Fatigue behavior under multiaxial stress states including notch effects and variable amplitude loading

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entitled

Fatigue Behavior under Multiaxial Stress States Including Notch Effects and Variable Amplitude Loading

by

Nicholas R. Gates

Submitted to the Graduate Faculty as partial fulfillment of the requirements for the
Doctor of Philosophy Degree in Engineering

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The University of Toledo
August 2016
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Fatigue Behavior under Multiaxial Stress States Including Notch Effects and Variable Amplitude Loading

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The University of Toledo
August 2016

The central objective of the research performed in this study was to be able to better understand and predict fatigue crack initiation and growth from stress concentrations subjected to complex service loading histories. As such, major areas of focus were related to the understanding and modeling of material deformation behavior, fatigue damage quantification, notch effects, cycle counting, damage accumulation, and crack growth behavior under multiaxial nominal loading conditions. To support the analytical work, a wide variety of deformation and fatigue tests were also performed using tubular and plate specimens made from 2024-T3 aluminum alloy, with and without the inclusion of a circular through-thickness hole. However, the analysis procedures implemented were meant to be general in nature, and applicable to a wide variety of materials and component geometries. As a result, experimental data from literature were also used, when appropriate, to supplement the findings of various analyses.

Popular approaches currently used for multiaxial fatigue life analysis are based on the idea of computing an equivalent stress/strain quantity through the extension of static yield criteria. This equivalent stress/strain is then considered to be equal, in terms of
fatigue damage, to a uniaxial loading of the same magnitude. However, it has often been shown, and was shown again in this study, that although equivalent stress- and strain-based analysis approaches may work well in certain situations, they lack a general robustness and offer little room for improvement. More advanced analysis techniques, on the other hand, provide an opportunity to more accurately account for various aspects of the fatigue failure process under both constant and variable amplitude loading conditions. As a result, such techniques were of primary interest in the investigations performed.

By implementing more advanced life prediction methodologies, both the overall accuracy and the correlation of fatigue life predictions were found to improve for all loading conditions considered in this study. The quantification of multiaxial fatigue damage was identified as being a key area of improvement, where the shear-based Fatemi-Socie (FS) critical plane damage parameter was shown to correlate all fully-reversed constant amplitude fatigue data relatively well. Additionally, a proposed modification to the FS parameter was found to result in improved life predictions in the presence of high tensile mean stress and for different ratios of nominal shear to axial stress. For notched specimens, improvements were also gained through the use of more robust notch deformation and stress gradient models. Theory of Critical Distances (TCD) approaches, together with pseudo stress-based plasticity modeling techniques for local stress-strain estimation, resulted in better correlation of multiaxial fatigue data when compared to traditional approaches such as Neuber’s rule with fatigue notch factor.

Since damage parameters containing both stress and strain terms, such as the FS parameter, are able to reflect changes in fatigue damage due to transient material hardening behavior, this issue was also investigated with respect to its impact on variable
amplitude life predictions. In order to ensure that material deformation behavior was properly accounted for, stress-strain predictions based on an Armstrong-Frederick-Chaboche style cyclic plasticity model were first compared to results from deformation tests performed under a variety of complex multiaxial loading conditions. The model was simplified based on the assumption of Masing material behavior, and a new transient hardening formulation was proposed so that all modeling parameters could be determined from a relatively limited amount of experimental data. Overall, model predictions were found to agree fairly well with experimental results for all loading histories considered.

Finally, in order to evaluate life prediction procedures under realistic loading conditions, variable amplitude fatigue tests were performed using axial, torsion, and combined axial-torsion loading histories derived from recorded flight test data on the lower wing skin area of a military patrol aircraft (tension-dominated). While negligible improvements in life predictions were obtained through the consideration of transient material deformation behavior for these histories, crack initiation definition was found to have a slightly larger impact on prediction accuracy. As a result, when performing analyses using the modified FS damage parameter, transient stress-strain response, and a 0.2 mm crack initiation definition, nearly all variable amplitude fatigue lives, for un-notched and notched specimens, were predicted within a factor of 3 of experimental results. However, variable amplitude life predictions were still more non-conservative than those observed for constant amplitude loading conditions. Although there are numerous factors which could have contributed to this non-conservative tendency, it was determined that some of the error may have resulted from inaccuracies in life prediction
curves, the modeling of material deformation behavior, the consideration of normal-shear stress/strain interaction effects, and/or linear versus nonlinear damage accumulation.

In addition to crack initiation, fatigue crack growth behavior was also of interest for all tests performed in this study. Constant amplitude crack growth in notched specimens was observed to be a primarily mode I process, while cracks in un-notched specimens were observed to propagate on maximum shear planes, maximum tensile planes, or a combination of both. Specialized tests performed using precracked tubular specimens indicated that the preferred growth mode was dependent on friction and roughness induced closure effects at the crack interface. As a result, a simple model was proposed to account for frictional attenuation based on the idea that crack face interaction reduces the effective stress intensity factor (SIF) by allowing a portion of the nominally applied loading to be transferred through the crack interface. Crack path/branching, growth life, and growth rate predictions based on the proposed model were all shown to agree relatively well with the experimentally observed trends for all loading conditions considered.

For notched specimen fatigue tests, although crack growth was observed to be mode I-dominated, constant amplitude crack growth rates under multiaxial nominal stress states were observed to be higher than those for uniaxial loading at the same SIF range. While T-stress corrections were able to account for this difference in some cases, growth rates for pure torsion loading still had the tendency to be higher than those for uniaxial loading. Additionally, using short crack models to account for stress concentration and initial crack geometry effects was found to improve growth rate correlations in the notch affected zone. For 90° out-of-phase loading conditions, small crack growth appeared to
have been dominated by the mode I loading from the axial component of the applied stress, but as cracks grew, they turned, and mode I SIF range alone was unable to successfully correlate crack growth rate data.

Finally, for variable amplitude crack growth, two state-of-the-art analysis models, UniGrow and FASTRAN, were used to predict crack growth behavior for the notched specimens tested in this study. UniGrow is based on the idea that residual stress distributions surrounding the crack tip are responsible for causing load sequence effects, while FASTRAN attributes these effects to varying degrees of plasticity induced closure in the crack wake. While both models were able to predict nearly all uniaxial constant amplitude crack growth lives within a factor of 3 of experimental results, they both produced conservative predictions under uniaxial variable amplitude loading conditions. For variable amplitude torsion and combined axial-torsion crack growth, however, the degree of conservatism in these predictions was found to reduce. This was attributed to an increase in experimental growth rates due to multiaxial stress states effects, which are not accounted for in either UniGrow or FASTRAN. By comparing differences in crack growth life between tests performed using full and edited versions of the same loading history, it was found that FASTRAN was generally better able to account for the effects of small cycles and/or changes in loading history profile. Additionally, initial crack geometry assumptions were found to have a fairly significant impact on analysis results for the specimen geometry considered in this study.
To my parents, John and Mary Kaye Gates, and to all of my ancestors before them, who have worked hard to provide me with the great opportunities I enjoy in my life today.
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Last but not least, I would like to thank all of my family and friends who have been there for me and encouraged me throughout this long and sometimes arduous chapter in my life. This is especially true of my parents, who have offered their unwavering love and support, and have set great examples for me as I have grown to where I am today.
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List of Abbreviations

1D............................One-Dimensional
2D............................Two-Dimensional
3D............................Three-Dimensional
5D............................Five-Dimensional

AEN ......................Axial Edited history Notched
ARN ......................Axial Real loading Notched
ARS ......................Axial Real loading Smooth

CCH ......................Corner Crack from a Hole
CEN ......................Combined Edited history Notched
CRN ......................Combined Real loading Notched
CRS ......................Combined Real loading Smooth
CTOD ....................Crack Tip Opening Displacement

EPFM ....................Elastic-Plastic Fracture Mechanics
ESED ....................Equivalent Strain Energy Density

FCGD ....................Fatigue Crack Growth Database
FEA ......................Finite Element Analysis
FS ......................Fatemi-Socie
fc ......................failure crack

LDR ......................Linear Damage Rule
LEFM ....................Linear Elastic Fracture Mechanics
LT ......................Long Transverse (Longitudinal)

MT ......................Middle Tension
MTS ......................Maximum Tangential Stress

NA ......................Notched Axial
NC ......................Notched Combined
NT ......................Notched Torsion

OP ......................Out-of-Phase

PCA ......................Pre-Cracked Axial
PCC ......................... Pre-Cracked Combined
PCT .......................... Pre-Cracked Torsion
PDMR .......................... Path Dependent Maximum Range

Q1,3 ......................... Quadrants 1 and 3
Q2,4 ......................... Quadrants 2 and 4

S-N .......................... Stress-Life
SA ............................ Smooth Axial
SC ............................ Smooth Combined
SCT .......................... Smooth Combined Triangular path
SENT .......................... Single Edge Notch Tension
SFE .......................... Stacking Fault Energy
SIF .......................... Stress Intensity Factor
ST ............................ Smooth Torsion
STSA ......................... Smooth Torsion with Static Axial
SWT .......................... Smith-Watson-Topper
s1 ............................ secondary crack 1
s2 ............................ secondary crack 2

TCD .......................... Theory of Critical Distances
TEN .......................... Torsion Edited history Notched
TRN .......................... Torsion Real loading Notched
TRS .......................... Torsion Real loading Smooth
TT ............................ Through crack in a Tube

VSE .......................... Virtual Strain Energy

XFEM .......................... Extended Finite Element Method

γ-N .......................... Shear Strain-Life
ε-N .......................... Strain-Life
# List of Symbols

- $\%EL$ .................. percent elongation at failure
- $2N_f$ ...................... reversals to failure
- $A$ ......................... Tanaka’s non-proportionality parameter
- $A_A$ ...................... uniaxial stress-life fatigue curve coefficient
- $A_A'$ ..................... pseudo stress-life fatigue curve coefficient
- $A_T$ ...................... shear stress-life fatigue curve coefficient
- $a$ .......................... crack depth
- $b$ .......................... fatigue strength exponent
- $b_o$ ........................ shear fatigue strength exponent
- $b^{(0)}$ ........................ cyclic hardening rate constants
- $C$ .......................... crack growth equation coefficient
- $C_c$ ........................ microstructural transformation rate constant
- $C_{ij}$ ...................... loading proportionality constant tensor
- $\bar{C}$ ........................ Tanaka’s non-proportionality tensor
- $c$ .......................... surface crack length/half length
- $c_o$ ........................ shear fatigue ductility exponent
- $c^{(0)}$ ........................ kinematic hardening constants
- $D$ .......................... notch depth
- $d_i$ ........................ inside diameter
- $d_o$ ........................ outside diameter
- $E$ .......................... modulus of elasticity/Young’s modulus
- $E$ .......................... notch contribution to crack length
- $F$ .......................... non-proportionality factor
- $G$ .......................... shear modulus
- $\mathcal{G}$ .................... energy release rate
- $H^{(i)}$ ........................ slopes of discretized stress-strain curve
- $h$ .......................... equivalent plastic modulus
- $K$ .......................... monotonic strength coefficient
$K_{CPA}$ - critical plane-based stress intensity factor
$K_{eff}$ - effective stress intensity factor
$K_F$ - elastic-plastic fracture toughness
$K_f$ - fatigue notch factor
$K_q$ - equivalent fatigue notch factor
$K_I$ - mode I (opening mode) stress intensity factor
$K_{II}$ - mode II (in-plane shear) stress intensity factor
$K_{III}$ - mode III (anti-plane shear) stress intensity factor
$K_{IC}$ - plane strain fracture toughness
$K_{max}$ - maximum stress intensity factor
$K_o$ - peak overload stress intensity factor
$K_{op}$ - crack opening stress intensity factor
$K_q$ - equivalent stress intensity factor
$K_{q,nom}$ - nominally applied equivalent stress intensity factor
$K_r$ - residual stress intensity factor
$K_t$ - stress concentration factor
$K_{tq}$ - equivalent stress concentration factor
$K_{th}$ - threshold stress intensity factor
$K_{tij}$ - stress concentration factor tensor
$K_e$ - inelastic strain concentration factor
$K_g$ - inelastic stress concentration factor
$K'$ - cyclic axial strength coefficient
$K_{IP}'$ - cyclic In-Phase strength coefficient
$K_{MP}'$ - cyclic $90^\circ$ OP strength coefficient
$K_o'$ - cyclic shear strength coefficient
$K^*$ - structural cyclic strength coefficient
$k$ - FS parameter material constant

$L$ - critical distance characteristic length
$L$ - average grain size
$L(i)$ - unit vector of $i$th backstress term
$l'$ - crack transition length

$M$ - number of backstress components
$m$ - non-proportional hardening rate constant
$m$ - crack growth equation exponent
$m'$ - FAISTRAN fracture toughness parameter

$N_f$ - cycles to failure
$N_{fl}$ - number of cycles at fatigue limit
$N_{f,CE}$ - cycles to failure from composite life prediction
$N_{f,SS}$ - cycles to failure from stable material response life prediction
$N_{f,T}$ - cycles to failure from transient material response life prediction
$N_s$ - number of cycles to reach stabilized material response
$n$ - monotonic strain hardening exponent
$n'$ - cyclic axial strain hardening exponent
n₀' ........................cyclic shear strain hardening exponent
n_H' ........................cyclic in-phase strain hardening exponent
n_NP' ........................cyclic 90° OP strain hardening exponent
n* ........................structural cyclic strain hardening exponent
\bar{n} ........................unit exterior normal to yield surface

P_a  .........................axial load amplitude
P_m  .........................mean axial load
p  ..........................equivalent plastic strain
p  ........................UniGrow driving force constant

q  .........................size of strain memory surface

R  ..........................minimum to maximum stress ratio
R_{\text{eff}}  ..................effective load ratio for crack growth retardation
R^2  .........................coefficient of determination
R_{N_i}  ......................non-proportional kinematic hardening parameters
R_{P_i}  ......................cyclic kinematic hardening parameters
R_{T_i}  ......................target kinematic hardening parameters

r_m  .......................mean specimen radius
r_y  .......................plastic zone size
r_{y(c)}  ..................current cycle plastic zone size
r_{y(o)}  ..................overload cycle plastic zone size
r^{(i)}  ..................transient kinematic hardening parameters
r_{o}^{(i)}  ................initial kinematic hardening parameter values

S  ........................nominal stress
S_A  .......................nominal axial stress
S_{A,a}  ...................nominal axial stress amplitude
S_{A,m}  ....................mean nominal axial stress
S_{A,Nf}  ...................fully-reversed axial fatigue strength at N_f cycles
S_{\text{net}}  ...............nominal stress based on net cross section
S_{Nf}  ......................effective fully-reversed stress amplitude
S_T  ........................nominal shear stress
S_{T,a}  ...................nominal shear stress amplitude
S_{T,m}  ....................mean nominal shear stress
S_{T,Nf}  ...................fully-reversed shear fatigue strength at N_f cycles
S_{vm,a}  ..................nominal von Mises stress amplitude
\bar{S}  ....................deviatoric stress tensor

T  ........................applied torque
T_a  .......................torque amplitude
T_i  .......................stress vector
T_m  .......................mean torque
l  ..........................specimen wall thickness
u_i  .......................displacement vector
\( W \) specimen width
\( w \) strain energy density

\( Y_{I}, Y_{II}, Y_{III} \) SIF geometry factor functions

\( \alpha \) average undeformed crack face asperity angle
\( \alpha_{1}, \alpha_{2} \) FASTRAN tensile constraint factors
\( \alpha_{\text{eff}} \) average effective crack face asperity angle
\( \alpha_{\text{NP}} \) non-proportional cyclic hardening coefficient
\( \bar{\alpha} \) total backstress tensor

\( \beta \) load level influence coefficient
\( \beta \) FASTRAN compressive constraint factor

\( \gamma_{a} \) shear strain amplitude
\( \gamma_{m} \) mean shear strain
\( \gamma'_{f} \) shear fatigue ductility coefficient

\( \Delta J \) cyclic J-integral range
\( \Delta K \) stress intensity factor range
\( \Delta \gamma_{\text{max}} \) maximum shear strain range
\( \Delta \tau \) shear stress range on maximum shear strain plane
\( \delta_{ij} \) Kronecker delta function

\( \varepsilon_{1}, \varepsilon_{2}, \varepsilon_{3} \) principal strain components
\( \varepsilon_{a} \) axial strain amplitude
\( \varepsilon_{ij} \) strain tensor
\( \varepsilon_{m} \) mean axial strain
\( \varepsilon_{q} \) equivalent strain
\( \varepsilon_{qa} \) equivalent strain amplitude
\( \varepsilon^{e} \) elastic strain
\( \varepsilon_{p}^{p} \) uniaxial plastic strain amplitude
\( \varepsilon'_{f} \) fatigue ductility coefficient
\( \bar{\varepsilon}^{p} \) plastic strain tensor

\( \theta \) angular location on hole perimeter
\( \theta_{c} \) crack propagation direction
\( \theta_{p} \) principal stress direction

\( \Lambda \) biaxial stress ratio \( (\sigma_{x}/\sigma_{y}) \)
\( \lambda \) nominal stress ratio \( (\tau/\sigma) \)

\( \mu \) coefficient of friction (sliding)

\( \nu \) Poisson’s ratio
\( \nu_e \) .................. elastic Poisson’s ratio
\( \nu_p \) .................. plastic Poisson’s ratio
\( \nu' \) .................. effective Poisson’s ratio

\( \rho \) .................. material characteristic length (Neuber)
\( \rho_e \) .................. effective crack tip radius
\( \rho^* \) .................. material elementary block size

\( \Sigma D_s \) .................. fatigue damage sum based on stable material response
\( \Sigma D_t \) .................. fatigue damage sum based on transient material response
\( \Sigma \varepsilon^p \) ................. total plastic strain incurred before material stabilization
\( \sigma_1, \sigma_2, \sigma_3 \) ........... principal stress components
\( \sigma_{a,Nf} \) .................. stress amplitude corresponding to strain amplitude at \( N_f \) cycles
\( \sigma_{ij} \) .................. stress tensor
\( \sigma_{n,\text{max}} \) ................. maximum stress normal to the maximum shear plane
\( \sigma_0 \) .................. fatigue/endurance limit
\( \sigma_q \) .................. equivalent stress
\( \sigma_{qa} \) .................. equivalent stress amplitude
\( \sigma_{qm} \) .................. equivalent mean stress
\( \sigma_{q,\text{max}} \) ................. maximum equivalent stress
\( \sigma_u \) .................. ultimate tensile strength
\( \sigma_{\text{vm,a}} \) ................. von Mises equivalent stress amplitude
\( \sigma_y \) .................. tensile yield strength
\( \sigma' \) .................. elastic stress
\( \sigma'_q \) .................. equivalent elastic (pseudo) stress range
\( \sigma'_f \) .................. fatigue strength coefficient
\( \sigma'_y \) .................. cyclic yield strength

\( \tau \) .................. shear stress
\( \tau'_f \) .................. shear fatigue strength coefficient

\( \chi^{(i)} \) .................. ratcheting rate constants
Chapter 1

Introduction

1.1 Introduction to Research Topics

Engineering fatigue is defined as the progressive and localized failure process that occurs in components and structures when they are subjected to dynamic and fluctuating stresses. Under fatigue loading circumstances, failure often occurs at stress levels that are considerably lower than the tensile or yield strength of a particular material. The importance of accounting for fatigue in mechanical design is emphasized by the reality that it is the most common cause of mechanical failure in metals, estimated to result in approximately 90% of all such failures (Callister, Jr., 2007). Since the first observance of mechanical failures due to fatigue in the mid-19th century, the emphasis placed on understanding fatigue damage mechanisms and making accurate fatigue life predictions has only continued to increase.

The fatigue failure process is most often described in terms of two distinct stages: crack initiation and crack propagation. Crack initiation represents the portion of fatigue life involved in nucleating and growing a fatigue crack to a small length, usually on the order of a few hundred microns or less. Crack propagation is then the period of time during which a crack grows from this initially small length to a critical length. When a
crack reaches this critical length, which is dependent on component geometry, loading conditions, and material properties, failure usually occurs suddenly and catastrophically.

For some cases, where relatively simple component geometries are subjected to simple loadings (e.g. constant amplitude pure axial or pure torsion loadings) there are a number of well-established and generally accepted techniques to estimate fatigue life. Among these are stress-life and strain-life approaches, which use an experimentally generated fatigue life curve for a given material to relate the stress or strain state within a component to a corresponding fatigue life. The extension of these techniques to more complex component geometries and loading cases, however, often results in fatigue life predictions that do not fall within an acceptable margin of error. Therefore, more robust models are needed for these cases. Examples of some factors which must be accounted for in a general fatigue life analysis include: multiaxial stress states, variable amplitude loadings, stress concentration effects, environmental effects, and component surface finish. Despite growing research over the years, there are still no widely accepted, or generally applicable, fatigue life prediction methodologies for such complex loading conditions.

The material presented in this study focuses primarily on the effects of multiaxial variable amplitude loadings on components containing stress concentrations. Multiaxial loadings occur when a component is subjected to an applied load along more than one axis. Such loadings, which include biaxial tension loading as well as combined axial-torsion loading, are very common in application and result in the presence of multiaxial stress states within a material. Variable amplitude loadings, which occur when applied loading conditions vary with time, are also very common and often unavoidable in
service loading situations. Finally, because stress concentrators, such as holes, grooves, fillets, splines, etc., result in localized regions of elevated stresses within a component, they serve as likely sites for fatigue cracks to initiate and their effects cannot be ignored.

Each of these conditions by itself adds its own degree of complexity to the task of performing an accurate fatigue life analysis. This is especially true when considering the variability that is inherent in even the simplest of fatigue life calculations. Although much literature exists for many of these different aspects individually, there are very few studies on synergistic effect of all three working together. However, when considering the majority of cases in industry where a component is subjected to cyclic loadings, all of these aspects are typically present.

