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A Dissertation

entitled

Modeling and Analysis of Synchronization Schemes for the
TDMA Based Satellite Communication System

by

Chong Wang

Submitted to the Graduate Faculty as partial fulfillment of the
requirements for the Doctor of Philosophy Degree in Engineering

___________________________________
Dr. Junghwan Kim, Committee Chair

___________________________________
Dr. Patricia R. Komuniecki, Dean
College of Graduate Studies

The University of Toledo

December 2012
An Abstract of

Modeling and Analysis of Synchronization Schemes for the TDMA Based Satellite Communication System

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Chong Wang

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The University of Toledo
December 2012

As a multiple access technique, time division multiple access (TDMA) is becoming more widespread in satellite communications. Although single-carrier (SC) modulation techniques has been mostly employed in the TDMA satellite communication system, multi-carrier modulation techniques, like orthogonal frequency division multiplexing (OFDM), are also promising candidates for the future high rate satellite communication due to its high bandwidth efficiency and robustness to fading. As is known, accurate synchronization is a key requirement for efficient data transmission in TDMA systems. Because of the channel impairment or propagation delay, the receiver may experience the carrier frequency offset (CFO) or symbol timing error (STE) that may result in difficulty of signal detection and estimation. Comparing with SC systems, OFDM systems are more sensitive to synchronization errors, because these errors will lead to inter-symbol interference (ISI) and inter-carrier interference (ICI). Although many synchronization algorithms have been disclosed in literature as the effective recovery methods to reduce detection error in the presence of CFO as well as STE, tradeoffs still have to be made between the estimation accuracy and estimation range (or
implementation complexity). This motivates further research to improve the performances of synchronization methods for the TDMA based satellite communication system.

The main objectives of this research are modeling and mathematical analyses of the synchronization methods under various channel impairment scenarios towards the practical applications in TDMA satellite communication systems. To achieve these objectives, this research firstly investigates the performance degradation due to CFO for both SC and OFDM systems. The performance of SC system impaired by CFO is mathematically analyzed and computer simulation is performed under additive white Gaussian noise (AWGN). For OFDM system in the presence of CFO, the performance of binary phase shift keying/quadrature phase shift keying (BPSK/QPSK)-OFDM impaired by CFO is evaluated and simulated under AWGN, Rayleigh flat fading, and frequency selective fading. Because of the lack of analyses in higher order modulation or modulation with non-rectangular decision region, this research derives the closed form of symbol error probability expressions for 8-ary phase shift keying (8PSK)-OFDM and 16-ary quadrature amplitude modulation (16-QAM)-OFDM impaired by CFO under AWGN. All of the corresponding simulation results validate the correctness of the theoretical analyses and symbol error probability derivations.

In order to compensate the effect of CFO to SC systems, numerous methods have been proposed as the effective architecture for CFR. Thus, comparison between various existing synchronization algorithms under certain condition has been performed based on modeling and mathematical analysis. By the results, the major impact factors of different CFR algorithms have been identified. Then, a novel CFR algorithm, which is derived
based on least square estimation method, is proposed and evaluated under AWGN. Finally, the proposed algorithm is simulated and compared with other exiting algorithms with respect to estimation range, estimation accuracy and computational complexity. The simulation results show that the proposed algorithm has the benefit of reduced computational complexity and high estimation accuracy.

In the case of OFDM systems, two novel data-aided (DA) carrier frequency and symbol timing recovery algorithms are proposed. These maximum likelihood (ML) based algorithms jointly estimate the CFO and STE for the OFDM systems. The performance of the proposed algorithms is evaluated and compared with existing estimation algorithms in terms of estimation range and accuracy under AWGN, Rayleigh flat fading and frequency-selective multipath fading, respectively. Simulation results confirm the effectiveness of the proposed algorithms under various levels of signal to noise ratios (SNRs) and the robustness of different lengths of cyclic prefix (CP) of the preamble.
Acknowledgements

I would like to express my deepest gratitude to my advisor, Dr. Junghwan Kim, for his valuable guidance and advice throughout my academic studies. I would also like to thank my committee members, Dr. Mansoor Alam, Dr. Mohsin Jamali, Dr. Thomas Stuart and Dr. Dong-Shik Kim for their valuable advice, excellent support, and guidance during my dissertation research.

My sincere appreciation is extended to Marcia King-Blandford for providing me the opportunity to work at the provost office. Without her support, it would have been difficult for me to complete my Ph.D study. It is really a great working environment where I have learned a lot. I would also like to appreciate the provost staffs: Barbara Monger, Dr. Penny Poplin Gosetti, Margaret F. Traband, Donna Belkofer for their kind help and encouragement.

Special thanks to my good friends Dr. Mike Orra, Dr. Joe Moening, Xueliang Lu, Pooja Raorane, Ayoade Oguntade, and Maulik Oza who not only assist me with the research work, but also help me quickly blend into American culture. It has been a great pleasure to meet them all.

Finally, I would like to express my deepest gratitude to my parent for their support and encouragement. Without their love, I would have never been possible to reach here.
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List of Abbreviations

8PSK .........................8-ary Phase Shift Keying
16-QAM ....................16-ary Quadrature Amplitude Modulation

AWGN .....................Additive White Gaussian Noise

BER .......................Bit Error Rate
BPSK ......................Binary Phase Shift Keying

CBTR ......................Carrier and Bit Timing Recovery
CFO .........................Carrier Frequency Offsets
CFR .........................Carrier Frequency Recovery
CPO ........................Carrier Phase Offsets
CHF ........................Characteristic Function
CDMA ......................Code Division Multiple Access
CAZAC ....................Constant Amplitude Zero Autocorrelation
CRLB ......................Cramèr-Rao Lower Bound
CP ..........................Cyclic Prefix

DA ........................Data-Aided
DD ..........................Decision Directed
DAB ........................Digital Audio Broadcasting
DOL ........................Direct On Line
DVB-S2 .....................Digital Video Broadcasting Satellite Second Generation
DVB-SH ...................Digital Video Broadcasting Satellite Services to Handhelds
DVB-T .....................Digital Video Broadcasting Terrestrial
DSB ........................Direct Satellite Broadcasting
DFT ........................Discrete Fourier Transform

EEV ........................Estimation Error Variance

FB ..........................Feedback
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>FF</td>
<td>Feed-forward</td>
</tr>
<tr>
<td>FT</td>
<td>Fourier Transform</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
</tr>
<tr>
<td>ICI</td>
<td>Inter-Carrier Interference</td>
</tr>
<tr>
<td>ISDB-T</td>
<td>Terrestrial Integrated Services Digital Broadcasting</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter-Symbol Interference</td>
</tr>
<tr>
<td>LSE</td>
<td>Least Square Estimation</td>
</tr>
<tr>
<td>LOS</td>
<td>Line Of Sight</td>
</tr>
<tr>
<td>M-PSK</td>
<td>M-ary Phase Shift Keying</td>
</tr>
<tr>
<td>MAPE</td>
<td>Maximum A Posteriori Estimation</td>
</tr>
<tr>
<td>MLE</td>
<td>Maximum Likelihood Estimation</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>MILSATCOM</td>
<td>Military Satellite Communications</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum Mean-Square Estimation</td>
</tr>
<tr>
<td>MCRB</td>
<td>Modified Cramér–Rao lower bound</td>
</tr>
<tr>
<td>MA</td>
<td>Multiple Access</td>
</tr>
<tr>
<td>NDD</td>
<td>Non Decision Directed</td>
</tr>
<tr>
<td>NDA</td>
<td>Non-Data Aided</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PN</td>
<td>Pseudo-Noise</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
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<tr>
<td>RDMA</td>
<td>Random Packet Division Multiple Access</td>
</tr>
<tr>
<td>RB</td>
<td>Reference Burst</td>
</tr>
<tr>
<td>RRC</td>
<td>Root Raised Cosine</td>
</tr>
<tr>
<td>S-DMB</td>
<td>Satellite Digital Multimedia Broadcasting</td>
</tr>
</tbody>
</table>
SNR.............................Signal to Noise Ratio
SDMA.........................Space Division Multiple Access
SER ..............................Symbol Error Rate
SSD ..............................Signal Space Decomposition
STE ..............................Symbol Timing Error
STR ..............................Symbol Timing Recovery

TDMA.........................Time Division Multiple Access
TB ..............................Traffic Burst

UW .........................Unique Word

VSAT ..........................Very Small Aperture Terminal
Chapter 1

Introduction

Satellite communication is to use the satellite as a relay to provide data exchanges between two or more earth stations (fixed or mobile) for long distance communication. The coverage area of satellite communication can greatly exceed that of a terrestrial system, and the transmission cost of a satellite is independent of the distance from the center of the coverage area. Moreover, satellite to satellite communication is very precise [1]. Therefore, satellite communication is increasingly used for global communications, as well as for radio and television broadcasting [2].

In the satellite communication system, there are mainly five principal forms of multiple access (MA) techniques, which allow several terminals connected to the same multi-point transmission medium to transmit over it and to share its capacity [3]. In frequency division multiple access (FDMA), all users share the satellite at the same time, but each user transmits its data at a unique allocated frequency. With time division multiple access (TDMA) all users are separated by time, presenting the sequential data transmission in transponder at the same carrier frequency. In code division multiple access (CDMA) technique all users transmit orthogonally coded spread spectrum signals that can be separated at the receiver by correlation with the transmitted code [4]. Space
division multiple access (SDMA) is a scheme where all concerned earth stations can use the same frequency at the same time within a separate space available for each link [3]. Random (Packet) division multiple access (RDMA) is a scheme where a large number of satellite users share the same transponder asynchronously by randomly transmitting short burst or packet divisions [3]. Among these five multiple access techniques, TDMA has been widely employed in several practical applications of satellite communication systems, because the TDMA signals can be divided by time, thus they are easily reconfigured for changing traffic demands, and resistant to noise and interference. Moreover, the entire bandwidth of the system is used for all the users, only one signal is presented in the transponder at one time, thus eliminating the carrier to inter-modulation noise and reducing the consequences of other non-linear impairments [5].

In the TDMA system, both single-carrier modulation and multi-carrier modulation can be employed [4, 6]. The application of single-carrier TDMA system can be found in the satellite systems such as very small aperture terminal (VSAT) satellite communication systems [7-8], digital video broadcasting satellite second generation (DVB-S2) [9-11], direct satellite broadcasting (DSB) [12] and high speed satellite TDMA ground system (HiSTARS) of KROEASAT-3 [13]. On the other hand, as a special case of multi-carrier modulation, orthogonal frequency division multiplexing (OFDM) modulation has been attracted much interest in many wireless and broadcast applications such as IEEE 802.11/WIFI, IEEE 802.16/WIMAX, 3GPP LTE [14], as well as European digital audio broadcasting (DAB), digital video broadcasting terrestrial (DVB-T), terrestrial integrated services digital broadcasting (ISDB-T), and digital subscriber line (xDSL) [15]. Simultaneously OFDM modulation is also becoming promising scheme for
satellite communication systems [16-18], and it has been adopted by digital video broadcasting satellite services to handhelds (DVB-SH) [19-21]. On the ETSI side, the ETSI S-UMTS group initiated several new activities to the study of OFDM-based air interfaces for 4G mobile satellite systems [22]. Moreover, the application of OFDM techniques in the physical layer of a satellite system devoted to the provision of satellite digital multimedia broadcasting (S-DMB) services has also been considered [23-24]. Furthermore, OFDM modulation has recently been explored in joint terminal engineering office (JTEO) research efforts for identifying the optimal air-interface for military satellite communications (MILSATCOM) networks through wideband global SATCOM (WGS) systems [16].

1.1 Motivation of Research

In a TDMA based satellite system, it is well known that perfect synchronization is crucial toward transmission quality, because the synchronization error causes severe inter-symbol interference (ISI) or inter-carrier interference (ICI) which eventually leads to a considerable performance degradation [25]. The major synchronization issues encountered in the TDMA based system are carrier synchronization, symbol timing synchronization, frame synchronization and network synchronization [25-26].

Carrier synchronization (or called carrier recovery) refers to the technique to estimate and compensate carrier frequency offsets (CFO) and carrier phase offsets (CPO) in the receiver. CFO/CPO may due to the mismatch of oscillators between the transmitter and receiver, or the nonlinear characteristic of the wireless channel, such as Doppler shift [27].
In TDMA-based satellite data transmission, because there is no discrete spectral component provided to the receiver at the symbol rate, the demodulator therefore has no direct knowledge of exactly when the received symbols start and end. Even when the incoming symbol rate is known, the receiver needs to deduce the phase of the received symbols relative to its own local symbol clock reference. Symbol timing synchronization (or called symbol timing recovery) is defined to estimate symbol timing error (STE) and identify the precise moment of time when individual symbols start and end [15].

Frame synchronization plays an important role in multi-users channels to identify the boundaries between different users and establish where the information is coming from and to where it must be routed. Network synchronization is also essential in two-way communication system. Bit arriving at a given multiplexer must be available at the right time so that the assigned time slots are correctly filled and no bits are lost [26].

No matter what modulation schemes are used, both STE and CFO will degrade the performance of TDMA system. Thus, it is important to estimate and compensate these synchronization errors to guarantee the reliable data transmission. Note that in the OFDM system, although the effects of ISI caused by STE can be significantly reduced by inserting cyclic prefix (CP), STE still causes the phase rotation of the sub-carriers in frequency domain even if STE is shorter than the CP. Thus, the estimation accuracy of STE is also critical to the performance of OFDM systems as CFO estimation.

Because the symbol duration for single-carrier modulation scheme is very short under high rate transmission, single-carrier modulation system is more vulnerable to ISI. On the other hand, OFDM can effectively overcome ISI by the insertion of CP. However, OFDM is much more sensitive to CFO than that of single-carrier modulation system,
because the CFO can destroy the orthogonality of sub-carriers and lead to severe ICI. Thus, both single-carrier and OFDM-based multi-channel systems have their own advantages and disadvantages. The selection of either of these schemes should depend on the specific application scenarios.

Since CFO can cause significant degradation to the performance of TDMA system, and most synchronization schemes cannot completely compensate this offset, it is also important to quantitatively identify the effect of CFO, which can establish criteria to evaluate different synchronization schemes.

Instead of transmitting data in a continuous stream, TDMA systems always operate in a burst mode with a set of known modulation symbols inserted to the user data packet at the transmission for synchronization. In this study, carrier frequency recovery (CFR) and symbol timing recovery (STR) schemes have been fully investigated and explored for the application of burst mode TDMA based satellite system employing both single-carrier and OFDM modulation schemes.

1.2 Background and Previous Works

Most of the available synchronization methods can be simply classified into two categories: either data-aided (DA) or non-data aided (NDA) (also known as blind) synchronization algorithms. In general, DA algorithm exploits the information from known pilot symbols or preamble training sequences, and it usually yields high estimation accuracy with the tradeoff of low implement efficiency [28]. On the contrary, NDA algorithm usually results in low estimation accuracy with computation overhead, but no need of extra data transmission. Moreover, based on the way how to estimate
synchronization errors, the synchronization schemes can also be classified as joint or separate estimation algorithms.

In the case of single-carrier modulation based TDMA system, preamble training sequence (DA method) is always used to provide synchronization information because of its superior estimation accuracy compared with NDA method. Much effort has been devoted to find optimum estimation algorithms for carrier and symbol timing recovery [29]. The reference [30] proposed a computationally efficient CFR algorithm based on least square method, which can yield good performance at high signal to noise ratio (SNR) region. M. Luise and R. Reggiannini [31] presented an efficient technique for fast CFR based on ML estimation theory, which proves to be closed to the Carmer-Rao lower bound (CRLB) in the absence of fading effects. Jingxin Chen [27] proposed a fast carrier recovery (CR) method using a short preamble, which is applicable to burst mode communication system. Michael P. Fitz innovated a practical planar filtered carrier frequency estimation algorithm by exploiting the ML estimator [32]. He extended his research [33] and proposed a new frequency estimator, which is suitable to the communication system that requires high speed, and recursive frequency estimation. It was found that it can obtain near optimum performance at the moderate signal to noise ratio. A new data aided estimation algorithm was also invented by Umberto Mengali and Aldo N. D’ Andrea [34] and its threshold is lower and acquisition range is larger than those of Fitz [33] and L&R algorithm [31] for a given estimation accuracy.

Towards accurate estimation and proper compensation of the synchronization errors in the data recovery of OFDM symbols, multiple efficient synchronization (and/or estimation) algorithms have been proposed [15, 35]. Moose [36] proposed a DA method
to estimate the frequency offset using two repeated OFDM symbols, but the estimation range of this ML-based algorithm is limited to the half of the sub-carrier spacing. Schmidl and Cox [37] presented a DA method using a preamble with two identical halves for joint symbol timing and carrier frequency synchronization. Since their method does not consider the effect of cyclic prefix (CP) toward its maximum likelihood (ML) estimator, the timing metric exhibits a ‘plateau’ [38] for whole duration of the CP, which inevitably yields large variance in the timing estimate. To reduce the variance of [37] caused by the CP, Minn [38] initially proposed a method in which Schmidl and Cox’s timing metric is averaged over a window of CP length. Although this method can alleviate the ‘plateau problem’ of [37], the estimation variance is still large due to the comparable timing metrics around the optimum timing. Later Minn [38-39] also proposed another method by modifying the Schmidl and Cox’s preamble with a new sequence, which contains four equal parts (but the last two parts with different sign). This method gives relatively smaller estimator variance but still exhibits large mean square error (MSE) in the multipath fading channel. To overcome the deficiency of Minn’s approach, Park [40] presented a novel DA timing offset estimation method using a preamble (symbol) which is consist of four parts, with two symmetric parts at the beginning and their respective conjugates at the end of the preamble. His method yields significantly smaller mean squared error (MSE). However, the frequency offset estimation cannot be obtained from Park’s method and the performance is seriously degraded under multipath fading condition. In order to improve the performance under fading channel, Seung [41] proposed a novel DA method, which uses a specially designed preamble to achieve small MSE under fading. In [42, 43], the super-imposed OFDM timing and frequency scheme
is proposed by including a pre-defined pseudo-noise (PN) sequence. The difference between these two methods is that the PN sequence used in [43] is in the form of constant amplitude zero autocorrelation (CAZAC) sequence and the preamble is same as that of [37]. Although this method has the advantage of better bandwidth utilization, it will require extra power to transmit the super-imposed PN sequence. Instead of utilizing the correlation of the identical parts in the preamble, Bin [44] developed a novel symbol timing error estimation method based on the cyclic structure of the OFDM symbol and the DFT-based channel estimator. This DA method [44] uses the noise subspace of channel estimates to find the starting point of the symbol. Although it was claimed that the frequency offset can be estimated first by using the identical method of Schmidl’s [37] because of the same training sequence used, no details are available on the way to deal with frequency offset in this method, since the frequency offset is recovered after the symbol timing synchronization in Schmidl’s method. As to the NDA estimation algorithm, most algorithms are based on cyclic correlation. Park [45] proposed a blind synchronization algorithm, which firstly exploits the second-order cyclostationarity of the received signals and then using the information in the cyclic correlation. Jan-Jaap [46] proposed a NDA joint synchronization method for OFDM system using a correlation with CP. However, the estimation range of this ML based algorithm is limited to the half of the sub-carrier spacing only. In [47], another cyclic correlation based STR method is proposed, and it is proved that the maximum of this method points to the correct timing offset, irrespectively of channel conditions. In addition, Ninh [48] also proposed a cyclic correlation based synchronization method. However, his CP consists two parts. One part is a copy of the head of the last symbol and the other part is a copy of the tail of that
symbol. Besides the cyclic correlation based algorithm, Daniel [49] proposed a ML symbol timing estimator by exploiting both the redundancy in the CP and available pilot symbol used for channel estimation. (Daniel also presented another NDA timing synchronization method using both the cyclic prefix and the pilot symbols.), but his estimator is highly sensitivity to the carrier frequency variation.

Considering the evaluation of multi-carrier system based on OFDM impaired by CFO, numerous results are available on the evaluation of error probability of OFDM systems. As is summarized in [50], mainly two different methods are widely used to quantify the effect of ICI caused by CFO: Gaussian approximation and characteristic function (CHF). In [51-53], the error probability is derived by approximating the ICI as a Gaussian process based on the central limit theorem. However, this approach is proved to be inaccurate for low SNR [50]. On the other hand, Sathananthan and Tellambura derived the exact Bit Error Rate (BER) or Symbol Error Rate (SER) for OFDM with CFO using characteristic function (CHF) of the ICI under additive white Gaussian noise (AWGN) [54]. The main weakness of [54] is that the results of the average error probability are expressed in the form of an infinite series expansions rather than a closed form error probability formula. In [55], the authors evaluate the error probability of OFDM impaired CFO by analyzing the CHF of the received signal’s real and imaginary components separately. However, the real and imaginary parts of the received signal are not independent. Recently, in [50], the authors follow the procedure presented in [54], and derive an exact closed form of BER/SER expression using the relationship between sum and product of cosine functions for OFDM systems with CFO. The results of [50] have been further improved by [56], because the argument of Q-function is assumed to be
positive in [50]. However, both of them only consider the OFDM system employing M-ary phase shift keying (MPSK) modulation schemes, and do not provide any analysis for the OFDM employing quadrature amplitude modulation (QAM) modulation scheme. As can be seen from the existing results for performance evaluation in the presence of CFO, practical evaluation of error probability requires approximation of infinite series and solution is limited to the applications of BPSK and QPSK cases with rectangular decision region mostly. For higher order modulation like 16-QAM and the cases with non-rectangular decision region like 8-ary phase shift keying (8PSK), there are only few results [54, 57] based on infinite series expansion. With respect to performance evaluation of single-carrier system impaired by CFO, it is reasonable to normalize the CFO as in the OFDM system for a fair comparison. In [58-60], the error probability of single-carrier system in the presence of CFO is evaluated based on the Gaussian approximation process for the ICI terms. However, the Gaussian approximation method has been proved to be less accurate when SNR is high.

Although many synchronization methods are currently available in the literature, most of them have been concentrated on very specific circumstances only. Hence, it is hard to generalize a synchronization method, which is robust under different circumstances. Moreover, many synchronization methods are highly dependent on particular parameters, which may introduce a tradeoff between the estimation accuracy and estimation range. Thus, initially it is necessary to provide a complete assessment on the synchronization methods to identify their limitations. Then further research is still needed to improve the synchronization performances in estimation accuracy, estimation range and implementation complexity for a robust application to different circumstances.
By the results, the reliable and efficient synchronization methods for the TDMA based satellite system can be determined finally.

1.3 Research Objective

The major objective of this research will focus on developing and evaluating the feasible synchronization schemes to mitigate the performance degradation of TDMA based satellite communication system under various critical synchronization impairments and different channel interferences.

To accomplish this major objective, extensive review has been initially performed to analyze the conventional synchronization methods used to estimate CFO for single-carrier modulation based TDMA system. And then the performance (in terms of estimation range and accuracy) of conventional synchronization methods is discussed and evaluated under AWGN channel by both mathematical derivation and computer simulation. The crucial factors for each conventional synchronization methods have been determined, and the tradeoff between estimation range and accuracy (or implementation complexity) is identified. Finally, this study develops an improved CFR algorithm to mitigate the performance degradation due to CFO. Then, the performance of proposed CFR algorithm has been evaluated and compared with the conventional CFR algorithms with respect to estimation range, accuracy and implementation complexity.

Since single-carrier modulation based TDMA systems suffer great performance degradation under fading effect, and the mobile satellite channel is dominant by multi-path fading introduced interference, this research also consider the application of multi carrier OFDM-TDMA system, because of its great immunity to fading and ISI. In order
to investigate the suitable synchronization methods in the OFDM system, this research
initially analyze the error performance of OFDM system impaired by CFO under various
channel environments. Due to the lack of existing performance evaluation for higher
order modulation schemes and modulation schemes with non-rectangular decision region,
this study also gives a fully investigation on error performance of 8PSK OFDM and 16-
QAM OFDM under AWGN. Secondly, the performance of conventional synchronization
methods for OFDM systems is discussed and evaluated with respect to estimation range
and accuracy, respectively. Finally, this study presents two novel robust joint CFR and
STR methods for multi-carrier OFDM systems toward the improved the estimation
accuracy. Then, the performance of proposed algorithm is compared with other exiting
joint CFR and STR methods through computer simulations and theoretical analyses.

1.4 Dissertation Organization

The rest of this dissertation is organized as follows.

Chapter two briefly describes TDMA based satellite system and its channel model.
In order to evaluate different synchronization methods, the estimation criteria and
classification of synchronization methods are also provided.

In Chapter three, the performance degradation of single-carrier modulation based
TDMA system impaired by CFO is mathematically evaluated firstly. Then, the
conventional CFR methods are numerically investigated with respect to estimation range
and accuracy. Based on the CFO estimation formula, the crucial parameters influencing
the performance of each CFR method are identified. Finally, this chapter explains the
proposed CFR through mathematical derivations.
Chapter four presents on the degradation of error probability due to CFO in the single-carrier modulation based TDMA system. The exiting synchronization algorithms are evaluated and compared through computer simulations. In addition, the pros and cons of different synchronization methods are presented based on the simulation results. Finally, the performance evaluation of proposed method is conducted through computer simulation. And its improved estimation accuracy at the high signal to noise ratio (SNR) region with reduced computational overhead will be illustrated.

