 7 km diameter ice floe only requires a wind speed of 5 m/s to cause a 2.42 MN load (sufficient to begin ice motion and breaking against the upward breaking ice cone). A 35 km diameter ice floe only requires a 1 m/s wind speed to create a 2.42 MN load. Because the average wind speed offshore from Cleveland is about 7 m/s, it is reasonable to assume that it will not be rare for the structure to encounter moving ice sheets and need to plow through them.
6.6 Combining the Ice Loads onto One Structure

Putting it all together, the combined load case on the structure can be considered. For a stationary ice sheet, a combination of loads would occur, made up of loads due to thermal ice pressure and arch effect (these should not be time dependant) and the load due to wind drag on the ice sheet (this may be time dependant, because the wind speed and direction could change with time). This load combination cannot exceed the load magnitude due to a moving ice sheet plus possible ice keel (if the wind is strong enough, the ice sheet and possible ice keel will simply break and begin moving, rather than allowing the load on the structure to increase further).

For a moving ice sheet, the load required to break the ice sheet will always be present, in addition to the load required to break through an ice ridge keel (which will not always be present). In either case, (moving or stationary ice) the maximum load is limited to the value required to break the ice sheet (plus possible ice keel). Because the moving ice sheet case subjects the foundation to more sharply changing loads, i.e. the sudden cracking of ice (shown by Fig. 3-3) as opposed to smoothly changing winds, the moving ice sheet case is judged to be the more severe load case, and will be the subject of the simulations presented in this work.

From [IEC, 2009] a triangular load profile can be generated using the calculated loads from above as the peak loads of the dynamic load profile. However, none of the sources used in this report suggest a way to apply the ice keel load as a dynamic load. From the equations given by [C-CORE 2008] the ice keel load is related to the penetration depth of the structure into the ice keel. Assuming that the ice keel is attached to the ice sheet and moves with it, the force vs. penetration depth can be converted to...
force vs. time for the moving ice ridge. Another option is to consider that the ice keel can crush against the tower in the same way as a flat ice sheet against a vertical tower. So we could have the force vs. penetration depth modulated by a sine wave or a saw tooth wave to add this dynamic effect. However, no work was found in the literature to support these assumptions with measured data. In this work, it is assumed that the load of the ice keel will be proportional to the penetration distance of the structure into the ice ridge. The ice keel force will go to zero once global failure of the keel occurs.
Chapter 7

Ice Load Implementation in FAST

7.1 Description of FAST Program

FAST [Jonkman, 2005] is an aeroelastic simulator that can calculate the extreme loads and fatigue loads for horizontal axis wind turbines, working for both two and three bladed varieties. FAST was developed by the National Renewable Energy Laboratory.

A detailed description of all the inner workings of FAST is beyond the scope of this paper, so only a qualitative description will be given. As described in the Users Guide [Jonkman, 2005], FAST operates by a time marching simulation to determine the structural and aerodynamic response of the wind turbine to the given wind conditions. FAST runs a combination of modal dynamics and multibody dynamics. The major components of the turbine (support platform, tower, base plate, nacelle, hub, blades, etc.) are modeled as separate bodies connected to each other through certain degrees of freedom. The blades, tower, and driveshaft are modeled as flexible bodies, and the other components being modeled as rigid bodies.

The tower is divided into an arbitrary number of equal length segments; these are used to integrate the elastic forces along the length of the tower. Each of these segments is subjected to a uniform load along the length of the segment; this is how FAST applies the wave loads along the length of the tower.
7.2 User Tower Loading Subroutine

In order to apply the desired ice loads to the wind turbine system in FAST, the user tower loading subroutine was used (henceforward referred to as UserTwrLd, this is the name of the subroutine in the FAST source code). Code can be inserted into the user tower load subroutine to generate the desired time dependant loads due to ice, and to apply these loads to whichever tower segment is desired. Because UserTwrLd only applies uniform loads distributed over at least one tower segment, point loads cannot be applied. Instead, any load we choose to apply to the tower must pass through the center of one of the tower elements (i.e. it must be evenly distributed over an element). In the case of a horizontal load, unless the desired point of application coincides exactly with the center of a tower element (this is unlikely), there will be an offset between where the force is supposed to act on the tower, and where it actually acts on the tower. This offset will change the bending moment caused by the horizontal load (the offset changes the lever arm of the bending moment). To correct this, a certain bending moment is applied to the tower segment in addition to the horizontal load. This bending moment is calculated to exactly balance the bending moment deficit or excess caused by inaccurately placing the horizontal force at the center of the tower element. The end result is that a horizontal load may be placed anywhere on the tower (not just at the center of a tower element), and the bending moment created at the base of the tower by that horizontal load will accurately reflect the desired location at which the horizontal load was placed.