One such example is the wing of an aircraft. In this case, the structure is subjected to both bending and shear loadings, resulting from lift and drag forces, engine weight, thrust, control surface manipulation, etc. This produces a multiaxial state of loading, and therefore stress, within the structure. Additionally, take-off events, landing events, and in-flight maneuvers, combined with the varying amounts of lift, drag, thrust, and turbulence experienced during any given flight, cause the magnitude of these loadings to continuously vary with time. Finally, the wing of an aircraft contains many notches which result in localized stress concentrations. Whether it’s a rivet hole, a radius in a wing spar, an access hole, etc., all of these are necessary design features, but they effectively reduce the fatigue life of the structure.

Even just based on this one example, it is easy to realize the great practical importance of studying the multiaxial variable amplitude fatigue behavior of notched members. In an ever developing world, with exponential advances in technology and
global commerce, the need to be able to produce safe and efficient designs is greater than ever. This is especially true for transportation industries, such as aerospace, where a catastrophic failure is not an option. A key element in producing such designs is to be able to confidently and accurately predict the various stages of a component’s fatigue life. This not only applies to new designs being developed, but also to existing structures where damage tolerant design philosophies have been employed. Being able to confidently assess and manage the fatigue damage of in-service components not only increases safety and reliability, but also has the potential to decrease maintenance costs by allowing structural inspections to be performed at more appropriate intervals.

1.2 Research Motivation and Objectives

The central objective of the research performed in this study was to be able to better understand and predict fatigue crack initiation and growth from stress concentrations which are subjected to complex multiaxial service loading histories. In doing so, emphasis was placed on combining modeling techniques for each aspect of the fatigue failure process into a single fatigue life analysis procedure which progressed in a logical and consistent manner. To preserve general applicability, an emphasis was also placed on implementing physically based modeling techniques, so that the actual damage mechanisms at work within a material were reflected in each analysis. Furthermore, the use of experimental data and empirical fitting parameters was kept to a minimum, as these are not always available when designing a component in an industry setting.

Given these goals, experimental data and observations generated from a wide variety of material deformation and fatigue tests, along with advanced computational
simulation techniques, were used to gain insight into the damage mechanisms at work within a material for a given set of loading conditions. By doing this, it enabled various life prediction techniques to be evaluated and/or proposed based on the actual physics of material failure rather than purely empirical results. Although all experimental work utilized specimens made from a single material, the analyses performed were meant to be general in nature, and applicable to a wide variety of materials and component geometries. As a result, experimental data from literature were also used, when appropriate, to supplement the findings of various analyses. A flow chart summarizing some of the different fatigue life analysis procedures investigated in this study is included in Figure 1.1. Areas where key contributions were made are indicated by different colored boxes and are explained in the figure. The research was focused on both crack initiation and crack growth stages of fatigue life.

1.3 Outline of Dissertation

This dissertation begins with an extensive literature review, summarized in Chapter 2, which establishes some of the theories, modeling techniques, and experimental results currently available for the individual topics of interest in this study. The review is ordered based on the progression of a general fatigue life analysis procedure. Discussions on material deformation behavior under multiaxial stress states, including non-proportional hardening, with and without notch effects are presented first. Next, information relevant to fatigue crack initiation is presented with respect to multiaxial fatigue damage parameters, multiaxial cycle counting methods, and cumulative damage rules. Crack initiation discussions, concluded with a review of studies considering the
synergistic effects of multiaxial variable amplitude fatigue behavior in the presence of stress concentrations, are then followed by crack growth discussions. The crack growth sections begin with a general overview of linear elastic and elastic-plastic fracture mechanics concepts. From there, topics reviewed include: mixed-mode crack growth, crack closure effects, crack coalescence and branching phenomena, short crack effects, and notch effects on crack growth. Finally, the chapter is concluded with a discussion on issues related to crack growth analysis under both uniaxial and multiaxial variable amplitude loading conditions.

Chapter 3 outlines the experimental program implemented in the current study. The experimental program is wide-ranging and was designed to supplement the various aspects of the computational and analytical work performed. Topics discussed include: material selection, specimen design and preparation, testing equipment, crack monitoring techniques, and the testing program itself. The testing program includes a variety of material deformation tests, along with constant and variable amplitude fatigue tests, performed with both notched and un-notched specimens. The testing program section reviews the various types of tests performed in this study and the information gained from each of these tests.

Given the importance of accurate stress-strain estimations in a fatigue life analysis, Chapter 4 first presents experimental results and analysis related to material deformation behavior. The chapter begins by presenting results from monotonic and cyclic deformation tests aimed at establishing baseline mechanical properties and constitutive relations for the testing material. Next, a cyclic plasticity model is presented from which material deformation behavior can be estimated for more complex loading
conditions. Emphasis is placed on developing simplified relations from which material constants for the model can be computed from a relatively limited amount of experimental data. Stress-strain predictions from this model, including the effects of transient material behavior due to both cyclic and non-proportional hardening, are then compared to experimental results from a number of constant and variable amplitude deformation tests.

The latter part of Chapter 4 focuses on local material deformation behavior for notched specimens. For the specimen geometry tested in this study, stress distributions surrounding the notch are first investigated based on results from linear elastic FEA. Then, nonlinear FEA results are used to evaluate the applicability of various notch stress-strain estimation models under a variety of uniaxial and multiaxial nominal stress states. Comparisons between nonlinear FEA solutions and local plasticity corrected stress-strain values are not only made for the specimens tested in this study, but also for two different notched specimen geometries taken from literature. Analyses are performed at the notch root, as well as at various distances away from the notch root, in order to evaluate the effect of stress gradients and stress redistribution on the resulting deformation behavior.

Moving on to crack initiation aspects, Chapter 5 presents experimental results and analysis related to constant amplitude fatigue tests performed using both un-notched and notched specimens. This chapter begins with an overview of the constant amplitude fatigue tests performed in this study, which includes the specific testing procedures implemented, along with the experimental results obtained from each test. Then, baseline stress-life and strain-life fatigue properties, derived from some of the more basic tests, are presented next. Once these fatigue properties were determined, the next section of
Chapter 5 focuses on evaluating constant amplitude fatigue life predictions based on commonly used equivalent stress- and strain-based analysis techniques. Analyses are performed for both un-notched and notched specimen tests. Similarly, critical plane-based fatigue life predictions are also compared to experimental results for both un-notched and notched specimens. Major areas of focus include load path effects on fatigue damage, stress gradient effects on fatigue damage at notches, and the consideration of mean stress effects in critical plane-based damage parameters. A modified version of the Fatemi-Socie damage parameter is also presented in this chapter in an attempt to improve the correlation of experimental fatigue data under a wide variety of loading paths.

Similar to Chapter 5, Chapter 6 presents experimental results and analysis related to crack initiation in both un-notched and notched specimen fatigue tests. This chapter, however, investigates specific aspects related to variable amplitude loading effects on fatigue damage. These include load history dependence of material deformation behavior, multiaxial cycle counting procedures, and cumulative damage rules. The variable amplitude loading history applied in this study, along with different variations of the history used in testing, are discussed first. Then, similar to the previous chapter, an overview of the variable amplitude testing program, including testing procedures and experimental results, is presented next. From there, fatigue life predictions based on commonly used equivalent stress- and strain-based analysis techniques are compared to experimental results for the different variable amplitude loading conditions. Finally, the ability of critical plane-based approaches to account for the synergistic effect of all of the different factors involved in a multiaxial variable amplitude fatigue life analysis is
evaluated, considering the insight gained from constant amplitude results, based on comparisons to experimental data.

After studying crack initiation behavior, Chapter 7 presents experimental results and analysis related to constant amplitude crack growth behavior for the specimens tested in this study. After verifying finite element-based modeling techniques used to generate SIF solutions for more complex crack geometries and loading conditions, cracking behavior is first investigated with respect to crack growth in un-notched specimens. Specific topics of interest include the effect of friction and mechanical interlocking on shear- and/or mixed-mode crack growth, as well as the mechanisms related to crack branching under such conditions. A model is then proposed which attempts to quantify some of these effects on resulting shear/mixed-mode crack growth behavior, and corresponding comparisons are made to experimental results. The latter part of Chapter 7, on the other hand, is concerned with mode I crack growth in notched specimens. Major areas of focus for these analyses include the effect of initial crack geometry and SIF solution on experimental crack growth rate correlations, as well as the effects of notches and multiaxial stress states on mode I crack growth.

Building on the results of Chapter 7, Chapter 8 investigates crack growth behavior under both uniaxial and multiaxial variable amplitude loading conditions. Experimental results and analysis are presented first for notched specimen fatigue tests. Due to the complexities involved in variable amplitude crack growth, however, analyses were performed using state-of-the-art crack growth models already available in existing software packages. Discussions on the results of these analyses are then provided with respect to both their agreement with experimental crack growth data, and with respect to
the similarities and differences observed between constant and variable amplitude crack growth behavior under different nominal loading conditions. Since variable amplitude crack growth in un-notched specimens violates some of the assumptions made by these models, however, only the experimental crack growth results from these tests are presented in the latter part of Chapter 8.

In addition to individual sections included at the end of Chapters 4-8, which provide a summary and some conclusions based on the specific analyses presented within each chapter, an overall summary of this dissertation is presented in Chapter 9. After reviewing the work performed, and offering some general conclusions based on the experimental data and analyses presented herein, some suggestions are also made on areas where improvements can be made with respect to the various fatigue life analysis procedures investigated. Due to time constraints, these issues could not be addressed in the current study, but could make excellent topics for future related research.
Figure 1.1  Flowchart summarizing different fatigue life analysis procedures investigated in this study. Areas of key contribution are indicated by different colored boxes and are explained on the right side of the figure.
Chapter 2

Literature Review

2.1 Overview

There are a number of different methodologies available for predicting the fatigue life of notched components subjected to multiaxial variable amplitude loading conditions. Regardless of the exact procedure selected, however, there are several steps that need to be performed before the end result is achieved. To help illustrate these steps, a flowchart is presented in Figure 2.1 which outlines the general process flow for a fatigue crack initiation analysis based on a variety of different life prediction methodologies. Although this figure is similar in concept to the flowchart presented in Figure 1.1, Figure 1.1 is meant to be specific to the particular crack initiation and crack growth analyses performed in the subsequent chapters of this dissertation. Figure 2.1, on the other hand, is a more generalized flowchart, for crack initiation aspects only, which outlines some of the more specific steps required for a wider variety of analysis procedures.

As seen in Figure 2.1, when considering crack initiation life, there are several types of parameters that can be used to calculate multiaxial fatigue damage. These include stress-based, strain-based, and energy-based models. In addition, there also exist a number of parameters, including those based on critical plane concepts, which relate
fatigue damage to some combination of stress, strain, or energy quantities. However, the input loading for a fatigue life analysis is typically known in the form of either an applied stress history or an applied strain history, but not both. Therefore, depending on which damage parameter is selected for a particular analysis, a good understanding of material constitutive behavior and non-proportional hardening models may be required in order to derive accurate strain response from a given stress history, or vice versa. This is especially true in cases where plastic deformation is present. Additionally, in the case of a notched component, the loading is most often known in terms of nominally applied values, and not representative of the stresses or strains controlling fatigue damage at the notch root. Therefore, additional models are needed to relate these nominal stresses and strains to the local deformation behavior at the notch.

Once appropriate stress and/or strain histories are obtained, a multiaxial cycle counting method is then needed to decompose the variable amplitude loading history into equivalent constant amplitude loading cycles from which fatigue damage can be calculated. Finally, once fatigue damage has been calculated for each counted cycle, a damage accumulation rule must be selected and applied in order to determine when failure will occur based on the combined damage of all cycles contained in the loading history.

Following the initiation of a fatigue crack, there is also the stage of fatigue life involved in growing the crack to a critical length, at which failure will occur. Since often times crack initiation and crack growth make up significant portions of a component’s total fatigue life, it is usually necessary to consider both in order to get an accurate representation of the entire fatigue failure process. When considering crack growth,
stresses and strains at the crack tip, which are influenced by the nominal loading, crack length, and component geometry, play the dominant role in driving fatigue damage. Additionally, crack growth rates can be affected by phenomena such as crack closure and mixed-mode crack extension, which can greatly increase the complexity of a crack growth analysis.

In order to provide proper background knowledge on these subjects, pertinent information from literature on each of the aforementioned aspects of the multiaxial variable amplitude fatigue life prediction of notched components was reviewed, and is presented, in the following sections. Although this is in no way meant to be an exhaustive review for any particular topic, the goal was to identify and summarize the most popular methods currently in use for each step of a complex fatigue life analysis problem. Additionally, reviewing the models in terms of comparisons with experimental data was also a goal, as this helps to identify the strengths and weaknesses of each approach.

2.2 Notation

In order to avoid ambiguity in mathematical expressions throughout the remainder of this dissertation, this section is included in order to explain the style of tensor notation used within.

When indicial notation is used to define a tensor operation, letter subscripts are used to define the tensor dimensions and standard summation convention applies when dummy indices are present. The Kronecker delta symbol, $\delta_{ij}$, is equal to 1 for $i = j$ and is equal to 0 otherwise. The permutation symbol, $\varepsilon_{ijk}$, would always be followed by three indices and should not be confused with a strain tensor, $\varepsilon_{ij}$. Furthermore, a symbol with a
bracketed letter superscript (e.g. \(x^{(i)}\)) indicates a number series of expressions and does not necessarily represent a tensor.

When tensors are denoted symbolically, this is done by using \textbf{bold} characters accented with a tilde, \(\sim\). A colon between two tensors is used to denote their inner product, also referred to as their dot or scalar product. The MacCauley brackets are denoted by \(\langle \rangle\) and represent the following mathematical expression: \(\langle x \rangle = (x + |x|)/2\). The norm of a tensor is denoted by double vertical brackets (i.e. \(\|\vec{x}\| = \sqrt{\vec{x}:\vec{x}}\)). The identity (or unit) tensor, denoted by \(\vec{I}\) in symbolic notation, is equivalent to the Kronecker Delta operator of the same rank. Finally, a prefix \(d\) denotes an increment or differentiation of the immediately following quantity.

### 2.3 Deformation Behavior

In most cases, being able to accurately determine, or predict, a material’s stress-strain response at critical locations within a component or structure is an essential first step in making reliable fatigue life predictions. If not done correctly, errors in this step can propagate and magnify throughout subsequent steps of a fatigue life analysis. Since usually only applied stresses OR strains are known at the beginning of a fatigue analysis, it is often necessary to estimate one from the other. This is especially true for complex multiaxial loading cases where many multiaxial fatigue damage parameters rely on both stress and strain quantities to compute fatigue damage.
2.3.1 Elastic Deformation

In the case of purely linear elastic loading, the derivation of a strain history from a given stress history, or vice versa, is a straightforward process. The following stress-strain relations from Hooke’s Law may be directly employed in a case where isotropic material behavior is assumed:

\[
\sigma_{ij} = \frac{E\nu}{(1+\nu)(1-2\nu)} \delta_{ij} \varepsilon_{kk} + 2G \varepsilon_{ij}
\]  

(2.1)

\[
\varepsilon_{ij} = \frac{\sigma_{ij}}{2G} - \frac{\nu}{E} \delta_{ij} \sigma_{kk}
\]  

(2.2)

where \( E \) is Young’s Modulus, \( \nu \) is Poisson’s ratio, \( G \) is the Shear Modulus, \( \delta \) is the Kronecker delta function, \( \sigma_{ij} \) is the stress tensor, and \( \varepsilon_{ij} \) is the strain tensor.

For a notched component subjected to purely elastic loading at the notch root, stress-strain calculation remains a relatively simple task. Local stresses or strains can be computed by multiplying nominal stresses, strains, or applied loads by appropriate concentration factors. Stress concentration factors, as typically defined, relate the nominally applied stress to the maximum equivalent stress value, in terms of either von Mises or maximum principal stress, at the notch root. However, multiple concentration factors can also be used to relate the nominal loading to each individual component of stress or strain at the notch. Stress and strain concentration factors may be determined using a number of techniques which include: direct measurement, analytical solutions, photoelasticity methods, and finite element analysis.

Since in most practical cases local stresses or strains cannot be obtained experimentally through direct measurement, typically using electrical resistance strain gages or photoelasticity analysis, an alternative approach for determining stress
concentration factors is by solving for local stresses analytically using theory of elasticity. Analytical stress concentration solutions for a variety of common, and relatively simple, geometries can be readily found in stress concentration handbooks such as (Pilkey and Pilkey, 2008).

However, with continual advances in manufacturing and machining capabilities, component geometries are becoming increasingly more complex. This makes it difficult or impossible to find stress concentration solutions using more traditional approaches such as handbooks. Additionally, handbooks typically only give concentration values at the maximum equivalent stress/strain location for a given notch configuration, and only for a limited number of applied loading conditions. Any information about stress distributions or stress state surrounding the notch is not available. Fortunately, advances in computational technology and numerical solving methods, coupled with the now widespread availability of commercial finite element analysis software packages, allow the stress and strain state to be calculated at any point within even the most complex geometries without the need to produce a physical model.

For these reasons, finite element analysis has now become the preferred method for the determination of stress concentration factors. Not only can any component geometry be analyzed, but the local stresses due to any combination of applied loadings can be determined as well. Stress and strain distributions surrounding a notch are automatically generated and concentration factors can be defined in whatever way is most convenient for subsequent analyses. This is especially useful when a component’s geometry is such that the definition of a nominal stress value, often required for traditional stress concentration factor solutions, is not easily defined. Instead, stress
concentration can be defined in terms of the nominally applied loadings and the resulting local stresses.

Once the local linear elastic stress or strain components have been determined for each applied loading, the principle of superposition can be used to determine the complete elastic stress state within a component for any combination of loadings. This can be expressed mathematically, in terms of both nominal stresses and applied loadings, by the following equation (Köttgen et al., 1995):

\[
\sigma_{ij} = \sum_{m=1}^{n} K_{t,ij}^{(m)} S^{(m)} = \sum_{m=1}^{n} C_{ij}^{(m)} L^{(m)}
\]

(2.3)

where \(\sigma_{ij}\) is the local elastic stress tensor, \(K_{t,ij}^{(m)}\) contains the stress concentrations factors relating each local stress component to the \(m^{th}\) nominal stress, \(S_m\), and \(C_{ij}^{(m)}\) is a tensor containing the proportionality constants relating each local stress component to the \(m^{th}\) applied load, \(L_m\). The total number of nominally applied loads or stresses is \(n\). Finally, after obtaining the desired local elastic stress or strain values, stress and strain components can be related using Hooke’s Law (Equations (2.1) and (2.2)) to obtain the complete local elastic stress-strain state.

### 2.3.2 Elastic-Plastic Deformation

Deriving accurate stress-strain response becomes much more complicated once a material’s yield strength has been exceeded. When yielding occurs, not only is the relationship between stresses and strains no longer linear, but changes in material constitutive behavior due to phenomena such as cyclic hardening or softening and non-proportional hardening may also need to be considered.
For simple cases such as uniaxial or pure torsion loading of a cyclically stabilized material, a simple relationship can be derived through the fitting of cyclic stress-strain data, generated through fatigue testing or incremental step tests, to relate elastic-plastic stresses and strains under these conditions. Ramberg and Osgood (1943) proposed such a relation using a power law to describe the elastic-plastic portion of the stress-strain curve. The Ramberg-Osgood relation, in terms of uniaxial stresses and strains, is expressed as follows:

\[
\varepsilon_a = \frac{\Delta \varepsilon}{2} = \frac{\Delta \sigma}{2E} + \left(\frac{\Delta \sigma}{2K'}\right)^{\frac{1}{n'}}
\]

(2.4)

where \(E\) is Young’s modulus, \(K'\) is the cyclic axial strength coefficient, and \(n'\) is the cyclic axial strain hardening exponent. The equation can also be rewritten in terms of shear quantities as follows:

\[
\gamma_a = \frac{\Delta \gamma}{2} = \frac{\Delta \tau}{2G} + \left(\frac{\Delta \tau}{2K_o'}\right)^{\frac{1}{n_o'}}
\]

(2.5)

where \(G\) is the shear modulus, \(K_o'\) is the cyclic shear strength coefficient, and \(n_o'\) is the cyclic shear strain hardening exponent.

For general multiaxial stress states and/or complex variable amplitude loading histories, however, phenomena such as Bauschinger effect, cyclic hardening/softening, non-proportional hardening, and material memory effect may also need to be accounted for. A popular method used to model material behavior under these more complex loadings is through the application of incremental cyclic plasticity models. These plasticity models, and the constitutive relations that govern them, are based on elasticity and plasticity theories, along with continuum mechanics concepts and experimental observations which describe the stress-strain response of a material under a given set of
loading conditions (Jiang, 1993). Numerous plasticity models and theories have been proposed over the years for application to multiaxial cyclic loading. While some are based on simple concepts and generalizations, others involve more complex mathematical formulations and/or require a large number of material constants as input (Lee et al., 2012).

For application to fatigue life analysis, it is desirable to have a relatively simple model that is capable of accurately modeling stress-strain response under cyclic loading. In this respect, the complexity of plasticity models can be considerably reduced by making some assumptions. For most metals at or near room temperature, it is a reasonable assumption that temperature and loading rate will not significantly affect the material’s constitutive behavior. Therefore, rate independent and isothermal material response is assumed in many cases. In addition, most metals can be assumed as uniform, homogeneous, and initially isotropic (a material which has been plastically deformed is no longer isotropic). Furthermore, although Young’s modulus, \( E \), and Poisson’s ratio, \( \nu \), are generally not constant after yielding, there is a minimal influence on their values as a result of small plastic deformations and they are typically considered to be constant (Jiang, 1993). Since these assumptions hold for the purposes of this study, the following discussion on plasticity models will consider only these cases.