In Chapter five, firstly, the symbol error performance of OFDM TDMA system is analyzed in the presence of CFO, and the symbol error probabilities of 16-QAM OFDM and 8PSK OFDM systems impaired by CFO under AWGN are mathematically derived. Then, multiple conventional DA/NDA CFR and STR methods are introduced. In the end, two new joint CFR and STR methods are proposed, and numerically evaluated in detail.

Chapter six provides the simulation results on the performance degradation of OFDM system in the presence of CFO and the mathematical results derived in Chapter five are verified. Then, the proposed joint CFR and STR methods are evaluated and compared with the conventional synchronization methods. Based on the simulation results, the proposed methods are proved to be robust under different interferences.

Finally, some concluding remarks and suggestions for future research are made in Chapter seven.
Chapter 2

Synchronization in TDMA based Satellite Communication Systems

As discussed in Chapter one, the synchronization is very important for the TDMA based satellite communication systems. In order to analyze the synchronization methods in both single-carrier TDMA and OFDM-based multi-carrier TDMA systems, it is necessary to establish the model of the TDMA based satellite system and its channel models firstly. Since the fixed satellite communication channel is primarily a line of sight (LOS) link for the uplink and downlink, the channel noise in LOS link is generally modeled as the AWGN. However, the channel environment for mobile satellite communication is much more complex. Transmission between a satellite and a mobile terminal on the ground is generally no longer just simple LOS link. The radio wave may encounter a multiplicity of obstacles in the path, subjecting the transmitted wave to reflections, diffraction, and scattering, resulting in a multiplicity of rays reaching the receive antenna [61]. Also, because the transmitter or receiver terminal is moving, the power received is also varying, resulting in signal fading [61].

As in statistics, there are various estimation methods, and each of them has its own advantage depending on the implementation scenarios. Thus, it is necessary to
classify the estimation methods into different categories at the beginning. Moreover, some estimation criteria, such as bias and error variance, must be taken into consideration for evaluating the performance of an estimator. However, it is generally expected that the estimator can be unbiased, and achieve a small error variance as much as it can. Therefore, CRLB usually establishes a fundamental lower limit to any unbiased estimator. Since no estimator can provide lower error variance than this bound, it can be served as an evaluation standard. Hence, any estimation method is aiming to improve its accuracy towards this bound. Unfortunately, this CRLB cannot be easily computed in many practical situations. Instead the modified CRLB (MCRB) is mostly preferred to reduce computational complexity [26].

2.1 Introduction of TDMA based Satellite Communication System

The TDMA based satellite communication system typically consists of a set of traffic stations (also referred to as terminals) and a master reference station as shown in Figure 2-1 [62].

Figure 2-1: Principle of TDMA based satellite system [62].
In this technique, a frame is divided into multiple time slots, which are shared among the terminals. A master reference station with visibility of the whole network maintains access discipline and scheduling during the frame. It transfers a reference burst to provide frame synchronization and signaling information and identifies the transmitting stations. Reference burst (RB) will be followed by the traffic bursts, which are transmitted by the corresponding traffic stations. These burst of TDMA frame contain a preamble sequence, which is used to provide the synchronization information. Figure 2-2 [62] gives the composition of reference and traffic bursts structure.

Reference burst (RB) is transmitted by a designated reference station to achieve the start of frame for all earth stations involved. The frame so-defined is divided into time slots, and each earth station is assigned a time slot in which to transmit its information. Then, each earth station sends its own traffic burst (TB), among which also includes preamble sequences for carrier recovery and symbol timing during one station. Since there are significant time delays between users, each user receives the reference burst with a different phase, and its traffic burst is transmitted with a correspondingly different phase within the time slot. There is therefore a need for guard times to take account of this uncertainty. Each time slot (TS) is therefore longer than the period needed for the
actual traffic burst, thereby avoiding the overlap of traffic burst even in the presence of these propagation delays. Then, a unique word (UW) sequence is followed to perform the frame and burst synchronization.

The analysis presented in this study concentrates on the carrier and symbol timing recovery methods. It is assumed that frame synchronization has been accomplished by suitable unique word detection schemes.

2.2 Model of TDMA based Mobile Satellite Channel

In the mobile satellite channel, the transmitted signal is not only corrupted by the classical AWGN, but also suffers from fading. In other words, the signal transmitted to the receiver contains not only a direct LOS radio wave, but also a large number of distracted, scattered, and reflected radio waves. Even worse in urban centers or mountain areas, the LOS wave is often blocked by obstacles, and a collection of differently delayed waves is all what is received by a mobile earth terminal. The fading in radio propagation can be categorized as large scale fading and small scale fading [63]. Large scale fading manifests itself as the average signal power attenuation or path loss due to motion over large areas; whereas small scale fading refers to the dramatic changes in the signal amplitude and phase that occur due to small changes in the spatial separation between the transmitter and receiver [64].

Small scale fading can be examined in either time or frequency domain. In the case of time domain, coherent time ($T_c$) is defined as the period of time over which the fading process is correlated. Thus, if coherent time ($T_c$) is smaller than symbol duration ($T_s$), the channel is fast fading. Otherwise, the channel is small fading for $T_c > T_s$. With
respect to the frequency domain, coherent bandwidth \((B_c)\) measures the frequency range over which the fading process is correlated, and it is defined as the frequency bandwidth over which the correlation function of two samples of the channel response taken at the same time but different frequencies falls below a suitable value. Hence, the channel is said to exhibit **frequency selective fading** if \(B_c < B_w\), whereas the channel is said to experience **flat fading** if \(B_c > B_w\), where \(B_w\) denotes the signal bandwidth \([65]\).

Based on the mobility effect, the mobile satellite signals can be affected by multipath, shadowing or blockage, among which multipath fading is by far the most critical problems. Since multipath propagation leads to rapid fluctuations of the phase and amplitude of the signal, it has a small-scale fading effect. Moreover, since fading is generated by path length differences between the multiple rays, it can be classified as **narrowband** or **wideband** depending on the characteristics and location of scatters with respect to the mobile. Narrowband implies that the channel under consideration is sufficiently narrow that its frequency response can be considered flat \([61]\), while the message bandwidth significantly exceeds the channel's coherence bandwidth in the wideband system. Thus, the frequency response of wideband system experiences frequency selective fading. For example, the channel for voice and low data rate (kHz range) mobile satellite systems can be considered as purely narrowband, while broadband multimedia mobile radio and satellite based mobile systems (information bandwidth is in the MHz to GHz range) have to be considered as wideband fading links \([61]\). In this research, only the multipath fading is focused for the mobile satellite link.

As is known, AWGN is always the dominant channel impairment in the mobile satellite channel. Let \(s(t)\) denotes the transmitted signal, which can be represented as
\[
    s(t) = \sum_{m} s_m g(t-mT)
\]

where \( s_m \) is a sequence of transmitted data symbol, \( g(t) \) is the impulse response of transmitter shaping filter. Then, the equivalent baseband received signal \( y(t) \) under only AWGN impairment can be expressed as [66]

\[
    y(t) = \sum_{l} a_l \exp[-j\psi_l(t)]s[t-\tau_l] + n(t)
\]

where

\[
\psi_l(t) = 2\pi[f_c \tau_l - \beta \cos \theta_l(t-\tau_l)]
\]

\( \beta \) represents the Doppler shift, \( \theta_l \) is the \( l \)th path wave arriving angle, \( n(t) \) is the AWGN noise with zero mean and double sided power spectral density (PSD) of \( N_0/2 \), \( f_c \) is the carrier frequency, \( a_l \) denotes the amplitude attenuation factor for path \( l \), whose propagation delay is given by \( \tau_l \). Thus, the channel impulse response of the complex multipath channel is given as

\[
    h(\tau, t) = \sum_{l} a_l \exp[-j\psi_l(t)]\delta(\tau-\tau_l)
\]

\( h(\tau, t) \) denotes the channel response at time \( t \) due to an impulse applied at time \( t-\tau \), and \( \delta() \) is the dirac delta function.

If the difference between different path delays is smaller than the symbol duration, the \( \tau_l \) in equation (2.3) are all approximately equal to \( \tau' \). Then, the channel impulse response can be expressed as

\[
    h(\tau, t) = \sum_{l} a_l \exp[-j\psi_l(t)]\delta(\tau-\tau') = h(t)\delta(\tau-\tau')
\]
Because the Fourier transform of equation (2.5) is $h(t)exp(-j2\pi f t')$, all frequency components in the received signal are subject to the same complex gain $h(t)$. In this case, the received signal is said to exhibit flat fading. When there is no dominant LOS link, the amplitude of the received signal is Rayleigh distributed and the phase of the received signal is normal distributed by the central limit theorem. Thus, the probability density functions of the amplitude and phase of $h(t)$ at any time $t$ are given as [66]

$$p_{|h(t)|}(x) = \frac{x^2}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$  \hspace{1cm} (2.6)

$$p_{\arg h(t)}(x) = \frac{1}{2\pi}, \quad -\pi \leq x \leq \pi$$ \hspace{1cm} (2.7)

When components of a signal travelling through different paths arrive at the receiver with delays significantly larger as compared to the bit or symbol duration, the signal will undergo significant amount of distortion across the information bandwidth. Then it results in frequency selective fading or wideband fading. The impulse response of a wideband channel model can be written as

$$h(\tau, t) = \sum_{l=0}^{Q-1} h_l(t)\delta[\tau - \tau_l]$$ \hspace{1cm} (2.8)

where $h_l(t)$ and $\tau_l(t)$ denote the time varying complex gain and excess delay of the path $l$. $Q$ is the total number of path. There are various methods to generate the time varying complex gain $h_l(t)$, such as filtering two zero mean independent white Gaussian noise with same variance. In addition, Clark proposed a simple and efficient method to generate $h_l(t)$, which can be expressed as [65]

$$h_l(t) = \sum_{i=1}^{N} \alpha_i e^{j\phi_i} e^{j(2\pi \frac{\nu}{\lambda} \cos \theta_i)} = \sum_{i=1}^{N} \alpha_i e^{j\phi_i} e^{j(2\pi \beta \cos \theta_i)}$$ \hspace{1cm} (2.9)
where $\lambda$ and $f$ represents the wavelength and frequency of the signal, $\theta_i$ denotes the angle between the direction of mobile movement and the $i$th scatter, $\alpha_i$ and $\phi_i$ are the complex amplitude and phase of the $i$th scatter respectively, $N$ is the approximate number of scatter, and $\beta$ represents the Doppler shift. The amplitude is assumed to be constant for all scatters in path $l$, then

$$\alpha_i = \sqrt{\frac{P_l}{N}} \quad (2.10)$$

where $P_l$ is the average power for the received signal $l$th path. Thus, the time varying complex gain and excess delay of the path $l$ can be expressed as [65]

$$h_l(t) = \sqrt{\frac{P_l}{N}} \sum_{i=1}^{N} e^{i \phi_i} e^{j (2\pi ft \cos \theta_i)} \quad (2.11)$$

### 2.3 Identification of Synchronization Methods

There are mainly four estimation methods used in the unknown parameter estimation: least square estimation (LSE), maximum likelihood estimation (MLE), minimum mean-square estimation (MMSE), and maximum a posteriori estimation (MAPE). Let $\psi$ represents a set of signal parameters to be estimated, the transmitted and received signal is represented by a set of $L$ symbols, then the received signal in equation (2.1) can be rewritten as

$$r = s(\psi) + n \quad (2.12)$$

where $r = [r_1, r_2, \ldots, r_L]$ and $s(\psi) = [s_1(\psi), s_2(\psi), \ldots, s_L(\psi)]$. LSE is trying to find the estimate parameter $\hat{\psi}$, which can minimize the square error of [28]

$$\hat{\psi}_{LSE} = \arg\min_{\psi} \left[ \sum_{i=1}^{L} [r_i - s_i(\psi)]^2 \right] \quad (2.13)$$
MLE is to choose an estimate of the unknown parameter \( \psi \), which gives the received signal a high probability of occurrence. Then, the maximum likelihood (ML) estimate is of the value which maximizes the conditional probability density function \( p(r \mid \psi) \). For example, the condition probability is defined from equation (2.12)

\[
p(r \mid \psi) = (\pi N_0)^{-L/2} \prod_{i=1}^{L} \exp \left\{ -\frac{[r_i - s_i(\psi)]^2}{N_0} \right\}
\]  

(2.14)

The ML estimate is given by [63]

\[
\hat{\psi}_{MLE} = \arg \max_{\psi} [p(r \mid \psi)]
\]  

(2.15)

It is obviously to see that the ML estimate also needs to minimize the term \( \sum_{i=1}^{L} [r_i - s_i(\psi)]^2 \). Therefore, ML estimate and LS estimate are equivalent.

MMSE describes the approach which minimizes the mean square error (MSE), which is given as [63]

\[
MSE = E[(\hat{\psi} - \psi)^2]
\]  

(2.16)

Thus, the MMSE is the estimate value that minimize the average function of

\[
\hat{\psi}_{MMSE} = \arg \min_{\psi} \left\{ E[(\hat{\psi} - \psi)^2 \ p(\psi \mid r)] \right\}
\]  

(2.17)

where \( p(\psi \mid r) \) is a posteriori probability density function of random parameter \( \psi \) with the given received signal \( r \). From the Bayesian theorem, the conditional probability density function \( p(\psi \mid r) \) can be denoted as [63]

\[
p(\psi \mid r) = \frac{p(r \mid \psi) p(\psi)}{p(r)}
\]  

(2.18)
The denominator of the posteriori distribution does not depend on the unknown parameter \( \psi \) and therefore plays no role in the estimation. The method of MAPE is to estimate \( \psi \) as the maximum of the posterior distribution of this parameter \[63\]

\[
\hat{\psi}_{\text{MAPE}} = \arg \max_{\psi} [p(\psi \mid r)]
\]  (2.19)

Thus, MAPE is more accurate than MLE because it incorporates a prior distribution during the estimation. However, it is noted that the MAPE is the same as MLE when the unknown parameter is the sample value of a uniformly distributed random variable.

In order to categorize synchronization methods, a detailed mathematical model of communication transmitter and receiver needs to be built at the beginning. The unknown signal parameter \( \psi \) may include frequency \( (f) \), phase \( (\theta) \), and symbol timing \( (\lambda) \). Thus, the probability density function (pdf) of \( r \) on the condition of unknown parameters can be given as \[28\]

\[
p(r \mid \psi) = \sum_{\text{all sequence of } s(\psi)} P[s(\psi)]p[r \mid s(\psi)]
\]  (2.20)

where \( P \) denotes the probability and \( p \) denotes pdf, and all sequence of \( s(\psi) \) means all possible transmitted signals. Then, the ML estimate of \( \psi \) is the value that maximizes the conditional probability in equation (2.20), but it is not necessary to do all at once.

There are mainly three basic categories for dealing with synchronization at the receiver. The first one includes DA and NDA. DA methods multiplex a special signal, named a pilot signal into the information signal. This special signal allows the receiver to extract and, thus to synchronize its local oscillator to the carrier frequency and phase of the received signal. The preamble sequence \( s_0(\psi) \) is known in the DA synchronization algorithms, so only one term of the sum of equation (2.20) remains \[28\].
because of \( P[s(\psi) = s_0(\psi)] = 1 \). The estimation rule thus reduces to maximizing the likelihood function [28]
\[
(\hat{\psi})_{DA} = \arg \max_{\psi} p[r | s(\psi) = s_0(\psi)]
\]

NDA is to derive the estimation parameters directly from the received signal. This approach has the distinct advantage that the total transmitter power can be allocated only to the transmission of the information bearing signal. However, a critical environment will lead to a serious performance degradation of the NDA algorithms. When the detected sequence \( s(\psi) \) is used as if it were the true sequence, one speaks of Decision Directed (DD) synchronization algorithms. When the error probability is low, \( s(\psi) \) is the true sequence \( s(\psi) \). Then only one term contributes to the sum of equation (2.20)
\[
\sum_{\text{all sequence of } s(\psi)} P[s(\psi)] p[r | s(\psi)] \approx p[r | s(\psi) = s(\psi)]
\]

Because of \( P[s(\psi) = s(\psi)] = 1 \), Thus
\[
(\hat{\psi})_{DD} = \arg \max_{\psi} p[r | s(\psi) = s(\psi)]
\]

Instead of using a decision directed scheme to obtain the carrier estimate, non decision directed (NDD) scheme may treat the data as random variables and simply average over these random variables prior to maximization. Class NDD algorithms are obtained if one actually performs (exactly or approximately) the averaging operation. For example, NDD for BPSK can be [28]
\[
p(r \mid \psi) = \prod_{n=0}^{N-1} \{ p[s_n(\psi) = s^1(\psi)]p[r \mid s_n(\psi) = s^1(\psi)] \]
+ \{ p[s_n(\psi) = s^2(\psi)]p[r \mid s_n(\psi) = s^2(\psi)] \}
\]

(2.25)

where \( s^1(\psi) \) is transmitted signal when information \( a_n = 1 \) and \( s^2(\psi) \) is transmitted signal when information \( a_n = 0 \).

From the operating point of view, the algorithms can be further categorized according to how the estimates are extracted from the received signal. Thus, another category of synchronizers is distinguished, error tracking (or feedback, or closed loop) and feedforward (or open loop) synchronizers. Feedforward (FF) directly estimates the unknown parameters from the received signal before it is corrected in the interpolator (for timing) or the phase rotator (for carrier recovery). The other category, error tracking, derives an estimate of the error and feeds a corrective signal back to the interpolator or phase rotator. Feedback structures inherently have the ability to automatically track slowly varying parameter changes. They are therefore also called error feedback (FB) synchronizers [26].

### 2.4 Evaluation Criteria for Synchronization Methods

From estimation theory, the quality of an estimator can be evaluated in terms of estimation bias, estimation error variance (EEV) and mean square error (MSE). In statistics, bias of an estimator is the difference between this estimator’s expected value and the true value of the parameter being estimated. By the definition, the bias of this estimator is defined to be [63]

\[
\text{Bias}[\hat{\psi}] = E[\hat{\psi}] - \psi = E[\hat{\psi} - \psi]
\]

(2.26)
An estimator is said to be unbiased if its bias is equal to zero for all values of parameter $\psi$. The estimation error variance is defined as [63]

$$\sigma^2 = E\{[\hat{\psi} - E(\hat{\psi})]^2\} \quad (2.27)$$

The MSE can be considered as the second moment of the error as variance. The MSE is the variance if the estimator is unbiased and is defined as [63]

$$MSE = E\{[\hat{\psi} - \psi]^2\} \quad (2.28)$$

The ultimate accuracy that can be achieved in synchronization operations is an important measurement for every actual synchronizer. Just as the channel capacity $C$ is a bound on the ideal channel, there exists a lower bound on the variance of the parameter estimate. Assuming the parameter estimate is unbiased, the variance of the estimation error is often used as a performance measure. All practical estimators have a variance larger than this bound, which can at best be approached but never reached.

Therefore, establishing this bound is an important goal since it provides benchmarks for evaluating the performance of synchronizers. Tools to approach this problem are available from the parameter estimation theory [26] in the form of CRLBs, which give fundamental lower limits to the variance of any parameter estimators. This bound is the reciprocal of statistical expectation, which reveals the estimation information of the channel, so that as the information goes up the bound becomes smaller, which make the use of the term seem reasonable, and is also known to be asymptotically achievable for a large enough number of observations, under mild regularity conditions. If the lower bound is attained, the used estimation is said to be ‘efficient’ [26].
As can be observed, in many cases of practical interest, the computation of true CRLBs is a demanding task. Assume that \( \hat{\psi} \) is any unbiased estimation of a parameter \( \psi \). Then a lower bound to the variance of the error \( [\hat{\psi} - \psi] \) is given as [26]

\[
CRB(\psi) = \frac{1}{E_r \left\{ \left( \frac{\partial \ln p(r|\psi)}{\partial \psi} \right)^2 \right\}}
\]

(2.29)

where \( r \) is received sequence, \( E_r \) denotes statistical expectation with respect to the subscripted variable, and \( p(r|\psi) \) is the conditional probability density function of \( r \) for a given \( \psi \). Unfortunately, in most cases of practical interest, the computation of above equation is impossible because either \( p(r|\psi) \) cannot be carried out analytically or the expectation poses insuperable obstacles.

In order to avoid the computational complexity related to the true CRLBs, MCRB have been derived in [26] as the following

\[
MCRB(\psi) = \frac{1}{E_{r,u} \left\{ \left( \frac{\partial \ln p(r|u,\psi)}{\partial \psi} \right)^2 \right\}}
\]

(2.30)

where \( u \) is the information on the received signal whose probability density function \( p(u) \) is known and does not depend on \( \psi \). As they have the same structure as the true CRLBs, they are referred to as MCRBs. MCRB is much easier to use. In fact, for the Gaussian channel, the probability density in equation (2.14) is well-known exponential function whose argument is a quadratic form in the difference between received sequence \( r \) and the transmitted sequence \( s \). Thus, the logarithm of \( p(r|u,\psi) \) is equal to this quadratic form and the expectation in (2.14) can be readily derived. The MCRBs are much easier to
evaluate than the CRBs, but are generally looser than the true CRB, especially at lower signal-to-noise ratio, which implies bad channel condition.
Chapter 3

Carrier Frequency Recovery (CFR) in Single-carrier Systems

Single-carrier based burst mode transmission technique has been widely applied in many applications such as satellite TDMA systems and terrestrial mobile cellular radio [7-13]. In many cases, synchronization errors, especially for CFO, can severely degrade the transmission performance. Thus, a preamble of known symbols is always appended to the beginning of each burst for CFR. This kind of synchronization algorithms is usually called DA algorithm. Although synchronization errors can also be compensated by some algorithms without the aid of a preamble (called NDA or blind algorithm), these algorithms have been proved to be less accurate in comparison with the DA cases [64].

Due to the propagation delay, Doppler frequency shift and the inaccuracy of the oscillator, CFO is a crucial degrading factor among all synchronization errors that are required to be estimated as accurate and fast as possible. Therefore, many CFR methods have been proposed for a more accurate and simpler estimation target. However, there still exist tradeoffs between the estimation accuracy and implementation complexity, and contradicts between the estimation accuracy and its range [29].
In this study, the DA CFR will be fully investigated under AWGN channel towards the application to the single-carrier TDMA based satellite communication system. And then a new CFR algorithm is proposed. This proposed algorithm is based on the least square standard. It follows the procedure proposed in [30], and extends the analysis to derive an accurate smoothing factor for the expression of multiplying received signals between more than one symbol delay. By applying the smoothing factor, performance of the proposed method can achieve the Cramer-Rao lower bound after a certain SNR threshold. In order to evaluate the performance of different carrier frequency recovery algorithms properly, three features are concerned: estimation accuracy, estimation range and implementation complexity. Since single-carrier scheme is very sensitively to the fading effect, it is usually employed in the channel where direct on line (DOL) is dominant. Thus the single-carrier TDMA system will be investigated under AWGN only in this study.

The remainder of this chapter is organized as follows. The first section will be concentrated on the synchronization model for the single-carrier TDMA architecture as well as its corresponding block components. Thereafter, the performance degradation will be analyzed in the presence of CFO. In Section 3.3, detailed descriptions of conventional feedforward data-aided CFR algorithms will be mathematically evaluated, and their performance degrading factors will be identified respectively. Finally, the proposed method based on least square error will be mathematically presented in Section 3.4.
3.1 Modeling and Description of Synchronization in Single-carrier TDMA System

As discussed earlier, the synchronization errors can greatly deteriorate the system performance, thus most single-carrier systems employ a complex scheme to achieve fast and accurate synchronization. Figure 3-1 shows a typical structure of overall carrier and symbol timing synchronization schemes for the single-carrier TDMA based satellite communication system.

![Block diagram of overall carrier and symbol timing recovery system.](image)

Figure 3-1: Block diagram of overall carrier and symbol timing recovery system.

As can be seen in Figure 3-1, the modulated signal is shaped by the root raised cosine (RRC) shaping filter, up-converted to the carrier frequency, time division multiplexed to a specific time slot, and then sent to the satellite. In the receiver, the incoming signal is noise corrupted and may be subject to the CFO/CPO and STE. Thus, the receiver must perform suitable synchronization process to accurately estimate and then compensate these impairment factors.
Towards this purpose, the received signal is first compensated by the coarse frequency recovery block (1) after passing through the matched filter. This is because the initial frequency offset can be large enough to accumulate phase rotation with the time, and most digital carrier and timing recovery algorithms work properly only in the presence of a small frequency offset. Hence, the frequency synchronization is usually implemented in two steps. A coarse frequency compensation, that does not require timing information, is performed firstly to reduce the frequency offset within a narrow range, and a relatively small frequency offset will be compensated in the frequency recovery block (3).

Since both frequency and phase recovery require the timing information during the estimation process, the symbol timing recovery as shown in block (2) must be applied before carrier recovery. Thus, the timing estimation algorithms must be tolerant to an arbitrary CFO/CPO.