What follows is a step by step explanation of how the modified subroutine works in FAST. When the fast program is running, the tower loading subroutine is called during
each time step to determine the load which occurs on each tower segment. The subroutine has the ability to retain information between subroutine calls, so it is possible to have the subroutine perform certain actions on only the first time it is called (rather than at each time step of the FAST simulation). So, on the first call to the user tower loading subroutine, an external text file is opened, and values from this file are taken into the program. The FAST program takes in other case specific data (geometry, tower and turbine distributed properties, wind conditions, etc) in the same way. The values taken in from the external text file are preserved in memory for use in subsequent calls to the subroutine. The values that come into the program are shown in Fig. 7-1.
Figure 7-1: Sample Ice Properties Input File
7.3 Generating a Dynamic Load Profile

During the first call to UserTwrLd, the horizontal and vertical loads on the tower from the ice sheet are calculated according to the procedure discussed in Chapter 3. The node at which the ice forces must be applied to the tower is also calculated; this is based on the user input of the water depth, which specifies where the ice sheet hits the tower. The results of these calculations are saved for use during subsequent calls to UserTwrLd. Note that the calculations from Chapter 3 give the maximum or peak loads caused by the ice sheet. In addition to calculating the peak load, the subroutine also calculates the bending moment deficit needed to account for the horizontal force not being located exactly at the center of a tower segment, as previously discussed. Furthermore, the bending moment caused by the vertical load being offset from the centerline of the tower is calculated. All four of these loads (horizontal and vertical forces, and the two bending moments) are computed during the first subroutine call and saved for use during subsequent calls. These loads are the maximum (peak) loads which will be applied to the structure.

The dynamic loads caused by the ice sheet fluctuate between 0% and 100% of these maximum loads [IEC, 2009]. During each time step, the magnitude of a sawtooth waveform is calculated, which varies between zero and one. The sawtooth waveform used in these simulations is based on the one recommended by the IEC standard. In these simulations, the triangular waveform ramps for the first 70% of each cycle, and ramps down during the remainder of the cycle.

Once the horizontal and vertical loads have been calculated, they may be scaled by the magnitude of the sawtooth wave and applied to the appropriate tower segment.
The moments required to compensate for the horizontal and vertical loads being offset from the center of the tower segment are also scaled by the sawtooth wave and applied to the proper tower segment. The user may specify at what time the loading begins (the loads do not necessarily begin at $t=0$). In this way, the time dependant loads from the ice sheet are applied to the tower.

In order to apply the load from the ice ridge keel to the structure, UserTwrLd uses the user specified information of ice speed and the time at which the ice ridge begins touching the foundation to calculate how far the monopile foundation has penetrated into the ice ridge. The ice ridge forces depend on the penetration distance of the foundation into the ridge. At each time step after the ice ridge begins hitting the foundation, the penetration distance and then the force are calculated, according to equations from Section 3.4. These calculations are repeated for each time step after the ridge first touches the foundation. The height of the part of the ice ridge keel which contacts the foundation (called “effective height” of the keel) changes with the penetration distance of the foundation into the ice ridge, as shown in Fig. 7-2.
It is assumed that the ice ridge keel load has a triangular distribution [C-CORE 2008]. Equivalently, the ice ridge load can be modeled as a point load acting at the one third of the effective height of the keel down from the waterline.

Once the magnitude and location of the ice ridge keel load has been determined, it must be applied to one or more tower segments. Because the location of the ice ridge keel load depends on the effective height of the ice ridge keel (which changes with time), the location of the ice ridge keel equivalent load will move with time. As the point of application of the ice load moves the load needs to be applied to different tower segments. If the ice ridge load were applied to only one tower segment, this would require an additional bending moment to be applied to that segment to correct for the ice ridge force location not being at the center of that segment; however, this will introduce a flaw. That is, as the center of the ice ridge load moves down the tower, the tower node to
which it should be applied will change. When the ice keel load jumps from one tower segment to another, the additional bending moment will also need to move to the new segment. When the ice keel load changes from being below the tower segment center to above the tower segment center, the direction of the additional bending moment will change. This change in direction will induce a disturbance in the tower; however this is not a real physical phenomenon, rather it is a modeling shortcoming. To eliminate this flaw, the ice ridge keel load is applied as two horizontal loads on two adjacent tower segments, instead of a horizontal load and a bending moment on one tower segment. This lets the ice ridge load cause the correct foundation base bending moment even if it is not at the center of a tower segment, and it also prevents non-physical shaking of the tower when the ice ridge load changes location on the tower.