Although there are many types of cyclic plasticity models available for stress-strain estimation (e.g. overlay, single surface, multi-surface, two-surface, endochronic, etc.), the focus of this literature review is on single yield surface models due to their popularity and robustness in cases involving both proportional and non-proportional loadings (Chaboche, 2008; Halama et al., 2012; Jiang, 1993; Jiang and Kurath, 1996).
These models assume that the present state of the material depends only on the present values of observable variables and a set of internal state variables. Regardless of the specific formulation used, however, there are several elements which are essential to any cyclic plasticity model.

The most basic component of any cyclic plasticity model is the yield function. This function defines the shape, size, and position of the yield surface, and determines whether or not plastic deformation occurs for a given state of stress. The yield function can be expressed in general terms as follows:

\[ f(\bar{\sigma}, \bar{\alpha}, \bar{\varepsilon}^p) = F(\bar{\sigma} - \bar{\alpha}) - g(\bar{\varepsilon}^p) \]  

(2.6)

Although their shape is generally assumed to be constant, yield surfaces are sometimes allowed to change in size (isotropic hardening) through a variable yield stress term, \( g(\bar{\varepsilon}^p) \), which is dependent on the accumulated plastic strain tensor. Additionally, the yield surface may also be allowed to translate (kinematic hardening) through a variable backstress term, \( \bar{\alpha} \). These changes in the yield surface allow for different characteristics of a material’s behavior to be accounted for during a particular loading path. When the value of the yield function, \( f \), is less than zero, the stress state is elastic. A plastic or neutral loading process, however, occurs at yield function values greater than or equal to zero. The two most commonly used yield functions are based on the von Mises (distortion energy) yield criterion and the Tresca (maximum shear stress) yield criterion.

In addition to a yield function, a flow rule is also needed in plasticity computations in order to relate the increment of plastic strain to the increment of stress when the trial stress state resides outside of the yield surface. The most commonly used flow rule for metallic materials is known as the associated flow rule, or normality
condition (Drucker, 1960, 1951). This states that the plastic strain increment is collinear with the exterior normal to the yield surface at the current stress state, and can be expressed as follows:

\[
d\bar{\varepsilon}^p = \frac{1}{h} (d\bar{\mathbf{S}}: \bar{\mathbf{n}}) \bar{\mathbf{n}}
\]  

(2.7)

where \( h \) is the plastic modulus function, \( d\bar{\varepsilon}^p \) is the increment in plastic strain tensor, \( d\bar{\mathbf{S}} \) is the increment in deviatoric stress tensor, and the unit exterior normal to the yield surface at the current stress state, \( \bar{\mathbf{n}} \), is defined as:

\[
\bar{\mathbf{n}} = \frac{\bar{\mathbf{S}} - \bar{\alpha}}{(\bar{\mathbf{S}} - \bar{\alpha}) : (\bar{\mathbf{S}} - \bar{\alpha})}
\]  

(2.8)

In order to ensure that the current stress state always remains on the yield surface during elastic-plastic deformation, a consistency condition needs to be satisfied. In other words, there should be no change in the value of the yield function from one elastic-plastic process to another. This can be expressed mathematically as follows:

\[
d\bar{\mathbf{S}}: \bar{\mathbf{n}} - d\bar{\alpha}: \bar{\mathbf{n}} - \sqrt{2} d(g(\bar{\varepsilon}^p)) = 0
\]  

(2.9)

For cases where the size of the yield surface is considered to be constant, the last term to the left of the equals sign may be dropped, thus reducing Equation (2.9) to:

\[
d\bar{\mathbf{S}}: \bar{\mathbf{n}} = d\bar{\alpha}: \bar{\mathbf{n}}
\]  

(2.10)

This implies that, during an elastic-plastic loading process, the projection of the backstress increment is the same as the projection of the deviatoric stress increment onto the exterior unit normal of the yield surface. By combining Equations (2.7) and (2.9), the plastic modulus function, \( h \), can be defined as follows:

\[
h = \frac{\bar{\mathbf{n}}: d\bar{\alpha}}{dp} + \sqrt{2} \frac{d(g(\bar{\varepsilon}^p))}{dp}
\]  

(2.11)
where the equivalent plastic strain increment, $d\psi_p$, is defined as:

$$d\psi_p = \sqrt{d\bar{\varepsilon}^p : d\bar{\varepsilon}^p}$$  \hspace{1cm} (2.12)

For the majority of plasticity models, the formulations given to this point are common to them all. The hardening rule, however, which specifies changes in the yield surface during plastic deformation, is generally what differentiates one plasticity model from another. There are two basic types of hardening rules: isotropic and kinematic. Isotropic hardening rules assume that the initial yield surface expands or contracts uniformly with the onset of plasticity, but does not translate with respect to its original origin. In general, isotropic hardening may be representative of processes such as cyclic hardening or softening, which occur in the transient periods of a loading history before cyclic stability has been achieved. On the other hand, kinematic hardening assumes a yield surface that is fixed in size and shape, but translates as plastic deformation occurs. Because of this translation, kinematic hardening rules are able to account for Bauschinger effect, where a decrease in yield strength is observed following a change in the direction of loading. Being able to account for this phenomenon is a key requirement for cyclic loading applications.

In reality, the characteristics of cyclic material deformation can be a mix between isotropic and kinematic hardening behaviors. Therefore, some plasticity models may take into consideration both types of hardening. However, because the period of initial transient material behavior is relatively short compared to the total fatigue life for many loading histories, it is often ignored in a fatigue life analysis. Since isotropic hardening is not characteristic of a cyclically stabilized material, kinematic hardening is typically the only type of hardening considered in stress-strain computation for these applications. In
other cases though, such as variable amplitude loading histories with periodic overloads and/or varying degrees of loading non-proportionality, transient behavior may become important. Therefore, some kinematic hardening rules have been adapted to consider transient hardening/softening behavior through the modification of the hardening modulus in the kinematic evolution (Jiang, 1993).

The basis for many modern single surface kinematic hardening rules is the work of Armstrong and Frederick (1966), which introduced a nonlinear hardening relation through a recovery term for strain memory effect. The translation of the yield surface was described by the following relation:

\[ d\tilde{\alpha} = a_a d\tilde{\varepsilon}^p - c_a \tilde{\alpha} dp \]  
(2.13)

where \(a_a\) is a constant and \(c_a\) is a scalar function dependent on the plastic strain path. Although an improvement over previous linear hardening functions, this model generally leads to large overestimations of cyclic ratcheting and ratcheting rates that are not consistent with experimental observations (Ohno and Wang, 1993). Additionally, using only the two material constants given above, the model is usually unable to accurately describe the shape of stress-strain hysteresis loops.

In an attempt to improve the Armstrong-Frederick hardening relation, Chaboche \textit{et al.} (1979; Chaboche, 1987) expressed a hardening rule in the form of a series expansion of backstress components, where each component takes the same form as the Armstrong-Frederick relation. The backstress tensor thus assumes the following form:

\[ \tilde{\alpha} = \sum_{i=1}^{M} \tilde{\alpha}^{(i)} \]  
(2.14)

where
\[ d\bar{\alpha}^{(i)} = c^{(i)} \left( r^{(i)} d\bar{\varepsilon}^p - W^{(i)} \bar{\alpha}^{(i)} dp \right) \quad (i = 1,2,\ldots,M) \] (2.15)

c^{(i)} and \( r^{(i)} \) are material constants, while \( W^{(i)} \) is a function of the current stress state.

With the inclusion of the additional material constants for each backstress component, this hardening rule is able to describe the shape of stress-strain hysteresis loops much better than the single Armstrong-Frederick relation. In the original formulation of the Chaboche model, \( W^{(i)} = 1 \). However, due to problems that could arise under certain loading conditions, a threshold term was later introduced (Chaboche et al., 1991) which takes on the following form:

\[ W^{(i)} = \left( 1 - \frac{\xi^{(i)}}{\bar{\alpha}^{(i)} : \bar{\alpha}^{(i)}} \right) \quad (i = 1,2,\ldots,M) \] (2.16)

The threshold value for the dynamic recovery of the \( i \)th backstress component, \( \bar{\alpha}^{(i)} \), is \( \xi^{(i)} \). With the inclusion of this threshold term, the hardening function is able to take on either a linear form (\( W^{(i)} = 0 \), when the backstress component is below a certain level, or the traditional nonlinear form modified by a value of \( W^{(i)} \) between 0 and 1. This, in turn, helps to reduce the overestimations of cyclic ratcheting predicted by previous models.

Since the development of the Chaboche model, many other nonlinear kinematic hardening formulations have been proposed over the years with the primary goal of obtaining better ratcheting prediction under complex loading conditions. However, most of these are still based on the superposition of several backstress components as originally proposed by Chaboche, but with modifications of the original form. A table comparing many of these proposed modifications was compiled by Halama et al. (2012) and has been reproduced in Table 2.1.
In the case of multiaxial stresses and strains, loading conditions may be described as either proportional or non-proportional. Proportional loading occurs when two or more types of cyclic loading occur at the same frequency and with the maximum and minimum values of each load component occurring at the same time. In this situation, the orientation of the principal loading directions remains fixed in time and the ratios of the various loads are constant. On the other hand, for non-proportional loading, loading frequencies and/or phasing of maximum and minimum values may vary independently for each applied load. These loading conditions result in a constant change in the orientation of the principal loading directions throughout the loading history. Figure 2.2 illustrates schematically the differences in stress path and principal loading directions between a proportional (in-phase) and 90° out-of-phase non-proportional loading cycle. Non-proportional loading may also occur in situations where loading components shift from a proportional loading condition with one principal loading direction, to another combination of proportional loads resulting in a different principal loading direction. In order to distinguish this type of behavior from non-proportionality due to out-of-phase loading paths, additional hardening caused by sudden changes in the principal loading direction, from one proportional loading path to another, is often termed cross-hardening.

It is a commonly observed phenomenon that non-proportional loading can result in a higher resistance to plastic flow than proportional loading in certain materials (i.e. non-proportional hardening). First observed by Taira et al. (1968), this additional hardening was later explained by Kanazawa et al. (1979) as an effect of slip plane interaction caused by changes in slip orientation as the maximum shear plane rotates during non-proportional loading. This can prevent the development of stable dislocation
structures within the material, which would normally exist under proportional loading, and results in additional cyclic hardening. The level of non-proportional hardening experienced by a material is dependent on the shape and sequence of the loading path, as well as the loading amplitude and material microstructure. The maximum level of non-proportional hardening occurs for $90^\circ$ out-of-phase loading conditions, which result in an elliptic or diamond shaped stress or strain path, depending on the type of waveform applied. These stress-strain paths result in complete rotation of the principal axes and activate slip planes in all directions (Shamsaei, 2010).

Although the effect of non-proportional hardening can be significant, not all materials experience this phenomenon. Doong et al. (1990) conducted a study using polycrystalline 1100 aluminum, OFHC copper, 304 stainless steel, and 310 stainless steel in order to evaluate the effects of microstructure and slip character on non-proportional hardening. It was found that the aluminum, which showed no additional hardening under non-proportional loading, has a high Stacking Fault Energy (SFE) and exhibits wavy slip. The stainless steels, on the other hand, showed large amounts of non-proportional hardening and were characterized by low SFE and planar slip. The OFHC copper fell between the aluminum and stainless steels in terms of both non-proportional hardening and SFE. Therefore, the sensitivity of a material to non-proportional loading can be related, qualitatively at least, to its stacking fault energy.

Building on these findings, Borodii and Shukaev (2007) noted that for a large variety of materials, the amount non-proportional hardening experienced by the material showed the tendency to increase with an increase in the amount of strain hardening observed in monotonic tension tests. By relating the amount of strain hardening to the
ratio of ultimate strength to yield strength, they proposed the following empirical relationship to approximate the maximum expected non-proportional hardening for a given material:

$$\log(\alpha_{NP}) = 0.705 \frac{\sigma_u}{\sigma_y} - 1.925$$  \hspace{1cm} (2.17)

The non-proportional cyclic hardening coefficient, $\alpha_{NP}$, is defined as follows:

$$\alpha_{NP} = \frac{\sigma_{eq,90^\circ OP}}{\sigma_{eq,IP}} - 1$$  \hspace{1cm} (2.18)

where $\sigma_{eq,90^\circ OP}$ is the $90^\circ$ out-of-phase equivalent stress amplitude and $\sigma_{eq,IP}$ is the in-phase equivalent stress amplitude at the same equivalent strain amplitude level. Realizing that this relation was limited due to its lack of strain level dependence, they suggested bounds of $\Delta\varepsilon_p/2 > 0.02\%$ and $\Delta\varepsilon/2 < 1\%$ for its applicability. Within this range of strain, it was found that Equations (2.17) and (2.18) were able to consistently predict non-proportional stress amplitudes within an error of $\pm10\%$ for the majority of the 20 materials investigated.

Similarly, Shamsaei and Fatemi (2010) observed that cyclic hardening materials also tend to exhibit non-proportional hardening, while cyclic softening materials typically do not. Considering that both cyclic hardening and non-proportional hardening are associated with SFE, they concluded that microstructural features and deformation mechanisms affect non-proportional hardening to much the same extent as cyclic hardening. A relationship was then proposed between the non-proportional cyclic hardening coefficient and the uniaxial monotonic and cyclic deformation properties of a material. The empirical relation is given as follows:

$$\alpha_{NP} = 1.6 \left( \frac{K}{K'} \right)^2 \left( \frac{\Delta\varepsilon}{2} \right)^2(n-n') - 3.8 \left( \frac{K}{K'} \right) \left( \frac{\Delta\varepsilon}{2} \right)^{n-n'} + 2.2$$  \hspace{1cm} (2.19)
where \( K \) and \( n \) are the monotonic strength coefficient and strain hardening exponent, respectively, and \( K' \) and \( n' \) are the cyclic strength coefficient and cyclic strain hardening exponent, respectively. They reported excellent correlation between Equation (2.19) and experimentally calculated non-proportional cyclic hardening coefficients for several materials found in literature.

Numerous methods have been proposed over the years to account for the effects of additional non-proportional hardening in the prediction of material stress-strain response. These methods fall into two major groups: empirical formulations and constitutive relations. Constitutive relations are generally considered within the context of cyclic plasticity models and use continuum mechanics concepts to relate the stress-strain response of a material to its internal state under a given loading path. Empirical relations, on the other hand, relate the shape of a loading path to a factor of non-proportionality. This factor of non-proportionality is then used in conjunction with the material’s non-proportional cyclic hardening coefficient to modify the stress-strain response of the material (Shamsaei et al., 2010a).

One of the more popular empirical methods was proposed by Kanazawa et al. (1979). This method considers slip band interaction, caused by changing maximum shear planes, by defining the factor of non-proportionality, \( F \), as the fraction of shear strain at 45° to the maximum shear strain range. When looked at from a Mohr’s circle of strain standpoint, the factor of non-proportionality is closely related to the ratio of the minor to major axes of an ellipse formed by completely enclosing a non-proportional strain path. Other empirical methods are similar in concept and include, among others, those proposed by Itoh et al. (1995) and Kida et al. (1997). Although the empirical methods are
simple and easy to implement, they can often result in inaccurate predictions of stress-strain response. For example, Shamsaei et al. (2010a) found that such methods often overestimate the stress response under in-phase strain-controlled loading paths featuring a gradual change in straining direction. In addition, they noted that empirical methods cannot account for the progression of cross-hardening when sudden changes in loading direction occur.

Constitutive models, although more difficult to implement, usually result in much better predictions of stress-strain response under complex loading conditions (Fatemi and Shamsaei, 2011). McDowell (1985) introduced a non-proportionality parameter into material constitutive relations using a two surface Mroz type plasticity model. The parameter related the non-proportionality of the loading to the time derivative of principal strain. Alternatively, Benallal and Marquis (1987) used the angle between the stress and plastic strain rate to consider the non-proportionality effect of the loading. This effect was then incorporated into the cyclic plasticity model, via the flow rule, through an increase in yield stress. Fan and Peng (1991) later proposed two parameters, based on the Benallal and Marquis non-proportionality factor, to distinguish between cross hardening due to sudden changes in loading direction, and the typical hardening associated with the non-proportionality of a loading path.

However, the non-proportionality parameter which has received the most attention following its introduction is that proposed by Tanaka (1994). Tanaka proposed a scalar non-proportionality parameter, $A$, based on the state of internal dislocation structures. This was achieved by considering non-proportionality to be a function of the
normalized plastic strain rate vector and the internal microstructure of the material, represented by the following expression:

\[ A = \sqrt{1 - \frac{n_{ab}C_{pqab}C_{pqca}n_{cd}}{C_{ijkl}C_{ijkl}}} \]  \hspace{1cm} (2.20)

where \( \mathbf{n} \) is the unit normal to the yield surface (Equation (2.8)) and \( \mathbf{C} \) is a fourth rank tensor that is initially zero and changes with the loading increment according to the following relation:

\[ d\mathbf{C} = C_c(n_{ij}n_{kl} - C_{ijkl})dp \]  \hspace{1cm} (2.21)

\( C_c \) is a material constant related to the rate at which microstructural changes occur with variations in the level of non-proportionality. Based on this parameter, consistent results were observed between predicted and experimentally observed stress-strain response for 316 stainless steel under a variety of loading paths (Tanaka, 1994). Additionally, Shamsaei et al. (2010a) reported satisfactory stress predictions in all cases for a number of discriminating strain paths using Tanaka’s non-proportionality parameter coupled with an Armstrong-Frederick type cyclic plasticity model. Tests were performed using 1050 quenched and tempered steel (showing no additional non-proportional hardening) as well as 304L stainless steel (showing significant non-proportional hardening).

### 2.3.3 Elastic-Plastic Notch Deformation

The previous section discussed the calculation of elastic-plastic stresses and strains under the framework of incremental cyclic plasticity models. While these models are directly applicable when stress or strain values are known for the desired analysis location, this is typically not the case for a component containing a notch. In notched components, it is usually the nominal loading that is known, and local stresses and/or
strains must be estimated from the notch geometry, material properties, and applied loading history. Finding a reliable method to calculate local elastic-plastic stress and strains from nominal values has been an area of much research interest over the years.

Perhaps the best way to obtain local stress-strain histories would be through the use of incremental nonlinear finite element analysis. Nonlinear FEA combines the finite element method with the cyclic plasticity constitutive relations discussed in the previous section in order to compute elastic-plastic stresses and strains at any point within a component. Although this method can produce highly accurate local stress-strain values (depending on the plasticity model employed), the process is computationally expensive and highly impractical in cases involving complex component geometry and/or long loading histories. Therefore, it is desirable to have a simpler method to relate nominal loading to local stresses and strains without sacrificing a significant amount of accuracy in the calculation. In most cases, this is achieved by applying a notch stress-strain estimation model, in conjunction with material constitutive relations, to calculate stresses and strains at only the critical locations within a component.

Typical notch stress-strain estimation models approximate elastic-plastic notch root stresses and strains by relating them to the stresses and strains that would exist at the notch root if the material were to theoretically remain linear elastic beyond its yield point. These theoretical elastic stresses and strains are often referred to as pseudo stresses, $\sigma^e$, and pseudo strains, $\varepsilon^e$, (Köttgen et al., 1995) and can be computed using the methods outlined in Section 2.3.1 for local linear elastic deformation behavior. Using pseudo stress or strain histories as a starting point for elastic-plastic notch analysis greatly simplifies the derivation of local stresses and strains.
Some of the earliest notch stress-strain estimation models to gain widespread popularity include those proposed by Neuber (1961) and Glinka (1985). Neuber’s rule, which was developed based on a notched prismatic body loaded in pure shear, equates the geometric mean of the elastic-plastic stress and strain concentration factors to the elastic stress concentration factor. The most common expression for Neuber’s rule is given as follows:

\[
\frac{(K_t S)^2}{E} = \sigma^e \varepsilon^e = \sigma \varepsilon
\]  

(2.22)

where \(K_t\) is the elastic stress concentration factor, \(S\) is the nominally applied elastic stress, \(E\) is Young’s modulus, \(\sigma^e\) and \(\varepsilon^e\) are local pseudo stress and strain respectively, and \(\sigma\) and \(\varepsilon\) are local elastic-plastic stress and strain at the notch, respectively. Glinka’s rule, also referred to as the Equivalent Strain Energy Density (ESED) rule, on the other hand, is based on the assumption that the strain energy density at the notch root is the same for both linear elastic and elastic-plastic material behavior as long as the notch plastic zone is surrounded by an elastic stress field. Equating elastic strain energy density to elastic-plastic strain energy density results in the following relation:

\[
\int_0^\varepsilon^e \sigma^e \, d\varepsilon^e = \int_0^\varepsilon^p \sigma \, d\varepsilon
\]  

(2.23)

Of these two notch stress-strain estimation rules, Glinka’s rule has been shown to agree better for plane strain conditions at the notch root, while Neuber’s rule tends to agree better for plane stress conditions (Sharpe et al., 1992; Stephens et al., 2000). Although modified versions of these models have been proposed, which attempt to unify their theoretical differences for application to all notch geometries (Zappalorto and Kujawski, 2015), these models are relatively recent and have not yet been extensively
evaluated. As a result, because of the conservative tendency of local stress-strain predictions, Neuber’s rule has proven to be the most popular method of deriving local stresses and strains for use in fatigue damage calculations. Consequently, it has been modified and adapted many times since it was first introduced in order to be applied to more complex multiaxial loading conditions (Tipton, 1991).