After the symbol timing synchronization, the received signal is fed into the frequency and phase recovery blocks (3) and (4). By applying proper carrier frequency and phase estimation algorithms, the impaired received signal is expected to be recovered from frequency and phase offsets. However, it still can be suffered from some residual frequency and phase errors, which will lead to severe problems in the symbol detection. Therefore, the carrier recovery system must do a further step to track the residual frequency and phase offsets in the phase tracking block (5).

As can be seen from figure 3-1, the TDMA based satellite requires multiple steps to fulfillment synchronization. Among all of these steps, carrier frequency recovery
(block 3 shown in figure 3-1) plays an important role in determining the overall system performance. Hence, only carrier frequency recovery will be concentrated in this study.

Due to its simple demodulation structure, MPSK modulation scheme is selected, and the channel noise $n(t)$ is assumed to be additive white and Gaussian distributed with two side noise power spectral density $N_0/2$. The motivation of using higher order of MPSK is to increase the bandwidth efficiency of the PSK. In a MPSK system, transmitted signals can be expressed as [68]

$$s(t) = \sqrt{\frac{E_s}{T}} \exp[j(2\pi f_c t + \theta_i)] g(t - iT), \quad 0 \leq t \leq T$$

(3.1)

where $E_s$ denotes the transmitted symbol energy, $f_c$ is the carrier frequency, $T$ is the symbol duration. The signal power $P$ can be calculated by $P = E_s/T$, $g(t)$ denotes the impulse response of the shaping filter, $i$ is the symbol index and $\theta_i$ carries the transmitted signal, which is selected from the signal set $\{0, \pi\}$ and $\{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$ for $M=2$ and $M=4$, respectively. Without loss of generality and for mathematical convenience, it is desirable to represent all band-pass signals and corresponding channels to the equivalent low-pass signals equivalents using complex envelope. As a consequence, the complex envelope representation of the MPSK signal can be denoted as

$$s(t) = \sqrt{\frac{E_s}{T}} \sum_i \exp(j \theta_i) g(t - iT)$$

(3.2)

And the received signal can be denoted as

$$y(t) = s(t) + n(t)$$

(3.3)

If the received signal suffers a frequency offset $\Delta f$, phase offset $\Delta \theta$ and timing error $\Delta t$, the received signal will rotate its phase with time by $\exp(j2\pi \Delta f t)$ and a constant $\exp(j \Delta \theta)$, and delay $\Delta t$. Then, the received signal under this offsets can be rewritten as
\[ y(t) = s(t - \Delta t) \exp[j(2\pi \Delta f t + \Delta \theta)] + n(t) \]  \hfill (3.4)

In the receiver, the receiver filter \( g_R(t) \) is selected to match the shaping filter \( g(t) \), then overall \( h(t) = g(t) \otimes g_R(t) \) (\( \otimes \) denotes the convolution operation of functions) satisfies the Nyquist criterion which is defined as

\[
h[kT] = \begin{cases} 1, k = 0 \\ 0, k \neq 0 \end{cases}
\]  \hfill (3.5)

And the variance \( \sigma_n^2 \) of \( n(t) \) is now defined as

\[
\sigma_n^2 = \frac{N_0}{2} \int_{-\infty}^{\infty} H(f) df = \frac{N_0}{2}
\]  \hfill (3.6)

where \( H(f) \) is the Fourier transform of \( h(t) \), and its integral is unity. Thus, the received signal after matching filter can be approximated as

\[ y(t) = \exp[j(2\pi \Delta f t + \Delta \theta)] \sum_i \exp(j \theta_i) h(t - iT - \Delta t) + n(t) \]  \hfill (3.7)

If the sampling frequency is \( f_s \), and sampling interval \( T_s \) is \( 1/f_s \), the samples of received signal equation (3.7) result in (\( n \) denotes the sample index)

\[ y[nT_s] = \exp[j(2\pi \Delta f nT_s + \Delta \theta)] \sum_i \exp[j \theta_i] h[nT_s - iT - qT_s] + n[nT_s] \]  \hfill (3.8)

where \( \Delta t = qT_s \). To recover the transmitted data, the optimum points are selected by taking the sample at every symbol period \( T \), Thus

\[ y[kT] = \exp[j(2\pi \Delta f kT + \Delta \theta)] \sum_i \exp[j \theta_i] h[kT - iT - qT_s] + n[kT] \]  \hfill (3.9)

where \( 0 \leq k \leq N-1 \) for transmitting a block of \( N \) modulated symbols.
3.2 Performance Degradation due to Carrier Frequency Offset (CFO) in Single-carrier System

Let $\varepsilon$ denotes the CFO normalized by $BW/N$, where $BW$ is the total occupied bandwidth [58]. Then, $\Delta f = \varepsilon / NT$. By replacing $\Delta \theta$ with $\theta$ to represent phase offset, the samples of received signal can be represented as

$$y[nT_s] = \exp[j\left(\frac{2\pi}{NT}e_nT_s + \theta\right)]\sum_i \exp[j\theta_i]h[nT_s - iT - qT_s] + n[nT_s]$$

(3.10)

let $\tau$ be the STE normalized to the symbol duration $\tau = \Delta \tau / T$, then the received signal after selecting the optimum point can be expressed as

$$y[kT] = \exp[j\left(\frac{2\pi}{N}e_k + \theta\right)]\sum_i \exp[j\theta_i]h[kT - iT - \tau T] + n[kT]$$

(3.11)

Assuming the phase offset and symbol timing error have been perfectly compensated, the samples of received signal in one symbol duration can be derived from equation (3.10)

$$y[kT] = \exp[j\left(\frac{2\pi}{N}k\varepsilon\right)]\exp[j\theta_i] + n[nT]$$

(3.12)

In the case of BPSK, $\theta_i$ is selected from the signal set $\{0, \pi\}$. If the symbol $\exp(j0)$ is transmitted, the conditional error probability can be calculated as

$$p(e | \varepsilon) = \frac{1}{N} \sum_{k=0}^{N} \Pr\{\cos\left(\frac{2\pi}{N}k\varepsilon\right) + n_R[k] \leq 0\}$$

(3.13)

where $n_R[k]$ is the real part of $n(k)$. Based on the statistics of $n_R[k]$, equation (3.12) can be derived as

$$p(e | \varepsilon) = \frac{1}{N} \sum_{k=0}^{N} Q\left(\sqrt{\frac{\cos^2\left(\frac{2\pi}{N}k\varepsilon\right)}{\sigma_n^2}}\right)$$

(3.14)
where $Q(\cdot)$ denotes the Q-function. From [26], the signal energy $E_s$ can be derived based on previous assumption of $E_s=1$ for BPSK, then $E_s/N_0=1/(2\sigma_n^2)$, and the conditional error probability of BPSK impaired by carrier frequency offset $\epsilon$ can be calculated as ($E_s=E_b$ for BPSK)

$$p(e | \epsilon) = \frac{1}{N} \sum_{k=0}^{N-1} Q\left(\sqrt{\frac{2E_b}{N_0}} \cos\left(\frac{2\pi}{N} k \epsilon\right)\right)$$

(3.15)

In order to get error probability of QPSK impaired by CFO, the conditional error probability can be first calculated as

$$p(e | \theta) = 1 - p(c | \theta)$$

(3.16)

where $p(c|\theta)$ denotes the conditional correct probability. Assuming the signal is transmitted with equal probability, the conditional correct probability is the same with any symbols. If the symbol $\exp(j\pi/4)$ is transmitted, the conditional correct probability detection can be computed by

$$p(c | \epsilon) = \Pr\{\cos[\frac{2\pi}{N} k \epsilon + \frac{\pi}{4}] + n_R[k] \geq 0; \sin[\frac{2\pi}{N} k \epsilon + \frac{\pi}{4}] + n_I[k] \geq 0\}$$

(3.17)

where $n_R[k]$ and $n_I[k]$ are the real and imaginary part of $n(k)$, respectively. Based on the statistics of $n_R[k]$ and $n_I[k]$, the above equation can be written as

$$p(c | \epsilon) = \left\{1 - \frac{1}{N} \sum_{k=0}^{N-1} Q\left(\sqrt{\frac{\cos^2\left(\frac{2\pi}{N} k \epsilon + \frac{\pi}{4}\right)}{\sigma_n^2}}\right)\right\} \times \left\{1 - \frac{1}{N} \sum_{k=0}^{N-1} Q\left(\sqrt{\frac{\sin^2\left(\frac{2\pi}{N} k \epsilon + \frac{\pi}{4}\right)}{\sigma_n^2}}\right)\right\}$$

(3.18)

From [26], the signal energy $E_s$ can be derived based on previous assumption $E_s=1/2$ for QPSK, then $E_s/N_0=1/(4\sigma_n^2)$, and the conditional error probability of QPSK impaired by carrier frequency offset $\epsilon$ can then be calculated as
p(e | c) = \frac{1}{N} \sum_{k=0}^{N-1} [\sqrt{\frac{4E_s}{N_0}} \cos(\frac{2\pi k e + \pi}{4})] + \frac{1}{N} \sum_{k=0}^{N-1} [\sqrt{\frac{4E_s}{N_0}} \sin(\frac{2\pi k e + \pi}{4})] -
\frac{1}{N} \sum_{k=0}^{N-1} [\sqrt{\frac{4E_s}{N_0}} \cos(\frac{2\pi k e + \pi}{4})] \times \frac{1}{N} \sum_{k=0}^{N-1} [\sqrt{\frac{4E_s}{N_0}} \sin(\frac{2\pi k e + \pi}{4})]

(3.19)

3.3 Conventional Carrier Frequency Offset Estimation Algorithms

This section presents a full description of the conventional data aided feedforward algorithms investigated, which can be simply classified into three categories: delay and multiply based, least square based and autocorrelation based [29]. In order to simplify the presentation, the symbol timing is assumed to be known for the development of the algorithm.

From equation (3.9), the baseband sampled received signal during the preamble can be rewritten as [26]

\[ y_k = s_k e^{j(2\pi Tk + \Delta \theta)} + n_k, \quad k = 1, 2, ..., L \]  

(3.20)

where \( L \) is the preamble size, \( s_k \) represents the preamble symbols, \( T \) is the symbol duration, \( \Delta \theta \) is an unknown random phase with uniform probability density in \([0, 2\pi]\) and \( n_k \) is the sample of AWGN with independent in-phase and quadrature components, each of which has the zero mean, variance of \( \sigma_n^2 / 2 \) and double-sided PSD \( N_0 / 2 \). Thus, the SNR is defined as \( P / \sigma_n^2 \). However, because of the property \( s_k s_k^* = 1 \) (where \( s_k^* \) means complex conjugate of \( s_k \)), the dependence on \( s_k \) can be removed by multiplying \( y_k \) by \( s_k^* \). Thus, equation (3.20) turns out to be [26]

\[ r_k = e^{j(2\pi Tk + \Delta \theta)} + n_k^* s_k^* = e^{j(2\pi Tk + \Delta \theta)} + v_k, \quad k = 1, 2, ..., L \]  

(3.21)

where \( v_k = n_k s_k^* \) is statistically equivalent to \( n_k \).
3.3.1 Delay and Multiply-Based Algorithms

Rearranging Equation (3.21), the received signal can be given as [26]

\[ r_k = e^{j(2\pi f T_k + \Delta \phi)} + v_k = e^{j(2\pi f T_k + \Delta \phi)}[1 + \nu_k] = d_k e^{j(2\pi f T_k + \Delta \phi + \phi_k)} \] (3.22)

where \( \nu_k = v_k e^{-j(2\pi f T_k + \Delta \phi)} = n_k s^* e^{-j(2\pi f T_k + \Delta \phi)} \) still has the same statistics as \( n_k \), and

\[ d_k e^{j\phi_k} = 1 + \nu_k \] (3.23)

Thus, \( \phi_k \) is approximately independent zero mean and Gaussian distributed with variance of \( \sigma_n^2/2P \). If \( \alpha_{k,M} \) defines the argument of the product \( r_{k+M}^* r_k \), the argument of \( \alpha_{k,M} \) can be given as [26]

\[ \arg(\alpha_{k,M}) = 2\pi M \Delta f T + \phi_{k+M} - \phi_k, \ k = 1,2,...,L-M \] (3.24)

where \( M \) is the \( M \) symbol delayed version of current symbol, \( \arg[] \) denotes the argument of a complex number. Hence the frequency offset can be approximately estimated by [26]

\[ \hat{\Delta f} = \frac{1}{2\pi MT} \sum_{k=1}^{L-M-1} \arg(\alpha_{k,M} r_k^*) \] (3.25)

Because of the periodical property of \( \arg[] \) function, the estimation range of this algorithm \( |f| \) is less than \( 1/(2MT)^{-1} \).

3.3.2 Least Square Based Algorithms

Kay proposed a carrier frequency recovery algorithm based on the least square standard [30]. Let \( \alpha_k \) defines the argument of the product \( r_k r^*_k \), then

\[ \alpha_k = \arg(r_k r^*_k) = 2\pi \Delta f T + \phi_{k+1} - \phi_k, \ k = 1,2,...,L-1 \] (3.26)

Since \( \phi_k \) is approximately independent zero mean and Gaussian distributed, the MLE estimation of \( \Delta f \), which is equivalent to the minimum variance unbiased estimator for the linear model of equation (3.26), can be calculated by minimizing [30]
\[ J = (\alpha - 2\pi \Delta f T)C^{-1}(\alpha - 2\pi \Delta f T) \]  

(3.27)

where \( \alpha = [\alpha_1 \alpha_2 \cdots \alpha_{L-1}]^T \), \( I = [1 \ 1 \ \cdots \ 1]^T \) and \( C \) is the \((L-1) \times (L-1)\) covariance matrix of \( \alpha \).

After some algebra, the frequency offset can be estimated by [30]

\[ \Delta \hat{f} = \frac{1}{2\pi T} \sum_{k=1}^{L-1} c_k \arg(r_{k+1} r_k^*) \]  

(3.28)

where \( c_k \) is a smoothing function given by

\[ c_k = \frac{3}{2} \frac{L}{L^2 - 1} \left[ 1 - \left( \frac{2k - L}{L} \right)^2 \right] = \frac{6k(L-k)}{L(L^2 - 1)} \]  

(3.28)

With this algorithm, the estimation range \(|f|\) is less than \(1/(2T)^{-1}\).

### 3.3.3 Autocorrelation Based Algorithms

L&R proposed a carrier frequency offset estimation algorithm based on the autocorrelation of the received signal. As derived in [31], the equivalent likelihood function is given by [31]

\[ \Lambda_L(\Delta f) = \left| \sum_{i=1}^{L} r_i e^{-j2\pi \Delta f T} \right|^2 = \sum_{k=1}^{L} \sum_{m=0}^{L} r_k^* r_{k+m} e^{-j2\pi \Delta f (k-m)} \]  

(3.30)

Taking the derivative of equation (3.30) with respect to frequency \( \Delta f \) and equating it to zero, the frequency offset can be calculated after some straightforward arithmetic [31]

\[ \Delta \hat{f} = \frac{1}{\pi T (M+1)} \arg(\sum_{k=1}^{M} R_k), \quad M = 1, \ldots, L - 1 \]  

(3.31)

where \( R_k \) denotes the estimated autocorrelation of the sequence \( r_h \), which is defined as

\[ R_k = \frac{1}{L-k} \sum_{i=k+1}^{L} r_i^* r_{i-k} \quad , \quad k = 0, 1, \ldots, L - 1 \]  

(3.32)

With this algorithm, the estimation range \(|f|\) is less than \(1/[(M+1)T]^{-1}\).
Chen proposed a very similar carrier frequency estimation algorithm to L&R in his (or her) work [27]. As derived in [27], the estimated frequency offset is given by

$$\Delta \hat{f} = \frac{1}{2\pi M} \arg(R'_M)$$  \hspace{1cm} (3.33)

With this algorithm, the estimation range $|f|$ is less than $1/(2MT)^{-1}$.

The approach adopted by Fitz also used autocorrelation of the sequence $r_k$ and the frequency offset is given as [32-33]

$$\Delta \hat{f} = \frac{1}{\pi TM(M + 1)} \arg(\sum_{k=1}^{M} R_k)$$  \hspace{1cm} (3.34)

The Fitz estimator is found to be unbiased in the range $|f| \leq 1/(2MT)^{-1}$ and achieves the MCRB, provided that $M$ equals to $L/2$. Its estimation accuracy degrades as $L$ decreases, however, the computation load gets lighter and estimation range gets wider.

The data-aided frequency estimation algorithm proposed by M&M looks like the combination of Fitz and Kay’s algorithms. It also starts by exploiting the correlations $R_k$, and frequency offset can be estimated by [34]

$$\Delta \hat{f} = \frac{1}{2\pi T} \sum_{k=1}^{M-1} c_k \arg(R_{k+1}R'_k)$$  \hspace{1cm} (3.35)

where the smoothing factor is defined as

$$c_k = \frac{3[(L-k)(L-k+1)-M(L-M)]}{M(2M^2-6ML+3L^2-1)}$$  \hspace{1cm} (3.36)

### 3.4 Proposed Carrier Frequency Offset Estimation Algorithms

From the results of investigation described in Section 3.3, it can be concluded that the frequency offset can be estimated by taking the argument of $r_{k+1}^*r_k$, $R_k$ or $R_{k+1}^*$. In
all of the conventional algorithms using the autocorrelation function $R_k$, the received signal and its delayed terms are commonly involved during the estimation process. Thus, a better estimation can be achieved with the tradeoff between estimation accuracy and implementation complexity. On the other hand, in those algorithms that are not based on autocorrelation algorithms, such as Kay algorithm. Estimators are derived based on least square standard to generate a smoothing factor for each argument of $r_{k+1}r^*_k$. However, Kay algorithm only involves the received signal and its one symbol delayed term in the estimation process for example. Although delay and multiply-based algorithms consider the received signal and all its delayed terms, it simply calculates the approximate of frequency offset without employing any estimation standard. Hence, additional improvement can be expected if the least square standard is applied to the delay and multiply-based algorithms. As indicated in Section 3.3.2, the smoothing factor $c_k$ of Kay algorithm plays a key role to improve the performance. Thus, a complete derivation process of $c_k$ will be provided first.

From equation (3.27), the frequency offset can be estimated by [30]

$$2\pi T\Delta f = \frac{I^T C^{-1} \alpha}{I^T C^{-1} I}$$

(3.37)

It is clear to see from equation (3.26) that $\alpha_k$ is a real moving average process with driving noise variance $\sigma_n^2/2P$ and coefficients $b_1=1, b_2=-1$ [30]. The covariance matrix takes on the tridiagonal form [30]

$$C = \frac{\sigma^2}{2P} \begin{bmatrix}
2 & -1 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 \\
0 & 0 & 2 & -1 & 0 \\
0 & 0 & 0 & 2 & -1
\end{bmatrix}$$

(3.38)

The inverse of $C$ with the $(i,j)$ element is given by [30]
\[ C_j^{-1} = \frac{2P}{\sigma^2} \left[ \min(i, j) - \frac{ij}{L} \right], \quad 0 \leq i, j \leq L - 1 \]  

(3.39)

where \( \min(i, j) \) denotes the minimum of \( i \) and \( j \). Thus

\[
C^{-1} = \frac{2P}{\sigma^2} \begin{pmatrix}
1 - \frac{1 \times 1}{L} & 1 - \frac{1 \times 2}{L} & 1 - \frac{1 \times k}{L} & 1 - \frac{1 \times (L-1)}{L} \\
1 - \frac{2 \times 1}{L} & 2 - \frac{2 \times 2}{L} & 2 - \frac{2 \times k}{L} & 2 - \frac{2 \times (L-1)}{L} \\
1 - \frac{3 \times 1}{L} & 2 - \frac{3 \times 2}{L} & 3 - \frac{3 \times k}{L} & 3 - \frac{3 \times (L-1)}{L} \\
1 - \frac{i \times 1}{L} & 2 - \frac{i \times 2}{L} & k - \frac{i \times k}{L} & k - \frac{i \times (L-1)}{L} \\
1 - \frac{(L-1) \times 1}{L} & 2 - \frac{(L-1) \times 2}{L} & k - \frac{(L-1) \times k}{L} & (L-1) - \frac{1 \times (L-1)}{L}
\end{pmatrix}
\]

(3.40)

The summation of \( k \)th column of \( C^{-1} \) can be calculated as

\[ C_k^{-1} = \sum_{i=1}^{k-1} i + (L - 1 - k)k - \frac{k(k-1)}{2} \frac{k(L-k)}{L} \]

(3.41)

Then the denominator of equation (3.37) is given by

\[ I^T C^{-1} I = \sum_{k=1}^{L-1} k(L-k) = \frac{L(L^2-1)}{12} \]

(3.42)

Thus, from equation (3.37), the frequency offset estimate can be rewritten as

\[ 2\pi T \Delta f = \frac{I^T C^{-1} \alpha}{I^T C^{-1} I} = \frac{1}{I^T C^{-1} I} \sum_{k=1}^{L-1} C_k^{-1} \alpha_k = \sum_{k=1}^{L-1} c_k \alpha_k \]

(3.43)

The smoothing factor \( c_k \) can be calculated as

\[ c_k = \frac{C_k^{-1}}{I^T C^{-1} I} = \frac{12}{L(L^2-1)} \frac{k(L-k)}{2} = \frac{6k(L-k)}{L(L^2-1)} \]

(3.44)

Similar to the discussion in Kay algorithm, let \( \alpha_{k,M} \) defines the argument of the product \( r_{k+M} r^*_k \), then
\[ \alpha_{k,M} = \arg\{r_{k+M}^*\} = 2\pi M \Delta f + \varphi_{k+M} - \varphi_k, \quad k = 1, 2, \ldots, L - M \]  

(3.45)

This estimation process is also a moving average with coefficients \( I \) and \(-I\). The MLE of \( \Delta f \) can be derived by minimizing

\[ J = (\alpha - 2\pi M \Delta f I) C_M^{-1} (\alpha - 2\pi M \Delta f I) \]  

(3.46)

where \( \alpha = [\alpha_1, \alpha_2, \alpha_{L-M}]^T \) and \( C_M \) is the \((L-M) \times (L-M)\) covariance matrix of \( \alpha \) with coefficients \( b_1=1, b_M=-1 \), and it is defined as

\[
C_M = \frac{\sigma^2}{2P} \begin{pmatrix}
2 & -1 & 0 & 0 \\
0 & 2 & -1 & 0 \\
-1 & 2 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix} 
\]  

(3.47)

The solution of equation (3.46) is given by

\[ 2\pi MT \Delta f = \frac{I^T C_M^{-1} \alpha}{I^T C_M^{-1} I} \]  

(3.48)

Since \( C_M^{-1} \) is not tridiagonal matrix, it cannot be simply evaluated by equation (3.39). By calculating \( C_2^{-1} \) for \( L=8, 10 \) and \( C_4^{-1} \) for \( L=12, 16 \), we can get

\[
C_2^{-1} = \frac{2P}{\sigma^2} \begin{pmatrix}
3/4 & 0 & 1/2 & 0 & 1/4 & 0 \\
0 & 3/4 & 0 & 1/2 & 0 & 1/4 \\
1/2 & 0 & 1 & 0 & 1/2 & 0 \\
0 & 1/2 & 0 & 1 & 0 & 1/2 \\
1/4 & 0 & 1/2 & 0 & 3/4 & 0 \\
0 & 1/4 & 0 & 1/2 & 0 & 3/4
\end{pmatrix}_{6 \times 6} \]

(3.49)
\[
C_2^{-1} = \frac{2P}{\sigma^2} \begin{pmatrix}
\frac{4}{5} & 0 & 3/5 & 0 & 2/5 & 0 & 1/5 & 0 \\
0 & 4/5 & 0 & 3/5 & 0 & 2/5 & 0 & 1/5 \\
3/5 & 0 & 6/5 & 0 & 4/5 & 0 & 2/5 & 0 \\
0 & 3/5 & 0 & 6/5 & 0 & 4/5 & 0 & 2/5 \\
2/5 & 0 & 4/5 & 0 & 6/5 & 0 & 3/5 & 0 \\
0 & 2/5 & 0 & 4/5 & 0 & 6/5 & 0 & 3/5 \\
1/5 & 0 & 2/5 & 0 & 3/5 & 0 & 4/5 & 0 \\
0 & 1/5 & 0 & 2/5 & 0 & 3/5 & 0 & 4/5
\end{pmatrix}_{8 \times 8}, \\
L = 10
\] (3.50)

\[
C_4^{-1} = \frac{2P}{\sigma^2} \begin{pmatrix}
\frac{3}{4} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\
0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 \\
0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{4} & 0 \\
0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{4} \\
\frac{1}{2} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\
0 & \frac{1}{2} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\
\frac{1}{4} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 & 0 \\
0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{3}{4} & 0 & 0 \\
0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{3}{4} & 0 \\
0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{3}{4}
\end{pmatrix}_{12 \times 12}, \\
L = 16
\] (3.51)
The summation of $k$th column of inverse matrix $C_M^{-1}$ can be summarized as

$$C_{k,M}^{-1} = \left[ k/M \left( [L/M] - \left[ k/M \right] \right) \right]$$

where $\left[ x \right]$ denotes the smallest integer not less than $x$. Then the denominator of equation (3.48) is given by

$$I^T C_M^{-1} I = \frac{[L/M]\left( [L/M]^2 - 1 \right)}{12}$$

Thus, from equation (3.37), the frequency offset estimate can be rewritten as

$$2\pi T M \Delta f = \frac{I^T C_M^{-1} \alpha}{I^T C_M^{-1} I} = \frac{1}{I^T C_M^{-1} I} \sum_{k=1}^{L-M} C_{k,M}^{-1} \alpha_k$$

The new smoothing factor $c_{k,M}$ can be calculated as

$$c_{k,M} = \frac{C_{k,M}^{-1}}{I^T C_M^{-1} I} \frac{12 \left[ k/M \right] \left( [L/M]^2 - 1 \right)}{[L/M] \left( [L/M]^2 - 1 \right)}$$

Thus, the frequency offset estimate can be rewritten as

$$\Delta f = \frac{1}{2\pi MT} \sum_{k=1}^{L-M} c_{k,M} \arg(r_{k,M} r_k^*)$$
Because of the periodical property of $arg[]$ function, the estimation range of the proposed algorithm $|f|$ is less than $1/(2MT)^{1/4}$. The improved performance of the proposed CFR estimator will be illustrated in the next Chapter.
Chapter 4

Performance Evaluation of Carrier Frequency Recovery in Single-carrier Systems

This chapter is devoted to the complete evaluation on the performance of carrier frequency recovery algorithms developed in Chapter 3. Firstly, the BER degradation due to carrier frequency offset is evaluated under AWGN. Secondly, the performance of various carrier frequency recovery algorithms will be evaluated by the estimation range and estimation accuracy. Then, the pros and cons of different carrier frequency recovery algorithms are identified. In Section 4.3, the proposed carrier frequency recovery algorithm is evaluated through simulation and compared with the MCRB. Finally, the performance comparison between conventional and proposed carrier frequency recovery algorithms is provided in Section 4.4.