In order to determine when the maximum ice ridge keel load occurs, UserTwrLd compares the keel load from the previous time step to that of the current time step. If UserTwrLd finds that the ice ridge keel load has decreased in magnitude, it means that the ice ridge keel load has reached its maximum which means that global failure of ice ridge keel has occurred (explained in Section 3.4). Once the global failure of the ice ridge occurs, UserTwrLd sets the ice ridge keel force to zero for all future time steps of the simulation.

7.4 Validating FAST Calculations with ADAMS

There exists software other than FAST that can be used to simulate what happens to the wind turbine; one such tool is named ADAMS [MSC Software, 2012] (Automatic Dynamic Analysis of Mechanical Systems). ADAMS works differently than FAST:
Where FAST simulates the blades and tower of the wind turbine as flexible bodies using one or two pre-calculated mode shapes, ADAMS simulates the blades and tower as an array of six-DOF rigid bodies, each with lumped mass and inertia properties. This array of bodies is connected together with joints of appropriate stiffness and damping (represented by stiffness and damping matrices), described in [Jonkman, 2007]. This configuration makes it unnecessary for ADAMS to use pre-calculated mode shapes like FAST. By using two different methods to both simulate the same wind turbine, the results can be compared and used to validate one method against the other. This is important because it allows us to validate the results from FAST (ADAMS is a widely used and accepted general purpose software not limited to wind turbines).

It is necessary validate the modified version of FAST to see that it works properly when simulating ice loads on offshore turbines. For an offshore turbine, the submerged portion of the structure will likely have very different stiffness properties from the above water portion of the tower. In this work, FAST simulates these two parts of the structure as one flexible beam with varying properties (recall that this flexible beam is limited to two bending mode shapes by FAST). To verify that FAST can correctly simulate the two parts of the tower as one beam with varying properties, we compare FAST predictions with the results from ADAMS, presented in Section 8.3.
Chapter 8

FAST Simulation Results

8.1 Results from Flat Ice Sheet

In this chapter, the FAST simulation results are presented, showing how the NREL 5MW Offshore Baseline turbine responded to loads from a 61.8cm thick moving ice sheet. Simulation results of the same turbine subjected to loads from a 92.7cm ice sheet and an 11.2 m deep ice keel are also shown. All tower and ice properties are the same as those given in Sections 6.2 and 6.3. The loads from the ice sheet are applied at 20m up from the tower base (this assumes a 20m water depth), and the loads from the ice keel are applied at one third of the keel effective height down from the waterline (this assumes a triangular load distribution for the keel load). The direction of motion of the ice is the same as the direction of the wind.

The wind speed used for all simulations was 11.4 m/s, which is the rated wind speed of the NREL 5MW Baseline turbine [Jonkman, 2009]. The rated wind speed is chosen because it is expected to give a large representative steady wind load (gusts, other unsteady winds, or winds not aligned with the turbine yaw direction are not examined in this work). The rated wind speed is where the turbine first begins producing its rated power of 5MW. If the wind speed were to increase above the rated wind speed, the blades would be pitched to reduce rotor load and avoid exceeding the 5MW rated power. So the rated wind speed was selected to give the largest loads on the rotor; this disregards the loads on the nacelle and tower, which do not reach a maximum at the rated wind speed.
Figure 8-1: Foundation Base Shear Force for 61.8 cm thick ice sheet, showing cylindrical structure and two conical structures