One of the most popular generalizations of Neuber’s rule to multiaxial loadings was proposed by Hoffmann and Seeger (1989). This approach first establishes a relationship between the applied loading and local stress and strain magnitude by replacing the uniaxial quantities in Neuber’s rule with equivalent stresses and strains as follows:

\[
\left( \frac{K_{tq}S}{E} \right)^2 = \sigma_q \varepsilon_q
\]

(2.24)

where \( K_{tq} \) is the ratio of the theoretical elastic von Mises equivalent notch stress to the nominal stress, \( S \). Nominal stress may be defined in any way which is convenient for the calculation. On the right-hand side of the equation, \( \sigma_q \) and \( \varepsilon_q \) are the local elastic-plastic von Mises stress and strain, respectively. Next, the principal stresses and strains at the notch root are related to the equivalent stresses and strains by applying Hencky’s flow rule, which effectively leads to a generalization of Hooke’s law based on an equivalent elastic-plastic modulus and effective Poisson’s ratio, in conjunction with some assumptions concerning the principal stress and strain components. The first assumption is that the principal stress direction remains fixed during loading, and is equal for both elastic and elastic-plastic conditions. The second assumption is that the ratio of local surface strains remains constant under both elastic and elastic-plastic conditions. These assumptions are represented by the following relations, respectively:
For use in cyclic loading situations, the stress and strain quantities are replaced by their respective stress and strain ranges. Reducing a cyclic loading history into a sequence of monotonic loadings, by using stress and strain ranges and expressing the cyclic stress-strain curve in terms of stress/strain range, this method then allows for the calculation of local stresses and strains at the notch root for any given fatigue loading history.

Moftakhar *et al.* (1995), however, noted some problems with the assumptions of Hoffmann and Seeger. Namely, that the constant surface strain ratio and the energy relations of Neuber are in contradiction when the complete set of equations is reduced to a uniaxial stress state. Additionally, they noted that the proportionality of notch tip strain components only holds for circumferential notches. As a result, the authors proposed that the equivalence of the fractional contribution of the largest principal notch tip stress and strain components to the total strain energy density be used instead. This can be expressed by the following equation:

\[
\frac{\sigma_1 e_1}{\sigma_{ij} e_{ij}} = \frac{\sigma_1 e_1}{\sigma_{ij} e_{ij}}
\]  

Combining this equation with material constitutive relations and either Neuber’s rule or the ESED rule, the authors showed that experimental results for notch root stresses and strains under a variety of loading conditions fell between the two predicted solutions.

Although these generalized Neuber or ESED approaches have been shown to provide reasonable correlation to experimental results, they are not without their shortcomings. Perhaps the most notable is that because of the use of Hencky’s flow rule,
with the assumption of a constant principal stress direction, they are only applicable when proportional, or nearly proportional, loading paths exist at the notch root. In many situations, however, this is not the case. When plastic deformation occurs at a notch, the stress path is almost always non-proportional, regardless of whether or not the nominal loading is proportional (Buczynski and Glinka, 2000).

To overcome this problem, Buczynski and Glinka (2000) coupled an incremental form of Neuber’s rule, equating the increments of total distortional strain energy density contributed by each pair of associated stress and strain components, with a cyclic plasticity model (Mroz and Garud in this case) in order to estimate local stresses and strains for general multiaxial loadings. Doing so allows for material memory effects and changes in constitutive behavior due to non-proportional hardening to be accounted for in an analysis as well. For the case of a circumferentially grooved round bar subjected to multiaxial cyclic loading, the authors found good agreement between predictions and experimental results for both notch tip stresses and strains using this approach. Additionally, later publications by Ince et al. (2014) and Ince and Glinka (2016) focus on combining this method with computational modeling techniques to allow for direct post-processing of linear elastic FEA results.

Different in concept from the generalizations of uniaxial notch stress-strain estimations models discussed to this point, a method was proposed by Barkey et al. (1994), under the framework of incremental plasticity, to estimate notch root stresses and strains under general, proportional and non-proportional, multiaxial loadings. Sometimes referred to as a pseudo-material approach, it works by first introducing a “structural yield surface” and a nominal stress-local strain relation. This relation can be derived through
nonlinear FEA, direct measurement, or the application of a uniaxial approximation formula such as Neuber’s rule or ESED. Notch root strains are then computed directly from the nominal loading through the use of a cyclic plasticity model. The directionality of work hardening caused by the notch is accounted for through the use of an anisotropic yield function. Directional yield strengths are defined in terms of the stress concentration factors for each local stress component and the material isotropic yield strength. Stress-strain predictions based on this formulation were found to be in good agreement, both qualitatively and quantitatively, with experimental data for large radius circumferentially grooved solid shaft specimens of 1070 steel under a number of proportional and non-proportional load paths.

Noting that the anisotropic yield criterion imposes certain limitations on the types of loading that can be analyzed (e.g. elastic coupling at the notch root between any two nominal stresses, as in combined axial and bending loading of a grooved bar, is not allowed), Köttgen et al. (1995) proposed pseudo stress and pseudo strain-based approaches for notch analysis which can be applied to any notch geometry and loading using standard isotropic plasticity models. This was again done through the introduction of a structural yield surface concept. The first step in such an analysis is to derive a pseudo stress-local notch strain curve for the pseudo stress approach, or a pseudo strain-local notch stress curve for the pseudo strain approach. This curve represents the stress-strain response of the structure and, again, can be derived through nonlinear FEA, direct measurement, or the application of a uniaxial approximation formula such as Neuber’s rule or ESED. Next, this curve, along with the pseudo stress or pseudo strain history, are input into any standard incremental cyclic plasticity model to calculate elastic-plastic
notch strains in the case of the pseudo stress approach, or elastic-plastic notch stresses in the case of the pseudo strain approach. From there, the elastic-plastic stresses or strains are input back into the plasticity model, along with the material cyclic stress-strain curve, to complete the computation of elastic-plastic notch stress and strain histories. A schematic showing the evolution of the pseudo stress approach is shown in Figure 2.3 in terms of relevant stress-strain curves. After comparing predictions from both approaches to FEA results for a variety of notch geometries, materials, and loading paths, Köttgen et al. concluded that the pseudo strain approach is well suited for application to sharper notches, while the pseudo stress approach may be more appropriate for cases involving components with mild notches at higher load levels.

Lee et al. (1995) proposed an almost identical approach to that of Köttgen et al. (1995), but used a two-surface kinematic plasticity model instead of a multi-surface model. Later, Gu and Lee (1997) extended the same procedure by using an endochronic plasticity model to calculate local stresses and strains.

McDonald and Socie (2011), however, cited some disadvantages of the pseudo-material approaches. Among these is the coupling of notch geometry to the true material response. Additionally, they noted that these approaches are not necessarily compatible with the more traditional models such as Nueber’s rule or ESED. To overcome these disadvantages, the authors proposed a generalized method of estimating notch root stresses and strains by adopting a directional alignment criterion (e.g. stress components aligned before and after deformation, strain components aligned, or mixed) and applying a magnitude condition (e.g. Neuber or ESED) as a modified boundary condition. The resulting method retains the advantages of a pseudo-material approach, but decouples the
geometrical and local material deformation behavior. Comparing predictions based on this model to experimental results for incremental step and box loading paths, the authors reported adequate predictions for both stress and strain values. The stress aligned case was found to most closely reflect the local strain paths observed in the experiments.

### 2.3.4 Stress and Strain Gradient Effects

One problem often encountered in a notch fatigue analysis is the consideration of stress and/or strain gradient effects. Normally, when stress concentration factors are defined for a particular notch geometry, the values are based on the point at the notch root with the largest concentration of stress or strain quantities. However, there usually exists a steep stress/strain gradient moving away from the notch root as well. This gradient is an important consideration in fatigue analyses because the mechanisms which cause fatigue damage take place within a finite volume of material, rather than at a single point. Therefore, the stress-strain values considered in a fatigue analysis should be some type of averaged values that take into consideration the stress/strain variation over this volume. Not accounting for gradient effects at the notch root can often result in overly conservative fatigue life predictions (Tipton and Nelson, 1997; Topper et al., 1969).

One of the simplest and most common ways to correct for gradient effects is by using a fatigue notch factor, \( K_f \), when computing local stresses and strains from a notch stress-strain estimation rule such as Neuber’s or ESED (Topper et al., 1969). The fatigue notch factor is defined experimentally as the ratio of un-notched to notched fatigue strength at a particular fatigue life and varies depending on material properties, applied loading conditions, and notch geometry (DuQuesnay et al., 1986).
An approximation for the value of $K_f$ at long life (usually at least $10^6$ cycles) was proposed by Neuber (1958), as follows:

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\rho/r}}$$  \hspace{1cm} (2.28)\\

where $\rho$ is an empirically determined material dependent characteristic length and $r$ is the notch root radius. A similar alternate equation was also proposed by Peterson (1959). If the material characteristic length required for Equation (2.28) is not available, it can be estimated for steel alloys, based on ultimate tensile strength, according to the following empirical relation (Kuhn and Hardrath, 1952):

$$\log \rho = -\frac{\sigma_u - 134}{586}$$  \hspace{1cm} (2.29)

where $\rho$ is in mm and $\sigma_u$ is in MPa.

Topper et al. (1969) reported better agreement between fatigue life predictions and experimental results when using $K_f$ with Neuber’s rule as opposed to using $K_t$. However, the implementation of a fatigue notch factor has a number of potential drawbacks. For example, Equation (2.28) requires $K_t$ to be defined in terms of nominal and local stresses, and the determination of the material characteristic length is done empirically. Therefore, for components where the nominal stress is not clearly defined, the notch root radius is considered to be zero, or for materials where the characteristic length constant is not available, the application of these equations becomes challenging. Furthermore, for multiaxial non-proportional loadings, where the combined stress concentration effect from each applied loading continuously varies throughout a cycle, the definition of fatigue notch factor can become vague.
A more recent approach to stress/strain gradient consideration, which expands on the concepts proposed by Neuber and Peterson, but overcomes the problems associated with the fatigue notch factor, is the Theory of Critical Distances (Susmel, 2008; Taylor, 2008). The Theory of Critical Distances (TCD) refers to a group of several methods, based on fracture mechanics concepts, which can be used to compute averaged stress/strain values at a notch. All of the TCD methods are based on a material dependent characteristic length (critical distance), $L$, in conjunction with linear elastic stress analysis results. This critical distance is closely related to the crack transition length found from a Kitagawa–Takahashi diagram (discussed later), which represents the length at which fatigue damage/failure switches from being controlled by the fatigue limit to being controlled by the threshold stress intensity factor for a given material (essentially defining the bounds for a non-propagating crack in terms of applied stress level and crack length). The following equation gives the formulation of $L$, where $\Delta K_{th}$ is the Mode I threshold stress intensity factor range at a given load ratio and $\Delta \sigma_o$ is the fatigue limit stress range at the same load ratio:

$$L = \frac{1}{\pi} \left( \frac{\Delta K_{th}}{\Delta \sigma_o} \right)^2$$

(2.30)

Although the different TCD approaches interpret the meaning of this critical distance differently when it comes to how stresses or strains are averaged, they have all been shown to generally yield similar fatigue strength values (Susmel, 2008; Taylor, 2008).

The two most common TCD approaches are the point method and the line method, although area-based and volume-based methods have also been proposed. The point method accounts for gradient effects by considering stress/strain values at a distance of $L/2$ from the maximum stress or strain location at the notch root, along a path
normal to the notch curvature. The line method, on the other hand, averages the stress and/or strain values along a line, of length $2L$, starting at the maximum stress location and moving away from the notch in a direction normal to its curvature. A schematic illustration of both of these methods, along with their corresponding equations and relevant nomenclature, is presented in Figure 2.4.

TCD approaches have been shown to work well for a number of applications in both fatigue and fracture analysis (Taylor, 2008). Among these is the extension of stress gradient consideration under uniaxial loading to situations involving multiaxial and/or variable amplitude loading conditions (Susmel, 2008; Susmel and Taylor, 2012). Although it was shown that the critical distance value is generally smaller under uniaxial loading than pure shear loading, the effect of assuming a value of $L$ in torsion equal to that under uniaxial loading was studied by Susmel and Taylor (2006). They found that experimental and predicted fatigue lives were mostly within a factor of ±25% error (for both point and line methods) when using this assumption. Additionally, since $L$ under torsion was generally larger than the assumed value, the predictions had a conservative tendency for torsion loading. Therefore, it was concluded that the TCD methods could reasonably be extended to constant amplitude multiaxial loading conditions by assuming a constant $L$ value, equal to that under fully-reversed uniaxial loading.

In multiaxial applications of TCD, mean stresses and out-of-phase loading effects on a given plane are accounted for through an appropriate multiaxial fatigue damage parameter. Two different interpretations of the critical distance path/plane orientation were then proposed. For the first interpretation, using the point method as an example, the location at which fatigue damage is assessed corresponds to the point a distance of
$L/2$ away from, and along a path perpendicular to, the notch root. The second interpretation, on the other hand, considers the point at a distance of $L/2$ along a path starting at the notch root and moving outward at an angle, $\theta^*$, corresponding to the plane of maximum shear or normal stress (depending on the cracking behavior) at the notch root. No comparisons to experimental results were available, however, to evaluate the suitability of these two different interpretations.

Another concern in the application of TCD concepts is that the critical distance value tends to increase with a decrease in fatigue life under constant amplitude loading conditions (Susmel and Taylor, 2011). This increase accounts for an increasing difference between the linear elastic and elastic-plastic stress-strain fields surrounding a notch as the amount of localized plasticity increases. As a result, it was proposed that a simple power law relation could be used to represent the $L$ vs. $N_f$ curve, and that the appropriate critical distance value and corresponding fatigue life could be calculated numerically using a recursive procedure.

However, in the case of multiaxial variable amplitude loadings, each cycle could potentially have a different $L$ value. Therefore, a more elaborate procedure was proposed (Susmel and Taylor, 2012) from which an appropriate single $L$ value can be determined for an entire variable amplitude loading history. This procedure essentially involves the same recursive solving technique that would be required for constant amplitude loadings, but instead uses an equivalent $N_f$ value in the material $L$ vs. $N_f$ curve. This equivalent value is calculated based on the damage sum found through the use of an appropriate multiaxial fatigue damage parameter and cumulative damage rule. Using a linear damage
rule as an example, the formulation for the equivalent cycles to failure, $N_{f,eq}$, can be expressed as follows:

$$N_{f,eq} = \frac{\sum_{i=1}^{j} n_i}{\sum_{i=1}^{j} \frac{n_i}{N_{f,i}}}$$ (2.31)

where $n_i$ is the number of cycles at a given load level, $i$, $N_{f,i}$ is the number of cycles to failure at load level $i$ (based on chosen damage parameter), and $j$ is the total number of load levels.

It should be noted, however, that in a study of notched uniaxial fatigue behavior (Susmel and Taylor, 2010), it was shown that if elastic-plastic behavior is taken into account explicitly during a fatigue life analysis, accurate life predictions can be obtained by considering the value of $L$ to remain constant regardless of fatigue life. This, in turn, would make the additional $L$ vs. $N_f$ relations (and thus the recursive solving technique) unnecessary for both constant and variable amplitude loading conditions.

### 2.4 Multiaxial Fatigue Damage Parameters

Once stress-strain histories are known for the fatigue critical location(s) within a component, relating the variation of these stresses and strains to the fatigue damage that occurs within a material is the most fundamental step in any fatigue life analysis. However, many different methodologies for fatigue damage calculation exist, and due to the number of variables involved, no approach has been found to produce consistently accurate fatigue life predictions regardless of the material and/or loading history under consideration. This is especially true when multiaxial stress states are present.
Early multiaxial fatigue damage parameters, sometimes referred to as classical approaches, were often focused on computing an equivalent stress quantity through the extension of static yield criteria. This equivalent stress is then considered to be equal, in terms of fatigue damage, to a uniaxial stress of the same magnitude. A corresponding life prediction can then be obtained from a uniaxial fatigue life curve. Examples of some stress-based multiaxial fatigue criteria include von Mises equivalent stress and maximum shear stress for ductile behaving materials, and maximum principal stress for brittle behaving materials. In the event that mean or residual stresses are present, an additional relation is needed to compute an equivalent mean stress for use in a uniaxial mean stress correction model. Common approaches for mean stress consideration include von Mises equivalent mean stress and the sum of the mean normal stress components, which is proportional to the hydrostatic stress (Socie and Marquis, 2000; Stephens et al., 2000).

Similar to the stress-based approaches, strain-based approaches were also proposed as multiaxial fatigue damage parameters. Analogous to equivalent stress criteria, equivalent strain quantities including octahedral shear strain, maximum shear strain, and maximum principal strain, have also been used to relate multiaxial fatigue damage to equivalent uniaxial loadings.

Although these classical approaches are simple in concept and easy to implement, they typically do not reflect the damage mechanisms at work within the material (Socie and Marquis, 2000). It has been shown in multiple studies that effective stress or strain parameters such as von Mises fail to bring together fatigue data even for the two simple cases of uniaxial and pure torsion loadings, for example in (Gladskyi and Fatemi, 2013; Sakane et al., 2011). Additionally, they generally cannot account for increased fatigue
damage observed under non-proportional loading conditions (Fatemi and Socie, 1989; Fatemi and Stephens, 1989; Lopez-Crespo et al., 2015; Tipton and Nelson, 1997) and have been shown to result in inadequate correlation of fatigue data from tests performed under different ratios of axial to shear stress (Fatemi and Socie, 1989; Fatemi and Stephens, 1989).

In an attempt to account for the effects of non-proportional loadings, however, modified equivalent stress-based approaches have also been proposed by Sonsino (1995) and Lee et al. (2007). Sonsino’s approach, which is only applicable when constant phase differences exist between sinusoidal axial and shear stress signals, accounts for increases in fatigue damage through the calculation of an effective shear stress quantity. This effective shear stress is derived based on the interaction of shear stress components on various interference planes, and is used to compute an effective von Mises stress amplitude. The approach of Lee et al., on the other hand, accounts for increases in fatigue damage by modifying the equivalent stress amplitude based on a path dependent non-proportionality factor and the material non-proportional hardening coefficient.

In addition to stress- and strain-based approaches, energy-based approaches may also be used to quantify multiaxial fatigue damage. Energy approaches relate fatigue damage to the products of stresses and strains and, therefore, are able to reflect variations in damage due to changes in material constitutive behavior. Several energy parameters have been proposed over the years including the plastic work per cycle and the total strain energy density per cycle models of Ellyin (1997) and Park and Nelson (2000), respectively. Although energy parameters are applicable to both proportional and non-proportional loadings, their output is a scalar quantity, which does not reflect the nature
of observed damage mechanisms. In effect, they fail to associate fatigue damage with a preferred failure plane, such as the maximum shear or the maximum principal stress/strain plane(s).

In order to overcome the shortcomings of classical multiaxial fatigue damage parameters, significant effort in the last few decades has been put into developing more sophisticated damage parameters which reflect the actual damage mechanisms of the fatigue failure process (Fatemi and Shamsaei, 2011; Socie and Marquis, 2000; You and Lee, 1996). Chief among these are critical plane approaches, which build on the idea that fatigue cracks tend to initiate on preferred planes within a material. Critical plane approaches are typically based on the idea of crack initiation occurring on or around either the maximum principal plane or the maximum shear plane(s). As a result, in addition to general applicability to both proportional and non-proportional multiaxial loadings, these approaches have the added benefit of being able to predict failure plane orientation, which is useful information if a subsequent crack growth analysis is to be performed. Similar to the classical fatigue damage parameters, critical plane approaches can be stress-based, strain-based, energy-based, or they can be based on some other combination of stresses and strains.

Stress-based parameters are best suited for the high cycle fatigue regime, where local stresses and strains remain elastic. After reviewing experimental data for combined loading cases, Findley (1959) suggested that the normal stress, \( \sigma_n \), acting on a shear stress plane would affect the allowable alternating shear stress, \( \Delta \tau/2 \), for materials exhibiting crack initiation on maximum shear planes. Cyclic shear stresses are considered to be primarily responsible for crack nucleation while normal stress assists in the development
and growth of microscopic fatigue cracks. A linear influence of normal stress was proposed as follows:

\[
\left( \frac{\Delta \tau}{2} + k \sigma_n \right)_{\text{max}} = \sqrt{1 + k^2} \tau_f' \left( 2N_f \right)^{b_o}
\]  

(2.32)

where \(k\) is a material dependent parameter reflecting the influence of normal stress on fatigue damage which typically varies between 0.2 and 0.3 for ductile behaving materials. Shear fatigue strength coefficient and shear fatigue strength exponent are represented by \(\tau_f'\) and \(b_o\) respectively, and \(N_f\) is the corresponding cycles to failure. The critical plane is considered to be the plane experiencing the highest value of the damage parameter.

Similar to the damage parameter of Findley, McDiarmid (1987) proposed that the maximum normal stress and shear stress acting on the plane experiencing the maximum range of shear stress could be related to fatigue life as follows:

\[
\frac{\Delta \tau_{\text{max}}}{2} + \frac{t_{A,B}}{2\sigma_u} \sigma_{n,\text{max}} = \tau_f' \left( 2N_f \right)^{b_o}
\]  

(2.33)

where \(k\) in the Findley parameter has been replaced by a function of the shear fatigue strength for case A or B cracking, \(t_{A,B}\), and the material ultimate tensile strength, \(\sigma_u\). The critical plane in the McDiarmid model is considered to be the plane experiencing the maximum range of shear stress, rather than the plane experiencing the maximum value of the damage parameter.