4.1 Performance Degradation Due to Carrier Frequency Offset

As discussed in Chapter 3, the frequency offset will accumulate the phase as the number of transmitted symbols is increased. Even a small frequency offset will severely destroy the performance. Thus, in order to fairly and consistently evaluate the performance degradation due to frequency offset for all single-carrier systems, the
frequency offset is normalized to $\epsilon = \frac{BW}{N}$. Table 4.1 shows the simulation scenarios throughout this section. Figures 4-1 and 4-2 show SER for BPSK and QPSK impaired by carrier frequency offset.

Table 4.1: Simulation scenarios for performance degradation due to carrier frequency offset.

<table>
<thead>
<tr>
<th>Bit rate</th>
<th>256 kbps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of samples per symbol</td>
<td>8</td>
</tr>
<tr>
<td>Shaping filter</td>
<td>Square root raised cosine filter</td>
</tr>
<tr>
<td>Roll-off factor</td>
<td>0.5</td>
</tr>
<tr>
<td>Channel</td>
<td>AWGN</td>
</tr>
<tr>
<td>Signal energy to noise spectral ratio $E_s/N_0$</td>
<td>0-15 dB</td>
</tr>
<tr>
<td>Modulation scheme</td>
<td>BPSK</td>
</tr>
<tr>
<td>Symbol rate</td>
<td>256 kbps</td>
</tr>
</tbody>
</table>

Figure 4-1: BPSK SER performance degradation due to carrier frequency offset.

As can be seen from figures 4-1 and 4-2, the performance gets worse as frequency offset gets larger. The simulated curves of selected frequency offsets match well with the theoretical formula derived in equations (3.15) and (3.19) of Chapter 3. Moreover,
comparing the performance between BPSK and QPSK, QPSK is more sensitive to the frequency offset than that of BPSK.

![Figure 4-2: QPSK SER performance degradation due to carrier frequency offset.](image)

**4.2 Performance Evaluation of Conventional Carrier Frequency Offset Estimation Algorithms**

Estimation range and estimation accuracy are two important features to evaluate an estimator performance. In the case of carrier frequency offset estimation, frequency offset should be normalized by the symbol rate $1/T$ for a general analysis. Thus, the estimation range can be evaluated by showing the expectation of normalized frequency offset estimate $E(\hat{\Delta f} T)$ versus its actual normalized frequency offset $\Delta f T$. Based on the estimation theory, there exist lower bounds for the error variance of any unbiased parameter estimator to evaluate the estimation accuracy under different assumptions. Among these bounds, the MCRB is always selected for the frequency offset estimator.
under AWGN channel, and it is derived by normalized to the squared symbol rate as [26]

$$T^2 \times MCRB[\hat{\Delta f}] = \frac{3}{2\pi^2 L^3} \frac{1}{E_s / N_0}$$  

(4.1)

The accuracy of an unbiased estimator can be determined by measuring how close the estimator error variance comes to this lower bound.

In this section, computer simulations have been performed to evaluate all algorithms discussed in Chapter 3. Without loss of generality, the following hypotheses have been made throughout this chapter: i) symbol timing is assumed to be perfect; ii) phase offset is constant during the preamble; iii) modulation scheme is QPSK with 0.5 roll-off Nyquist filter; iv) frequency offset is assumed to be $0.02/T$.

### 4.2.1 Performance Evaluation of Delay and Multiply Based Algorithms

As discussed in Section 3.3.1, the performance of delay and multiply frequency offset estimation algorithm not only depends on the preamble size $L$, but on the length of $M$. Since the preamble size $L$ does not affect the estimation range as indicated in Section 3.3.1, the performance of estimation range will only be evaluated by varying the length of $M$ with fixed preamble size. Figure 4-3 shows the expectation of normalized frequency offset estimation, $E(\hat{\Delta f}T)$, versus $\Delta fT$ using delay and multiply algorithm without any AWGN effect. Note that the ideal carrier frequency offset estimation should follow the line with $E(\hat{\Delta f}T) = \Delta fT$. As can be seen from figure 4-3, the operational range reduces with the increasing of length $M$, and the estimation range of delay and multiply algorithm is verified to be limited by $\pm 1/(2MT)^{1/2}$. Moreover, the estimated frequency offset exhibits no bias during the estimation range.
Figure 4-3: Normalized mean frequency offset estimation using delay and multiply algorithm with different length of $M$ (SNR=20dB).

Since both the preamble size $L$ and length $M$ can affect the estimation accuracy for delay and multiply algorithm, figures 4-4 and 4-5 illustrate the performance of estimation accuracy by varying $L$ or $M$, respectively. The modified Cramer-Rao bounds corresponding to the various preamble sizes are also indicated for comparison purpose. From figure 4-4, the estimation accuracy gets better as $M$ gets larger. No matter which $M$ is selected, the performance exhibits large deviation apart from the MCRB bound at very low SNR region. When SNR gets better, the performance gets closer to the MCRB bound, but can never achieve the bound. As is intuitively shown in figure 4-5, estimation accuracy improves as preamble size $L$ increases. For a fixed size of $M$, increasing the preamble size $L$ will cause more performance degradation compared with the respective MCRB bounds.
Figure 4-4: Normalized frequency offset estimation error variance using delay and multiply algorithm with different length $M$ ($\Delta fT=0.02$).

Figure 4-5: Normalized frequency offset estimation error variance using delay and multiply algorithm with different preamble size $L$ ($\Delta fT=0.02$).
4.2.2 Performance Evaluation of Least Square Based Algorithms

As is known from Section 3.3.2, Kay algorithm is based on the least square estimation standard, and preamble size $L$ is the only parameter that can determine the performance of Kay algorithms. Thus, figure 4-6 shows the expectation of normalized frequency offset estimates $E(\hat{\Delta f})$ as a function the actual frequency offset $\Delta fT$ using Kay algorithm with fixed preamble size. It is clear to see from figure 4-6, the estimation is unbiased over the estimation range, which is verified to be $\pm 1/(2T)^{\frac{1}{2}}$.

![Figure 4-6: Normalized mean frequency offset estimation using Kay algorithm.](image)

Figure 4-6 illustrates the estimation accuracy of Kay algorithm with different preamble size of $L$. The modified Cramer-Rao bounds corresponding to the various preamble sizes are also indicated for comparison. From figure 4-7, the estimation accuracy can be improved by increasing the preamble size $L$. The performance exhibits large deviation from the respective MCRB bounds at very low SNR region in general.
However, when SNR gets larger ($E_s/N_0$ at approximately 12 dB), the error variance of Kay algorithm can achieve the MCRB bound. Thus, the major improvement of Kay algorithm over delay and multiply algorithm is the reduced error variance at high SNR regions due to adding the smoothing factor $c_k$ in the estimation process.

![Normalized frequency offset estimation error variance using Kay algorithm with different preamble size $L$ ($\Delta f T=0.02$).](image)

**Figure 4-7:** Normalized frequency offset estimation error variance using Kay algorithm with different preamble size $L$ ($\Delta f T=0.02$).

### 4.2.3 Performance Evaluation of Autocorrelation Based Algorithms

In this section, autocorrelation based carrier frequency recovery algorithms are evaluated. Figure 4-8 show the normalized mean frequency offset estimation, $E(\hat{\Delta f T})$, versus $\Delta f T$ using L&R algorithm.
Figure 4-8: Normalized mean frequency offset estimation using L&R algorithm with different length of $M$ ($SNR=20dB$).

The curves of figure 4-8 are linear and cross x axis with unit slope in a large range, which confirm that the estimates are practically unbiased. From Section 3.3.3, the preamble size $L$ does not show any effect on estimation range of L&R. As expected, the frequency offset estimation range is limited to $1/[(M+1)T]^{-1}$, which confirms the result discussed in Section 3.3.3. Moreover, increasing the length of $M$ will reduce the estimation range of L&R algorithm.

Figures 4-9 and 4-10 illustrate the estimation accuracy of L&R algorithm by varying $L$ or $M$, respectively. The modified Cramer-Rao bounds corresponding to the various preamble sizes are also indicated for comparison.
Figure 4-9: Normalized frequency offset estimation error variance using L&R algorithm with different length $M$ ($\Delta fT=0.02$).

Figure 4-10: Normalized frequency offset estimation error variance using L&R algorithm with different preamble size $L$ ($\Delta fT=0.02$).
As can be seen from figure 4-9, the estimation accuracy improves with the increasing of length $M$. From [29], the error variance that approaches to the MCRB bound only when the optimal length of $M$ is selected to be around half of the preamble size $L/2$. However, the estimation range will be reduced as the length of $M$ increases. Since $\Delta f_T=0.02$ is considered in this study, the length of $M$ is chosen up to 8 to guarantee an unbiased estimation. In addition, it is clear to see from figure 4-10 that the estimation accuracy increases when the preamble size gets larger. For a fixed length of $M$, increasing the preamble size $L$ leads to degradations in estimation accuracy.

The normalized mean frequency offset estimation, $E(\hat{\Delta f_T})$, as a function of actual frequency offset $\Delta f_T$ using Chen algorithm without AWGN is shown in figure 4-11. As expected, the estimation range is unbiased and limited to $1/(2MT)^{-1}$.

Figure 4-11: Normalized mean frequency offset estimation using Chen algorithm with different length of $M$ (SNR=20dB).
As to the comparison with the modified Cramer-Rao bounds, the estimation accuracy of Chen algorithm is illustrated in figures 4-12 and 4-13 by varying $M$ or $L$, respectively. As can be seen from figure 4-12, the estimation accuracy improves with the increasing of preamble size $M$. With the preamble size $L=128$, the error variance cannot approach to the MCRB bound when $M$ is increased to 8. Although increasing $M$ can improve the estimation accuracy, the estimation range will also be decreased and the estimation turns out to be biased. Furthermore, as can be seen from figure 4-12, the estimation accuracy is improved when the preamble size $L$ is increased. For a fixed length of $M$, the error variance of the estimator deviates much from its corresponding MCRB bound as the preamble size $L$ gets larger.

![Figure 4-12: Normalized frequency offset estimation error variance using Chen algorithm with different length $M$ ($\Delta fT=0.02$).](image)

Figure 4-12: Normalized frequency offset estimation error variance using Chen algorithm with different length $M$ ($\Delta fT=0.02$).
Figure 4-13: Normalized frequency offset estimation error variance using Chen algorithm with different preamble size $L$. ($\Delta f T = 0.02$).

Similar to the discussion in L&R and Chen algorithms, figure 4-14 shows the normalized mean frequency offset estimation, $E(\hat{\Delta f} | T)$, versus $\Delta f T$ using Fitz algorithm. As expected, all of the curves are linear and cross x axis with unit slope from the origin to the frequencies $1/(2MT)^{1}$, which confirms that the estimates are practically unbiased in the estimation range $\pm 1/(2MT)^{1}$. Moreover, the preamble size $L$ does not show any effect on normalized mean frequency offset estimation for fixed length of $M$. Also increasing the length of $M$ could reduce the estimation range as shown in figure 4-14. Figures 4-15 and 4-16 illustrate the estimation accuracy of Fitz algorithm by varying $L$ or $M$, respectively. It is clear that the error variance gets closer to the MCRB bound with the increasing of length $M$. Moreover, as shown in figure 4-16, the error variance is reduced with the increasing of preamble size $L$. 
Figure 4-14: Normalized mean frequency offset estimation using Fitz algorithm with different length of $M$ ($SNR=20dB$).

Figure 4-15: Normalized frequency offset estimation error variance using Fitz algorithm with different length $M$ ($\Delta fT=0.02$).
In the case of M&M frequency estimation algorithm, figure 4-17 shows the normalized mean frequency offset estimation, $E(\hat{\Delta}f T)$, versus $\Delta f T$ using M&M algorithm without AWGN. As expected, the curves are linear and cross x axis with unit slope from the origin to the frequencies $\pm 1/(2MT)^{-1}$.

Figures 4-18 and 4-19 show the estimation accuracy of M&M algorithm by varying $L$ or $M$, respectively. The same trend is found as shown in other autocorrelation based algorithms. Figure 4-18 shows that the error variance reduces as $M$ gets larger. Moreover, the estimation accuracy can be improved by increasing the preamble size $L$. 

Figure 4-16: Normalized frequency offset estimation error variance using Fitz algorithm with different preamble size $L$ ($\Delta f T=0.02$).
Figure 4-17: Normalized mean frequency offset estimation using M&M algorithm with different length of $M$ ($SNR=20dB$).

Figure 4-18: Normalized frequency offset estimation error variance using M&M algorithm with different length $M$ ($\Delta fT=0.02$).
4.3 Performance Evaluation of Proposed Carrier Frequency Offset Estimation Algorithms

As was observed from Section 4.3, autocorrelation based carrier frequency recovery algorithms exhibit better estimation accuracy than others under low SNR. However, as SNR gets better, all of the autocorrelation based algorithms cannot approach MCRB bound for frequency offset $\Delta fT=0.02$. On the other hand, least squared based Kay algorithm can achieve MCRB bound when SNR is higher than a certain threshold. Moreover, compared with other algorithms, autocorrelation-based algorithm requires more computational overhead. In this section, the proposed carrier frequency offset estimation algorithm will also be evaluated in terms of the estimation range and estimation accuracy.

Figure 4-19: Normalized frequency offset estimation error variance using M&M algorithm with different preamble size $L$ ($\Delta fT=0.02$).
4.3.1 Performance Evaluation of the Proposed Algorithm with respect to Estimation Range

Figure 4-20 shows the expectation of normalized frequency offset estimation, $E(\Delta \hat{T})$, versus $\Delta fT$ using the proposed algorithm. As can be seen from figure 4-20, the estimated frequency offset exhibits no bias for a certain range, which gets narrower with the increase of length $M$. Thus, the estimation range of the proposed algorithm is verified to be limited by $\pm 1/(2M)^{1/4}$. Note that the proposed algorithm is identical to Kay algorithm when $M$ equals to 1.

Figure 4-20: Normalized mean frequency offset estimation using proposed algorithm with different length of $M$ (SNR=20dB).

4.3.2 Performance Evaluation of Proposed Algorithm with respect to Estimation Accuracy

From Section 3.4, both the preamble size $L$ and length $M$ can affect the estimation accuracy. Figures 4-21 and 4-22 illustrate the estimation accuracy of the proposed
algorithm by varying $L$ or $M$, respectively. The modified Cramer-Rao bounds corresponding to the various preamble sizes are also indicated for comparison.

![Normalized frequency offset estimation error variance using proposed algorithm with different length of $M$ (ΔfT=0.02)](image)

Figure 4-21: Normalized frequency offset estimation error variance using proposed algorithm with different length of $M$ ($ΔfT=0.02$)

From figure 4-21, the estimation accuracy is improved as $M$ gets larger for lower SNR region. However, the error variance of the proposed algorithm can approach to the MCRB when SNR is larger than a certain value. Although the estimation accuracy is improved by increasing the length of $M$, it does not increase the implementation complexity. As can be seen from Section 4.3, the proposed method does not require large computational load for large length of $M$. As is intuitively shown in figure 4-22, estimation accuracy improves as preamble size $L$ increases for fixed length of $M$. No matter which $M$ is selected, the performance exhibits large deviation from the MCRB
bound at very low SNR region. When SNR gets better, all of the error variance with different length of $M$ can achieve its corresponding MCRB bounds.

![Graph showing the normalized frequency offset estimation error variance using proposed algorithm with different preamble size $L$ ($\Delta fT = 0.02$)](image.png)

Figure 4-22: Normalized frequency offset estimation error variance using proposed algorithm with different preamble size $L$ ($\Delta fT = 0.02$)

### 4.4 Performance Comparisons of Carrier Frequency Offset Estimation Algorithms

As can be concluded from Sections 4.2 and 4.3, the performance of carrier frequency estimators depends on multiple parameters, which include the preamble size $L$, the length of $M$ (except Kay algorithm), and the SNR. In order to fairly evaluate the estimation range of different carrier frequency offset estimation algorithms, the preamble size $L$ and length of $M$ are chosen to be $L=128$, $M=8$. Figure 4-23 shows the expectation of normalized frequency offset estimation, $E(\hat{\Delta f})$, versus $\Delta fT$ for all algorithms discussed so far.
By careful examination of figure 4-23, Kay algorithm exhibits a much larger operation range than others. Moreover, it is clear to see that L&R algorithm has an estimation range of about 12.5% of 1/T, which is approximately twice of those of Chen, Fitz, M&M and M&D. Note that the proposed algorithm has essentially the same estimation range as Chen, Fitz, M&M, M&D. Therefore, compared with most carrier frequency offset estimation algorithms except L&R and Kay, the proposed algorithm does not show any degradation in estimation range under the same condition.

In figures 4-24 to 4-26, estimation error variance as of function of $E_s/N_0$ are plotted to investigate the estimation accuracy of all of the algorithms for different frequency offsets. The frequency offsets are chosen to be 5%, 2%, and 1% of the symbol rate. To guarantee an unbiased estimation, the parameter $M$ is selected to be 4, 8, and 16 for each offsets, respectively.
Figure 4-24: Performance comparison of normalized frequency offset estimation error variance ($\Delta fT=0.05$, $L=128$, $M=4$)

Figure 4-25: Performance comparison of normalized frequency offset estimation error variance ($\Delta fT=0.02$, $L=128$, $M=8$)
Figure 4-26: Performance comparison of normalized frequency offset estimation error variance ($\Delta f_T=0.01$, $L=128$, $M=16$)

As can be observed from figures 4-24 to 4-26, Kay algorithm consistently departs much from the MCRB lower bound at the low SNR region, although it yields the largest operating range. The D&M and proposed algorithms get closer to the MCRB bound than the Kay algorithm at low SNR region. When SNR gets improved, the proposed algorithm is overlapped with the MCRB bound, but the D&M algorithm can never achieve it. Compared to the proposed algorithm with other autocorrelation based algorithms, the proposed algorithm does not show better estimation accuracy at the low SNR region. However, as the SNR gets better, the proposed algorithm gets very close to the MCRB bound, but the autocorrelation based algorithms still show much deviations from the MCRB bound.
Another important issue to choose an estimation algorithm is its computational complexity. Table 4-2 shows the comparison of computational complexity for all of the algorithms discussed.

Table 4.2: Comparison of the computational complexity.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Category</th>
<th>Computational complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>D&amp;M</td>
<td>Delay and Multiply</td>
<td>$L-M$</td>
</tr>
<tr>
<td>Kay</td>
<td>Least square</td>
<td>$L-1$</td>
</tr>
<tr>
<td>L&amp;R</td>
<td>Autocorrelation</td>
<td>$L(2M-L-1)/2$</td>
</tr>
<tr>
<td>Chen</td>
<td>Autocorrelation</td>
<td>$L(2M-L-1)/2$</td>
</tr>
<tr>
<td>Fitz</td>
<td>Autocorrelation</td>
<td>$L(2M-L-1)/2$</td>
</tr>
<tr>
<td>M&amp;M</td>
<td>Autocorrelation</td>
<td>$L(2M-L-1)/2$</td>
</tr>
<tr>
<td>Proposed</td>
<td>Least square</td>
<td>$L-M$</td>
</tr>
</tbody>
</table>

The computational complexity is compared in terms of the number of required complex-valued multiplications. As can be seen from table 4-2, the autocorrelation based algorithms exhibit the same computational complexity, which is much higher than other algorithms. Thus, their improved estimation accuracy is due to the increased computational load. Moreover, the proposed algorithm shows the lowest computational load, which is the same as that of the D&M algorithm. However, the proposed algorithm can achieve much better estimation accuracy as discussed earlier.
Chapter 5

Carrier Frequency and Symbol Timing Recovery in Multi-carrier Systems

As a special multi-carrier modulation technique, OFDM has been successfully applied to a wide variety of digital communications applications over the past several years. The basic principle of OFDM is to divide a high rate bit stream into the low rate stream that is transmitted over a number of orthogonal subcarriers simultaneously. Thus the symbol duration is increased, and then the symbol bandwidth is reduced for each sub channel, which results in a much narrower sub-channel bandwidth than the coherent bandwidth. Although the whole channel is still severely affected by the multipath delay, each sub channel can be considered as flat. Hence, OFDM is much more resistant to fading than signal-carrier systems. Moreover, increased symbol duration leads to small ISI, which is also significantly reduced in the OFDM system by introducing cyclic prefix (CP). CP is added to preserve the orthogonality property over the duration of useful part of signal. Although CP causes a loss in signal to noise ratio (SNR), it is a worth tradeoff for combating interference. On the other hand, the OFDM system also has some drawbacks. First of all, it is more sensitive to carrier frequency offsets than that of single-carrier system. Because of the Doppler shift in the channel and instability of transmitter
and receiver oscillators, the carrier frequency offset will cause severe ICI due to the
destruction of orthogonality between subcarriers. In addition, due to the delay of signal,
the receiver is unknown to the starting point of each OFDM symbol. It is very important
to identify the optimum FFT window to overcome ISI. Furthermore, since the OFDM
symbol is the summation of $N$ different symbols, it can have high peaks when phases are
accumulated constructively. Hence, the OFDM system has a high peak-to-average power
ratio (PAPR) that can cause unwanted saturation in the power amplifiers, leading to in-
band distortion and out-of-band radiation.

Towards accurate estimation and proper compensation of these synchronization
errors in the data recovery of OFDM symbols, multiple efficient synchronization (and/or
estimation) methods have been proposed. Most of these available methods can be
classified into two categories: joint or separate estimation of symbol timing error and
frequency offset, or DA and NDA (also known as blind) estimation algorithms. In
general, the DA algorithm exploits the information from known pilot symbols or training
sequences, and it can yield high estimation accuracy with the tradeoff of low spectral
efficiency. On the contrary, the NDA algorithm usually results in low estimation
accuracy with computational overhead, but no need of extra data transmission.

This chapter will give a full investigation on the carrier frequency and symbol
timing recovery schemes of OFDM system. Section 5.1 presents the model of
synchronization schemes in the OFDM system. Since OFDM is very sensitive to carrier
frequency offset, the performance degradation due to carrier frequency offset is firstly
investigated in Section 5.2. The symbol error probability for OFDM with BPSK, QPSK,
8PSK and 16-QAM will be derived. Sections 5.3 and 5.4 will evaluate the existing
symbol timing recovery and carrier frequency offset estimation algorithms, respectively. Section 5.5 proposes two improved methods for joint estimation of symbol timing error and carrier frequency offset in OFDM system.