Figure 8-1 shows that the cylindrical structure loading has higher frequency than the conical structure loading. For the cylinder, the frequency of the ice load is the same as the lowest tower natural frequency; this loading frequency is specified by [IEC, 2009]. The reason why IEC specifies this loading for cylindrical structures is that the tower shakes at its natural frequency; each time the tower vibrates, it presses against the edge of the ice sheet and induces crushing of the ice, which, in turn, induces the ice force on the tower. For the ice cone, edge of the ice sheet will ride up (or down) the side of the cone, until the ice sheet is deflected out of the plane of the water surface enough to cause bending failure in the ice. This bending failure will occur at some distance from the ice cone. As the ice moves, the edge of the ice sheet where the crack occurred will once
again touch the ice cone and ride up the side of the cone. This causes a new crack to form and the cycle to repeat. Depending on how fast the ice is moving, and how far away the cracks form from the cone, the frequency of the load peaks from the ice will change (the peaks in the force occur at the moment when the ice sheet cracks). This process does not depend on the natural frequency of the tower, because the natural frequency of the tower is much higher than the breaking frequency of the ice.

A transient can be seen in the peaks of the cylindrical structure loading; this is likely a result of the tower having to adjust to a change in average load.

The cylindrical structure setup produces huge ice forces, peaking around 10 MN, much higher than either of the conical structures. The ice cones give peak shear forces of about 3 MN for the upward breaking cone, and about 2.25 MN for the downward breaking cone. The significantly lower loads due to the ice cones when compared to the no-cone setup should make it obvious that any offshore structure subject to these ice conditions (a 61.8 cm thick ice sheet) will incorporate an ice cone.
Figure 8-2: Foundation Base Shear Force for 61.8 cm thick ice sheet, for the two ice breaking cones.

Figure 8-2 shows the same 61.8cm ice sheet case, but excludes the cylindrical structure case. For the upward breaking cone, the combined ice and wind loads peak at a value about 330% larger than the wind only loads. For the downward bending cone, the ice plus wind loads peak about 260% higher than the wind only loads. Small oscillations can be seen in the load, even for the no ice condition. These are not caused by the ice; rather they are a normal component of the wind turbine operational loads. The rotor frequency is 12.1 RPM (0.202 Hz), the tower natural frequency is about 0.4 Hz, but these oscillations appear to be at 0.28 Hz. The oscillations appear to die off, so it is likely that these are part of a startup transient, caused when the structure starts in a neutral position and then is subjected to the wind at the beginning of the simulation. These oscillations are
not caused by the ice loading, and are orders of magnitude smaller than the ice effects, so they are not a significant concern.

Note here that the downward breaking cone gives lower foundation base shear force than the upward breaking cone.

Figure 8-3: Foundation Base Bending Moment for 61.8 cm thick ice sheet, for the cylinder and both ice cones

Figure 8-3 shows that the cylindrical structure produces a vastly larger foundation base bending moment than either cone structure. This should remove all doubt that ice cones should be used on offshore structures in Lake Erie.
Figure 8-4: Foundation Base Bending Moment for a 61.8cm thick ice sheet, for the two ice breaking cones

Figure 8-4 shows that the foundation base bending moment from the combined ice and wind case peak 48% and 56% higher than the wind only case, for the upward and downward breaking cones, respectively. The downward breaking cone gives larger foundation base bending moment than the upward breaking cone. However, the downward breaking cone gave a smaller foundation base shear force than the upward breaking cone. Note that the foundation base shear force is the same as the horizontal ice load applied at the waterline. If the horizontal ice loads for both cone shapes are applied at the same distance from the foundation base (which they are), then the larger load must create the larger bending moment. How can this be explained? The answer is that in
addition to horizontal loads, each ice cone is subjected to vertical loads from the ice, as shown in Fig. 8-5.

![Figure 8-5: Location of Horizontal and Vertical Loads on the Ice Breaking Cones](image)

The vertical load for the upward breaking cone presses down on the side of the cone, giving a moment in the opposite direction from the moment caused by the horizontal load (this mitigates the moment caused by the horizontal load). For the downward breaking cone (which submerges the ice instead of lifting it), the vertical load pushes up on the cone, giving a moment in the same direction as the moment from the horizontal load. This enhances the moment caused by the horizontal load. Consequently the downward breaking cone experiences a larger total foundation base bending moment than the upward breaking cone.
Figure 8-6 shows that the tower top displacement is not very much altered by the 61.8cm ice sheet. The tower top deflection is increased by ten to fifteen percent; the downward breaking cone creates more tower top deflection than the upward breaking cone because this cone shape creates larger foundation base bending moments than the upward breaking cone.
As can be seen from Fig. 8-7, the blade root bending moment is not significantly changed at all by the presence of ice sheet loads on the structure (notice that all three lines overlap). This means that ice loads should not affect the design of the blades or influence their fatigue life.
8.2 Results from Flat Ice Sheet plus Ice Ridge Keel

Figure 8-8: Foundation Base Shear Force for 92.7 cm thick ice sheet and 11.2 m deep keel, for the upward cone and downward cone.