In cases where significant plasticity is present, strain-based critical plane models are more appropriate than their stress-based counterparts. A popular strain-based damage parameter for materials exhibiting failure on shear planes was proposed by Brown and Miller (1973). Similar to the stress-based models of Findley and McDiarmid, but with strains in place of stresses, Brown and Miller related fatigue damage to the alternating
shear strain, $\Delta \gamma / 2$, and normal strain range, $\Delta \varepsilon_n$, occurring on the plane experiencing the maximum range of shear strain. The Brown-Miller parameter, and its corresponding fatigue life equation, is given as follows (Kandil et al., 1982):

$$\frac{\Delta \gamma_{max}}{2} + S \Delta \varepsilon_n = A \frac{\sigma'f}{E} (2N_f)^b + B \varepsilon'_f (2N_f)^c$$  \hspace{1cm} (2.34)

where $S$ is a material dependent parameter reflecting the influence of normal strain on fatigue damage, $\sigma'f$ and $b$ are the axial fatigue strength coefficient and fatigue strength exponent, respectively, $\varepsilon'_f$ and $c$ are the axial fatigue ductility coefficient and fatigue ductility exponent, respectively, and constants $A$ and $B$ are defined as follows:

$$A = 1 + v + S(1 - v)$$ \hspace{1cm} (2.35)

$$B = 1 + v_p + S(1 - v_p)$$ \hspace{1cm} (2.36)

where $v$ is elastic Poisson’s ratio and $v_p$ is Poisson’s ratio for fully plastic conditions, usually taken to be 0.5. Since the Brown-Miller parameter is based solely on strain quantities, however, it cannot account for the effects of mean stresses without an additional mean stress correction model.

As mentioned previously, multiaxial fatigue damage parameters based on only stress or only strain terms cannot reflect changes in material constitutive behavior due to phenomena such as cyclic and/or non-proportional hardening. Therefore, to overcome this limitation, critical plane approaches using both stress and strain quantities have also been proposed. One such parameter, which has become popular for application to materials exhibiting crack initiation and/or microcrack growth on maximum shear planes, is the Fatemi-Socie (FS) critical plane parameter (Fatemi and Socie, 1988). This parameter, based on a similar concept to previously discussed models, assumes that while alternating shear strain is the primary driving force behind fatigue crack initiation, the
maximum normal stress on the crack plane affects the nucleation and growth of small cracks by influencing the amount of friction and interlocking between opposing crack faces.

This concept is supported through biaxial fatigue data generated by Socie and Shield (1984) under several different multiaxial loading paths for Inconel 718. These paths all featured the same maximum shear and normal strain amplitudes, but different levels of mean normal stress on the maximum shear strain plane. By studying crack growth data for cracks up to around 2 mm in length, the crack growth rate was found to increase with increasing maximum normal stress, leading to shorter observed fatigue lives. The effect was found to be relatively small for cracks on the order of the material grain size, but increased with crack length.

Similarly, when studying the effects of static mean stress on shear-mode crack growth in tubular specimens of 1045 steel, Kaufman and Topper (2003) found that by increasing the tensile normal stress on the maximum shear plane, fatigue life continually decreased until a critical level of mean stress was applied. Conversely, by simultaneously applying axial and hoop stresses, the effect of compressive mean stress, normal to both maximum shear planes, was found to increase fatigue life. After studying crack front and fracture surface asperity profiles, this behavior was again attributed to varying levels of friction and mechanical interlocking between opposing crack faces. These findings are consistent with those reported in other studies, e.g. (Fatemi and Kurath, 1988), as well. Therefore, the inclusion of the maximum normal stress per cycle in the FS parameter not only predicts an increase in fatigue damage due to non-proportional loading, but it also
accounts for the effects of mean stress in a manner that holds physical significance, consistent with these experimental results.

The FS parameter predicts fatigue life in terms of shear fatigue properties based on the following equation:

\[
\frac{\Delta \gamma_{\text{max}}}{2} \left( 1 + k \frac{\sigma_{n,\text{max}}}{\sigma_y} \right) = \frac{\tau_f^i}{G} (2N_f)^{b_0} + \gamma_f^i (2N_f)^{c_0}
\]

where \( \Delta \gamma_{\text{max}} \) is the maximum range of shear strain experienced on any plane, \( \sigma_{n,\text{max}} \) is the maximum normal stress occurring on the same plane for the cycle of interest, \( \sigma_y \) is the material yield strength, and \( k \) is a material dependent parameter reflecting the influence of normal stress on fatigue damage (different from \( k \) in the Findley parameter). The maximum normal stress is normalized by yield strength as a means of preserving the unitless feature of strain.

The right-hand side of Equation (2.37) represents the shear strain-life curve for the material under consideration. In the event that shear fatigue properties are not available for damage calculation, the right side of this equation may alternatively be expressed in terms of uniaxial fatigue properties as follows (Stephens et al., 2000):

\[
FS = \left[ (1 + \nu_e) \frac{\sigma_f^i}{E} (2N_f)^b + (1 + \nu_p) \varepsilon_f^i (2N_f)^c \right] \left[ 1 + k \frac{\sigma_f^i}{2\sigma_y} (2N_f)^b \right]
\]

where \( \nu_e \) is elastic Poisson’s ratio, \( \nu_p \) is Poisson’s ratio for fully plastic conditions, usually taken to be 0.5, and all other fatigue properties correspond to the fully-reversed uniaxial strain-life equation.

Another key aspect of this model is that because the normal stress term is multiplied by the shear strain range, the FS parameter assumes that cyclic shear strain must be present in order for fatigue damage to occur. This is not the case for the previously
discussed critical plane parameters (e.g. Findley and McDiarmid) where, in certain situations, the summation of shear and maximum normal stress terms may lead to the prediction of fatigue damage on planes where only a static axial stress exists.

Overall, the FS parameter, based on either shear or uniaxial fatigue properties, has been shown to correlate experimental and predicted fatigue lives relatively well for a variety of materials and loading conditions. These include mild and high strength steels, stainless steels, superalloys, and aluminum alloys under various proportional and non-proportional loading paths with and without the inclusion of mean stresses, e.g. (Fatemi and Kurath, 1988; Firat, 2012; Gao et al., 2010; Han et al., 2002; Kim and Park, 1999; Lopez-Crespo et al., 2015; Morrow, 1988; Park and Nelson, 2000).

For materials exhibiting crack initiation and/or microcrack growth on maximum principal planes, the Smith-Watson-Topper (SWT) parameter (Smith et al., 1970) has gained acceptance as a suitable model for use with the critical plane concept. The SWT parameter relates fatigue damage to the product of the maximum principal strain amplitude, $\Delta \varepsilon_1 / 2$, and the maximum normal stress on the same plane, $\sigma_{n,max}$, as follows:

$$\frac{\Delta \varepsilon_1}{2} \sigma_{n,\text{max}} = \frac{\sigma_f'^2}{E} (2N_f)^2b + \sigma_f'e_f' (2N_f)^{b+c}$$

(2.39)

Similar to the FS parameter, the inclusion of the maximum normal stress term can implicitly account for the presence of mean stress. Numerous studies have reported good correlation between predicted and experimental fatigue lives for tensile failure mode materials, under a variety of multiaxial loading conditions, when using the SWT parameter, e.g. (Firat, 2012; Zhao and Jiang, 2008). However, noting that the SWT parameter can result in overly non-conservative life predictions in the presence of large compressive mean stresses, Kujawski (2014) proposed a deviatoric formulation of the
parameter, which only considers distortion energy for damage calculation. While producing results similar to the traditional parameter for positive to moderately negative mean stresses, the deviatoric SWT parameter was shown to produce significant improvements in fatigue life predictions for $R$ ratios less than approximately $-2$ (Kujawski, 2014; Kujawski and Sree, 2014).

Other energy-based critical plane models use a combination of both shear and tensile work to quantify fatigue damage. One such model, proposed by Liu (1993), is a generalization of a uniaxial parameter based on virtual strain energy (VSE). It includes both elastic and plastic work components and considers work done on specific planes within the material. The virtual strain energy model considers two possible failure modes for multiaxial loading: tensile failure or shear failure. Assuming tensile failure, the VSE parameter, and corresponding fatigue life curve, take the following form:

$$\Delta W_I = (\Delta \sigma_n \Delta \epsilon_n)_{max} + (\Delta \tau \Delta \gamma) = \frac{4\sigma_f'^2}{E}(2N_f)^{2b} + 4\sigma_f' \epsilon_f' (2N_f)^{b+c} \quad (2.40)$$

Damage is calculated by first identifying the plane containing the maximum amount of tensile work and then adding the shear work on that same plane. For shear failure, the process is similar: the plane with the maximum amount of shear work is identified and added to the corresponding tensile work on that plane. This is expressed as follows:

$$\Delta W_{II} = (\Delta \sigma_n \Delta \epsilon_n) + (\Delta \tau \Delta \gamma)_{max} = \frac{4\tau_f'^2}{G}(2N_f)^{2b_o} + 4\tau_f' \gamma_f' (2N_f)^{b_o+c_o} \quad (2.41)$$

Whichever assumed failure mode results in the largest VSE is the plane on which failure is expected to occur. One drawback to this method, however, is that the VSE must be modified with a separate correction term in order to account for the effects of mean stress on fatigue damage.
In order to account for the effects of mean stress directly within an energy-based damage parameter, Chu (1995a) and Glinka et al. (1995) proposed critical plane models containing terms for both the maximum shear and maximum normal stress on a particular plane. These models are expressed by Equations (2.42) and (2.43), respectively:

\[
\Delta W^* = \left( \sigma_{n,\text{max}} \frac{\Delta \varepsilon}{2} + \tau_{n,\text{max}} \frac{\Delta \gamma}{2} \right)_{\text{max}}
\]

\[
\Delta W^* = \frac{\Delta \tau \Delta \gamma}{2} \left( \frac{\sigma'_f}{\sigma'_f - \sigma_{n,\text{max}}} + \frac{\tau'_f}{\tau'_f - \tau_{n,\text{max}}} \right)
\]

Although similar to Liu’s VSE parameter, in that all of the models combine the effect of tensile and shear work on a given plane, the models of Chu et al. and Glinka et al. do not consider tensile and shear failure modes separately. Instead, the plane on which the combined effect of tensile and shear work is maximum (i.e. the plane on which the damage parameter is maximized) is considered to be the critical plane.

Regardless of the theory and/or physical basis of any multiaxial fatigue damage parameter, however, there are certain characteristics that the parameter should possess in order to help ensure that it is robust and generally applicable to a wide variety of fatigue life analyses. These include the ability to account for varying stress states, mean stresses, changes in material constitutive behavior due to cyclic and/or non-proportional hardening, and other effects such as load interaction and path dependence. A parameter that can successfully incorporate all of these features has the best chance of being successful in even the most complex fatigue loading conditions, which often exist in multiaxial service loading applications.
2.5 Multiaxial Cycle Counting Methods

Once an appropriate stress and/or strain history has been derived for a particular fatigue life analysis, one of the next obstacles to overcome is identifying individual loading cycles within the history. Once cycles have been identified, an appropriate fatigue damage parameter can then be used to calculate the fatigue damage resulting from each counted cycle. For uniaxial or equivalent stress/strain histories, there are many methods available for such a task. ASTM Standard E1049 (2009) lists level-crossing counting, peak counting, simple-range counting, range-pair counting, and rainflow counting as some of the more prominent. Of these, rainflow cycle counting has emerged to be the most commonly used method. This is because from a physical standpoint, the rainflow algorithm is able to identify closed hysteresis loops within a loading history.

For loading histories composed of multiple stress and/or strain components, cycle counting is not as straightforward. Because load components remain in a constant ratio to one another under linear elastic proportional loading conditions, a uniaxial cycle counting technique may be applied to any loading component, or an equivalent stress history, and the range and mean values of all other stress-strain components can subsequently be determined through appropriate scaling of the counting variable. For multiaxial histories resulting in plastic deformation and/or non-proportional loading, however, stress and strain components may vary nonlinearly, out-of-phase, or at different frequencies with respect to one another (asynchronous). In the case of asynchronous and/or random loadings, which are often encountered in service loading histories, there may be no clear definition of a cycle. As a result, the amplitudes and mean stresses of the
different loading components, required as input for fatigue damage calculation, can change depending on the load component used for cycle counting.

To overcome the ambiguity of cycle counting for general multiaxial loading histories, Bannantine and Socie (1991a, 1991b) proposed a procedure for variable amplitude fatigue life analysis based on a modification of uniaxial rainflow counting for use with the critical plane concept. In this approach, an appropriate shear or tensile failure-based critical plane damage parameter is first chosen (e.g. FS or SWT parameter) depending on the type of cracking behavior exhibited by the material being analyzed. If the cracking behavior is unknown, both parameters may be considered in the analysis. Next, shear and normal components of stress and strain are projected onto various candidate planes on which fatigue life analysis is to be performed. Once stress-strain histories are obtained for all candidate planes, cycle counting is performed twice (if two parameters are considered) using standard uniaxial rainflow counting techniques: once for the shear-based parameter and once for the tensile-based parameter. The loading component on which cycle counting is performed depends on the driving quantity for each damage parameter (e.g. shear strain for FS and normal strain for SWT).

Once cycles are identified for the counting variable, additional quantities required for damage calculation (e.g. $\alpha_{n,\text{max}}$ for FS and SWT) are then identified for each counted cycle. Some discussions on challenges involved in the identification of secondary quantities are provided in (Langlais et al., 2003; Shamsaei, 2010). These mainly focus on issues related to preserving the relationship between the primary counting variable and the secondary variables throughout the progression of the cycle counting algorithm. Finally, damage can be computed for each cycle and on each plane using each damage
parameter. Once the damage from each cycle has been summed for each candidate plane using an appropriate damage accumulation rule, the critical plane can then be identified as the plane experiencing the largest amount of damage, and the minimum fatigue life can be computed. For materials with unknown cracking behavior, the minimum of the two predicted lives may be considered as the fatigue life estimate. Figure 2.5 provides an overview of the various steps involved in the Bannantine-Socie cycle counting procedure.

Because the orientation of the critical plane in a complex multiaxial variable amplitude loading history is often difficult to determine prior to cycle counting, a drawback to this approach is that many planes must be analyzed in order to determine the one which will produce the minimum fatigue life. It should also be noted that considering the critical plane to be the plane experiencing the maximum amount of accumulated damage is different than what is used in the traditional definition of some damage parameters, for instance where the critical plane is considered to be the maximum shear or normal stress/strain range plane. However, considering the maximum damage plane as the critical plane has been shown, at least in some studies, to be a more reliable approach for fatigue life prediction (Socie and Marquis, 2000; Chen et al., 2006).

An alternative multiaxial cycle counting method, proposed by Wang and Brown (1995, 1996a), relies on identifying reversals from a modified von Mises or Tresca equivalent stress or strain history. Once reversals have been identified, fatigue damage can then be calculated using any parameter. The cycle counting procedure is summarized by the following steps:

1. Compute equivalent stress and strain histories for the entire loading block (e.g. Equations (2.44) and (2.45) for von Mises criterion)
\[
\sigma_q^*(t) = \frac{3}{\sqrt{2}} \hat{S}^*(t):\hat{S}^*(t)
\]

\[
\varepsilon_q^*(t) = \frac{1}{1 + \nu'} \frac{3}{\sqrt{2}} \hat{\varepsilon}^*(t):\hat{\varepsilon}^*(t)
\]

where \( \hat{S}^* \) and \( \hat{\varepsilon}^* \) are the deviatoric relative stress and strain tensors (e.g. \( \hat{\varepsilon}^* = \varepsilon^* - \frac{1}{3} Tr(\hat{\varepsilon}^*)\hat{I} \)) and \( \nu' \) is the effective Poisson’s ratio, defined as follows:

\[
\nu' = 0.5 - \frac{(0.5 - \nu)\sigma_q^*}{E\varepsilon_q^*}
\]

where \( \nu \) is Poisson’s ratio under linear elastic conditions.

2. Identify the point at which the highest equivalent stress or strain value is reached. Rearrange the loading history to start with this reference point.

3. Compute relative stress and strain histories by subtracting the stress and strain tensors at the new reference (starting) point from each point in the loading history (e.g. \( \hat{\varepsilon}^*(t) = \varepsilon(t) - \varepsilon^R \)), thus making the starting point in the history equal to zero. Recompute equivalent stresses and strains to obtain a relative equivalent stress-strain history.

4. Find the maximum of the relative equivalent history. A half reversal is counted by combining the sequence of data points where the relative equivalent stress or strain is increasing up until the peak value identified.

5. Using each point at which equivalent stress or strain starts to decrease as a new reference point, construct new “blocks” of data points that start and end with the same equivalent stress or strain value. A trailing block, consisting of all data points following the first reversal, is also created.
6. Repeat steps 3-6 for each loading block created until no reversals remain in the loading history.

An example showing the progression of the Wang-Brown cycle counting methodology is given in Figure 2.6. Once all reversals have been identified, damage may be calculated for each reversal by considering an equivalent stress or strain approach, or by using another approach, such as a critical plane parameter. When using a critical plane parameter, the points between which each reversal was identified from the equivalent stress/strain history are used as the start and end points for each reversal from the original stress and strain tensors. The critical plane may then be identified, and damage calculated, for each stress-strain excursion between the identified reversal points. After summing damage for each reversal, a final fatigue life prediction is obtained.

Although using a critical plane approach with Wang-Brown cycle counting seems relatively straightforward, it presents a number of issues in its implementation. To begin, the critical plane identified for each reversal is not necessarily the same plane, but damage is always calculated on the critical plane for each reversal. Therefore, when summing damage for each reversal, the total damage may reflect the damage on a large number of planes. As a result, this approach cannot predict a failure plane and instead must assume a random crack orientation. Although it was argued by Wang and Brown (1996b) that a changing critical plane can account for changes in crack growth path throughout the loading history, this is inconsistent with the overall coplanar nature often observed for crack initiation and small crack growth. Despite these drawbacks, Wang and Brown (1996b) reported good agreement between their approach, the Bannantine-Socie approach, and experimental results for variable amplitude proportional loadings when
using a strain-based critical plane damage parameter for En15R steel. For non-proportional variable amplitude loadings, they found the Wang-Brown approach to give life predictions mostly within a factor of 1.5 of experimental lives, while the Bannantine-Socie approach gave non-conservative predictions by as much as a factor of 5.

Pointing out the fact that Wang-Brown cycle counting is unable to account for load path influence on fatigue damage, Dong *et al.* (2010) proposed the concept of calculating a path dependent maximum range (PDMR) of effective stress. The path dependent effective stress may then be used as an equivalent stress-based fatigue damage parameter which takes into account loading non-proportionality by considering the length of the loading path for each reversal in equivalent stress space. However, the process of identifying cycles for which to compute the path length is very similar to that proposed by Wang and Brown. The main difference between the two methods is in the selection of the initial reference point. Dong *et al.* point out that because the Wang-Brown method selects the maximum equivalent stress value (always positive) as the initial reference point for reversal identification, it has the potential to miss the maximum stress range in the history in certain cases. Because the PDMR method selects the maximum range of equivalent stress as the first reversal, however, this issue is avoided. Additionally, the equivalent stress quantity used for cycle counting by Dong *et al.* is not necessarily von Mises stress. Instead, an equivalent shear stress is defined as $\sqrt{\beta} \tau$, where $\beta$ is a measure of fatigue strength equivalency between normal and shear stress-based fatigue tests within a life range of interest.

Meggiolaro and de Castro (2012a) also proposed a modified version of the Wang-Brown cycle counting method with the aim of overcoming some of the previously
discussed shortcomings. Their two major modifications include the method of identification of the starting point for cycle counting, and a reduction of deviatoric stress or strain components to a five-dimensional (5D) Euclidean sub-space. The transformation of stresses or strains into the 5D space (which may include fewer dimensions for simplified stress or strain states) greatly reduces the computational complexity of the original Wang-Brown counting method by eliminating the need to compute a relative equivalent history for every new starting point identified in the cycle counting procedure. The revised procedure for identifying the initial starting point involves first identifying the maximum range of the equivalent stress or strain quantity in the entire loading history (represented by the longest chord in the 5D sub-space). Then, of the two extreme values defining this range, the one with the greatest distance from the origin in the stress or strain space is chosen as the starting point. Selecting the starting point in this manner ensures that the maximum range in the history is always captured.

In addition to modifications to the Wang-Brown cycle counting method, Meggiolaro and de Castro (2012a, 2012b) also offered some refinements to improve the application of cycle counting methods for use with critical plane approaches. They noted that for some shear crack orientations, there may be two shear components acting on the critical plane. Since both components can have a significant influence on crack initiation and growth, fatigue damage calculations should consider their combined effect. However, cycle counting approaches such as Bannantine-Socie usually ignore the effect of one component, considering one to be dominant compared to the other, or they are applied to each shear component individually. Therefore, it is possible that they will result in non-conservative predictions when both shear components are significant. However, an
effective shear stress or strain acting on a plane is not easily defined. The magnitude of the two components cannot simply be used for cycle counting because of the sign problem this would create. To address this issue, Meggiolaro and de Castro suggested that the shear stress or strain path should be drawn in a 2D shear space for each reversal, so that an effective shear stress/strain range can be identified using an enclosing surface method. Enclosing surface methods evaluated in their study included, among others, the Minimum Ball and Minimum Circumscribed Ellipse methods.