5.1 Model and Description of Synchronization in Multi-carrier Systems

Figure 5.1 shows the general block diagram of OFDM system. In the transmitter, \( N_{sd} \) high speed serial data symbols are first converted into parallel data. After inserting \( N_{cp} \) pilots for channel estimation, and padding zeros for up-sampling, \( N_{sd}+N_{cp} \) symbols are modulated on \( N_{so} \) subcarriers by \( N \) points FFT operation, where \( N_{so} \) denotes the number of actual occupied subcarriers. The maximum number of carrier used by OFDM is limited by the size of FFT, which is given by \( N_{so} \leq N/2-2 \) for real valued time signal and \( N_{so} \leq N-1 \) for complex valued time signal. In order to generate real time OFDM signal, the transmitted signal must be complex conjugate pairs and symmetric to the Nyquist frequency, which is half of the sampling frequency, and equal to \( N/2 \) in the sample index. Since the DC component (0th sample index), and the Nyquist frequency (\( N/2 \)th sample index) cannot have complex conjugate, these two point cannot be used as subcarriers to transmit data. If the OFDM is transmitted in complex time signal to maximize the bandwidth efficiency, it requires quadrature techniques to process real and imaginary parts respectively, because hardware can only accept real time signal. In this case, only DC component cannot be uses [69].
As a result, the $n$th time samples of the transmitted baseband OFDM signal, assuming $N_g$ samples of CP is included, can be expressed as [70]

$$
x_m[n] = \frac{1}{N} \sum_{k=0}^{N-1} X_m[k] \exp\left[ j \frac{2\pi}{N} k(n - N_g) \right], \quad 0 \leq n \leq N + N_g - 1
$$

(5.1)

where $k$ denotes the subcarrier index, $m$ denotes the OFDM symbol index, $X(k)$ is the modulated data symbol in frequency domain, which is available on $N_{\text{so}}$ subcarriers. $x(n)$ denotes the transmitted time domain OFDM signal. After passing through a frequency selective multipath fading channel with channel impulse response $h[n]$, the $n$th samples of the received signal $r[n]$ is represented by

$$
r_m[n] = (x_m[n - \theta] \otimes h_m[n]) \exp\left( j \frac{2\pi}{N} \epsilon n \right) + z_m[n]
$$

(5.2)

where $\otimes$ denotes cyclic convolutions, $\theta$ denotes the timing error in the unit of sample, and $\epsilon$ is the carrier frequency offset normalized by the subcarrier space. $z_m[n]$ is the sample of additive white Gaussian noise with zero mean and variance $\sigma^2$ per dimension.

The channel impulse response $h_m[n]$ is defined as

$$
h_m[n] = \sum_{\tau=0}^{Q-1} h_m[\tau] \delta[n - \tau]
$$

(5.3)
where \( h_m[\eta] \) represents a complex gain of the \( \eta \)th path, \( \tau_\eta \) represents the \( \eta \)th path delay, and \( Q \) is the maximum delay channel tap. To simplify the representation of the channel delays, the channel response \( h_m[n] \) can be represented by the delay of samples, and then the channel impulse response can be rewritten as
\[
h_m[n] = \sum_{\eta=0}^{N-1} h_m[\eta] \delta[n-\eta]
\] (5.4)

where \( D \) is the maximum delay in samples, and the time domain coefficients \( h_m[\eta] \) are modeled as zero mean with variance of \( \sigma^2_{h_m[\eta]} + \sigma^2_{h_m[\eta+1]} + \ldots + \sigma^2_{h_m[D-1]} = 1 \). Hence, the received signal after passing the channel is
\[
r_m[n] = \exp(j \frac{2\pi}{N} n) \sum_{\eta=0}^{N-1} h_m[\eta] v_m[n - \theta - \eta] + z_m[n]
\] (5.5)

The transmitted signal can be recovered by taking the FFT of the received signal, which is given as
\[
Y_m[l] = \sum_{n=0}^{N-1} r_m[N_g + n] \exp(-j \frac{2\pi}{N} l n)
\]
\[
= \sum_{n=0}^{N-1} \left[ \exp\left(j \frac{2\pi}{N} n + N_g \right) \sum_{\eta=0}^{N-1} h_m[\eta] v_m[n - \theta + N_g - \eta] \right] \exp\left(-j \frac{2\pi}{N} l n \right)
\]
\[
= \sum_{n=0}^{N-1} \left[ \exp\left(j \frac{2\pi}{N} n + N_g \right) \sum_{\eta=0}^{N-1} h_m[\eta] v_m[n - \theta + N_g - \eta] \right] \exp\left(-j \frac{2\pi}{N} l n \right) + \sum_{n=0}^{N-1} z_m[n + N_g] \exp\left(-j \frac{2\pi}{N} l n \right)
\]
\[
= Y_m[l] + Z_m[l], \quad 0 \leq l \leq N - 1
\] (5.6)

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\[ Y_n[I] = \sum_{n=0}^{N-1} \exp \left[ j \frac{2\pi}{N} \varepsilon (n + N_g) \right] \sum_{\eta=0}^{N-1} h_m[\eta] \]

\[ = \frac{1}{N} \sum_{k=0}^{N-1} X_m[k] \exp \left[ j \frac{2\pi}{N} [n - \theta - \eta] k \right] e(-j \frac{2\pi}{N} l n) \]

\[ = \frac{1}{N} \sum_{k=0}^{N-1} X_m[k] \sum_{\eta=0}^{D-1} h_m[\eta] \exp(-j \frac{2\pi}{N} \eta k) \exp(-j \frac{2\pi}{N} \theta k) \]

\[ \sum_{n=0}^{N-1} \exp \left[ j \frac{2\pi}{N} (k + \varepsilon - l) n \right] \exp(j \frac{2\pi}{N} \varepsilon N_g) \] (5.7)

By using the formula \( \sum_{n=0}^{N-1} u^n = \frac{1-u^N}{1-u} \) and \( 1 - \exp(j2\phi) = -j2 \sin \phi \exp(j \phi) \), \( Y_m[I] \) can be calculated as

\[ Y_n[I] = a_m[I] \exp(-j \frac{2\pi}{N} \theta l) X_m[I] S_0 + \]

\[ \sum_{k=0, \, k \neq l}^{N-1} a_m[k] \exp(-j \frac{2\pi}{N} \theta k) X_m[k] S_{k-l} \] \( 0 \leq k \leq N-1 \) (5.8)

where \( a_m[l] \) is defined as

\[ a_m[l] = \sum_{\eta=0}^{D-1} h[\eta] \exp(-j \frac{2\pi}{N} \eta l) \] (5.9)

The sequence \( S_{l+k} \) except \( S_0 \) is defined as the coefficients of ICI terms between \( l \)th and \( k \)th subcarriers. Taking the effect of CP into consideration, \( S_{k} \) can be calculated as

\[ S_k = \sum_{n=0}^{N-1} \exp \left[ j \frac{2\pi}{N} (k + \varepsilon) n \right] \exp(j \frac{2\pi}{N} \varepsilon N_g) \]

\[ = \exp(j \frac{2\pi}{N} \varepsilon N_g) \sin[\pi(k + \varepsilon)] \frac{\sin[\frac{\pi(k + \varepsilon)}{N}]}{N \sin[\frac{\pi(k + \varepsilon)}{N}]} \cdot \exp \left[ j \pi(\frac{N-1}{N})(k + \varepsilon) \right] \] (5.10)
The objective of OFDM symbol timing and carrier frequency synchronization is to estimate and compensate \( \theta \) and \( \epsilon \) in the receiver. Figure 5-2 shows the block diagram of a typical OFDM receiver synchronization process. Carrier frequency offset is usually estimated in three steps. Firstly, a coarse decimal frequency offset is estimated with the coarse frequency recovery. Secondly, the integer part of carrier frequency offset is obtained in the frequency domain. Finally, a tracking loop structure is always utilized to estimate the fine decimal part of the carrier frequency offset to guarantee the estimation accuracy. Symbol timing recovery is usually realized in two steps. First of all, a coarse symbol timing recovery is implemented in the timing domain, and then the fine symbol timing recovery is executed in the frequency domain to ensure a more accurate estimation.

![Figure 5-2: Block diagram of synchronization process in the OFDM receiver.](image)

In this study, the blocks of coarse frequency offset and coarse timing error estimations will be investigated assuming the effect of sampling frequency mismatch is negligible. For the effective development of an improved synchronization method with respect to the estimations of interest, it is viable to investigate the performance degradation due to synchronization errors and the characteristics of existing synchronization algorithms.

### 5.2 Effect of Carrier Frequency Offset on the Performance of OFDM Systems

By overlapping spectra of multiple subcarriers, the OFDM system provides high spectral efficiency. However, this benefit can only be achieved when the orthogonality
among subcarriers is maintained. Since the CFO can significantly destroy the orthogonality between the subcarriers and induce phase error accumulation over the successive symbols, the estimation accuracy of CFO becomes more important for the OFDM system. Thus, it is important to evaluate the performance of OFDM system impaired by CFO for the practical applications in the beginning.

Since OFDM is more sensitive to carrier frequency offsets than that of single-carrier system, this section will give a full investigations on the symbol error probability expression of OFDM impaired by carrier frequency offset. As can be seen from equation (5.8), carrier frequency offset can cause severely degradation to the OFDM system due to the destruction of the subcarriers orthogonality. By adding the CP to the OFDM symbol, the received signal can be free of ISI, if the channel maximum excess delay is shorter than the CP. Although adding CP can eliminate the ISI caused by symbol timing error, it also requires additional transmitted energy. Thus, a SNR loss due to the insertion of CP needs to be introduced, which is given as [71]

\[ SNR_{loss} = -10 \log_{10} \left( \frac{T_s}{T_g} \right) \]  \hspace{1cm} (5.11)

where \( T_s \) is the OFDM symbol duration and \( T_g \) is cyclic prefix duration. Assuming perfect timing error compensation, the received signal under AWGN can be derived from equation (5.8)

\[ Y_t = a_t X_t S_0 + \sum_{k=0,k \neq t}^{N-1} a_k X_k S_{k-t} + Z_t \]  \hspace{1cm} (5.12)

where

\[ a_t = \sum_{\eta=0}^{D-1} h_\eta \exp(-j \frac{2\pi}{N} \eta t) \]
And the second term in equation (5.12) denotes the ICIIs caused by carrier frequency offset.

As discussed earlier, mainly two methods are widely used to quantify the effect of ICI caused by carrier frequency offset. Among them, the method that approximates the ICI as a Gaussian process leads to pessimistic results for high signal to noise ratios, the method based on the CHF of the ICI is employed here to evaluate the symbol error probability of OFDM system in the presence of carrier frequency offset. Without loss of generality, the transmitted data symbols are assumed to be independent and identically distributed, and then the symbol error probability can be derived by considering the 0th subcarrier \((k=0)\) only [54].

### 5.2.1 Effect of Carrier Frequency Offset on the Performance of BPSK/QPSK OFDM

For BPSK modulation, the transmitted signal \(X_k\) is chosen from the set \(\{-1, 1\}\) with equal probability. By the definition, the CHF of the real part of \(Y_l\) can be expressed as [54]

\[
\phi_{\Re(Y_l)}(\omega) = \exp \left[ j \omega \Re\{S_0\} - \frac{\omega^2 \sigma^2}{2} \right] \prod_{l=2}^{N} \cos[\omega \Re\{S_l\}] \quad (5.13)
\]

Since the product of cosine can be expressed as a sum of cosine [50]

\[
\prod_{k=1}^{M} \cos \phi_k = \frac{1}{2^{M-1}} \sum_{k=1}^{2^{M-1}} \cos(\Phi^T e_k) \quad (5.14)
\]

where \(\bar{\Phi} = (\phi_1, \phi_2, \ldots, \phi_M)^T\), \(e_k\) is the \(k\)th column of a more general \(M \times 2^{M-1}\) matrix \(E_M\), of which \(k\)th column is essentially the binary representation of the number \(2^{M-k}\), where
binary zeros are replaced with $-j$s. Then the CHF of the real part of $Y_l$ can be further simplified as [50]

$$
\phi_{R(Y_l)}(\omega) = \frac{1}{2^{N-2}} \exp \left[ j \omega R(S_0) - \frac{\omega^2 \sigma^2}{2} \right] \times \sum_{k=1}^{2^{N-2}} \cos \left[ \omega \mathcal{R}(S^Te_k) \right]
$$

$$
= \frac{1}{2^{N-1}} \sum_{k=1}^{2^{N-2}} \left[ \exp \left( j \omega \alpha_k - \frac{\omega^2 \sigma^2}{2} \right) + \exp \left( j \omega \beta_k - \frac{\omega^2 \sigma^2}{2} \right) \right]
$$

(5.15)

where $S=(S_1, S_2, ..., S_{N-1})^T$, $\alpha_k=\text{Re}\{S_0+S^Te_k\}$ and $\beta_k=\text{Re}\{S_0-S^Te_k\}$. Since the signal is binary, an error occurs if the real part of $Y_l$ is smaller than zero. Then, the symbol error probability of BPSK OFDM impaired by carrier frequency offset under AWGN can be expressed as [50]

$$
P_s(\varepsilon) = \frac{1}{2^{N-1}} \sum_{k=1}^{2^{N-2}} \left[ Q\left(\sqrt{2\gamma} \alpha_k \right) + Q\left(\sqrt{2\gamma} \beta_k \right) \right]
$$

(5.16)

where $\gamma$ denotes the energy per symbol to noise spectral density ratio ($E_s/N_0$) (note that $E_s=1$, $\sigma^2=N_0/2$), $Q(\cdot)$ is the Gaussian Q-function. In the Rayleigh fading channel, the conditional symbol error probability of BPSK OFDM can be calculated as [56]

$$
P_s(\varepsilon) = \frac{1}{2} \frac{1}{2^{N}} \sum_{k=1}^{2^{N-2}} \left\{ \text{sgn}(\alpha_k) \sqrt{2\sigma_R^2 \alpha_k^2 \gamma} + \text{sgn}(\beta_k) \sqrt{2\sigma_R^2 \beta_k^2 \gamma} \right\}
$$

(5.17)

where $\sigma_R$ denotes the mode of Rayleigh distribution. Furthermore, the symbol error probability of BPSK OFDM under the frequency selective channel is given by [56]

$$
P_s(\varepsilon) = \frac{1}{2} \frac{1}{2^{N}} \sum_{k=1}^{2^{N-2}} f_1 \left[ \frac{c_\alpha \text{Re}\{S_k\} + \mu_k \gamma}{1 + \left[ c_{\alpha \alpha} \text{Re}\{S_k\} + \mu_k \gamma + \nu_k \gamma \right]} \right] + 
$$

$$
\frac{c_\alpha \text{Re}\{S_k\} - \mu_k \gamma}{1 + \left[ c_{\alpha \alpha} \text{Re}\{S_k\} - \mu_k \gamma + \nu_k \gamma \right]} f_2 \gamma}
$$

(5.18)
where $f_1 = \text{sgn}(\text{Re}\{S_1 + \mu_k\})$, $f_2 = \text{sgn}(\text{Re}\{S_1 - \mu_k\})$, $(a_1, a_2, ..., a_N)^T = F_D h$ with $h = (h_1, h_2, ..., h_D)$, $F_D$ denotes the first $D$ columns of the $N \times N$ discrete Fourier transform matrix, $a = (a_2, a_3, ..., a_N)^T$, $P_k = \text{diag}(S_2, S_3, ..., S_N) e_k$ with $\text{diag}(\cdot)$ being a diagonal matrix, $C_a a_1 = (C_a a_2, C_a a_3, ..., C_a a_N)^T$, $\mu_k = \text{Re}\{P_k^* C_a a_1\}$, $\nu_k = P_k^* (C_a a_2 - C_a a_1 C_a h_1) P_k^*$.

In the case of QPSK OFDM, the transmitted signal is selected from the sets $\{\pm 1 \pm j\}$ with equal probability. After some similar algebraic manipulations as discussed in BPSK OFDM, the symbol error probability of QPSK OFDM under AWGN is given as [50]

$$P_s(e) = 1 - \frac{1}{2^{2N-2}} \sum_{k=1}^{2^{N-2}} \sum_{n=1}^{2^{N-2}} \sum_{m=1}^{4} Q(-\sqrt{2\gamma} \varphi_{kn}[1,m]) Q(-\sqrt{2\gamma} \varphi_{kn}[2,m])$$

(5.19)

where $\varphi_{pq}[p,q]$ is the $(p,q)$th element of the $2 \times 4$ matrix $\psi$ defined as $\psi = (C_1, C_2, C_3, C_4)$, and $C_1 = S_A^1 - S_B^1 + S_A e_k + S_B e_n$, $C_2 = S_A^1 - S_B^1 - S_A e_k - S_B e_n$, $C_3 = S_A^1 - S_B^1 + S_A e_k - S_B e_n$, $C_4 = S_A^1 - S_B^1 - S_A e_k + S_B e_n$, $S_A^l = (\text{Re}\{S_i\} \text{Im}\{S_i\})^T$, $S_B^l = (\text{Im}\{S_i\} - \text{Re}\{S_i\})^T$ for all $l = 0, 1, ..., N-1$. In the Rayleigh fading channel, the conditional symbol error probability of QPSK OFDM impaired by carrier frequency offset can be derived as [50]

$$P_s(e) = \frac{3}{4} - \frac{1}{2^{2N-2}} \sum_{k=1}^{2^{N-2}} \sum_{n=1}^{2^{N-2}} \sum_{m=1}^{4} \left\{ \frac{2\sigma_k^2 \varphi_{kn}[1,m]}{1 + 2\sigma_k^2 \varphi_{kn}[1,m]} \right\}^\gamma \left[ 1 - \frac{1}{\pi} \left( \varphi_{kn}[1,m] \sqrt{\frac{1}{1 + 2\sigma_k^2 \varphi_{kn}[1,m]}} \right) \right] +$$

$$\left[ \frac{2\sigma_k^2 \varphi_{kn}[2,m]}{1 + 2\sigma_k^2 \varphi_{kn}[2,m]} \right]^\gamma \left[ 1 - \frac{1}{\pi} \left( \varphi_{kn}[2,m] \sqrt{\frac{1}{1 + 2\sigma_k^2 \varphi_{kn}[2,m]}} \right) \right]$$

(5.20)
5.2.2 Effect of Carrier Frequency Offset on the Performance of 8PSK OFDM under AWGN

By the definition, the signal space diagram of 8PSK is shown in figure 5-3 [55]. The transmitted signal $X_k$ is chosen from the set \{\(I_{(n-1)/2}e^{jn/M}, n=1,3,...,M/2-1\}\} with equal probability [55].

![Signal space diagram for 8PSK](image)

Figure 5-3: Signal space diagram for 8PSK.

By taking the signal space decomposition (SSD) approach, the symbol error probability of 8PSK can be given as [55]

$$
P_s = 1 - \frac{1}{2}(P_{D(x,y)l_0} + P_{D(u,y)l_0}) \approx P_{D(x,y)l_0} + P_{D(\tau,y)l_0}
$$

(5.21)

where $D(x,y)$ denotes the region from the axis $x$ to the axis $y$ with anticlockwise direction. Thus, the symbol error probability of 8PSK under AWGN can be further expressed as [68]

$$
P_s \approx P_{\text{Re}(Y_0)<0|l_0} + P_{\text{Re}(Y_0)>0|l_1}
$$

(5.22)

where $Y_0$ is the received symbol at the first subcarrier as denoted in equation (5.12). From equation (5.12), if the noise free received signal $U$ under AWGN is defined as [54]
Let us consider the first subcarrier with the transmitted symbol $X_0 = I_0$. The CHF of $U$ (denoted as $\Phi_{U|I_0}$) can be calculated as

$$
\phi_{U|I_0}(\omega) = \exp\left[j\Omega^T S_\alpha^0 \cos\left(\frac{\pi}{8}\right)\right] \exp\left[-j\Omega^T S_\beta^0 \sin\left(\frac{\pi}{8}\right)\right] \times
$$

$$
\prod_{l=1}^{N-1} \cos(V_l^T S_\alpha^l) \cos(V_{2l}^T S_\alpha^l) \cos(V_l^T S_\beta^l) \cos(V_{2l}^T S_\beta^l)
$$

(5.24)

where $\Omega S_\alpha^l$ and $\Omega S_\beta^l$ are respectively defined as

$$
\Omega^T S_\alpha^l = \omega_l \Re\{S_l\} + \omega_q \Im\{S_l\}
$$

(5.25)

$$
\Omega^T S_\beta^l = \omega_l \Im\{S_l\} - \omega_q \Re\{S_l\}
$$

(5.26)

and $\Omega = (\omega_i, \omega_q)^T, S_\alpha^l = (\Re\{S_l\}, \Im\{S_l\})^T, S_\beta^l = (\Im\{S_l\} - \Re\{S_l\})^T$ for all $l=0, 1, ..., N-1$.

By further simplifying (5.24) using (5.14), the CHF of $U$ can be expressed as

$$
\phi_{U|I_0}(\omega) = \frac{1}{2^{4(N-1)}} \exp\left[j\Omega^T S_\alpha^0 \cos\left(\frac{\pi}{8}\right)\right] \exp\left[-j\Omega^T S_\beta^0 \sin\left(\frac{\pi}{8}\right)\right] \times
$$

$$
\sum_{q=1}^{2^{N-2}} \sum_{p=1}^{2^{N-2}} \sum_{m=1}^{2^{N-2}} \sum_{k=1}^{2^{N-2}} \cos(V_q^T S_a e_q) \cos(V_{2q}^T S_a e_{2q}) \cos(V_q^T S_\beta e_q) \cos(V_{2q}^T S_\beta e_{2q})
$$

(5.27)

where $S_a = (S_a^1, S_a^2, ..., S_a^{N-1})$ and $S_\beta = (S_\beta^1, S_\beta^2, ..., S_\beta^{N-1})$. Then, by using Euler’s formula, equation (5.27) can be further calculated as

$$
\phi_{U|I_0}(\omega) = \frac{1}{2^{4(N-1)}} \sum_{q=1}^{2^{N-2}} \sum_{p=1}^{2^{N-2}} \sum_{m=1}^{2^{N-2}} \sum_{k=1}^{2^{N-2}} \exp\left[j\Omega^T \left(S_a^0 \cos\left(\frac{\pi}{8}\right) - S_\beta^0 \sin\left(\frac{\pi}{8}\right) + C_d e_q\right)\right]
$$

(5.28)
where matrix $C_1=(V_1S_{e_1} V_2S_{e_2} V_1S_{e_3} V_2S_{e_4})$ and $d_n (n=1,2,..., 2^4)$ is the $n$th column of a $4\times 2^4$ matrix, of which $n$th column is essentially the binary representation of the number $2^4-n$, where binary zeros are replaced with $-I$s.

From the property of CHF, if a random variable has pdf, the CHF is its Fourier transform (FT) [63]. Taking the FT of equation (5.28), the pdf of $U$ (denoted as $P_{U|I_0}$) can be derived as

$$p_{U|I_0}(\text{Re}\{U\}, \text{Im}\{U\}) = \frac{1}{2^{4(N-3)}} \sum_{q=1}^{2^N-2} \sum_{p=1}^{2^N-2} \sum_{m=1}^{2^N-2} \sum_{n=1}^{16} \{\delta(\text{Re}\{U\} - A_{q,p,m,k}^1[1,n]) \times \delta(\text{Im}\{U\} - A_{q,p,m,k}^1[2,n])\}$$

(5.29)

where $A_{q,p,m,k}^1[i,j]$ is the $(i, j)$ element of the $2\times 16$ matrices $A^1$, which is defined as

$$A^1 = [S_\alpha \cos(\frac{\pi}{8}) - S_\beta \sin(\frac{\pi}{8}) + C_1 d_1, S_\alpha \cos(\frac{\pi}{8}) - S_\beta \sin(\frac{\pi}{8}) + C_1 d_2, ...]$$

(5.30)

Similarly, if the symbol $X_0=I_1$ is transmitted on the first subcarrier, the pdf of $U$ (denoted as $P_{U|I_1}$) can be calculated by substituting $A^1$ in equation (5.29) with $A^2$, which is defined as

$$A^2 = [S_\alpha \cos(\frac{3\pi}{8}) - S_\beta \sin(\frac{3\pi}{8}) + C_1 d_1, S_\alpha \cos(\frac{3\pi}{8}) - S_\beta \sin(\frac{3\pi}{8}) + C_1 d_2, ...]$$

(5.31)

where $C_2=(V_3S_{e_1} V_4S_{e_2} V_3S_{e_3} V_4S_{e_4})$, $V_3=[\cos(3\pi/8)+\sin(3\pi/8)]/2$, $V_4=[\cos(3\pi/8)-\sin(3\pi/8)]/2$. 