When the ice keel is present, the surface ice sheet will have an increased thickness (here 92.7 cm instead of previous 61.8 cm) due to the buckling and re-freezing which forms the ice ridge. The foundation base shear force for the combined ice and wind case peaks at values 1060% and 1180% larger than the wind only case, for the upward and downward breaking cones, respectively. The thicker ice sheet causes a lower breaking frequency to occur in the ice sheet: compare the approximately 25 second breaking period of this case to the approximately 17.5 sec breaking period for the 61.8 cm ice sheet. In addition to the ice sheet loads, the ice rubble keel below the ice sheet adds a load superimposed on top of the ice sheet load. This rubble keel load slowly increases starting at 80 seconds, and at
about 155 seconds, suddenly vanishes. This is when the ice keel load has increased enough to cause global failure in the ice keel; the entire keel breaks and can no longer exert its load on the structure. In the troughs of the triangular wave-form, the loads from either cone shape are the same; at this point the ice sheet loads are zero and only the ice keel load is present. The ice keel gives the same loads to both cone shapes because it is assumed that it hits the structure below the cone under the water, where the two structures have the same shape.

Figure 8-9: Foundation Base Bending Moment for 92.7cm thick ice sheet and 11.2m deep keel, for the upward cone and downward cone.
Figure 8-9 shows the foundation base bending moment for the worst case scenario: thick ice plus ridge and keel. The foundation base bending moment is the highest bending moment loading along the tower height and thus it can show if the whole structure will be knocked over (that is, if the structure fails, it will fail at its base (at the seabed) through bending). This figure shows that the bending loads from the ice ridge and keel are very severe, reaching levels about 150% and 200% higher than the wind only loads, for the upward and downward breaking cones, respectively. For the same reasons as discussed before, the downward breaking cone gives larger bending moments than the upward breaking cone.
Figure 8-10: Tower Top Displacement for 92.7cm thick ice sheet and 11.2m deep keel, for the upward cone and downward cone.

Figure 8-10 shows that, during the same simulation, the tower top displacement is somewhat affected by the ice ridge and keel: it receives at between 15 and 20 cm of additional displacement in addition to the 50cm displacement from wind loads alone. Note that the downward breaking cone gives greater overall displacement than the upward breaking cone; this is due to the greater bending moment caused by ice on the downward breaking cone compared to the upward breaking cone.
Figure 8-11: Blade Root Bending Moment for 92.7 cm thick ice sheet and 11.2 m deep keel, for the upward cone and downward cone.

The blade root bending moment is not significantly altered by the presence of the 92.7cm ice sheet or ice keel. The only thing discernible is the disturbance near the time of 155 seconds, when the ice ridge undergoes global failure. The blade root bending moment changes are due to changes in the tower top displacement caused by the varying ice loads. However, this is a relatively small disturbance compared to the magnitude of the overall load on the blade.
8.3 Alternative Foundation Types

The foregoing results all show an offshore wind turbine tower supported by a monopole foundation. However, other foundation designs exist, such as a gravity base foundation. Such designs may be easier to construct in the Great Lakes, and are worthy of consideration in this work. In FAST, the monopole foundation is approximated as a one dimensional beam with certain mass and stiffness properties which can change along the length of the beam. In order to simulate a gravity foundation with the existing modeling capabilities of FAST, the gravity foundation would also have to be approximated as a one dimensional beam. This would require detailed information about the exact configuration of the foundation, which was not available to this research. However, the utility of this technique can be evaluated by simply altering the properties of the monopole foundation which comes with the 5MW NREL model.