Because the path for a given reversal must be known before an effective shear stress/strain range can be determined, for complex multiaxial variable amplitude loading histories, it is first necessary to define where a reversal begins and ends. For this, Meggiolaro and de Castro (2012a) suggested using their modified Wang-Brown cycle counting method. Therefore, the procedure for calculating fatigue life under variable amplitude loading conditions using a shear-based critical plane approach would include the following steps. First, stress and strain histories are transformed into normal and shear components acting on candidate planes for fatigue life analysis (similar to the Bannantine-Socie method). Next, for each candidate plane, reversals are identified using modified Wang-Brown cycle counting. Then, shear stress or strain paths are constructed, and effective shear values are computed for each identified reversal using an enclosing surface method. Finally, damage may be calculated for each reversal using an appropriate fatigue damage parameter, and then summed to determine total fatigue life. It should be noted that because this approach projects loading components onto candidate planes before damage calculation, a unique failure plane orientation will be identified.
The aforementioned cycle counting approaches have all essentially been adaptations of the rainflow cycle counting technique to multiaxial loading conditions. A more recent non-rainflow counting method, which relates the maximum damage parameter value within a loading block to the block’s total fatigue damage, was proposed by Anes et al. (2014). This method was named as a virtual cycle counting method because it does not rely on hysteresis loop identification to define cycles. Instead, the virtual cycle count is essentially a scaling factor which equates the damage from a given loading history to the damage produced by a single cycle of the highest equivalent stress amplitude. Although originally proposed for use with an equivalent shear stress criterion proposed by the same authors (stress scale factor equivalent shear stress), the virtual cycle count concept is represented in a more general sense by the following equation:

\[ v_{CC} = \frac{\sum |\sigma_{q,\text{peak/valley}}|}{2\sigma_{q,max}} \]  

(2.47)

where \( \sigma_{q,\text{peak/valley}} \) is the maximum/peak or minimum/valley equivalent stress value identified between two consecutive zero stress points and \( \sigma_{q,max} \) is the maximum equivalent stress value in the loading block.

When using virtual cycle counting with the stress scale factor equivalent shear stress, Anes et al. reported better fatigue life correlations than when using the Bannantine-Socie (with FS and SWT parameters) or Wang-Brown (with Brown-Miller parameter) approach for 11 different multiaxial loading paths applied to 24CrMo4 quenched and tempered steel specimens. One issue not addressed, however, is the computation of the virtual cycle counting parameter in the absence of zero stress points (e.g. uniaxial tension-tension loading). In this case, the sum of peak and valley stresses would be equal to the maximum equivalent stress, and other reversals in the history
would be ignored. The virtual cycle counting technique is also not applicable to critical plane approaches, as the method produces only a scalar quantity based on equivalent stress.

### 2.6 Cumulative Damage Rules

After fatigue damage has been computed for each cycle in a variable amplitude loading history, the total damage accumulated over the entire history must be summed in order to predict when failure will occur. Since the concept of cumulative fatigue damage was first suggested by Palmgren (1924), and expressed in a linear mathematical form by Miner (1945), many different rules have been proposed over the years in an attempt to properly quantify the progression of fatigue failure. Most of these subsequently proposed damage rules attempt to correct for various degrees of nonlinear fatigue damage accumulation observed in experimental studies.

Two comprehensive reviews of cumulative damage rules were published in the late 1990s by Fatemi and Yang (1998; Yang and Fatemi, 1998) which review over 200 different publications related to the topic. In these reviews, cumulative damage theories are divided into six general categories: linear damage rules, nonlinear or two-stage linearized damage curves, life curve modifications to account for load interaction effects, approaches based on crack growth concepts, continuum damage mechanics approaches, and energy-based methods. Advantages and disadvantages are then discussed with respect to each type of approach.

Despite the variety of approaches available, there is no single cumulative damage rule that has been found to result in consistently accurate fatigue life predictions.
regardless of material and/or loading conditions. As a result, due to its simplicity, the Palmgren-Miner linear damage rule (LDR) remains, to this day, the most widely used method of fatigue damage summation (Fatemi and Shamsaei, 2011). Additionally, advances in both uniaxial and multiaxial fatigue life prediction techniques have revealed that some of the deficiencies typically attributed to LDR may not actually be due to the damage rule itself, but instead stem from inadequacies in other areas of fatigue life analysis. For example, the nonlinearity in some damage rules is introduced in order to account for an increase in damage accumulation rate due to the presence of a growing crack. However, an alternative approach to account for this effect is to define a sufficiently small crack initiation length so that crack growth effects will not have a strong influence on the predicted initiation life.

The inability of LDR to account for nonlinear damage accumulation due to load sequence effects is another argument often made against its robustness. However, it has been shown (Jiang and Kurath, 1997a) that by including the effects of transient material response into cyclic plasticity modeling and damage calculation, predicted damage values at failure based on LDR were significantly improved over values obtained without the consideration of transient effects. Results were based on data from two step high-low strain-controlled fatigue tests of 304 stainless steel and 1070 steel. Thus, load sequence effects may be better accounted for through more advanced cyclic plasticity modeling techniques.

Another deficiency sometimes attributed to LDR is an inability to account for deformation history dependence, which results from changes in material constitutive behavior brought about by cyclic hardening/softening. However, this can be more
appropriately accounted for through a better choice of damage parameter. A parameter which considers both stress and strain terms will be able to reflect changes in material constitutive behavior and give a more accurate representation of fatigue damage at the individual cycle level. In the same study by Jiang and Kurath (1997a), LDR was combined with two different critical plane damage parameters to predict cumulative damage sums at failure. For the 304 stainless steel, the average damage sum at failure calculated using Findley’s parameter (stress-based) was 3.35, while the Fatemi-Socie parameter (stress and strain-based) improved predictions to an average value of 1.03. A damage sum at failure equal to unity represents a perfect life prediction.

As a final point worth mentioning, the application of nonlinear damage rules in cases involving truly variable amplitude loading histories is much more complex than for cases involving simple histories composed of blocks of varying magnitude constant amplitude cycles. In order to account for load sequence effects in a cycle-by-cycle fatigue life analysis using a nonlinear damage rule, sequence must be preserved in the cycle counting procedure. However, the possibility for loading histories to contain nested cycles makes defining a cycle sequence a non-trivial task. Since this issue is not relevant when considering linear damage accumulation, the application of LDR can greatly simplify a variable amplitude fatigue life analysis.

2.7 Synergistic Studies

Despite the abundance of literature available on the many individual aspects of multiaxial variable amplitude fatigue analysis of notched components, a very limited number of studies have been performed which attempt to combine all of these different
aspects into a single fatigue life analysis procedure. This is, of course, despite the significance of such types of analyses for application to practical engineering problems, where these complex loading conditions often coexist.

In order to better understand the fatigue failure process under such conditions, a large scale fatigue testing program, organized by the Society of Automotive Engineers (SAE) in the 1980s, was carried out through a cooperative effort between several universities and industry partners. The goal was to develop, understand, and apply technology to solve multiaxial fatigue problems. The SAE Biaxial Testing Program, as it was called, focused on fatigue life analysis of un-notched tubular and notched shaft specimens made from 1045 medium carbon steel in both normalized (Phase I) (Leese and Socie, 1989) and induction hardened (Phase II) conditions (Cordes and Lease, 1999). The testing program involved a variety of constant amplitude and variable amplitude loading histories for bending, torsion, and combined bending-torsion loadings. Phase I of the program evaluated the effects of multiaxial stress states and the presence of notches on resulting fatigue life predictions, while Phase II extended this work to include the effects of hardness gradients, microstructural transformations, and residual stress fields resulting from the induction hardening process.

Various multiaxial fatigue life prediction methodologies were evaluated by different researchers throughout the program. Tipton and Nelson (1989) studied the applicability of equivalent stress-based approaches to multiaxial fatigue life predictions of the SAE notched shaft specimens. They concluded that their usefulness is limited due to an inability to account for differences in fatigue life between in-phase and out-of-phase loadings tested at the same equivalent stress amplitude. They also noted that because of
their dependence on fatigue notch factor, they are often difficult to apply in the absence of experimental data.

Similarly, Tipton and Fash (1989) evaluated multiaxial fatigue life predictions using the following strain-based and critical plane-based approaches: maximum principal strain amplitude, maximum shear strain amplitude, von Mises effective strain amplitude, ASME Boiler and Pressure Vessel Code Section III, Brown and Miller approach, Lohr and Ellison approach, Garud’s plastic work approach, and Mowbray’s hydrostatic stress correction approach. Local notch strain values were obtained through a combination of elastic-plastic FEA and measured strain gage data. It was concluded that for the same applied loads, out-of-phase loading was less damaging than in-phase loading, but traditional strain-based approaches resulted in non-conservative life predictions for out-of-phase loadings. Critical plane and plastic work approaches, however, resulted in better life predictions than the others.

Some overall conclusions that were drawn from the initial work performed during the SAE Biaxial Testing Program indicated that improvements were needed with respect to non-proportional fatigue damage calculation, the ability to predict shear versus tensile-dominated damage processes, and robustness in damage accumulation rules. Despite the fact that a limited number of variable amplitude combined bending-torsion fatigue tests were also performed during the testing program, neither of the aforementioned summary books contains complete fatigue life analyses for these loadings. However, in a separate publication, Bannantine and Socie (1991a) performed fatigue life analysis for the SAE thin-wall tubular specimens under these proportional and non-proportional variable amplitude loading histories. They reported satisfactory fatigue life predictions when
using their cycle counting procedure combined with the FS and SWT critical plane damage parameters.

Additionally, in a study conducted by Han et al. (2002), the ability of several different multiaxial fatigue damage parameters to correlate fatigue data under complex loading conditions was evaluated. Specimens of SNCM630 alloy steel were subjected to irregular variable amplitude multiaxial loadings, but no notch effects were considered. When coupled with Bannantine-Socie rainflow cycle counting and the Palmgren-Miner linear damage rule, it was shown that the Fatemi-Socie parameter, as well as a critical plane-based weighted energy parameter, produced the best fatigue life correlations for all 12 constant and variable amplitude load paths considered. Both parameters were able to correlate nearly all of the data within a factor of 2 of experimental results.

More recently, when evaluating the effectiveness of their critical distance approach to multiaxial variable amplitude fatigue life analysis of notched components, Susmel and Taylor (2012) coupled the point method of TCD, as described in Section 2.3.4, with a Modified Wöhler Curve approach for multiaxial damage calculation. This approach assumes that fatigue life under multiaxial loadings can be directly estimated through the shear stress amplitude and normal stress relative to a critical plane (maximum resolved shear stress plane). The resulting fatigue damage is then calculated as a function of the applied loading, as well as the fully-reversed uniaxial and torsion stress-life fatigue constants. The life prediction capabilities of this method were evaluated through comparisons to experimental results obtained from 124 fatigue tests performed on circumferentially notched cylindrical specimens of C40 carbon steel under variable amplitude uniaxial, in-phase, and out-of-phase biaxial loading conditions. Some
comparisons were also made to experimental results from literature. The authors found that fatigue life predictions using their method were highly accurate, and most often fell within the uniaxial constant amplitude scatter bands for a given material, regardless of the loading history or notch severity. A drawback to this approach, however, is that several experimental fatigue life curves are required, under various loading conditions, to calibrate the Modified Wöhler Curve method. Additionally, an important issue highlighted by the authors was the dependence of predicted fatigue lives on the critical value of the cumulative damage sum. The need for a correct definition of this value was emphasized, as it is difficult to determine without appropriate experimental data.

Brighenti and Carpinteri (2012) also evaluated notch fatigue for arbitrary multiaxial loading histories by incorporating the effect of stress gradients into a damage mechanics approach. An endurance function, based on the stress and deviatoric stress invariants, was used to quantify damage accumulated at each point in the loading history until failure was predicted to occur. Since the approach is based on a continuous accumulation of damage within a volume of material, it does not rely on a critical plane interpretation or any cycle counting method in order to determine fatigue life. Fatigue life was analyzed for un-notched specimens of EN24T steel under various proportional and non-proportional constant amplitude loading paths, and most predictions were found to be within a factor 3 of the experimental results. Some medium carbon steel notched specimen data for different notch radii and biaxiality ratios were also analyzed, and predicted fatigue limits were found to be in reasonable agreement with those observed experimentally. A disadvantage of this approach, however, is that 11 different model parameters, determined through a genetic optimization algorithm, must be calibrated to
experimental data for a generic multiaxial stress history. Additionally, although the authors expressed the applicability of their approach to multiaxial variable amplitude loadings, no quantitative results were presented for such cases.

2.8 Crack Growth Aspects

To this point, the focus of this literature review has been on the crack initiation stage of fatigue life. In certain applications, such as when a safe-life design philosophy has been employed, crack initiation life is the basis for defining a component’s service life, and is the only stage of fatigue life considered. However, as mentioned previously, it is common for both crack initiation and crack growth to make up significant portions of a component’s total fatigue life. Therefore, while this may be an appropriate design strategy in some cases, it can lead to premature retirement and/or excessive component cost/weight in others. Considering both crack initiation and crack growth stages of fatigue life provides a more accurate representation of the entire fatigue failure process, and can help to increase the mechanical efficiency of a structure. This is especially true in cases where a fail-safe or damage-tolerant design philosophy has been adopted.

The growth life of a fatigue crack is generally divided into different stages based on its length. Although these stages are somewhat vaguely defined, they help to identify which factors are influencing the propagation of a crack at a particular length scale. In general, a fatigue crack can be considered to gradually transition through stages of crack formation, microstructurally small crack growth, physically small crack growth, and long crack growth. The amount of time spent in each stage can vary depending on the material, microstructure, and applied loading conditions (McDowell and Dunne, 2010).
The crack formation process encompasses both the nucleation of a fatigue crack, from dislocation mechanisms within an individual grain, and a small amount of crack growth through the domain of influence which triggered its nucleation. The term crack formation, as used here, is different from the crack initiation stage described in earlier sections. Crack initiation is typically used to refer to an arbitrarily defined crack length used as a convenient way to report fatigue lives in experimental studies. The crack formation stage, on the other hand, relates to a length scale closely tied to the microstructural attributes of a given material (e.g. grain, pore, or inclusion size). Although the length defined for crack initiation may be similar to that for crack formation, they are generally not the same. Crack formation environments typically are on the order of 100 microns or less (McDowell and Dunne, 2010).

Following crack formation, the microstructurally small stage encompasses crack growth to a length of several (i.e. 3-10) times the material grain size or secondary phase spacing. In this stage, cracks can be influenced significantly by the microstructure of a material. From there, the crack enters the physically small (or short) crack growth stage where it is long compared to the microstructure scale, but is still subject to heterogeneous growth characteristics due to its plastic zone being on the order of either a grain size, or the size of other significant microstructural barriers. Long crack behavior is not fully realized until the crack is of sufficient length to ensure that similitude conditions, in terms of both small scale yielding and damage, are met. This typically occurs in the length range of several hundred microns to a millimeter or so, depending on the material and applied loading. Once a crack is considered long, traditional crack growth models can
then be applied to predict growth rates for a given crack geometry and loading condition.
More details on long crack growth are presented in the following sections.

In some cases, such an emphasis is placed on crack growth life that it is the only stage of fatigue life considered. In these, so-called, fracture mechanics-based approaches to fatigue life analysis, a crack is assumed to always exist within a component from the time of manufacture. These cracks may manifest themselves in a variety of ways, some of which include: material defects, manufacturing flaws such as deep machining grooves, voids in welds, etc. These initial flaws are generally considered to take the place of the previously described crack formation stage. From there, crack growth is analyzed through the remaining stages in order to estimate a component’s total fatigue life. Fracture mechanics approaches are the basis of damage-tolerant design philosophies, in which the objective is to detect cracks in key structural components before they grow to a critical length. Periodic inspections are necessary in damage-tolerant design and the inspection intervals, as well the estimated residual fatigue life, are closely linked to the level of confidence in crack growth life estimates.

Although some guidelines are given in references such as the United States Air Force Damage Tolerant Design Handbook (Gallagher et al., 1984), one of the largest difficulties in a fracture mechanics analysis lies in determining the assumed “equivalent initial flaw size.” This value is not only dependent on material, but can also vary for each component or location considered. Because a large percentage of a component’s total fatigue life is often spent in small crack growth, small changes in this initial crack length can have a large impact on the resulting life prediction. Additionally, crack growth calculations, in particular at small crack lengths, are very sensitive to changes in material
properties, stress level, and loading sequence (Iyyer et al., 2007). In practice, it is often desirable to back calculate equivalent initial flaw sizes from full-scale fatigue test data, but this can be time and cost prohibitive in many cases. Variability in crack growth rates for microstructurally small cracks, brought about by the inherent randomness of polycrystalline materials at the microstructural scale, further hinders accurate crack growth analysis in this regime.

Because the advanced modeling techniques required to accurately predict crack formation and growth behavior at the microstructural level are still in development, it is often desirable to evaluate the total fatigue life of a component using a two-stage approach. The first stage, crack initiation, has already been discussed. As long as the crack initiation definition is defined sufficiently small, this stage can be used in place of the crack formation and microstructurally small crack growth stages through the application of empirical fatigue life curves generated using traditional experimental techniques. However, the effect of microstructural variability on fatigue processes will still manifest itself in the form of scatter in the experimentally observed crack initiation lives.

From there, crack growth analysis can be carried out starting from the physically short crack regime. For these reasons, both crack initiation and crack growth stages of fatigue life were evaluated in this study. The following sections review some of the more important fracture mechanics concepts to consider in the crack growth regimes of interest. Emphasis is placed on issues important to the analysis of crack growth from stress concentrations and/or where multiaxial and variable amplitude loadings play
significant roles. Other, more basic, concepts are briefly reviewed as well in order to present background information and terminology relevant to subsequent discussions.

2.8.1 Linear Elastic Fracture Mechanics Concepts

Linear Elastic Fracture Mechanics (LEFM) is by far the most studied area of fracture mechanics. This is especially true with respect to the mode I crack extension of long cracks. Because of this, only a brief description outlining key LEFM concepts, and terminology relevant to the discussions in subsequent sections, is given herein. For more in-depth discussion on LEFM and other fracture mechanics concepts, textbooks such as (Anderson, 2005) can be referenced.

There are three basic modes by which cracks can extend or propagate. These modes, termed modes I, II, and III, are illustrated in Figure 2.7 and refer to opening mode extension, in-plane shearing or sliding mode extension, and anti-plane shearing or tearing mode extension, respectively. Mode I crack growth is driven by remote tensile loads, while modes II and III are driven by remote shear loads. The basic quantity used in LEFM to quantify the crack tip driving force for any given crack extension mode is the stress intensity factor (SIF). The SIF is related to energy release rate (representing elastic energy per unit crack surface area required for crack extension) and will vary depending on the applied loading, crack length, crack geometry, and component geometry. The basic expressions for SIF with respect to each loading mode are given as follows:

\[ K_I = Y_I \sigma \sqrt{\pi a} \]  \hspace{1cm} (2.48a)

\[ K_{II} = Y_{II} \tau \sqrt{\pi a} \]  \hspace{1cm} (2.48b)

\[ K_{III} = Y_{III} \tau \sqrt{\pi a} \]  \hspace{1cm} (2.48c)
where $a$ is the crack length, $\sigma$ and $\tau$ are the remote normal and shear stresses acting on the crack plane, respectively, and $Y_I$, $Y_{II}$, and $Y_{III}$ are geometry and crack length dependent correction factors which account for the redistribution of remote stresses due to the presence of the crack. Geometry factors for many common component and specimen geometries have been quantified over the years and can be found in handbooks such as (Gallagher et al., 1984; Murakami, 1987; Tada et al., 2000).

LEFM crack growth data are typically reported in the form of a plot showing crack growth rate, expressed as the length of crack extension per cycle, versus $\Delta K$ for a given crack extension mode. These data are generally plotted in a log-log scale, which results in the appearance of three distinct regions of crack growth. These regions, appropriately referred to as regions I, II, and III, are shown schematically in Figure 2.8. Region I is the near threshold regime and contains the range of data where a decrease in $\Delta K$ below a specific material dependent threshold value, $\Delta K_{th}$, will result in a non-propagating crack condition. Region II, often referred to as the Paris region/regime, corresponds to a stable crack growth condition in which $\Delta K$ can be related to crack growth rate, $da/dN$, by a simple power law relation (often referred to as the Paris Law):

$$
\frac{da}{dN} = C(\Delta K)^m
$$

(2.49)

The experimentally determined constants $C$ and $m$ are material dependent and can also vary with applied loading conditions and/or growth mode. The final region, region III, corresponds to the rapid and unstable growth experienced by a crack as it approaches its critical length for failure.

Although LEFM concepts apply for a majority of crack growth situations, there are limitations to their applicability. Because of the elastic stress distributions assumed as
the basis for LEFM, it is important that excessive plasticity is not present when these concepts are being applied. Generally, this means that the length of a crack should be long compared to its plastic zone size, and that the stress state surrounding a crack should not result in a general yielding condition. Typical guidelines for situations in which LEFM can be safely applied to cyclic loading analyses include: a crack length (or half crack length) at least four times larger than the cyclic crack-tip plastic zone radius, a net nominal stress less than 80% of the material’s yield strength, and a specimen thickness and uncracked ligament of at least eight times the plastic zone radius (Stephens et al., 2000).

2.8.2 Elastic-Plastic Fracture Mechanics Concepts

When LEFM conditions are violated, alternative approaches must be used to relate the local stress state at a crack to its corresponding growth rate. Although methods to extend LEFM concepts beyond small-scale yielding conditions have been proposed (Dugdale, 1960; Irwin, 1961), they are still limited in their applicability. More preferred analysis approaches, which have been developed specifically for elastic-plastic conditions, fall under the classification of Elastic-Plastic Fracture Mechanics (EPFM) concepts.

One of the most popular EPFM parameters is the J-integral. Formulated by Rice (1968), Eshelby (1956), and Hutchinson (1968), the J-integral in EPFM is analogous to the SIF used in LEFM applications. Similar to the SIF, the J-integral is related to the energy release rate required for crack extension. It is computed through a path-independent contour integral starting from the lower crack surface and looping around the crack tip to the upper crack surface. Because of its path independence, the calculation of
J-integral can be simplified by choosing a contour that is sufficiently far from the crack tip, so that only elastic stresses and displacements are considered.