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Since the real and imaginary parts of the AWGN are independent and identically distributed with a respective variance of $\sigma^2$, the conditional probabilities in equation (5.22) can be further calculated as

$$P_{\text{im} | Y_e < 0 | I_o} = P(U + Z_0 < 0 | I_o = \exp(j\frac{\pi}{8}), U)$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{[(x - \text{Im}\{U\})]^2}{2\sigma^2}\right) dx = Q\left(\frac{\text{Im}\{U\}}{\sigma}\right)$$  \hspace{1cm} (5.32)

and

$$P_{\text{Re} | Y_e < 0 | I_i} = P(U + Z_0 < 0 | I_i = \exp(j\frac{3\pi}{8}), U)$$

$$= Q\left(\frac{\text{Re}\{U\}}{\sigma}\right)$$  \hspace{1cm} (5.33)

Averaging equations (5.32) and (5.33) with respect to the pdf given in (5.29) yields the symbol error probability of 8PSK OFDM as

$$P_s(\varepsilon) = \frac{1}{4(N-1)^2} \sum_{q=1}^{2^{N-2}} \sum_{p=1}^{2^{N-2}} \sum_{m=1}^{2^{N-2}} \sum_{k=1}^{2^{N-2}} \sum_{n=1}^{16} \left( Q\left(\sqrt{2}\mathbf{A}_{q,p,m,k}^1[2,n]\right) + Q\left(\sqrt{2}\mathbf{A}_{q,p,m,k}^2[1,n]\right) \right)$$  \hspace{1cm} (5.34)

### 5.2.3 Effect of Carrier Frequency Offset on the Performance of 16-QAM OFDM under AWGN

Recently M-QAM has been successfully implemented in xDSL system as well as next generation wireless access and wireless LAN (WLAN) [73]. Since square-constellation M-ary QAM does not require complicate demodulation scheme, and exhibits much better performance than other QAM constellations [72], it is more frequently applied recently. Figure 5-4 shows the constellation diagram of square 16-QAM [63], the transmitted signal is chosen from the set \{±1±j, ±1±3j, ±3±j, ±3±3j\} with
equal probability. Assuming the transmitted symbol $X_0$ to be in the first quadrant, $X_0 \in \{1+j, 1+3j, 3+j, 3+3j\}$, the CHF of $U$ can be calculated as

$$
\phi_U(\omega) = \left\{ \frac{1}{2} \exp(j\Omega^T S^a) + \frac{1}{2} \exp(j3\Omega^T S^a) \right\} \times
\left\{ \frac{1}{2} \exp(-j\Omega^T S^0) + \frac{1}{2} \exp(-j3\Omega^T S^0) \right\} \times \prod_{l=1}^{N-1} \cos(\Omega^T S_{\alpha}^l) \cos(2\Omega^T S_{\alpha}^l) \cos(\Omega^T S_{\rho}^l) \cos(2\Omega^T S_{\rho}^l) \tag{5.35}
$$

Substituting equation (5.14) into equation (5.35), the CHF of $U$ can be expressed as

$$
\phi_U(\omega) = \frac{1}{2^{n(N-2)}} \left\{ \frac{1}{2} \exp(j\Omega^T S^a) + \frac{1}{2} \exp(j3\Omega^T S^a) \right\} \times
\left\{ \frac{1}{2} \exp(-j\Omega^T S^0) + \frac{1}{2} \exp(-j3\Omega^T S^0) \right\} \times
$$
\[ \sum_{q=1}^{2^{N-2}} \sum_{p=1}^{2^{N-2}} \sum_{m=1}^{2^{N-2}} \sum_{k=1}^{2^{N-2}} \cos(\Omega^T S_a e_q) \cos(2\Omega^T S_a e_p) \cos(\Omega^T S_b e_m) \cos(2\Omega^T S_b e_k) \]

(5.36)

where \( S_a = (S_a^1 \ S_a^2 \ ... \ S_a^{N-1}) \) and \( S_b = (S_b^1 \ S_b^2 \ ... \ S_b^{N-1}) \). Then, by using Euler’s formula, equation (5.36) can be further calculated as

\[
\phi_{\nu}(\omega) = \frac{1}{2^{N(N-1)}} \sum_{q=1}^{2^{N-2}} \sum_{p=1}^{2^{N-2}} \sum_{m=1}^{2^{N-2}} \sum_{k=1}^{2^{N-2}} 16 \left\{ \exp \left[ j\Omega^T (S_a^0 - S_b^0 + C d_n) \right] + \exp \left[ j\Omega^T (3S_a^0 - 3S_b^0 + C d_n) \right] + \exp \left[ j\Omega^T (3S_a^0 - 3S_b^0 + C d_n) \right] \right\}
\]

(5.37)

where matrix \( C = (S_a e_q \ 2S_a e_p \ S_b e_m \ 2S_b e_k) \) and \( d_n (n=1,2,...,2^4) \) is the \( n \)th column of a \( 4\times4 \) matrix, of which \( n \)th column is essentially the binary representation of the number \( 2^4-n \), where binary zeros are replaced with -1s.

As discussed in [54], to derive the correct decision probability of 16-QAM, let us designate a function \( \psi(a, b) \) in terms of a product of the error functions as

\[
\Psi(a, b) = Q\left( \frac{a - \text{Re}\{U\}}{\sigma} \right) Q\left( \frac{b - \text{Re}\{U\}}{\sigma} \right)
\]

(5.38)

where \( Q(\cdot) \) denotes the Q-function. The correct decision region for 16-QAM with \( X_0 = 1 + j \) lies in the area \( D_{11} = \{a+jb| \ 0 \leq a \leq 2, \ 0 \leq b \leq 2\} \) in the complex plane. Thus, the conditional correct decision probability under AWGN (Z) can be expressed as [54]

\[
P_c(U + Z \in D_{11} \mid X_0 = 1 + j, U) = \Psi(0,0) - \Psi(0,2) - \Psi(2,0) + \Psi(2,2)
\]

(5.39)

Similarly for other three symbols in the 1st quadrant, we can calculate as

\[
P_{c,1} = P(U + Z \in D_{21} \mid X_0 = 3 + j, U) = \Psi(0,2) - \Psi(2,2)
\]

(5.40)

\[
P_{c,2} = P(U + Z \in D_{31} \mid X_0 = 1 + 3j, U) = \Psi(2,0) - \Psi(2,2)
\]

(5.41)
\[ P_{c,4} = P(U + Z \in D_{s,4} | X_0 = 3 + 3j, U) = \Psi(2,2) \]  

(5.42)

where \( D_{s,2} = \{a+jb| \ a \geq 2, \ 0 \leq b \leq 2\} \), \( D_{s,3} = \{a+jb| \ 0 \leq a \leq 2, \ b \geq 2\} \), and \( D_{s,4} = \{a+jb| \ a \geq 2, \ b \geq 2\} \).

Then, the total average correct probability can be given as [54]

\[ P_{c,av} = \left[ P_{c,1} + P_{c,2} + P_{c,3} + P_{c,4} \right] / 4 = \frac{1}{4} \Psi(0,0) \]  

(5.43)

Therefore, the average symbol error probability of 16-QAM can be expressed as [54]

\[ P_e = 1 - P_{c,av} \]  

(5.44)

By the definition of CHF, if a random variable has pdf, the CHF is its Fourier transform (FT) [63]. Taking the FT of (5.37), the pdf of \( U \) can be written as

\[
p(\text{Re}\{U\}, \text{Im}\{U\}) = \frac{1}{2^{2(N-1)}} \sum_{p=1}^{2^{N-2}} \sum_{m=1}^{2^{N-2}} \sum_{k=1}^{2^N} \sum_{n=1}^{2^N} \left\{ \delta(\text{Re}\{U\} - A_{q,p,m,k}^1 \{1, n\}) \times \right. \\
\left. \delta(\text{Im}\{U\} - A_{q,p,m,k}^1 \{2, n\}) + \delta(\text{Re}\{U\} - A_{q,p,m,k}^2 \{1, n\}) \times \right. \\
\left. \delta(\text{Im}\{U\} - A_{q,p,m,k}^2 \{2, n\}) + \delta(\text{Re}\{U\} - A_{q,p,m,k}^3 \{1, n\}) \times \right. \\
\left. \delta(\text{Im}\{U\} - A_{q,p,m,k}^3 \{2, n\}) + \delta(\text{Re}\{U\} - A_{q,p,m,k}^4 \{1, n\}) \times \right. \\
\left. \delta(\text{Im}\{U\} - A_{q,p,m,k}^4 \{2, n\}) \right\} 
\]  

(5.45)

where \( \delta(x) \) is the Dirac delta function, \( A_{q,p,m,k}^{i,j} \) is the \((i,j)\) element of the \( 2 \times 16 \) matrices \( A^i \), which are defined as

\[
A^1 = (S_{a}^0 - S_{1}^0 + \text{Cd}_1 S_{a}^0 - S_{2}^0 + \text{Cd}_2 \ldots S_{\alpha}^0 - S_{1\alpha}^0 + \text{Cd}_{\alpha}) 
\]  

(5.46)

\[
A^2 = (S_{a}^0 - 3S_{2}^0 + \text{Cd}_1 S_{a}^0 - 3S_{3}^0 + \text{Cd}_2 \ldots S_{\alpha}^0 - 3S_{1\alpha}^0 + \text{Cd}_{\alpha}) 
\]  

(5.47)

\[
A^3 = (3S_{a}^0 - S_{2}^0 + \text{Cd}_1 3S_{a}^0 - S_{3}^0 + \text{Cd}_2 \ldots 3S_{\alpha}^0 - S_{2\alpha}^0 + \text{Cd}_{\alpha}) 
\]  

(5.48)

\[
A^4 = (3S_{a}^0 - 3S_{2}^0 + \text{Cd}_1 3S_{a}^0 - 3S_{3}^0 + \text{Cd}_2 \ldots 3S_{\alpha}^0 - 3S_{2\alpha}^0 + \text{Cd}_{\alpha}) 
\]  

(5.49)
Averaging equation (5.43) with respect to the pdf given in equation (5.45) yields the symbol error probability of square 16-QAM based OFDM as

\[
P_s(\varepsilon) = 1 - \frac{1}{4} \sum_{q=1}^{2^{N-2}} \sum_{p=1}^{2^{N-2}} \sum_{m=1}^{2^{N-2}} \sum_{n=1}^{16} \left\{ Q(-\sqrt{2\gamma}A_{q,p,m,k}^1[1,n]) \times 
Q(-\sqrt{2\gamma}A_{q,p,m,k}^2[2,n]) + Q(-\sqrt{2\gamma}A_{q,p,m,k}^3[2,n]) \times 
Q(-\sqrt{2\gamma}A_{q,p,m,k}^4[2,n]) \right\}
\]

(5.50)

5.3 **Conventional Carrier Frequency and Symbol Timing Recovery Methods**

As is known, carrier frequency offset contributes to the significant degradation of the performance of OFDM system. If there is no frequency offset, the second term of equation (5.12) turns to be zero, because \( S_l \) is equal to zero when \( l \) is not equal to one. This second term will introduce ICI and destroy the orthogonality of OFDM sub-carriers, which is the reason why OFDM is more sensitive to frequency offset than single carrier system. Moreover, it is clear to see from equation (5.8) that the symbol timing error \( \theta \) cannot only rotate the phase of received signal, but also introduce ISI. In order to perfectly demodulate the received signal, symbol timing recovery must be performed to guarantee the optimum starting point of the FFT window. In this section, the conventional carrier frequency and symbol timing recovery methods will be briefly introduced. As noted, most of the available synchronization techniques can be simply classified into DA and NDA methods (or joint and independent methods).
5.3.1 Data-aided (DA) Carrier Frequency and Symbol Timing Recovery Methods

Schmidl’s [37] method can be considered as one of the most popular DA joint carrier frequency and symbol timing synchronization methods. In [37], the preamble sequence consists of the two identical halves (each half of the preamble sequence is denoted as $A_{N/2}$) of length $N/2$. By denoting its preamble sequence as $P_{sch}=[A_{N/2}, A_{N/2}]$, and the corresponding timing metric is expressed as [37]

$$M_{sch}[d] = \frac{|P_{sch}[d]|^2}{(R_{sch}[d])^2} \quad (5.51)$$

where $d$ is a time index corresponding to the first sample in a window of $N$ samples, and

$$P_{sch}[d] = \sum_{k=0}^{N/2-1} r_m[d+k] r_m^*[d+k + \frac{N}{2}] \quad (5.52)$$

$$R_{sch}[d] = \sum_{k=0}^{N/2-1} |r_m[d+k + \frac{N}{2}]|^2 \quad (5.53)$$

Then, the estimation of the timing error is identified as the maximum point of $M_{sch}[d]$ [37]

$$\theta = \text{argument}_{\theta} \max(M_{sch}[\theta]) \quad (5.54)$$

Since the preamble symbol contains two identical parts in the Schmidl method, the phase difference between these two halves is $\phi=2\pi\varepsilon N/2N=\pi\varepsilon$, the frequency offset can be estimated as argument [37]

$$\varepsilon = \frac{1}{\pi} \text{arg}(P_{sch}[d_{opt}]) \quad (5.55)$$

where $d_{opt}$ denotes the optimum timing point obtained in equation (5.55). Because of the periodicity of the function $\exp(\pi\varepsilon)$, the frequency offset can only be estimated with the limit of $|\varepsilon|<1$. 

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The timing metric of Schmidl’s method results in a ‘plateau’ which leads to a large estimation variance with regard to the starting point of the OFDM symbol [38], and the acquisition range for the carrier frequency offset is one subcarrier spacing due to the period of the $\text{arg}(\cdot)$ function in equation (5.55). In order to reduce Schmidl’s plateau, Minn [38] initially proposed an average timing metric over a window of length ‘$N_g+1$’ samples. Then the Minn’s timing metric is given as

$$M_{\text{minn1}}[d] = \frac{1}{N_g+1} \sum_{k=-N_g}^{0} M_{\text{sch}}[d+k]$$  \hfill (5.56)

Then the estimations of the timing error and frequency can be expressed by substituting $M_{\text{sch}}[d]$ with $M_{\text{minn1}}[d]$ in equations (5.54) and (5.55).

Later, Minn used a modified training sequence in the form of $P_{\text{minn2}} = [B_{N/4},B_{N/4},-B_{N/4},-B_{N/4}]$ (each part of the preamble sequence is denoted as $B_{N/4}$), which yields the timing metric expressed as [38]

$$M_{\text{minn2}}[d] = \frac{|P_{\text{minn2}}[d]|^2}{(R_{\text{minn2}}[d])^2}$$  \hfill (5.57)

where

$$P_{\text{minn2}}[d] = \sum_{k=0}^{1} \sum_{l=0}^{N/4-1} r_{m}^*[d+kN/2+l]r_{m}[d+kN/2+l+N/4]$$  \hfill (5.58)

$$R_{\text{minn2}}[d] = \sum_{k=0}^{1} \sum_{l=0}^{N/4-1} \left| r_{m}[d+kN/2+l+N/4] \right|^2$$  \hfill (5.59)

The estimation of timing error is [38]

$$\theta = \text{argument}_{\theta} \max_{\theta}(M_{\text{minn2}}[\theta])$$  \hfill (5.60)
The preamble symbol of Minn’s method contains four identical parts, the phase difference between these four parts is \( \phi = \frac{2\pi e N}{4N} = \frac{\pi e}{2} \), then, the frequency offset can be estimated as [38]

\[
\epsilon = \frac{2}{\pi} \arg(P_{\text{minn}}[d_{\text{sys}}])
\]

Because of the periodicity of the function \( \exp(\pi e/2) \), the frequency offset can only be estimated with the limit of \(|e|<2\).

From the timing metric of Minn’s method, the adjacent values around the optimum timing point have nearly all the identical sum of correlated products, except of a few uncorrelated pairs. This is the only difference between the optimum timing point and its adjacent points, which shows very slight variation around the peak value. This fact could significantly reduce the estimator performance. In order to improve the estimator accuracy, Park [40] seeks the method to enlarge the difference between the peak value of the timing metric and other neighboring points. This method can maximize the uncorrelated pairs for the points around the optimum timing position. Thus, Park’s preamble sequence is given as \( P_{\text{park}} = [C_{N/4}, D_{N/4}, -C^*_{N/4}, -D^*_{N/4}] \), where \( C_{N/4} \) represents samples of length \( N/4 \) generated by IFFT of a PN sequence, and \( C^*_{N/4} \) represents a conjugate of \( C_{N/4} \), and \( D_{N/4} \) is designed to be symmetric with \( C_{N/4} \). The timing metric of Park’s method is defined as [40]

\[
M_{\text{park}}[d] = \frac{|P_{\text{park}}[d]|^2}{(R_{\text{park}}[d])^2}
\]

where

\[
P_{\text{park}}[d] = \sum_{k=0}^{N/2-1} r_m[d-k] r_m[d+k]
\]
The estimation of timing error is argument \([40]\)

\[
\theta = \text{argument} \max_\theta (M_{park}[\theta]) \quad (5.65)
\]

In \([43]\), Ren found the major problem of Schmidl method resulting from the relatively small difference between the optimum timing point and its adjacent values in its timing metric. He tried to improve the symbol timing estimator accuracy by weighting the preamble sequence with a PN sequence, which can guarantee the maximum correlated pairs in the optimum timing point, and minimum uncorrelated pairs for other points. The preamble sequence of Ren is defined as \(P_{ren} = p_k P_{sch}\), where \(p_k\) is the PN sequence weighted factor of the \(k\)th sample of the original preamble. Although \(P_{sch}\) is still in the same form as denoted in Schmidl method, the generation process is different. \(P_{sch}\) is generated by IFFT of a constant amplitude zero autocorrelation (CAZAC) sequence, which results in a constant envelop of preamble. Thus, by multiplying the preamble sequence with the corresponding PN sequence, the weighted factor can be removed. The timing metric is given as \([43]\)

\[
M_{ren}[d] = \frac{P_{ren}[d]^2}{(R_{ren}[d])^2} \quad (5.66)
\]

where

\[
P_{ren}[d] = \sum_{k=0}^{N/2-1} p_k p_{k+N} r_m^*[d+k]r_m[d+k+N/2] \quad (5.67)
\]

\[
R_{ren}[d] = \sum_{k=0}^{N/2-1} |r_m[d+k]|^2 \quad (5.68)
\]

The estimation of the timing error is identified as the maximum point of \(M_{ren}[d]\) \([43]\)
\[ \theta = \text{argument} \max_{\theta} (M_{\text{ren}}(\theta)) \] (5.69)

Because the repetitive structure of the preamble is destroyed by multiplying the PN sequence as a weighting factor, the performance of Ren method is severely degraded under fast fading channel. Therefore, Seung [41] proposed a new preamble sequence, which is denoted as \( P_{\text{seu}} = [E_{N/2} F^*_{N/2}] \), where \( E_{N/2} \) represents the sequence of the length generated by the IFFT of the CAZAC sequence and \( F^*_{N/2} \) is a complex conjugate of the time reversed version of \( E_{N/2} \). Then, the timing metric of Seung is defined as [41]

\[ M_{\text{seu}}[d] = \frac{|P_{\text{seu}}[d]|^2}{|R_{\text{seu}}[d]|^2} \] (5.70)

where

\[ P_{\text{seu}}[d] = \sum_{k=0}^{N/2-1} r_m^*[d-k]r_m[d+k+1] \] (5.71)

\[ R_{\text{seu}}[d] = \sum_{k=0}^{N/2-1} \left| r_m[d+k - \frac{N}{2}] \right|^2 \] (5.72)

The estimation of the timing error is identified as the maximum point of \( M_{\text{ren}}[d] \) [41]

\[ \theta = \text{argument} \max_{\theta} (M_{\text{seu}}(\theta)) \] (5.73)

5.3.2 Non-data-aided (NDA) Carrier Frequency and Symbol Timing Recovery Methods

Since OFDM technique always employs cyclic prefix for combating ISI, it can also be used to estimate timing error by exploring its correlation. Jan-Jaap [46] developed a NDA joint carrier frequency and symbol timing estimation method using the correlation of the cyclic prefix, thus reducing the need for a pilot. Since this method is also based on the ML estimation, the timing metric is given as [46]
\[ \Lambda(\theta, \epsilon) = |\gamma[\theta]| \cos(2\pi\epsilon + \angle \gamma[\theta]) - \rho \phi[\theta] \]  
(5.74)

ML estimation yields maximum with respect to frequency offset when the cosine term in equation (5.74) equals to one, then
\[ \Lambda(\theta, \epsilon) = |\gamma[\theta]| - \rho \phi[\theta] \]  
(5.75)

where
\[ \gamma[\theta] = \sum_{k=0}^{\theta+N-1} r_m[k] y_m[k + N] \]  
(5.76)

\[ \phi[\theta] = \frac{1}{2} \sum_{k=0}^{\theta+N-1} |r_m[k]|^2 + |y_m[k + N]|^2 \]  
(5.77)

\[ \rho = \frac{SNR}{SNR+1} \]  
(5.78)

Thus, the joint ML estimation of the symbol timing error becomes [46]
\[ \theta_{ML} = \text{argument} \max_{\theta} \{|\gamma[\theta]| - \rho \phi[\theta]\} \]  
(5.79)

The frequency offset can be estimated as [46]
\[ \epsilon = \frac{1}{2\pi} \text{arg}(\gamma[d_{opt}]) \]  
(5.80)

Because of the periodicity of the function \( \cos(2\pi \epsilon + \angle \gamma[\theta]) \), the frequency offset can only be estimated with the limit of \( |\epsilon| < 1/2 \).

### 5.4 Proposed Joint Carrier Frequency and Symbol Timing Recovery Methods

As discussed in the previous sections, most synchronization methods for OFDM system are based on correlation of a given preamble. However, some of these methods do not consider the effect contained in the CP, which can contribute to the severe
degradation to the estimation accuracy. In this research, two improved and effective methods for DA joint carrier frequency and symbol timing recovery are presented. These MLE based methods consider the effect of CP. By exploring the multivariate Gaussian distribution within the OFDM preamble symbol, the accuracy of synchronization error estimation can be significantly enhanced due to its smaller estimation variance.

5.4.1 Preamble Sequence and Its Property

There are two kinds of preamble sequence employed in this research. The first one is generated by simply transmitting a PN sequence on the even numbered subcarriers, while zeros are used on the odd numbers of subcarriers in the frequency domain [37]. Thus, the preamble sequence after IFFT with CP is in the form of $P_1 = [A_{cp}, A_{N/2}, A_{N/2}]$. The second preamble sequence can be realized by directly transmitting four identical parts with different sign in the last two parts in the time domain. Then, the samples of the preamble sequence with CP are designed to be of the form $P_2 = [A_{cp}, A_{N/4}, A_{N/4}, -A_{N/4}, -A_{N/4}]$. Although the formats of preamble sequences $P_1$ and $P_2$ are similar to those of Schimdl and Minn’s, they do not consider the CP in their estimation process.

Let’s assume that the transmitted signal is affected by AWGN only. The transmitted signal $x[n]$, in one half of the preamble $P_1$ (denoted by $A_{N/2}$) or one quarter of the preamble (denoted by $A_{N/4}$), is a linear combination of independent, identically distributed random variables. If the number of subcarriers is sufficiently large, half of transmitted signal $x[n]$ can then be approximated as a complex Gaussian process whose real and imaginary parts are independent from each other. Then, the SNR can be defined as [46]
\[ SNR = \frac{\sigma_s^2}{\sigma_n^2} \]  

(5.81)

where \( \sigma_s^2 = E\{ |x[n]|^2 \} \), and noise variance \( \sigma_n^2 = E\{ |z[n]|^2 \} \).

Figure 5.2 shows the structure of OFDM preamble signal with CP in the time domain for the firstly proposed preamble sequence \( P_1 \).