It was arbitrarily selected to increase the mass and stiffness of the bottom portion of the tower (bottom 30m of the tower which is the monopole foundation) by a factor of ten; this is meant to approximate a gravity foundation structure. It is important to note that the tower mode shapes must be recalculated (using B-Modes software from NREL) when the tower properties are changed. The simulation was re-run in FAST with the new tower; this allowed a comparison between the new foundation, modeled as a hypothetical 10x mass and stiffness monopole, with the original monopile foundation. This shows how well this technique could work to simulate an actual gravity foundation. The results are shown in Fig. 8-12. Only the tower top displacement is shown; foundation base bending and foundation base shear are the same regardless of tower properties.
Figure 8-12: Tower Top Displacement for original and 10x mass and stiffness foundation; run in both FAST and ADAMS

Figure 8-12 shows that the tower with the stiffer foundation segment responds much less strongly to the ice. However, by comparing the ADAMS and FAST results, it can be seen that the two software packages agree less for the stiffer tower than for the original un-stiffened tower. ADAMS should be more accurate, because it does not use mode shapes to simulate the tower as FAST does; rather ADAMS simulates the tower as a series of six-degree-of-freedom connected elements. Also ADAMS includes torsion deformation of the blades. So the deviation of FAST predictions from those of ADAMS means that for the stiffer tower, FAST predictions decease in accuracy.

For very rigid foundations, such as gravity foundations, an alternative simulation model in FAST is to consider the foundation as a rigid body of known mass, while the
tower is clamped on top of it. In this case the ice loads must be applied to the rigid body (which in turn is connected through matrices of stiffness and damping values to the ground).

The design of a gravity base foundation was outside the scope of this research and the assumed stiffness and mass to model a gravity foundation may be unrealistic to determine reliably the validity of FAST predictions for a gravity base foundation. This question cannot be fully answered unless actual foundation properties are acquired.
Chapter 9

Foundation Design Implications

In this work, the ice loads were applied to a structure with a 6m diameter monopile foundation equipped with an 11.04 m waterline diameter ice cone. It was found that the bending moment at the base of this foundation was increased significantly by the presence of ice. The bending moment at the base of the foundation causes a certain amount of bending stress. In this chapter, hypothetical foundation structures were examined which are designed to have the same bending stress (when subjected to wind and ice loads) as the original 6m diameter monopile structure subjected only to wind loads. These hypothetical foundations are only meant to show the influence of ice loads on the foundation design, not to design the perfect foundation structure. To create a new hypothetical foundation which bears the additional ice loads but does not increase the bending stress felt at the base of the foundation, the diameter of the foundation was increased. It is assumed in this work that the bending stress is much larger than the other stress components (e.g. shear stress) so only bending stress will be examined here.

The hypothetical foundation will be a monopole equipped with an ice cone, and it will keep the same ratio of underwater monopole shaft diameter to cone waterline diameter as the original 6m diameter monopole foundation. (That is, the cone waterline diameter will be fixed at 1.84 times the underwater monopole shaft diameter). These dimensions are illustrated in Fig. 9-1 for an upward breaking cone. By keeping the ratio the same when changing the size of the foundation, the cone is kept appropriately sized to the monopole.
The original 6m diameter monopole foundation examined in this thesis has a 6 cm wall thickness [Jonkman June 2006]. The same wall thickness will be used in the new hypothetical foundation.

From the FAST simulation results, it was found that the foundation base bending moment due solely to wind loads to be about 78,400 kN*m. Using the 6 m monopole diameter and 6 cm wall thickness, second moment of area of the tower base cross section for the original foundation can be calculated according to Eq. 9.1, and the maximum bending stress at that cross section according to Eq. 9.2 is found to be about 48 MPa. For reference, steel has a yield stress of about 250 MPa.

\[
I_{\text{cylinder}} = \frac{\pi}{4} (r_{\text{outer}}^4 - r_{\text{inner}}^4)
\]

(9.1)

\[
\sigma_{\text{bending}} = \frac{M \cdot y}{I}
\]

(9.2)

In Eqs. 9.1 and 9.2, \(r_{\text{outer}}\) is the outer radius of the cylindrical monopole foundation, \(r_{\text{inner}}\) is its inner diameter, \(M\) is the bending moment at the tower base cross section.
section, $y$ is the radius to the outer fibers of the tower base cross section, and $I$ is the second moment of area of that cross section.

When the most severe ice conditions (the 50 year ice ridge consisting of 92.7cm ice and an 11.2 m deep keel) are applied to the 6 m diameter tower equipped with an upward breaking ice cone having an 11.04 m cone waterline diameter, a total bending moment (including wind effects) at the base of the foundation of about 240,500 kN*m is found, which is about 200% more than the wind only case. This induces a bending stress at the base of the foundation of about 131 MPa.