Although originally developed for nonlinear elastic material behavior, which differs from nonlinear elastic-plastic behavior upon unloading, J-integral concepts have been successfully extended to cyclic EPFM situations by relating the J-integral range, $\Delta J$, to crack growth rate. The formulation for $\Delta J$ is given as follows:

$$
\Delta J = \Delta J_e + \Delta J_p = \int_\Gamma (\Delta w dy - \Delta T_i \frac{\partial \Delta u_i}{\partial x} ds) \quad (2.50)
$$

where $\Delta w$ is the strain energy density range, $\Delta T_i$ is the stress range vector, $\Delta u_i$ is the displacement range vector, $ds$ is an increment along the integration path, and subscripts $e$ and $p$ refer to elastic and plastic components, respectively. For linear elastic conditions, J-integral can be related to energy release rate, $G$, and $K$ as follows:

$$
\Delta J = \Delta J_e = \Delta G = \frac{1}{E} (\Delta K_I^2 + \Delta K_{II}^2) + \frac{\Delta K_{III}^2}{2G} \quad \text{plane stress} \quad (2.51a)
$$

$$
\Delta J = \Delta J_e = \Delta G = \frac{(1 - \nu^2)}{E} (\Delta K_I^2 + \Delta K_{II}^2) + \frac{\Delta K_{III}^2}{2G} \quad \text{plane strain} \quad (2.51b)
$$

where $E$ is the modulus of elasticity, $G$ is the shear modulus, and $\nu$ is Poisson’s ratio.

From this relation, it can be seen that because $\Delta K = (K_{max} - K_{min})$, $\Delta J \neq (J_{max} - J_{min})$.

Although J-integral concepts have been used successfully to correlate crack growth rates for a wide range of loading conditions and plasticity levels, there are still situations for which the applicability of the approach is questionable. These include situations where the stress-strain response of a material varies significantly from loading to unloading, such as when a material undergoes significant cyclic hardening, and in situations where crack closure is significant under large-scale yielding conditions.
Specific guidelines for the applicability of $\Delta J$ to fatigue problems, however, have not yet been established.

In addition to the J-integral, crack tip opening displacement (CTOD) has also been suggested as an EPFM parameter for situations involving mode I crack extension (Wells, 1961). This approach assumes that the displacements at a crack tip can be directly related to crack growth rate. This was later proven when unique relationships between CTOD, J-Integral, and SIF were established. Within the limits of small scale yielding, SIF and CTOD are related by the following expression:

$$CTOD = \frac{4 K_I^2}{\pi \sigma_y E}$$  \hspace{1cm} (2.52)

A more general expression, derived through application of the strip-yield model, relates CTOD, J-integral, and SIF through the expression:

$$CTOD = \frac{K_I^2}{m \sigma_y E'} = \frac{J}{m \sigma_y}$$ \hspace{1cm} (2.53)

where $m$ is a constant equal to approximately 1.0 for plane stress conditions and 2.0 for plane strain. The modulus term, $E'$, is equal to $E$ for plane stress and $E/(1 - \nu^2)$ for plane strain conditions. The CTOD approach, however, has not received the attention that the J-integral has, due in part to the difficulty in obtaining accurate CTOD values for complex crack geometries.

### 2.8.3 Mixed-Mode Crack Growth

Crack growth mechanisms and their influence on crack growth rate have been researched extensively over the years for uniaxial loadings and mode I crack extension. This is because for a majority of component geometries and loading conditions, cracks
tend to align themselves to grow on planes of maximum tensile stress. For example, cracks initiated at a mechanical notch have been shown to nucleate and grow a short distance, usually comparable to the length of a few grains, on planes of maximum shear, but almost always turn so that long crack growth occurs on planes of maximum tensile stress (Qian and Fatemi, 1996a; Tanaka, 2010; Tanaka et al., 2006; Zhang and Fatemi, 2010).

There are certain situations, however, in which cracks will grow under the influence of more than one loading mode. Long cracks in un-notched/smooth specimens, for example, have been shown to propagate on maximum shear planes, maximum tensile planes, or a combination of both (Socie and Marquis, 2000). The preferred growth plane can depend on the material, the nominally applied loading, and/or the loading magnitude. This same behavior has also been observed for cylindrical specimens containing a circumferential notch (Murakami et al., 2003; Tanaka, 2012; Tschegg, 1983). In general, cracks in the low cycle fatigue regime tend to grow on maximum shear planes, while cracks at longer lives tend to branch and transition into mode I growth. While this type of behavior is most commonly documented for specimens subjected to pure torsion and/or combined axial-torsion loading, similar behavior has also been observed in specimens subjected to biaxial tension loading at various strain ratios (Ackermann et al., 2015).

2.8.3.1 Mixed-Mode Crack Growth Rate

When a crack grows in a mixed-mode manner, the SIF ranges for each loading mode are not additive. Instead, crack growth rate must be determined through an equivalent SIF range, which can be computed based on one of a number of available
theories. One popular equivalent SIF formulation, based on crack tip displacements, was proposed by Tanaka (1974) and takes the following form:

\[ \Delta K_q = \left[ \Delta K_I^4 + 8\Delta K_{II}^4 + \frac{8\Delta K_{III}^4}{1 - \nu} \right]^{0.25} \]  

This formulation has been shown to provide good correlation of experimental mixed-mode crack growth rates for various ratios of \( K_{II}/K_I \) (Zhang and Fatemi, 2010).

Another equivalent SIF range can be computed based on the strain energy release rate of a planar crack under plane stress conditions. By relating energy release rate to SIF for each crack extension mode and summing their individual contributions, the following relationship is derived:

\[ \Delta K_q = \left[ \Delta K_I^2 + \Delta K_{II}^2 + (1 + \nu)\Delta K_{III}^2 \right]^{0.5} \]  

Equivalent SIF values may then be used with the Paris equation and mode I crack growth constants to compute mixed-mode crack growth rate.

For consideration of small cracks growing under mixed-mode conditions, it has been suggested that strain-based intensity factors may be more suitable than their stress-based counterparts. Similar to a critical plane approach for crack initiation, Socie et al. (1987) proposed the following strain intensity factor expression, which considers shear and normal strain components acting on the plane of maximum shear strain range:

\[ \Delta K_q(\varepsilon) = \left[ (Y_{II} G\Delta \gamma_{\max})^2 + (Y_I E\Delta \varepsilon_n)^2 \right]^{0.5}\sqrt{\pi a} \]  

When applying the equivalent strain intensity factor to mixed-mode small crack growth data for SAE 1045 steel and Inconel 718, crack growth rates were found to correlate fairly well for a variety of loading conditions.
Similar to the model of Socie et al., Reddy and Fatemi (1992) also introduced an effective strain intensity factor, which is based on the Fatemi-Socie multiaxial fatigue damage parameter. Using shear strain and maximum normal stress on the maximum shear strain range plane as driving parameters, the expression takes the following form:

\[
\Delta K_{CPA} = G \Delta \gamma_{max} \left( 1 + k \frac{\sigma_{n,max}}{\sigma_y} \right) \sqrt{\pi a}
\]  

(2.57)

where \( k \) is a material constant found by fitting uniaxial and torsion fatigue data. The critical-plane based strain intensity factor range, \( \Delta K_{CPA} \), was found to produce good correlation of small fatigue crack growth rates under a variety of biaxial loading conditions for both SAE 1045 steel and Inconel 718. More recently, Shamsaei and Fatemi (2014) used this parameter to evaluate in-phase and 90° out-of-phase small crack growth data for 304 stainless steel and 1050 medium carbon steel in both normalized and quenched and tempered conditions. Crack growth rates for the different loading conditions were found to correlate reasonably well using the parameter, but improvements were made with the addition of an extra term to account for the wider range of planes experiencing a high percentage of damage during out-of-phase loading.

Finally, unlike SIF values in LEFM, because the J-integral is a representation of energy release rate, \( \Delta J \) contributions from each loading mode can simply be added to obtain an effective driving force parameter for mixed-mode crack growth situations. Doing so, however, assumes that a crack should grow in a self-similar manner (i.e. will retain a coplanar and constant shape) (Anderson, 2005). For linear elastic conditions, Equation (2.51) shows that summing contributions of J-integral for each loading mode results in an expression of the same form as the equivalent SIF given by Equation (2.55).
2.8.3.2 Mixed-Mode Crack Growth Direction

In addition to crack growth rate, crack growth direction must also be considered in situations involving mixed-mode crack growth, as it can change due to changes in local stress distributions at the crack tip. As a result, several models have been proposed in an attempt to predict crack growth direction under mixed-mode loading conditions.

One of the most widely used crack growth direction theories is the Maximum Tangential Stress (MTS) criterion. Developed by Erdogan and Sih (1963), it assumes that a crack will propagate in a direction, $\theta_c$, that maximizes the normal stress acting on a plane extending radially outward from the tip of the crack. Using the Westergaard stress equations, which describe the elastic stress field surrounding a crack tip for each mode of loading, and solving for the maximum tangential stress in the polar coordinate system, the following expression can be used to determine the crack growth direction for mixed-mode I and II loading:

$$K_I \sin \theta_c + K_{II}(3 \cos \theta_c - 1) = 0$$  \hspace{1cm} (2.58)

Although the MTS criterion has been shown, in some cases, to produce growth direction predictions that agree well with experimental observations (Zhang and Fatemi, 2010), there are also studies which show disagreement with MTS predictions (Tanaka, 1974).

Another widely used growth direction criterion is the Minimum Strain Energy Density Criterion (S-criterion) (Sih, 1974). According to this criterion, a crack will grow in a direction which minimizes the local strain energy density factor value surrounding a crack tip. The general expression for strain energy density factor is as follows:

$$S = \frac{1}{\pi} \left( a_{11}K_I^2 + 2a_{12}K_IK_{II} + a_{22}K_{II}^2 + a_{33}K_{III}^2 \right)$$  \hspace{1cm} (2.59)
where coefficients $a_{ij}$ are functions of growth angle, elastic modulus, and Poisson’s ratio. Like the MTS criterion, the S-criterion has been shown to agree well with some experimental results, but does not give satisfactory predictions for all loading conditions. Additionally, although it is simple to implement in complex loading cases, some questions have been raised over the years regarding its theoretical basis (Qian and Fatemi, 1996a).

In order to predict crack growth direction in cases where $J$-integral is used as a crack driving parameter, Hellen and Blackburn (1975) suggested that a crack would extend along the direction of a vector defined as follows:

$$\mathbf{J} = J_{I} \mathbf{i} + J_{II} \mathbf{j}$$

(2.60)

where

$$J_k = \int_{\Gamma} (w n_k - T_i u_{i,k} \, ds) \quad k = I, II$$

(2.61)

and $w$ is the strain energy density, $n_k$ is the $k$ component of the unit outward normal to the integration contour, $T_i$ is the stress vector, $u_i$ is the displacement vector, and $ds$ is an increment along the integration path. Although satisfactory predictions have been obtained (Dai and Zheng, 1987) based on this criterion for mode I-dominated crack growth, Gdoutos (1990) reported predictions which deviated significantly from experimentally observed crack paths in cases where mode II loading was dominant.

Several other criteria, in addition to those mentioned in this section, have also been proposed for both crack growth rate and direction under mixed-mode crack growth conditions, although they have not received the same attention. Despite the significance of mixed-mode crack growth in practical engineering applications, there are still no models available which are capable of producing satisfactory growth rate and direction.
predictions for all loading conditions. For additional details on mixed-mode crack growth aspects, a more comprehensive review on the topic was published by Qian and Fatemi (1996b).

2.8.3.3 Multiaxial Stress State Effects on Crack Growth

In addition to the possibility for mixed-mode crack growth, the crack growth rate for mode I cracks growing under multiaxial nominal loadings has been shown to vary from data obtained under uniaxial loading conditions. Brown and Miller (1985) studied the growth of through-thickness mode I fatigue cracks in thin rectangular plates of AISI 316 stainless steel under various biaxial stress states. They found that the presence of a negative tangential stress (T-stress) component, acting parallel to the crack growth plane, increased crack tip plastic zone size and crack driving force, resulting in faster mode I crack growth when compared to uniaxial loading. The magnitude of this effect was shown to be dependent on the T-stress amplitude as well as the degree of local plasticity. As a result, they proposed an equation to adjust crack growth rate for changes in crack tip plastic zone size, due to the presence of T-stress, based on the Dugdale (1960) strip-yield model:

\[
\frac{da/dN}{da/dN_1} = \frac{\sec\left\{\pi/\left[\sqrt{(4\sigma_u/\Delta\sigma)^2 - 3(\Lambda - 1)^2 + (\Lambda - 1)}\right]\right\} - 1}{\sec(\pi\Delta\sigma/4\sigma_u) - 1}
\]

(2.62)

where \(da/dN\) is the experimental mode I crack growth rate, \(da/dN_1\) is the T-stress corrected crack growth rate, \(\Lambda = \sigma_x/\sigma_y\) is the biaxial stress ratio (\(\sigma_y\) being normal to the crack plane and \(\sigma_x\) being the T-stress), and \(\sigma_u\) is the material ultimate tensile strength.

More recently, Tanaka et al. (2006) carried out fatigue crack growth tests using thin-walled tubular specimens of low-carbon steel containing a circular hole. Tests were
performed under uniaxial, torsion, torsion with static tension, and in-phase axial-torsion loading conditions. In all cases, mode I crack growth was observed, with crack growth rates for torsion and combined axial-torsion loadings higher than those corresponding to uniaxial loading. Accelerated crack growth was attributed to excessive plasticity at the crack tip, which was intensified due to presence of T-stress in the torsion and combined loading situations. By comparing crack growth rates based on cyclic J-integral range, calculated from a relation between load and displacement, they determined that this was an appropriate parameter for correlating crack growth rates in the presence of significant plasticity for both uniaxial and combined loading conditions.

Aside from variations in mode I crack growth rate due to different proportional multiaxial loading paths, similar differences between crack growth rates under in-phase and 90° OP axial-torsion nominal loading conditions have also been observed by Shamsaei and Fatemi (2014) and Fatemi et al. (2014). Shamsaei and Fatemi reported higher 90° OP crack growth rates for short shear-mode cracks in un-notched tubular specimens made from 1050 steel and Inconel 718, while Fatemi et al. reported lower 90° OP growth rates for long mode I cracks growing in notched tubular specimens made from 7075-T6 aluminum alloy. While these effects on growth rate are opposite, they were attributed to the same phenomenon. Under 90° OP non-proportional loading conditions, due to the rotation of principal stress axes, a larger range of planes within the material experience a high percentage of the maximum crack driving force when compared to in-phase loading at the same equivalent stress level.

For the short shear-mode cracks, it was argued that this led to higher 90° OP growth rates due to a higher probability for large driving forces to be aligned with
favorably oriented slip systems. For the long cracks, on the other hand, the large range of planes experiencing a high percentage of the maximum principal stress amplitude was assumed to be the cause of increased crack meandering and torturous crack paths observed in 90° OP tests. It was then argued that this crack meandering resulted in increased crack face interaction and roughness induced crack closure, which effectively restricted crack growth and lowered growth rates for the 90° OP loading conditions. For some additional information and considerations pertaining to crack growth under non-proportional loading conditions, the reader is referred to (Zerres and Vormwald, 2014).

2.8.4 Crack Closure

In many situations, crack growth rates vary from the traditionally reported long crack data when loading conditions change. For example, in a uniaxial fatigue test, if the loading range is kept constant and the $R$ ratio is decreased, a corresponding decrease in crack growth rate is typically observed as well. The existence of crack closure phenomena, first described in 1970 by Elber (1970), was later determined to be the probable cause of this type of loading dependence. Crack closure can generally be classified as any process by which the effective stress intensity at the tip of a crack is reduced due to premature contact of crack faces. There are many different mechanisms which can result in the development of crack closure, and actual closure levels experienced by a crack may be a combination of one or more of these mechanisms. The most common of types of crack closure are briefly described in the following paragraphs.

Perhaps the most relevant type of crack closure for mode I crack growth is plasticity induced closure, which results from compressive residual stresses in the wake of a growing crack. These residual stresses, brought about by elastic constraint
surrounding the zone of plastically deformed material at a crack tip, cause the crack faces to prematurely contact at a positive remote stress value. In general, the influence of plasticity induced closure effects increases with an increase in crack length due to the generation of larger plastic zone sizes at the crack tip.

Roughness induced closure is a type of crack closure in which the premature contact of crack faces upon unloading is caused by the presence of crack face asperities. It is a function of crack path meandering, crack tip plasticity, and the extent of mode II crack tip displacement. The effects of roughness induced closure are most significant at lower stress intensity ranges and load ratios where the CTOD is small. Small plastic zone sizes, which encourage single shear mechanisms of crack advance, and microstructures which are coarse grained and/or promote crystallographic crack paths also promote this type of crack closure (Ritchie, 1988).

Oxide or corrosion debris induced crack closure is the result of the formation of oxide layer(s) and/or the buildup of corrosion debris between crack faces due to moist environments and crack face fretting, respectively. Oxide induced closure has the largest effect on crack growth in highly oxidizing environments, in the near threshold regime where CTOD is small, for low load ratios, and in situations where roughness and/or plasticity induced closure effects are large (Hussain, 1997; Ritchie, 1988). A rougher crack surface and increased periods of crack contact caused by these other closure mechanisms will promote the development of fretting debris through abrasion.

Transformation induced closure is another type of crack closure that is sometimes experienced in materials which undergo loading induced phase transformations under cyclic loading conditions. Phase changes which result in a large increase in volume in the
plastically deformed crack tip region will result in residual compressive stresses that act to increase the effect of plasticity induced closure (Hussain, 1997).

Due to the nature of crack closure, the effects of any closure mechanism on mode I crack growth rate tend to decrease as the load ratio is increased. This is because as the loading becomes more tensile-dominated, the crack experiences larger opening displacements at the minimum load, which act to prevent crack face contact even when closure mechanisms are present. Therefore, the commonly observed mean stress dependence of crack growth rate is often attributed to crack closure. In order to correct for closure effects, the SIF level at which point crack faces first come into contact, $K_{op}$, must be determined. Then, an effective SIF range can be computed as follows:

$$\Delta K_{eff} = K_{max} - K_{op}$$

This effective SIF value can then be used in a Paris Law equation to calculate the crack growth rate independent of load ratio. Accounting for closure effects has been shown to successfully collapse experimental crack growth rate data for a number of nominal load ratios into a single curve (Tanaka, 1989).

One problem with accounting for closure effects through an equation such as Equation (2.63), however, is that it is difficult to obtain appropriate crack opening stresses/closure levels. This is due, in part, to the fact that the combined effect of the various closure mechanisms can vary independently with crack length, loading conditions, material properties, and testing environment. Therefore, it is difficult to predict crack opening levels for general loading cases, even if experimental data is available from standard crack growth tests. Additionally, due to the nature of the compliance curve from which closure levels are typically measured, there is a significant
range of loads at which a crack is partially closed. This makes a consistent and physically based definition for crack opening load difficult to define. Although standards have been developed to help reduce variability in opening load definition, these do not ensure that the resulting $K_{op}$ is the true value which reflects the actual changes in crack driving force. Compliance measurements themselves have also been the subject of scrutiny due to scatter that exists from a lack of sensitivity when crack opening displacements are measured remote from the crack tip (Anderson, 2005).

Shear- or mixed-mode crack growth situations present an additional set of challenges to the quantification of crack closure effects. This is due to the fundamental differences that exist between crack extension modes. For shear-mode crack growth, crack closure mechanisms still act to reduce the effective crack driving force at the crack tip, but the effects are generally not dependent on load ratio. This is because shear-mode crack extension is not a function of crack opening displacements and, ideally, crack surfaces are always in contact. Instead, crack closure mechanisms in shear-mode crack growth serve to reduce effective crack driving forces by causing an increase in friction and mechanical interlocking between opposing crack faces. More information regarding the role of crack face interaction in shear-mode crack growth is discussed in the next section.

2.8.5 Crack Coalescence and Branching

As previously mentioned, there sometimes exists a discrepancy between the preferred crack growth mode for un-notched and notched specimens. Cracks growing from notches, even if initiated on maximum shear planes, typically always turn to grow in mode I on planes of maximum tensile stress, while cracks in un-notched specimens have
been shown to propagate on maximum shear planes, maximum tensile planes, or a combination of both. The preferred plane depends on the material, the nominally applied loading, and/or the loading magnitude.

One explanation offered for the discrepancy between crack growth in un-notched and notched specimens is that a fundamental difference in crack growth mechanism exists. In the un-notched specimens, the uniformly stressed gage section allows for a large number of microcracks to develop and form “crack networks” across the specimen surface. The growth of these cracks is driven primarily by far field stresses and plasticity of the gage section. Each small crack can grow to a different length and at a different rate due to microstructural variations, shielding, and other interaction with adjacent cracks in the network. These cracks grow individually and remain relatively small until near fracture where they coalesce to form a failure crack which retains the same orientation of the individual cracks. Shamsaei and Fatemi (2014) observed that the development of microcrack networks in un-notched specimens is more prominent in the low cycle fatigue regime due to increased plasticity in the gage section activating more slip systems. This observation agrees with the higher tendency for macroscopic shear-mode crack growth in this regime. They also observed a larger number of microcracks initiating in more ductile behaving materials, as opposed to higher strength materials, for solid cylindrical specimens, as opposed to tubular specimens, and for in-phase axial-torsion loadings, as opposed to 90° out-of-phase loadings.

For the notched specimens, on the other hand, only a small number of dominant cracks nucleate at the location(s) of maximum stress around the notch, typically on the maximum shear plane. With the absence of microcrack networks, unable to develop in
the lower stressed material surrounding the notch, the cracks quickly turn into the mode I direction and grow independently, driven by the stress and plasticity fields at the crack tip. Therefore, cracks initiated from notches generally grow in a continuous manner and lack the retardation or acceleration effects observed for cracks in un-notched specimens due to crack interaction. This is also the case for cracks growing in un-notched specimens tested at lower stress amplitudes. As early as the 1950s, Marco and Starkey (1954) noted a difference between these two types of crack growth mechanisms and termed the growth of long cracks through microcrack coalescence a type R crack system, whereas crack growth dominated by the propagation of a single crack was termed a type S crack system.