![Observation interval](image)

Figure 5-5: Structure of proposed OFDM preamble \( P_1 \)

A length of \( 2N+\kappa \) observation interval that includes a whole OFDM symbol is used as in [46]. If \( \theta \) is the symbol timing error in the unit of sample, this observation interval can be divided into multiple smaller subsets of samples as \( I = \{ \theta, \ldots, \theta+N-1 \} \), \( Q = \{ \theta+N, \ldots, \theta+N/2-1 \} \), \( I' = \{ \theta+N/2, \ldots, \theta+N/2+\kappa-1 \} \), \( Q' = \{ \theta+N/2+\kappa, \ldots, \theta+N-1 \} \) and \( I'' = \{ \theta+N, \ldots, \theta+N+N/2-1 \} \). The set \( I \) denotes the CP copied from set \( I'' \). Because the preamble sequence contains two identical halves, sets \( Q \) and \( I' \) also have the same sample indices as sets \( Q' \) and \( I'' \), respectively. If the received signal in the observation interval is defined as \( \vec{r} = (r[1], r[2], \ldots, r[2N+\kappa])^T \), only the samples in the union set \( C = I \cup I' \cup I'' \cup Q \cup Q' \) have their correlated parts, thus,

\[
\forall k \in I, E\{ r[k] r^*[k+m] \} = \begin{cases} 
\sigma_s^2 + \sigma_n^2 & m = 0 \\
\sigma_s^2 e^{-2\pi i m/2} & m = N/2 \\
\sigma_s^2 e^{-i \pi m} & m = N \\
0 & \text{otherwise}
\end{cases}
\]  

(5.82)
\[ \forall k \in Q, E\{r[k]r^*[k + m]\} = \begin{cases} 
\sigma_s^2 + \sigma_e^2 & m = 0 \\
\sigma_e^2 e^{-j\pi/2} & m = N/2 \\
\sigma_e^2 e^{-j3\pi/2} & m = 3N/4 \\
0 & \text{otherwise} 
\end{cases} \quad (5.83) \]

Figure 5.3 depicts the structures of OFDM preamble signal with CP for the second proposed preamble sequence \( P^2 \) in the time domain.

\[ \forall k \in I, E\{r[k]r^*[k + m]\} = \begin{cases} 
\sigma_s^2 + \sigma_e^2 & m = 0 \\
\sigma_e^2 e^{-j\pi/2} & m = N/4 \\
\sigma_e^2 e^{-j3\pi/2} & m = 3N/4 \\
0 & \text{otherwise} 
\end{cases} \quad (5.84) \]

The set \( I \) denotes the CP copied from set \( I^i \). Because the preamble sequence contains four correlated parts, set \( I \) has its correlated sample indices as sets \( I^i, I^j, I^k, I^l \). Similarly, set \( Q \) also has the correlated sample indices as sets \( Q^i, Q^j, Q^k \). If the received signal in the observation interval is defined as \( \mathbf{r} \), only the samples in the union set \( C = I^i \cup I^j \cup I^k \cup I^l \cup Q^i \cup Q^j \cup Q^k \) have their correlated parts, Thus,

\[ \forall k \in Q, E\{r[k]r^*[k + m]\} = \begin{cases} 
\sigma_s^2 + \sigma_e^2 & m = 0 \\
\sigma_e^2 e^{-j\pi/2} & m = N/4 \\
\sigma_e^2 e^{-j3\pi/2} & m = 3N/4 \\
0 & \text{otherwise} 
\end{cases} \quad (5.85) \]
5.4.2 Mathematical Model for Symbol Timing Recovery

Let us define the log-likelihood function, $\Lambda(\theta, \varepsilon)$, which is the logarithm of the probability density function (pdf) $f(\bar{r})$ of the observed samples in received signal $\bar{r}$ on the condition of given symbol timing error $\theta$ and carrier frequency offset $\varepsilon$

$$\Lambda(\theta, \varepsilon) = \log[f(\bar{r} | \theta, \varepsilon)]$$

(5.86)

Then, using the correlation properties of the observations in the received signal when preamble sequence $P_j$ is transmitted, the log-likelihood function can be written as

$$\Lambda(\theta, \varepsilon) = \log[\prod_{k=1} f(r[k], r[k + N/2], r[k + N]) \prod_{k \neq Q} f(r[k], r[k + N/2]) \prod_{k \neq C} f(r[k])]$$

$$= \log[\prod_{k=1} \frac{f(r[k], r[k + N/2], r[k + N]) \prod_{k \neq Q} f(r[k], r[k + N/2]) \prod_{k \neq C} f(r[k])}{f(r[k]) f(r[k + N/2]) f(r[k + N])}]$$

(5.87)

where $f(\cdot)$ denotes the pdf. Note that the third term in equation (5.87) is independent of $\theta$ and $\varepsilon$, and the numerators of the first and the second terms are in the form of 3-D and 2-D complex Gaussian distributions, respectively. The denominator of equation (5.87) consists of multiple 1-D complex Gaussian density functions, which can be generally defined as

$$f(r[k]) = \frac{1}{\sqrt{2\pi(\sigma_r^2 + \sigma_r^2)}} \exp\left(-\frac{1}{2} \frac{|r[k]|^2}{\sigma_r^2 + \sigma_r^2}\right)$$

(5.88)

From the multi-rate Gaussian distribution, the joint pdf of the Gaussian random variable $\bar{x} = [x_1, x_2, \ldots, x_n]$ is defined as [63]

$$f(\bar{x}) = \frac{1}{(2\pi)^{n/2} (\det M)^{1/2}} \exp[-\frac{1}{2} (\bar{x} - \bar{m}_x)^T M^{-1} (\bar{x} - \bar{m}_x)]$$

(5.89)
where \( \bar{m}_x \) denotes the mean of these Gaussian distributed variables, \( M \) is the covariance matrix, \( M^T \) denotes the inverse of \( M \) and \( (\cdot)^T \) denotes the matrix transpose. By the definition, the covariance matrix is expressed as

\[
M = \begin{bmatrix}
\sigma_1^2 & \mu_{12} & \cdots & \mu_{1n} \\
\mu_{12} & \sigma_2^2 & \cdots & \mu_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\mu_{1n} & \mu_{2n} & \cdots & \sigma_n^2
\end{bmatrix}
\]

(5.90)

where the joint central moment \( \mu_{ij} \) is defined as

\[
\mu_{ij} = E[(x_i - m_i)(x_j - m_j)]
\]

(5.91)

It is convenient to define a normalized covariance

\[
\rho_{ij} = \frac{\mu_{ij}}{\sigma_i \sigma_j}, \quad i \neq j
\]

(5.92)

where \( \rho_{ij} \) satisfies the condition \( 0 \leq \rho_{ij} \leq 1 \). Thus, in the case of the second term of equation (5.87), it is a two-dimensional case. The received vector can be expressed as, \( \mathbf{r} = (r[k] \ r[k+N/2])^T \). And the corresponding covariance matrix is

\[
M = \begin{bmatrix}
\sigma_s^2 & \rho \sigma_s \sigma_2 \\
\rho \sigma_s \sigma_2 & \sigma_2^2
\end{bmatrix} = \begin{bmatrix}
\sigma_s^2 + \sigma_2^2 & \rho(\sigma_s^2 + \sigma_2^2) \\
\rho(\sigma_s^2 + \sigma_2^2) & \sigma_s^2 + \sigma_2^2
\end{bmatrix}
\]

(5.93)

The inverse of this covariance matrix is

\[
M^{-1} = \frac{1}{(\sigma_s^2 + \sigma_2^2)(1-\rho^2)} \begin{bmatrix}
1 & -\rho \\
-\rho & 1
\end{bmatrix}
\]

(5.94)

where \( \rho \) is defined as [46]

\[
\rho = \frac{\mu_{12}}{\sigma_1 \sigma_2} = \frac{E(r[k]r[k+N/2])}{\sigma_s^2 + \sigma_2^2} = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_2^2} = \frac{\text{SNR}}{\text{SNR}+1}
\]

(5.95)

The joint pdf of the Gaussian random variable \( \mathbf{r} \) is then defined as
\[
f(\vec{r}_i) = \frac{1}{(2\pi)^{\frac{\nu}{2}} (\det M)^{\frac{1}{2}}} \exp[-\frac{1}{2} (\vec{x} - \vec{m}_s)^T M^{-1} (\vec{x} - \vec{m}_s)]
\]
\[
= \frac{1}{(2\pi)(\det M)^{\frac{1}{2}}} \exp[-\frac{1}{2} (\vec{r}_i)^T M^{-1} (\vec{r}_i)]
\]
\[
= \frac{1}{(2\pi)(\det M)^{\frac{1}{2}}} \exp[-\frac{1}{2} \left| r[k] \right|^2 - 2\rho \Re \{e^{j\pi} r[k] r^*[k + N/2] + r[k + N/2]^2 \} \left(\sigma_i^2 + \sigma^2 \right) (1 - \rho^2)]
\]
(5.96)

Thus,
\[
\prod_{k\in Q} \frac{f(r[k], r[k + N/2])}{f(r[k])f(r[k + N/2])} \propto |\gamma_1(\theta)| - \rho \phi_1(\theta)
\]
(5.97)

where the symbol ‘ \(\propto\) ’ denotes “proportional to”, and
\[
\gamma_1(\theta) = \sum_{k=0}^{\theta+N/2-1} r[k] r^*[k + N/2]
\]
(5.98)
\[
\phi_1(\theta) = \frac{1}{2} \sum_{k=0}^{\theta+N/2-1} \left| r[k] \right|^2 + \left| r[k + N/2] \right|^2
\]
(5.99)

Thus ML estimate for the second term of equation (5.87) can be denoted as
\[
\Lambda (\prod_{k\in Q} \frac{f(r[k], r[k + N/2])}{f(r[k])f(r[k + N/2])} | \theta, \phi) = |\gamma_1(\theta)| - \rho \phi_1(\theta)
\]
(5.100)

The first term of equation (5.87) is a three-dimensional case. The received vector can be expressed as \(\vec{r}=(r[k] r[k+N/2] r[k+N])^T\). And the covariance matrix is defined as
\[
M = \begin{bmatrix}
\sigma_1^2 & \rho_{12} \sigma_1 \sigma_2 & \rho_{13} \sigma_1 \sigma_3 \\
\rho_{12} \sigma_1 \sigma_2 & \sigma_2^2 & \rho_{23} \sigma_2 \sigma_3 \\
\rho_{13} \sigma_1 \sigma_3 & \rho_{23} \sigma_2 \sigma_3 & \sigma_3^2
\end{bmatrix}
\]
\[
= \begin{bmatrix}
\sigma_i^2 + \sigma^2 & \rho (\sigma_i^2 + \sigma^2) & \rho (\sigma_i^2 + \sigma^2) \\
\rho (\sigma_i^2 + \sigma^2) & \sigma_i^2 + \sigma^2 & \rho (\sigma_i^2 + \sigma^2) \\
\rho (\sigma_i^2 + \sigma^2) & \rho (\sigma_i^2 + \sigma^2) & \sigma_i^2 + \sigma^2
\end{bmatrix}
\]
(5.101)
The inverse of this covariance matrix is,

\[
M^{-1} = \frac{1}{(\sigma^2_1 + \sigma^2)(\rho - 2\rho^2 + 1)} \begin{bmatrix}
\rho + 1 & -\rho & -\rho \\
-\rho & \rho + 1 & -\rho \\
-\rho & -\rho & \rho + 1
\end{bmatrix}
\]  

(5.102)

where

\[
\rho = \frac{\mu_{12}}{\sigma_1 \sigma_2} = \frac{\mu_{13}}{\sigma_1 \sigma_3} = \frac{\mu_{23}}{\sigma_2 \sigma_3} = \frac{E(r[k] r[k + N/2])}{\sigma^2_1 + \sigma^2} = \frac{\sigma^2_2}{\sigma^2_1 + \sigma^2} = \frac{\text{SNR}}{\text{SNR} + 1}
\]  

(5.103)

The joint pdf of the Gaussian random variable \( \mathbf{\mathbf{r}} \) is now defined as,

\[
f(\mathbf{\mathbf{r}}) = \frac{1}{(2\pi)^{1/2} (\det M)^{1/2}} \exp[-\frac{1}{2} (\mathbf{\mathbf{r}})^T M^{-1} (\mathbf{\mathbf{r}})]
\]

\[
= \frac{1}{(2\pi)(\det M)^{1/2}} \exp[-\frac{1}{2(\sigma^2_1 + \sigma^2)(1 - 2\rho^2 + \rho)} (\rho + 1) r[k] r^*[k + N/2] + (\rho + 1) [r[k + N/2]]^2 - 2\rho \text{Re} \{e^{j\pi} r[k] r^*[k + N/2] \}
\]

\[
- 2\rho \text{Re} \{e^{j\pi} r[k + N/2] r^*[k + N] \} - 2\rho \text{Re} \{e^{j\pi} r[k] r^*[k + N] \} \}
\]  

(5.104)

Thus,

\[
\prod_{k \neq l} \frac{f(r[k], r[k + N/2], r[k + N])}{f(r[k]) f(r[k + N/2]) f(r[k + N])} \propto |\gamma_2(\theta)| + |\gamma_3(\theta)| + |\gamma_4(\theta)| - \rho \phi_2(\theta) - \rho \phi_3(\theta) - \rho \phi_4(\theta)
\]  

(5.105)

where

\[
\gamma_2(\theta) = \sum_{k=0}^{\theta + N_2 - 1} r[k] r^*[k + N/2]
\]  

(5.106)

\[
\phi_2(\theta) = \frac{1}{2} \sum_{k=0}^{\theta + N_2 - 1} (|r[k]|^2 + |r[k + N/2]|^2)
\]  

(5.107)

\[
\gamma_3(\theta) = \sum_{k=0}^{\theta + N_2 - 1} r[k + N/2] r^*[k + N]
\]  

(5.108)
\[
\phi_i(\theta) = \frac{D}{2} \sum_{k=0}^{\theta + N/2 - 1} (|r[k+N/2]|^2 + |r[k+N]|^2)
\]
\[\text{(5.109)}\]
\[
\gamma_i(\theta) = \sum_{k=0}^{\theta + N-1} r[k]r^*[k+N]
\]
\[\text{(5.110)}\]
\[
\phi_i(\theta) = \frac{D}{2} \sum_{k=0}^{\theta + N-1} (|r[k]|^2 + |r[k+N]|^2)
\]
\[\text{(5.111)}\]

Thus, the corresponding ML timing estimate becomes
\[
M_{\text{proposed}} = \sum_{i=1}^{4} \left[ |\gamma_i(\theta)| - \rho \phi_i(\theta) \right]
\]
\[\text{(5.112)}\]

In the case of transmitting preamble sequence \(P_2\), the log-likelihood function of received signal can be written as
\[
\Lambda(\theta, \varepsilon) = \log \left\{ \prod_{k \in i} f(r[k], r[k+N/4], r[k+N/2], r[k+3N/4], r[k+N]) \prod_{k \in C} f(r[k]) \right\}
\]
\[
= \log \left\{ \prod_{k \in i} \frac{f(r[k], r[k+N/4], r[k+N/2], r[k+3N/4], r[k+N])}{f(r[k]), f(r[k+N/4]), f(r[k+N/2]), f(r[k+3N/4]), f(r[k+N])} \right\}
\]
\[
= \log \left\{ \prod_{k \in Q} \frac{f(r[k], r[k+N/4], r[k+N/2], r[k+3N/4])}{f(r[k]), f(r[k+N/4]), f(r[k+N/2]), f(r[k+3N/4])} \right\}
\]
\[\text{(5.113)}\]

Note that the third term in (5.113) is independent of \(\theta\) and \(\varepsilon\), and the numerators of the first and the second terms are in the form of 5-D and 4-D complex Gaussian distributions, respectively. From the multivariate Gaussian distribution, the joint pdf of the Gaussian
random variable $\tilde{r}_z = f[r(k), r(k+N/4), r(k+N/2), r(k+3N/4)]$ can be derived from equation (5.89). In this case, the covariance matrix and is defined as

$$M = \begin{bmatrix}
\sigma_z^2 + \sigma^2 & \rho(\sigma_z^2 + \sigma^2) & \rho(\sigma_z^2 + \sigma^2) & \rho(\sigma_z^2 + \sigma^2) \\
\rho(\sigma_z^2 + \sigma^2) & \sigma_z^2 + \sigma^2 & \sigma_z^2 + \sigma^2 & \rho(\sigma_z^2 + \sigma^2) \\
\rho(\sigma_z^2 + \sigma^2) & \rho(\sigma_z^2 + \sigma^2) & \sigma_z^2 + \sigma^2 & \rho(\sigma_z^2 + \sigma^2) \\
\rho(\sigma_z^2 + \sigma^2) & \rho(\sigma_z^2 + \sigma^2) & \rho(\sigma_z^2 + \sigma^2) & \sigma_z^2 + \sigma^2 \\
\end{bmatrix} \quad (5.114)$$

The inverse of this covariance matrix is

$$M^{-1} = \frac{1}{(\sigma_z^2 + \sigma^2)(2\rho - 3\rho^2 + 1)} \begin{bmatrix}
2\rho + 1 & -\rho & -\rho & -\rho \\
-\rho & 2\rho + 1 & -\rho & -\rho \\
-\rho & -\rho & 2\rho + 1 & -\rho \\
-\rho & -\rho & -\rho & 2\rho + 1 \\
\end{bmatrix} \quad (5.115)$$

Then the joint pdf of the Gaussian random variable $\tilde{r}_z$ is

$$f(\tilde{r}_z) = A \exp\left[-\frac{1}{2(\sigma_z^2 + \sigma^2)(2\rho - 3\rho^2 + 1)} \left\{ (2\rho + 1)|r[k]|^2 + (2\rho + 1)|r[k+N/4]|^2 \\
+ (2\rho + 1)|r[k+N/2]|^2 + (2\rho + 1)|r[k+3N/4]|^2 \right\} - 2\rho \Re\{e^{j\pi\rho/2}r[k]\}r^*[k+N/4] \\
- 2\rho \Re\{e^{j\pi\rho/2}r[k]\}r^*[k+N/2] \right\} \\
- 2\rho \Re\{e^{j\pi\rho/2}r[k+N/4]\}r^*[k+N/2] \right\} - 2\rho \Re\{e^{j\pi\rho/2}r[k+N/4]\}r^*[k+3N/4] \\
- 2\rho \Re\{e^{j\pi\rho/2}r[k+N/4]\}r^*[k+N/2] \right\} \right) \quad (5.116)$$

where $\rho$ is defined as in equation (5.95). Thus, the ML estimate of second term in (5.113) will be

$$\prod_{k \in Q} \frac{f(r[k], r[k+N/4], r[k+N/2], r[k+3N/4])}{f(r[k])f(r[k+N/4])f(r[k+N/2])f(r[k+3N/4])} - \infty \sum_{i=1}^{\infty} \left| \gamma_i(\theta) \right| - \rho \phi(\theta) \quad (5.117)$$

where
\[ \gamma_1(\theta) = \sum_{k=0}^{\theta+N-1} r[k] r^*[k+N/4] \] (5.118)

\[ \phi_1(\theta) = \frac{1}{2} \sum_{k=0}^{\theta+N-1} |r[k]|^2 + |r[k+N/4]|^2 \] (5.119)

\[ \gamma_2(\theta) = \sum_{k=0}^{\theta+N-1} r[k] r^*[k+N/2] \] (5.120)

\[ \phi_2(\theta) = \frac{1}{2} \sum_{k=0}^{\theta+N-1} |r[k]|^2 + |r[k+N/2]|^2 \] (5.121)

\[ \gamma_3(\theta) = \sum_{k=0}^{\theta+N-1} r[k] r^*[k+3N/4] \] (5.122)

\[ \phi_3(\theta) = \frac{1}{2} \sum_{k=0}^{\theta+N-1} |r[k]|^2 + |r[k+3N/4]|^2 \] (5.123)

\[ \gamma_4(\theta) = \sum_{k=0}^{\theta+N-1} r[k+N/4] r^*[k+N/2] \] (5.124)

\[ \phi_4(\theta) = \frac{1}{2} \sum_{k=0}^{\theta+N-1} |r[k+N/4]|^2 + |r[k+N/2]|^2 \] (5.125)

\[ \gamma_5(\theta) = \sum_{k=0}^{\theta+N-1} r[k+N/4] r^*[k+N] \] (5.126)

\[ \phi_5(\theta) = \frac{1}{2} \sum_{k=0}^{\theta+N-1} |r[k+N/4]|^2 + |r[k+N]|^2 \] (5.127)

\[ \gamma_6(\theta) = \sum_{k=0}^{\theta+N-1} r[k+3N/4] r^*[k+N] \] (5.128)

\[ \phi_6(\theta) = \frac{1}{2} \sum_{k=0}^{\theta+N-1} |r[k+3N/4]|^2 + |r[k+N]|^2 \] (5.129)
Similarly, the first term of equation (5.113) is a five-dimensional case. Using the multivariate Gaussian distribution, the ML estimate of first term in equation (5.113) can be calculated as

\[
\prod_{k=1}^{N_{kr}} \frac{f(r[k], r[k + N/4], r[k + N/2], r[k + 3N/4], r[k + N])}{f(r[k])f(r[k + N/4])f(r[k + N/2])f(r[k + 3N/4])f(r[k + N])}
\]

\[
\propto \sum_{i=1}^{10} \left[ \alpha_i(\theta) - \rho \beta_i(\theta) \right]
\]  

(5.130)

Since the definitions of \( \alpha_i(\theta) \) and \( \beta_i(\theta) \) are straightforward and can be easily obtained as in equation (5.117), they are not given here also due to repeating task. Hence, the corresponding ML timing metric becomes

\[
M_{\text{proposed2}} = \sum_{i=1}^{6} \left[ \gamma_i(\theta) - \rho \phi_i(\theta) \right] + \sum_{i=1}^{10} \left[ \alpha_i(\theta) - \rho \beta_i(\theta) \right]
\]

(5.131)

5.4.3 Mathematical Model for Carrier Frequency Offset Estimation

The phase difference between the two identical halves in the preamble sequence of the first proposed method is \( \pi \varepsilon \) and the phase difference between the cyclic prefix and its copied part is \( 2\pi \varepsilon \). The frequency offset can be estimated from \( \arg[\gamma_1(\theta_{ML})]/\pi \), \( \arg[\gamma_1(\theta_{ML})]/\pi \), \( \arg[\gamma_2(\theta_{ML})]/\pi \), \( \arg[\gamma_3(\theta_{ML})]/\pi \), \( \arg[\gamma_4(\theta_{ML})]/2\pi \), where \( \theta_{ML} \) is the point of best symbol timing. However, if the frequency offset is large enough, it will cause a phase rotation greater than \( \pi \). Then, it is impossible to recover the true frequency offset. If the last term \( \arg[\gamma_d(\theta_{ML})]/2\pi \) is included for frequency offset estimation, it will decrease estimation range from \( \pm 1 \) times of subcarrier spacing to \( \pm 0.5 \) times of it, but the estimation accuracy will be enhanced as a tradeoff. This is not included in the calculation.
to guarantee an estimation range comparable to the others (Schmidl [37] and Minn1 [38]). Hence, this proposed method can estimate the frequency offset within one subcarrier spacing with enhanced estimation accuracy. Then the corresponding frequency offset estimate is given as,

$$\varepsilon_{proposed} = \frac{1}{3} \sum_{i=1}^{3} \arg[\angle \gamma_i(\theta_{ML})]/\pi$$

(5.132)

Similar as discussed in the first proposed method, the phase difference between the four identical halves in the preamble sequence can be $\pi\varepsilon/2$, $\pi\varepsilon$, $3\pi\varepsilon/2$ and $2\pi\varepsilon$. The frequency offset can be estimated from $2\arg[\gamma_1(\theta_{ML})]/\pi$, $2\arg[\gamma_4(\theta_{ML})]/\pi$, $2\arg[\gamma_6(\theta_{ML})]/\pi$, $2\arg[\gamma_5(\theta_{ML})]/\pi$, $2\arg[\gamma_8(\theta_{ML})]/\pi$, $2\arg[\gamma_9(\theta_{ML})]/\pi$ and $2\arg[\gamma_{10}(\theta_{ML})]/\pi$, where $\theta_{ML}$ is the point of best symbol timing. Then the corresponding frequency offset estimate is given as,

$$\varepsilon_{proposed2} = \frac{2}{3} \sum_{i=1,4,6} \arg[\angle \gamma_i(\theta_{ML})]/\pi + \frac{1}{2} \sum_{i=1,5,8,10} \arg[\angle \alpha_i(\theta_{ML})]/\pi$$

(5.133)
Chapter 6

Performance Evaluation of Carrier Frequency and Symbol Timing Recovery Methods in Multi-carrier Systems

In this Chapter, the performance of carrier frequency and symbol timing recovery method is evaluated via computer simulations and results are compared to see the degrading effects under AWGN, Rayleigh flat fading and multi-path Rayleigh fading, respectively. Except for some special cases in the symbol error probability analysis, table 6-1 shows the simulation parameters throughout this chapter. All results were obtained by averaging over independent Monte-Carlo trials.

Table 6.1: Simulation Parameters for OFDM System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT size $N$</td>
<td>64</td>
</tr>
<tr>
<td>Number of data subcarriers $N_{sd}$</td>
<td>48</td>
</tr>
<tr>
<td>Number of pilot subcarriers $N_{cp}$</td>
<td>4</td>
</tr>
<tr>
<td>Number of subcarriers $N_{so}$</td>
<td>52</td>
</tr>
<tr>
<td>OFDM data symbol duration $T$</td>
<td>3.2us</td>
</tr>
<tr>
<td>Subcarrier spacing $1/T$</td>
<td>0.3125MHz</td>
</tr>
<tr>
<td>Sample frequency $N/T$</td>
<td>20MHz</td>
</tr>
<tr>
<td>Allowed guard interval</td>
<td>$T/4$, $T/8$</td>
</tr>
<tr>
<td>Cyclic prefix period $T_g$</td>
<td>0.8us, 0.4us</td>
</tr>
<tr>
<td>OFDM completed symbol duration $T+T_g$</td>
<td>4us, 3.6us</td>
</tr>
<tr>
<td>Modulation scheme</td>
<td>MPSK ($M=2,4,8$) and 16-QAM</td>
</tr>
</tbody>
</table>
In this research, the frequency selective multipath Rayleigh fading channel is modeled as an exponentially decaying power delay profile with root mean squared width equal to 100ns (corresponding to two samples), and the channel is modeled to simulate 5 independent paths with Rayleigh fading and the maximum delay spread is 250ns.

6.1 Performance Degradation due to Carrier Frequency Offset in Multi-Carrier Systems

As analyzed in Chapter 5, the exact formula of symbol error probability for OFDM impaired by CFO involves the matrix of $E_m$, which is the size of $V \times 2^V I$. Thus, it is impossible to directly get the SER performance for large number of sub-carriers without doing any approximation. Careful inspection of all of the SER formulas derived in Chapter 5, only some of the sub-carriers introduce significant interference. Therefore, a tradeoff can be made between the computational complexity and mathematical exactness by truncating the number of ICI terms involved in the formulae in calculating the exact results [50]. In order to verify the exact SER performance analyzed in Chapter 5, firstly only 8 subcarriers are considered to show the exact BER performance and then the performance of OFDM system using large FFT size specified in table 6-1 will be provided by approximation approach.

6.1.1 Performance Degradation of BPSK/QPSK OFDM due to Carrier Frequency Offset

Figure 6-1 presents the probability of symbol error for BPSK OFDM over AWGN channel with $N=8$ and $N_c=2$. 