To keep the same bending stress as the wind only case while applying the ice loads to the structure, a larger foundation is needed. Keep in mind that increasing the diameter of the foundation will also increase the ice loads, so designing a new hypothetical foundation is an iterative process. Keeping the same ratio of monopile diameter to ice cone waterline diameter as the previous foundation, it takes a foundation with a 10.44 m monopile diameter and a 19.2 m cone waterline diameter to bear the loads from the 92.7 cm ice sheet and the 11.2 m deep keel while only experiencing a foundation base bending stress of 48 MPa. This is an increase in both monopile diameter and cone waterline diameter of 74% over the original foundation.

Alternately, the same cone waterline diameter and tower diameter can be retained; instead the wall thickness of the foundation structure can be increased. If a new wall thickness of 17.5 cm is chosen (increased from the original 6 cm, almost tripled), the foundation can bear the most severe ice loads while maintaining a foundation base bending stress of 48 MPa.
It is up to the foundation designer to decide what a feasible foundation design is; however, it can be seen that it is possible to imagine hypothetical foundations capable of bearing the most severe ice loads in Lake Erie while keeping the bending stresses equal to those caused only by the wind loads. These hypothetical foundations are admittedly very simple and crude models (a real foundation would likely have more complex geometry than just a hollow constant diameter cylinder) but they show that such foundation design is possible without resorting to absurd materials or dimensions.
Chapter 10

Conclusions

10.1 Steps to Achieve Goal

The goal of this work was to study the effects of ice loading on an offshore wind turbine, particularly in Lake Erie.

Following the IEC standard, methods were gathered to calculate each of the ice loads. This includes loads caused by both moving ice and ice fixed to the structure. The IEC standard also permits a dynamic load profile to be generated from the load calculations. The required environmental information for the load calculations was collected from governmental sources, such as the NOAA, Great lakes Environmental Research Laboratory, and National Research Council Canada. This information included ice thickness, ice strength, ice movement speed, etc. The methods for calculating loads acting on an offshore wind turbine were then implemented in the NREL software called FAST. The FAST program (which natively simulates a wind turbine subject to wind loads) was modified to read in the ice properties supplied by the user, and to internally calculate the ice loads and generate a dynamic load profile, which was applied to the tower during simulation. The FAST simulation results were examined to determine the effects the ice loads have on the wind turbine structure.
10.2 Environmental Data Quality

One problem encountered during this work was the poor quality of the available ice thickness measured data. These data were limited in precision, so much so that a statistical analysis to determine a once per 50 year ice thickness event was impossible. Instead, an ice growth model was used, which predicts the ice thickness based on the air temperature history over the lake. Suitable air temperature measurements were found from the National Data Buoy Center (part of NOAA) which allowed a usable estimation of the ice thickness to be made.

10.3 Results

It was found that the once per 50 year ice thickness event is about 61.8cm, and in the case of an ice ridge, 92.7 cm with a 11.2m deep rubble keel.

Using the above data, FAST simulations were run to allow a comparison between the loads on an offshore wind turbine subjected only to wind loads and an offshore wind turbine subjected to combined wind and wave loads. It was found that the ice loads are very significant; compared to the wind only case, the 61.8cm ice sheet case gives a foundation base shear force increase of about 330% and 260% more than the wind only case, for the upward and downward breaking cones, respectively. The foundation base bending moment reaches values about 50% more (for either ice cone shape) than the wind only case. In the case of the 92.7cm ice sheet with rubble keel, the results are that the foundation base shear force increased to be 1180% and 1060% more than the wind only case, for the upward and downward breaking cones, respectively. The total
foundation base bending moment reached values 150% and 200% more than the wind only case. for the upward and downward breaking cones, respectively. These loads are for the ice cone tower configuration; without an ice cone, the foundation base shear force can be thirteen times the magnitude of the case with no ice, and the bending moment with an ice sheet can increase to three times the wind only operating load for the 61.8 cm thick ice sheet. Hence, a vertical-walled cylindrical structure is impractical and an ice cone structure will most likely be used on any offshore turbines in Lake Erie.