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It is easily to see from figure 6-1, the SER performance gets worse, when CFO gets larger. The theoretical SER performance of BPSK OFDM is worse than that of BPSK when CFO is not utilized (curves 1 and 2 in figure 6-1), which verifies the SNR loss denoted in equation (5.11). Furthermore, the simulated results of BPSK OFDM impaired by CFO match well with the mathematical formula in equation (5.16) for all cases (curves 4 and 5, 6 and 7, 8 and 9 in figure 6-1). Thus, the correctness of the theoretical derivation shown in equation (5.16) is proved.

Similarly, figures 6-2 shows the probability of symbol error the QPSK OFDM system over AWGN channel. As observed from the simulation results, the simulation results match well with the mathematic formula denoted in equation (5.19), which verifies the exactness of the analysis results. Comparing between OFDM with BPSK and
OFDM with QPSK, the OFDM with QPSK suffers much more degradation than OFDM with BPSK under same CFO, because the CFO not only destroys the orthogonality of the subcarrier for OFDM but also the orthogonality of the two dimensional orthonormal basis function of QPSK.

![Figure 6-2: Probability of symbol error for QPSK OFDM over AWGN channel (N=8, L=2).](image)

Figures 6-3 and 6-4 show the probability of symbol error the OFDM system employing BPSK and QPSK over Rayleigh flat fading channel, respectively. As can be seen from the simulation results, the simulation results match well with the mathematic formula denoted in equation (5.17) and equation (5.20) for OFDM/BPSK and OFDM/QPSK, which verifies the exactness of the analysis results. Moreover, there is no significant degradation when CFO is lower than 0.1 times of sub-carrier spacing. This suggests that the performance of OFDM is not sensitive to CFO over Rayleigh flat fading.
channel when compared with AWGN channel. Furthermore, comparing between OFDM with BPSK and OFDM with QPSK, the OFDM with QPSK still suffers much more degradation than OFDM with BPSK under same carrier frequency offset over Rayleigh flat fading channel.

Figure 6-3: Probability of symbol error for BPSK OFDM over Rayleigh flat fading channel with CP ($N=8, N_g=2$).
Figures 6-4 and 6-6 show the probability of symbol error for the OFDM system employing BPSK over multi-path frequency selective Rayleigh fading channel having 2 and 5 taps, respectively. The uniform power delay profile characteristic is assumed for the fading channel. As can be seen from the simulation results, SER decreases as the SNR value increases. When SNR gets larger, the SER gets saturated. Moreover, the simulation results match well with the mathematical formula denoted in equation (5.18) for OFDM/BPSK, which verifies the exactness of the analysis results. Furthermore, there is no significant degradation for different fading taps. This suggests that the performance of OFDM is not sensitive to frequency offset for different fading taps.
Figure 6-5: Probability of symbol error for BPSK OFDM over frequency selective Rayleigh fading channel (Tap=2, N=8).

Figure 6-6: Probability of symbol error for BPSK OFDM over frequency selective Rayleigh fading channel (Tap=5, N=16).
As analyzed earlier, the simulated SER performance matches well the theoretical analysis results. The last part of this section provides an approximate SER performance for the OFDM system denoted in table 6-1 due to the complexity for large number of sub-carriers. By investigating the ICI terms in equation (5.10), only first 5 terms show comparable large values when CFO is less than 0.5. Thus, the analysis results are obtained by using first five terms. Figures 6-7 to 6-8 depict the SER performance for OFDM systems employing BPSK and QPSK over AWGN with large FFT size specified in table 6-1, respectively.

Figure 6-7: Probability of symbol error for BPSK OFDM over AWGN channel with large FFT size.
Since only 52 subcarriers are actually used for signal transmission, the SER for OFDM without CFO can be derived by adding another SNR loss $10\log_{10}(52/64)$. As can be seen from figures 6-7 and 6-8, the simulated SER performance matches well the theoretical analysis results when CFO is not introduced (curves 2 and 3). Comparing figures 6-7, 6-8 with figures 6-1, 6-2, the performance is degraded due to the additional SNR loss $10\log_{10}(52/64)$. Moreover, in case when CFO is introduced (curves 4 to 9), the simulation results is still showing a closed match to their theoretical counterparts. Thus, for large FFT size, the performance of OFDM employing BPSK and QPSK impaired by CFO over AWGN channel can be approximately evaluated using equations (5.16) and (5.19), respectively.

Figure 6-8: Probability of symbol error for QPSK OFDM over AWGN channel with large FFT size.
6.1.2 Performance Degradation of 8PSK OFDM due to Carrier Frequency Offset

As discussed in BPSK/QPSK OFDM, the performance degradation of 8PSK OFDM will be initially evaluated with smaller number of subcarriers \( N \) to check the validity of our derivation. Figure 6-9 shows the computer simulation results of 8PSK OFDM over AWGN channel with and without CFO for \( N=8 \) and \( N_g=2 \) (length of CP).

![Graph showing the performance of 8PSK OFDM](image)

Figure 6-9: Probability of symbol error for 8PSK OFDM over AWGN channel (\( N=8, N_g=2 \)).

As can be seen from figure 6-9, the performance of 8PSK OFDM shows little degradation compared to the single carrier 8PSK scheme due to SNR loss caused by the insertion of the CP. The theoretical symbol error probability of (single carrier) 8PSK is defined as [63]

\[
P_s = 2Q\left(\sqrt{2\gamma \sin\left(\frac{\pi}{8}\right)}\right)
\]  

(6.1)
where $\gamma$ denotes the energy per symbol to noise spectral density ratio ($E_s/N_0$) (note that $E_s=1$, $\sigma^2=N_0/2$). The SNR loss due to the insertion of the CP can be qualified by $\text{SNR}_{\text{loss}}=-\log_{10}(N_g/N)$ [71]. Thus, the symbol error probability of 8PSK OFDM without CFO can be derived by replacing $\gamma$ in equation (6.1) with $\gamma-\text{SNR}_{\text{loss}}$. From simulation results shown in figure 6-9 (curves 2 and 3), it is clear that the simulated curves match well with theoretical formula, and the use of OFDM degrades the performance by approximately 1dB compared with the single carrier case at $E_s/N_0$ of $10^{-3}$. Figure 6-9 also shows that the BER degrades with increase of CFO, and the theoretical curves for all CFO derived in this study match well with the simulated results, which verify the correctness of our theoretical analysis. Furthermore, when the CFO is larger than 5% of subcarrier space, it will significantly destroy the performance. Thus, it is critical to compensate the CFO as much as possible to guarantee the performance of OFDM employing 8PSK.

From equation (5.11), it can also be noted that only the first few subcarriers of the ICI term $S_k$ introduces large interference when CFO is small. Thus, an approximate symbol error probability can be obtained for large size of subcarriers by truncating the number of ICI terms [50]. The, figure 6-10 shows the symbol error probability of OFDM employing 8PSK with small CFO for the OFDM system specified in table 6-1.

Because only 52 subcarriers are actually used for signal transmission in the standard, the symbol error probability of OFDM without CFO can be derived by replacing $\gamma$ in equation (6.1) with $\gamma-\text{SNR}_{\text{loss}}+10\log_{10}(52/64)$. Examining figure 6-10, the simulated curve closely matches with the theoretical formula (curves 2 and 3) when CFO is not introduced, hence thus the exactness of our symbol error probability expression for
8PSK OFDM (with CP) without CFO has been verified. In case when CFO is introduced (curves 4 to 9), we can observe that simulation results are a little deviated from the theoretical counterparts and they get severer as CFO gets larger. This deviation may attribute to the truncation of ICI terms taken for the error probability calculation. Thus, for larger number of subcarrier, tradeoff between the computation complexity and mathematical exactness of symbol error probability can be a viable option.

Figure 6-10: Probability of symbol error for 8PSK OFDM over AWGN channel with large FFT size.

6.1.3 Performance Degradation of 16-QAM due to Carrier Frequency Offset

The performance degradation of 16-QAM OFDM is firstly evaluated with smaller number of subcarriers \( N \) to check the validity of our derivation. Figure 6-11 shows the computer simulation results of equation (5.50) over AWGN channel with and without CFO for \( N=8 \) and \( N_g=2 \) (length of CP).
As can be seen from figure 6-11, the performance of square 16-QAM based OFDM shows little degradation than that of single carrier 16-QAM scheme due to SNR loss caused by the insertion of the CP. The theoretical symbol error probability of (single carrier) 16-QAM is defined as [63]

\[ P_s = \frac{3}{2} x - \frac{9}{16} x^2 \quad (6.2) \]

where \( x \) is written as

\[ x = 2Q\left( \sqrt{\frac{\gamma}{5}} \right) \quad (6.3) \]

The SNR loss due to the insertion of the CP is given by \( SNR_{loss} = -\log_{10}(N_g/N) \). Thus, the \( P_s(\varepsilon) \) of OFDM employing 16-QAM without CFO can be derived by replacing \( \gamma \) in equation (6.2) with \( \gamma - SNR_{loss} \). From simulation results shown in figure 6-11 (curves 2 and 6.9).
3), it is clear that the simulated curves match well with theoretical formula, and the use of OFDM (with CP) degrades the performance by approximately 1 dB compared with the single carrier case at $E_s/N_0$ of $10^{-2}$. Figure 6-11 also shows the effect of CFO clearly. As expected, error performance gets worse as CFO increases, and the theoretical curves for all CFOs derived in this study match well with the simulated results (curves 4 and 5, 6 and 7, and 8 and 9), which verify the correctness of our theoretical derivation shown in equation (6.2). Furthermore, when the CFO is larger than 5% of subcarrier spacing, $P_s(\varepsilon)$ will be saturated. Thus, it is critical to compensate the CFO as much as possible to guarantee the required performance of 16-QAM based OFDM.

In addition, figure 6-12 shows the simulation results of symbol error probability of 16-QAM based OFDM with CFO for the OFDM system specified in table 6-1. Note that only 52 subcarriers are actually used for signal transmission, the $P_s(\varepsilon)$ of OFDM for IEEE 802.11a standard without CFO can be derived by replacing $\gamma$ in equation (6.2) with $\gamma - SNR_{loss} + 10\log_{10}(52/64)$.

Examining figure 6-12, the simulated curve closely matches with the theoretical formula (curves 2 and 3) when CFO is not introduced, hence thus the exactness of our symbol error probability expression for 16-QAM based OFDM (with CP) without CFO has been verified. In case when CFO is introduced (curves 4 to 9), we can observe that simulation results are a little deviated from the theoretical counterparts and they get severer as CFO gets larger. This deviation may attribute to the truncation of ICI terms taken for the error probability calculation. Thus, for larger number of subcarrier, tradeoff between the computation complexity and mathematical exactness of symbol error probability can be a viable option. Hence, this symbol error probability expression shown
in equation (5.50) can also provide an approximate evaluation for large number
subcarriers with CFO.

![Graph](image)

Figure 6-12: Probability of symbol error for 16-QAM OFDM over AWGN with large
FFT size.

6.2 Performance Evaluation of Conventional Symbol Timing
Recovery Methods

Given the mathematic analysis in the previous chapter, performance of the
existing symbol timing recovery algorithms are evaluated via computer simulations and
results are mainly compared with Schmidl’s [37], Minn’s [39], Park’s [40], Ren’s [43],
and Seung’s [41], exploring the degrading effects over AWGN, Rayleigh flat fading and
multi-path frequency selective Rayleigh fading, respectively.
6.2.1 Timing Metrics

Figure 6-13 and figure 6-14 show the timing metric as a function of SNR for the cases of $N_g=N/4$ (figure 6-13) and $N_g=N/8$ (figure 6-14), respectively. The correct symbol timing is indexed as $\theta$ in the both figures. Note that the timing error is normalized by the total number of samples in one symbol duration. Some figures in the timing metric are not symmetrical is due to the implementation of different estimation algorithms.

Figure 6-13: Comparison of timing metric of existing estimators ($N_g=N/4$).

In figure 6-13, Schmidl’s method creates a ‘plateau’ for the whole interval of cyclic prefix. The timing metric of Minn 1st method can reduces this plateau, but still yields comparable metric values around the correct timing metric. Minn 2nd method uses a relatively complicated preamble sequence to reduce the plateau of Schmidl’s timing metric, however, it shows strong, comparable sub-peaks of timing metric, especially when $N_g=N/4$ (figure 6-14), which may significantly degrade synchronization.
performance. Although Park, Ren, and Seung methods exhibit impulse shape like peaks at the optimum timing point, they still show sub-peaks at half of the symbol duration position, which could also cause performance degradation to the estimation algorithms.

![Figure 6-14: Comparison of timing metric of existing estimators (N_g=N/8).](image)

6.2.2 MSE Performance over AWGN

The mean square error (MSE) reflects the variance of the estimators. Since Minn 2nd method exhibits high sub-peak when N_g=N/4, the performance of the estimators is evaluated by MSE over AWGN channel for N_g=N/8 in figure 6-15.
Figure 6-15: MSE of the timing offset estimation over AWGN channel.

As can be seen, the Minn 1st method outperforms the Schmidl method, because the plateau in Schmidl timing metric can significantly degrade the performance. Compared with Schmidl and Minn 1st methods, Minn 2nd shows smaller MSE when SNR is larger than 5 dB. Moreover, when SNR is larger than 10 dB, it can be seen that the MSE of Park, Ren and Seung methods is very low (they are not visible because of limited number of simulation symbols) which is due to the impulse like timing metrics around correcting timing. Therefore, the performance of symbol timing estimator over AWGN channel is highly dependent on the slope of the timing metrics at the correct timing point.
6.2.3 MSE Performance over Rayleigh Flat Fading

Figure 6-16 shows the MSE performance of the estimators over Rayleigh flat fading. The simulation results clearly show the performance degradation caused by fading as compared with the corresponding curves for the AWGN (compare the scale of MSE). Examining figure 6-16, Schmidl method still shows the worst performance among these estimation methods. Moreover, the Minn 2nd method outperforms the other methods when SNR is high. When SNR is low, Minn 1st method shows better estimation accuracy than others. As can be seen from figure 6-16, Park, Ren, and Seung methods do not exhibit better performance over flat Rayleigh fading channel.

Figure 6-16: MSE of the timing offset estimation over Rayleigh flat fading channel.
6.2.4 MSE Performance under Multi-path Frequency Selective Rayleigh Fading

Figure 6-17 presents the MSE of the estimators over Rayleigh frequency selective multipath fading channel. Examining the simulation results, Park, Ren, and Seung methods do not perform well over multipath frequency selective fading. Minn 1st has slightly improved estimation variance over Schmidl method, especially at moderate and high SNR values. Moreover, simulation results show Minn 2nd still shows the best estimation variance.

![Figure 6-17: MSE of the timing offset estimation over frequency selective Rayleigh fading channel.](image)

6.3 Performance Evaluation of Conventional Carrier Frequency Offset Estimation Methods

In this section, the performance of the conventional carrier frequency offset estimation methods are evaluated by their means and MSEs, and is explored the
degrading effects over AWGN, Rayleigh flat fading and multi-path frequency selective Rayleigh fading, respectively. Note that the estimation range of the carrier frequency offset estimators can be evaluated through S-curve, which is generated by showing the actual value versus its estimated value.

6.3.1 S-curve

Figure 6-18 shows S-curves of the conventional carrier frequency offset estimation methods.

![S-curves comparison](image)

Figure 6-18: Comparison of S-curves of the conventional frequency offset estimators.

It is clearly to see that the frequency offset estimation ranges of Schimil, Minn 1st, and Ren are only limited one of sub-carrier spacing due to the periodicity of the angle function, whereas Minn 2nd has twice of the estimation range than those of them. At the
same period of time, the more the repeated sequence is, the larger the frequency offset estimation range will be, but the worse the estimation accuracy will present.

### 6.3.2 MSE Performance under AWGN

Figure 6-19 shows the MSE of the conventional carrier frequency offset estimation methods over AWGN channel with 0.45 sub-carrier space frequency offset. It is clearly to see that Minn 2nd method does not perform well in frequency offset estimation, which means that there is a tradeoff between estimation range and estimation accuracy. Compared with Schmidl and Minn 1st methods, Ren method exhibits lower MSE, especially at high SNR region.

![Figure 6-19: MSE of the frequency offset estimation over AWGN channel.](image)

Figure 6-19: MSE of the frequency offset estimation over AWGN channel.
6.3.3 MSE Performance under Rayleigh Flat Fading

The performance of the conventional frequency offset estimator over Rayleigh flat fading channel is shown in figure 6-20. It can be observed that Minn 2nd method still presents large error variance over Rayleigh flat fading channel, which is in accordance with the trend over AWGN channel. Moreover, Schmidl and Minn 1st methods achieve lower MSE than Ren method, which denotes that the estimation accuracy of Ren method is sensitive to Rayleigh flat fading interference.

![Figure 6-20: MSE of the frequency offset estimation over Rayleigh flat fading channel.](image)

6.3.4 MSE Performance under Multi-path Rayleigh Frequency Selective Fading

The MSE of the conventional frequency offset estimation is illustrated in figure 6-21. The performance of frequency offset estimation is severely degraded over multi-path frequency selective channel. Moreover, Minn 1st and Schmidl methods presents much
lower error variance, while the MSE of Ren method degrades significantly over multi-path frequency selective Rayleigh fading channel.

Figure 6-21: MSE of the frequency offset estimation over frequency selective Rayleigh fading channel.

6.4 Performance Evaluation of Proposed Joint Symbol Timing and Carrier Frequency Recovery Methods

In this section, the performance of the proposed methods as described in Section 5.4 are investigated through computer simulation and compared with the conventional synchronization methods. Note that, channel statistic s and SNR are assumed to be known in the receiver. From the previous analysis for the conventional synchronization methods, Schmidl and Minn 2nd exhibit relatively better performance than others for both symbol timing error and frequency offset estimation. Thus, only these two methods
are selected to compare with the proposed methods due to the limited space in one figure for clear observation.

### 6.4.1 Timing Metric and S-curve of the Proposed Methods

The timing metric of the proposed methods is evaluated and compared with the conventional methods without any interference in figure 6-22. It is clear to see the proposed methods have impulse like timing metric. When compared with Park, Ren, and Seung methods shown in figure 6-22, the proposed methods does not exhibit any sub-peaks within one period symbol duration. Thus, it is expectable that the proposed methods can show better performance than the conventional methods in the symbol timing error estimation.

![Figure 6-22: Timing metric of the proposed estimators.](image-url)
Similar as discussed in the conventional frequency offset estimation, figure 6-23 firstly shows the s-curve of the proposed methods. Since the proposed 1st and 2nd method also employ two repeat and four repeat sequences in its preamble, the estimation ranges are the same as those of Schmidl and Minn 2nd. Based on the analysis results, it is expected that proposed 2nd method can have larger estimation range than proposed 1st method with the trade of estimation accuracy.

Figure 6-23: S-curves of the proposed estimators.

6.4.2 MSE Performance of the Proposed Methods under AWGN

Figures 6-24 and 6-25 show the MSE of the timing error and frequency offset for the proposed methods over AWGN channel. Both of the proposed methods outperform the Schmidl and Minn 2nd methods in timing error estimation, while the proposed 2nd shows more improvement than the proposed 1st method. With respect to frequency offset estimation, the proposed 2nd method does not present better performance than Minn 2nd.
as proposed 1st method. However, the performance of the proposed 2nd method is significantly improved over Minn 2nd.

Figure 6-24: MSE of the proposed timing error estimators over AWGN channel.
Figure 6-25: MSE of the proposed frequency offset estimators over AWGN channel.

6.4.3 MSE Performance of the Proposed Methods under Rayleigh Flat Fading

The MSE performance of the timing error and frequency offset for the proposed methods over Rayleigh flat fading channel is presented in Figures 6-26 and 6-27. Simulation results indicate that the proposed methods perform better than others for timing error estimation, although the improvement is not significant. Moreover, it is also observed that the proposed 2nd outperforms Minn 2nd for frequency offset estimation, while the MSE of the proposed 1st method estimation is closed to that of Schmidl method.
Figure 6-26: MSE of the proposed timing error estimators over Rayleigh flat fading channel.

Figure 6-27: MSE of the proposed frequency offset estimators over Rayleigh flat fading channel.
6.4.4 MSE Performance of the Proposed Methods under Multi-path Rayleigh Frequency Selective Fading

Figures 6-28 and 6-29 show the MSE of the timing error and frequency offset for the proposed methods over multi-path frequency selective fading channel. Examining the simulation results, both of the proposed methods can achieve much better performance than the conventional methods for both timing error and frequency offset estimation. With respect to symbol timing error estimation, the proposed methods outperform the Schmidl and Minn 2nd methods, while the proposed 2nd method not only show significantly improvement over Minn 2nd, but also exhibits less MSE than Schmidl in high SNR value region.

Figure 6-28: MSE of the proposed timing error estimators over multi-path Rayleigh frequency selective fading channel
Figure 6-29: MSE of the proposed frequency offset estimators over multi-path Rayleigh frequency selective fading channel
Chapter 7

Conclusions and Future Research

7.1 Conclusions

In this research, the synchronization schemes both for single-carrier and multicarrier modulation were modeled and analyzed towards application in TDMA-based satellite communication system. Firstly, the symbol error probability expressions for SC system and 8PSK/16QAM OFDM system impaired by CFO were mathematically derived. Secondly, one novel CFR algorithm for SC system and two novel joint STR and CFR algorithms were proposed. Finally, the computer based simulations were conducted to verify the symbol error probability and evaluate various synchronization algorithms.

In the assessment of symbol error probability due to CFO, the performance gets worse when CFO is increased as expected. The QPSK modulation in SC system is more sensitive to CFO than that of BPSK. In the case of OFDM system in the presence of CFO, this research focused on the higher order modulation and modulation with non-rectangular region. By exploiting the CHF of ICI and using the relationship between the sum and product of cosine function, the pdf of the received OFDM signal can be easily obtained by taking the FT of its CHF. All of the symbol error probabilities are expressed as a finite sum of products of Q-functions instead of infinite series. Finally, the computer
simulation results confirm that the theoretical expression is accurate enough to evaluate the symbol error probability for the smaller number of subcarriers. When number of subcarrier is large, a tradeoff between the computation complexity and analytical exactness can be a viable option.

This research also investigated and evaluated the existing CFR algorithms for the SC TDMA systems with perfect symbol timing and identified the design parameters that can yield better estimation range and estimation accuracy. By the results, Kay algorithm has the largest estimation range, and Fitz, Chen and M&M algorithms have a much narrower operating range. Moreover, Kay algorithm shows the lowest error variance under high SNR. However, when AWGN is dominant, all other four algorithms stay close to MCRB except Kay algorithm, which means Kay algorithms does not exhibit accurate estimation under low SNR region. In addition, increasing the preamble length could improve the performance, and the tradeoff between estimation range and estimation accuracy was also pointed out. Since the proposed CFR algorithm can be considered as a method which extra adds smoothing factor to D&M algorithm by using least square standard, the proposed algorithm shares the same estimation range as D&M algorithm. Because of the smoothing factor, the estimation accuracy is improved under low SNR region comparing with Kay algorithm. Although it still deviates from MCRB at low SNR region, the proposed algorithm shows exact match with MCRB when SNR gets larger than a certain threshold.

With regards to CFR and STR in OFDM system, two novel synchronization methods that can jointly estimate both STE and CFO are proposed. These ML-based algorithms are developed by taking the effect of cyclic prefix (CP) into consideration
along with the training sequence of Schmidl toward improved estimation accuracy. The proposed algorithm is thoroughly evaluated and its performance is compared with existing DA joint synchronization algorithms (Schmidl, Minn 1st, and Minn 2nd) in terms of the estimation range and accuracy. Although this ML-based estimation algorithm is derived under additive white Gaussian noise (AWGN), it is also proved to be feasible with frequency selective multipath Rayleigh fading by utilizing a modifying factor. The corresponding computer simulation results show that: For both AWGN and frequency selective multipath Rayleigh fadings, the proposed algorithm yields the lowest MSE for both symbol timing error and carrier frequency offset estimation compared with others of Schmidl’s and Minn 1st's, if the same training sequence is utilized for estimation. Even when compared with the results of Minn 2nd, in which special training sequence is utilized, the proposed algorithm still show comparable performance at higher SNRs for symbol timing estimation. Although the estimation range of the carrier frequency offset of the proposed method is narrower than that of Minn 2nd (with special training sequence), the estimation variance shows significant improvement, especially under frequency selective multipath Rayleigh fading effect.

7.2 Future Research

The research detailed herein has provided improvement for the synchronization schemes of TDMA based satellite communication systems wherein both of the single carrier and multi carrier modulation schemes have been taken into account. The results obtained for synchronization schemes in single carrier system have shown much promise in the AWGN channel environment. Further efforts should be applied towards the
application for other channel conditions such as flat fading and frequency selective fading. Moreover, current research for the theoretical performance of OFDM system impaired by carrier frequency offset only considered lower order modulation schemes. The performance degradation for higher order modulation schemes impaired by carrier frequency offset would require further analysis towards practical applications.

The computer based simulations have yielded a preliminary insight into the proposed synchronization algorithms. However, hardware implementation must be sought in order to obtain further accuracy and resolution into the system performance. In addition, this research will further explore synchronization schemes by using other computational intelligence methods such as evolutionary computation, neural networks, and fuzzy systems.
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