As a measure to test the capability of FAST for simulating other foundation types (for example, gravity foundations) a hypothetical foundation case was run. The FAST model of the NREL 5MW Offshore Baseline turbine has a monopile foundation which is modeled as the lower portion of the tower. To simulate a gravity base foundation, a hypothetical monopile foundation with a tenfold increase in mass and stiffness of the bottom portion of the model was used. The mode shapes for this new structure were recalculated for a FAST simulation. This stiffer tower was simulated in both FAST and ADAMS, with ADAMS being used to verify the accuracy of the FAST model. It was found that FAST can run the stiffer tower, (no computational or numerical problems occurred) however, the difference between the FAST and ADAMS results were larger for the stiffer monopile foundation compared to the original monopile foundation. This implies that FAST are not accurate when simulating foundations that are not similar to the flexible one-dimensional beams on which the FAST tower model is based. However, without realistic stiffness and mass data for a gravity base foundation, the accuracy of FAST simulation cannot be fully explored.
In case of a monopile tower, simple calculations showed that, while ice loads on
the tower/foundation structure can be much larger than the loads from the wind, these
loads can be supported by reasonably sized monopole structures.

10.4 Future Work

Currently, the capability exists to simulate an offshore turbine with a monopile
foundation subject to ice loads in FAST, with a monopole foundation cantilevered to the
lakebed. Future work in this area should seek to determine the validity of FAST to
simulate different types of foundations (such as gravity base foundations) and incorporate
realistic lakebed properties.
References


93
Appendix A

Equations for coefficients $A_1$, $A_2$, $A_3$, $A_4$, $B_1$ and $B_2$, used in Ice Load Calculations

This appendix shows how $A_1$, $A_2$, $A_3$, $A_4$, $B_1$ and $B_2$ were calculated for use in Eqs. 3.2 and 3.3, taken from [Ralston, 1977], used in calculating the horizontal and vertical loads from an ice sheet on an ice cone. In these equations, $\alpha$ is the ice cone angle measured from the horizontal, in radians, and $\mu$ is the ice to cone dynamic friction coefficient, and $\rho = A/R$, where $A$ is the distance from the center of the ice cone where the ice sheet cracks, and $R$ is the waterline diameter of the ice cone. Eq. A.10 allows the value of $\rho$ to be found.

\[ A_1 = \frac{1 + (2.711\rho \cdot \ln(\rho))}{3(\rho - 1)} \quad (A.1) \]

\[ A_2 = 0.075(\rho^2 + \rho - 2) \quad (A.2) \]

\[ A_3 = \frac{0.9}{4\cos(\alpha)} \left( 1 + \frac{\mu \cdot E(\sin(\alpha))}{\tan(\alpha)} \right) - \frac{\mu \cdot 0.9}{4\tan(\alpha)} (f(\mu, \alpha) \cdot g(\mu, \alpha)) \quad (A.3) \]

\[ A_4 = \frac{\tan(\alpha)}{1 - \mu \cdot g(\mu, \alpha)} \quad (A.4) \]

\[ B_1 = \frac{h(\mu, \alpha)}{\frac{\pi}{4} \sin(\alpha) + \frac{\mu \cdot \alpha}{\tan(\alpha)}} \quad (A.5) \]
Equations A.1 through A.6 use functions $f$, $g$, and $h$, shown in Eqs. A.7, A.8, and A.9. These use functions $F$ and $E$, which are the complete elliptic integrals, of the first kind, and the second kind, respectively.

\[ f(\mu, \alpha) = \sin(\alpha) + \mu \cdot \cos(\alpha) \cdot F(\sin(\alpha)) \]  
(A.7)

\[ g(\mu, \alpha) = \left[ \frac{1}{2} + \frac{\alpha}{\sin(2\alpha)} \right] \left[ \frac{\pi \cdot \sin(\alpha)}{4} + \frac{\mu \cdot \alpha}{\tan(\alpha)} \right] \]  
(A.8)

\[ h(\mu, \alpha) = \cos(\alpha) - \frac{\mu}{\sin(\alpha)} [E(\sin(\alpha)) - \cos^2(\alpha) \cdot F(\sin(\alpha))] \]  
(A.9)

The parameter $\rho$ used in Eqs. A.1 and A.2 is given by the solution to Eq. A.10.

\[ \rho - \ln(\rho) + 0.0830 \cdot (2\rho + 1)(\rho - 1)^2 \cdot \left( \frac{\rho_w g D^2}{\sigma_b t} \right) = 1.369 \]  
(A.10)