Knowing by which mechanisms and on which planes a crack will grow is essential to performing accurate crack growth analysis. There are many parameters that can influence mixed-mode crack growth behavior, some of which include loading magnitude and R ratio, loading sequence, material strength, and crack closure (Qian and Fatemi, 1996b). To account for these effects, several prediction models or correlation parameters for both mixed-mode growth direction and growth rate, which were reviewed in a previous section, have been proposed. However, these models are most effective in the absence of crack coalescence and cannot account for differences in crack path for such cases. For example, the MTS criterion would predict a mode II crack initiated under pure torsion loading to immediately branch at an angle of 70.5° to the initial crack growth direction and grow in mode I regardless of the applied loading magnitude (Doquet and Bertolino, 2008a; Murakami and Takahashi, 1998). This prediction is not in agreement with the shear-mode crack growth observed in many un-notched specimens from initiation up until fracture.
Owing to the complexity of the task, little research is available on models which attempt to quantify when or if cracks will grow by type R mechanisms, type S mechanisms, or whether or not these two mechanisms are even responsible for the differences in un-notched and notched specimen crack paths. To simplify this problem for the following discussion, it will be assumed that naturally occurring fatigue cracks always tend to initiate on planes of maximum shear stress. This is an assumption supported by both the physics of fatigue crack initiation and by large amounts of experimental evidence (Shamsaei and Fatemi, 2014). This then leaves two possibilities for subsequent crack growth; the crack can either branch and grow in mode I on maximum tensile plane(s), or the crack can remain coplanar and grow in mode II or mixed-mode conditions until failure.

Concerning the first possibility, there is a reasonable amount of literature available describing the processes governing the transition from stage I to stage II crack growth. Murakami and Takahashi (1998) studied the behavior of small mode II cracks growing under near threshold conditions for pure torsion loading of medium carbon steel solid cylindrical specimens. They observed three different conditions for crack growth from a precrack. Cracks either started to grow in both modes I and II from the precrack tip, with the mode II cracks stopping after a short distance to give way to the more dominant mode I cracks, or only mode I cracks were observed to grow from the precrack tip, or the crack propagated in mode II for a short distance before eventually branching to grow in mode I. Crack branching was attributed to a higher threshold SIF value for modes II and III as compared to mode I. They argued that this led to an arrest of shear-mode cracks as they dropped below their threshold value, presumably due to an increase
in roughness induced crack closure effects, which gave way to mode I growth when cracks were still above the mode I threshold SIF value (Murakami et al., 2003). They concluded that the mode I SIF of the branched cracks governed the fatigue limit for small cracks in torsion and were able to predict the fatigue limit using an extension of the square root of area parameter (Murakami and Takahashi, 1998).

Makabe and Socie (2001) studied crack growth in precracked specimens of 4340 steel under torsion loading with and without the influence of static axial stresses. They observed that cracks grew longer in mode II before branching as the applied shear strain and/or static axial stress increased. This was attributed to decreased contact and friction between opposing crack faces and the resulting change in slip band density surrounding the crack tip. The eventual branching mechanism was described as a zig-zagging between perpendicular slip bands generated in the vicinity of the crack tip. It was concluded that in order for a crack to continue to grow in shear-mode, the driving force for shear deformation at the crack tip must be large enough to overcome the friction between crack faces.

Considering the effects of crack face friction, Tong et al. (1995a) proposed a model to describe changes in local stress intensity factors during shear-mode crack growth based on an idealized crack face asperity angle and coefficient of friction. The model was developed for a two-dimensional edge crack growing nominally in pure mode II and relates theoretical crack sliding displacements computed using FEA to local displacement components due to wedging of crack face asperities. The local displacements are then used to compute a local reduction in mode II SIF due to friction and a local increase in mode I SIF due to wedging. Comparing the effective mode II SIF
to the local mode I SIF provides a means of predicting crack branching. The model predictions were not compared directly to experimental results, but the predicted trends agreed with experimental observations. In an extension of the same work (Tong et al., 1995b), the model was applied to mixed-mode I and II loadings and could be extended to other crack geometries as well, provided the appropriate weight functions and crack face loading distributions are known.

More recently, Künkler et al. (2008) predicted the two-dimensional crack propagation behavior of short cracks under uniaxial loading in the region of stage I to stage II transition using a microstructure sensitive modeling technique. They found that mode II growth occurs primarily on single slip systems within individual grains. Once the crack reaches a length where its plastic zone is large enough to activate additional slip systems in neighboring grains, the crack begins to transition to mode I growth by switching from a single slip mechanism to a double slip propagation mechanism. In double slip crack growth, shear displacements on the two different slip systems cause a crack tip opening which promotes mode I crack extension. The crack length at this transition was considered dependent upon crack geometry, grain size, and the orientation of slip systems in neighboring grains.

Pokluda et al. (2014) reviewed research on shear-mode crack behavior in metallic materials. It was concluded that the main factor driving the branching of mode II cracks is crack face friction and wedging caused by surface asperities. These asperities result from microstructural level crack meandering due to differing orientations of slip systems, and their average inclination angle was shown to vary based on the crystallographic structure of the material. When mode II sliding displacements occur between opposing
crack faces, the wedging effect of these asperities acts to simultaneously reduce the effective value of the mode II SIF and introduce a local mode I SIF component. When the conditions are such that the local mode I SIF value exceeds the mode I threshold, crack branching was considered to occur. The authors applied a simple crack branching criterion to predict whether or not crack branching would occur in the threshold region before a crack became non-propagating. They compared the predictions to experimental data of four different metals and reported good agreement.

While the aforementioned studies deal with describing or predicting the development of a mode I branch from a mode II crack in the near threshold regime, far less literature is available concerning the conditions for continuous coplanar shear-mode propagation of a crack. Tschegg (1983) discussed this issue in a study of mode III crack growth behavior in circumferentially grooved cylindrical specimens of 4340 steel. The change in fracture mode was found to coincide with the intersection of the mode I and mode III crack growth curves. At lower stress intensity values, where crack branching and factory roof fracture surfaces were observed, it was speculated that the mode I loading of favorably oriented microcracks in the crack tip plastic zone would produce higher crack growth rates than the effective mode III loading of the main crack. Therefore, whichever loading mode produced the highest crack growth rate values for a given combination of loading conditions and crack length was considered to control the crack growth process.

Using a similar “maximum growth rate” approach, Doquet and Bertolino (2008a) addressed the fracture mode issue by considering a local approach based on elastic-plastic crack tip stresses and strains derived via nonlinear incremental FEA. Two multiaxial
fatigue damage parameters, one predicting tensile-dominated failure (Smith-Watson-Topper) and one predicting shear-dominated failure (Fatemi-Socie) were then used to predict crack path based on which mode would produce the most damage (based on the shortest predicted life and highest growth rate). The damage parameter values were averaged over an arbitrary length (between 0.020 and 0.100 mm) along a radial line emanating from the crack tip along either plane experiencing the maximum normal strain or maximum shear strain range. A transition in growth mode from tensile-dominated to shear-dominated was successfully predicted at higher applied loading, but no quantitative comparisons were made to experimental results. A major drawback of this approach, however, is the need to perform nonlinear FEA over an entire cycle for each crack length and loading level being considered.

Tanaka (2012), in an investigation on shear crack growth in circumferentially grooved specimens of a stainless and a carbon steel, speculated that a criterion for coplanar versus branched crack growth could be related to the shear strain range ahead of the crack tip. For higher loading levels, it was suggested that increased plasticity at the crack tip could lead to the initiation of local shear oriented microcracks ahead of the crack tip, which would encourage coplanar crack growth through local coalescence. However, no quantitative analysis was provided.

2.8.6 Physically Small Crack Growth

Aside from variations in crack growth rate observed for microstructurally small cracks, which are not discussed in this study, growth rates for physically small cracks have also been shown to deviate from long crack data. Typically, growth rates for physically small mode I cracks are much higher than those for long cracks at the same
nominal SIF range (El Haddad et al., 1979; Pearson, 1975). This is true, in many cases, even when LEFM restrictions have not been violated. Often termed “short/small crack behavior,” these higher growth rates are commonly attributed to a combination of plastic zone size effects, and a decrease in crack closure due to the difference in length scale between short and long cracks (Anderson, 2005; Stephens et al., 2000). Understanding small crack behavior has been the subject of much investigation over the years due to its significance in fatigue applications. Because cracks typically spend the majority of their growth lives as small cracks, even small errors in growth rate predictions can have a significant impact on the resulting life predictions.

In the near threshold region, the relationship between short crack behavior and stress range can be visualized using a Kitagawa-Takahashi diagram (Kitagawa and Takahashi, 1976). Shown schematically in Figure 2.9, this diagram is useful in determining when the conditions required for a non-propagating crack will transition from being fatigue limit-controlled (line B-B) to SIF-controlled (line A-A). The theoretical transition between these two conditions, represented by the intersection of lines A-A and B-B, is often regarded as an important fracture mechanics parameter. In fact, the Theory of Critical Distances, discussed previously, is based on this quantity, which can be computed using Equation (2.30). However, it can be seen from the figure that actual crack behavior tends to make a gradual transition between these two conditions, and in reality there is no clearly defined transition length. Although the smaller crack lengths represented in Figure 2.9 are generally still considered to be microstructurally small, for most materials, the beginning of the physically small growth
regime will begin before the crack reaches a length of \( a_2 \). Therefore, some degree of short crack behavior must be considered for most physically small crack growth analyses.

One method of accounting for the higher growth rates experienced by small cracks is to simply extrapolate the region II Paris Law equation down to region I crack growth levels. Since small cracks, in most instances, still exhibit a threshold behavior similar to long cracks, this approach tends to result in conservative growth rate predictions for small cracks. However, in some cases, small crack growth rates can also exceed predictions, even with the extended Paris equation (Stephens et al., 2000). In these cases, non-conservative results would be obtained. The chance for non-conservative predictions, coupled with its general inaccuracy, makes this approach undesirable for application to in-depth small crack growth analyses.

An alternative approach to account for small crack behavior is to artificially extend the crack length used in SIF computation. Although empirical in nature, El Haddad et al. (1979) showed that by increasing the crack length by a constant amount, equal to the characteristic length defined by Equation (2.30), resulting SIF values were able to successfully correlate short crack growth rates with long crack data. Experimental results were obtained from tests of G40.11 steel for a variety of loading levels, and up to crack lengths of 1 mm.

In an attempt to provide a more physically sound approach to small crack growth analysis, McEvily et al. (1991) proposed a modified LEFM model which takes into account large scale plasticity effects, crack closure, and crack growth threshold. Plasticity effects were accounted for directly in the SIF solution using Irwin’s approach.
and a modified crack length, extended by a distance equal to its plastic zone size radius according to the Dugdale (1960) strip-yield model:

\[ \Delta K = \left[ \frac{\pi \rho_e}{4} + Y_I \frac{\sqrt{2 \pi a}}{2} \left( 1 + \sec \frac{\pi \sigma_{\text{max}}}{2 \sigma_y} \right) \right] \Delta \sigma \]  

(2.64)

where \( \rho_e \) is a material constant considered to be the effective crack tip radius, \( Y_I \) is the mode I geometry factor, \( a \) is the crack length (or half crack length), and \( \sigma_y \) is the material yield strength. Threshold and crack closure effects were then considered through the crack growth law as follows:

\[ \frac{da}{dN} = A \left[ (K_{\text{max}} - K_{op}) - K_{\text{eff,th}} \right]^2 \]  

(2.65)

where \( K_{\text{eff,th}} \) is the effective crack growth threshold (in the absence of crack closure effects), \( A \) is a material constant dependent on material and environment, and \( K_{op} \) is the SIF value at which the crack first opens, defined as follows:

\[ K_{op} = (1 - e^{-\kappa l}) K_{op,\text{max}} \]  

(2.66)

In this equation, \( \kappa \) is a constant which determines the rate of crack closure development, \( l \) is the length of a newly formed fatigue crack (measured from the tip of the initial crack or stress raiser), and \( K_{op,\text{max}} \) is the SIF level at which crack opening occurs for a macroscopic crack. In a comparison with experimentally observed small crack growth data for 1045 steel, the authors reported reasonable crack growth rate predictions at various stress levels using their model.

2.8.7 Notch Affected Crack Growth

Similar to small cracks, cracks growing from a notch tend to behave differently than cracks growing in a uniformly stressed region. This difference is greatest at the
notch root, and gradually diminishes as the crack grows away from the notch. Since a
large portion of the fatigue life of a component can be spent in the small crack growth
regime, however, being able to accurately predict the behavior of a crack growing
through a notch affected zone is often an essential step in a crack growth analysis.

There are several characteristic effects of notches that can influence the growth
behavior of a crack. Among these, the most prominent are the stress/strain concentration
effect and stress/strain gradients produced in the vicinity of a notch. The stress
concentration effect will produce a localized zone of elevated stresses, which will result
in increased driving forces for a notch affected crack. For a blunt notch or low levels of
applied loading, however, peak stresses may not exceed the yield strength of the material
and local deformation will remain elastic. For a sharp notch or higher loading levels, on
the other hand, local stresses at the notch root can exceed the yield strength of the
material and produce a zone of local plastic deformation in addition to the surrounding
notch elastic zone. When plasticity occurs at a notch, this zone can experience residual
stress effects, as well as cyclic hardening or softening. Residual stresses can then lead to
mean stress effects on crack growth, while cyclic hardening or softening can locally alter
the properties of a material, including its fracture resistance. Figure 2.10 illustrates
schematically a crack, including its own region of crack-tip plasticity, growing through
local elastic and elastic-plastic stress-strain fields at a notch.

In addition to the stress concentrations and stress gradients produced by a notch, a
notch can also serve to locally change the stress state within a component. For example, a
notched component subjected to a uniaxial tension loading will undergo a multiaxial
stress state in the notch affected zone. Outside of the notch zone stresses will only be in
the direction of the loading, while in the notch zone there will be an additional component of stress perpendicular to the applied loading (Stephens et al., 2000).

These notch effects, acting alone or in a synergistic manner, cause cracks growing from a notch to behave differently than similar cracks growing in the absence of a notch. As can be seen from Figure 2.11, which qualitatively compares trends between notch affected crack growth and long crack growth, cracks growing from a notch tend to exhibit higher growth rates than long cracks at the same nominal SIF range. For a crack growing through a purely elastic notch zone, growth rates are initially higher than those for long cracks and are always increasing. As the crack grows, the difference between the notch crack and long crack growth rate diminishes before the two curves eventually merge. For a crack growing through an elastic-plastic notch zone, however, although crack growth rates start out significantly higher than those for long cracks, growth rates initially decrease as the crack length increases. Eventually, growth rates peak at a minimum before increasing again to merge with the long crack curve after a sufficient amount of growth has occurred (Stephens et al., 2000). This trend is attributed to the fact that, as a crack grows, notch induced plasticity generally decreases at a faster rate than crack tip plasticity increases. It should also be noted that for sharper notches, where steep stress gradients exist, growth rates can drop below the threshold SIF range before sufficient crack tip plasticity has been generated to sustain crack growth. These conditions can result in a crack growing for a short distance before eventually becoming a non-propagating crack (Shin and Smith, 1988).

Because cracks initiate from notches, notch effects must often be considered for multiple stages of crack growth. For example, microstructurally and physically short
crack growth stages, as well as early stages of long crack growth, can all occur when a crack is still growing through the notch affected zone. The fact that microstructurally and physically short cracks both exhibit anomalous growth rate behavior when compared to long crack data, even with respect to un-notched specimens, only adds to the complexity of predicting crack growth from a notch. Additional considerations, such as notch effects on crack closure, may also need to be taken into account.

Although LEFM techniques have been fairly well established over the years for predicting the growth of long fatigue cracks, techniques for predicting crack growth behavior under the influence of a notch are not as clear. In order to develop a reliable and generally applicable model to predict crack growth from a notch, all of the previously mentioned factors should be considered, as well as effects due various material behaviors. As a result, many different methodologies have been proposed over the years by a number of researchers to address these issues, albeit mostly for uniaxial loading cases. These include, among others, extensions of effective crack length models, effective stress intensity models, effective strain intensity models, and elastic-plastic fracture mechanics models such as J-integral. Further extending these approaches from the uniaxial loading cases, for which they were developed, to multiaxial loading cases presents an even greater challenge. The following sections will highlight just a few of the more prominent models from each of the different methodologies.

2.8.7.1 Effective Crack Length Models

Similar to the short crack approach of El Haddad et al., Smith and Miller (1977) accounted for notch effects by suggesting that a notch causes a crack to effectively act as if it were longer than its actual length. Based on an in-depth stress analysis for cracks
growing from elliptical notches of various geometries in finite and semi-infinite plates, they developed a notch contribution to crack length parameter, \( e \), which takes into consideration both notch size and shape by including notch depth and root radius terms as follows:

\[
e = \begin{cases} 
7.69 \cdot l \sqrt{D/r} & \text{for } l < 0.13\sqrt{Dr} \\
D & \text{for } l \geq 0.13\sqrt{Dr}
\end{cases}
\]

where \( l \) is the actual crack length from the notch root, \( D \) is the notch depth, \( r \) is the notch root radius, and \( L = l + e \) is the effective crack length. The effective crack length may then be used with typical LEFM expressions to compute an effective SIF.

Although the theoretical notch contribution to crack length varies nonlinearly with notch geometry, the authors chose to use a single linear equation as an estimation for all cases. In doing this, a number of concerns were taken into consideration. For example, the proposed approximation both underestimates and overestimates the exact nonlinear corrections in order to compensate for errors over the whole range of crack growth within the notch affected zone. Also, the best fit for the equation was obtained at small crack lengths, as a majority of fatigue life is spent in this regime. Finally, the function was made to be simple and easily applied to a wide range of notch geometries. As a crack grows sufficiently long, the notch contribution becomes equal to the depth of the notch, thus suggesting the notch has no influence on the growth of the crack, other than acting as part of its length.

In order to test the applicability of their crack growth model, Smith and Miller compared the crack growth lifetime calculated using the model, from an initial small starter crack length to an arbitrary crack length of 28 mm, to experimental results.
Comparisons were made for both center and edge notched specimens of various geometries. Results were found to be in very good agreement with experimental results. In most cases, predicted lives fell within 15-20% of the experimentally determined lives.

2.8.7.2 Effective Stress Intensity Models

Although LEFM parameters, as commonly used, are insufficient to describe crack growth from notches, with some modification, an effective stress intensity factor can be used to account for notch effects on small crack growth. For example, Dowling (1979) proposed a model based on two individual expressions for SIF, dependent on crack length. This model accounts for notch effects on small cracks by modifying the remotely applied stress term by an appropriate stress concentration factor and correcting for changes in local crack geometry prior to SIF calculation. The model can be represented by the following equations:

\[ K_{\text{small}} = 1.12(K_t S)\sqrt{\pi l} \quad (2.68) \]

\[ K_{\text{long}} = Y_l S\sqrt{\pi (l + D)} \quad (2.69) \]

where \( K_t \) is the elastic stress concentration factor, 1.12 is a correction factor for the free edge of a notch, \( S \) is the nominally applied stress, \( l \) is the crack length from the notch root, and \( D \) is the notch depth.

The crack length at which the transition is made from the small crack solution to the long crack solution is found as the intersection of the two equations. Solving for this intersection results in the following expression for crack transition length:

\[ l' = \frac{D}{(1.12 K_t / Y_l)^2 - 1} \quad (2.70) \]
This model was shown to be in fairly good agreement with experimental results for both sharp and blunt notch configurations (Dowling, 1979). However, since the small crack stress intensity is based on the elastic stress concentration factor, it may not accurately describe local crack growth conditions in situations where notch root plasticity is significant.

Another, more recent, effective stress intensity factor model was proposed by Kujawski (2008). Based on an estimate for the elastic notch-tip stress field distribution, an expression was extended for application to problems with short crack growth from an elastic-plastic notch. The notch-tip stress field expression, describing elastic stress concentrations starting at the notch root \((x = 0)\) and moving outward (with increasing \(x\)), is based only on \(K_t\) and notch root radius, \(r\), and takes the following form:

\[
k_t(x) = f K_t \left[ \left(1 + 2 \frac{x}{r}\right)^{-0.5} + \left(1 + 2 \frac{x}{r}\right)^{-1.5} \right]
\]

(2.71)

Where \(f\) is defined as follows:

\[
f = \begin{cases} 
1 + \frac{\tan \left( \frac{\pi}{2K_t} \right)}{2.8} \left( \frac{x}{r} - 0.2 \right) & \text{for } \frac{x}{r} > 0.2 \\
1 & \text{for } \frac{x}{r} \leq 0.2
\end{cases}
\]

(2.72)

In order to apply this expression to cases involving notch tip plasticity, Neuber’s rule was used. An expression for the strain field at the notch is obtained by implementing Equation (2.22) in conjunction with a stress-strain relationship for the desired material. Finally, a stress intensity factor expression derived by Irwin (1960) for elastic notches was adapted to elastic-plastic notches by substituting strain concentration factor, \(k_e(x)\), in place of stress concentration factor, \(k_t(x)\):

\[
K_q = \lim_{r \to 0} k_e(x) S \sqrt{\pi (r + l)}
\]

(2.73)
where $x$ is taken to be equal to the crack length from the notch root, $l$. Using this expression for stress intensity factor, crack growth rates are able to be predicted using traditional LEFM approaches and crack growth laws. However, no comparison was made in the study between theoretical crack growth results and experimental data.

### 2.8.7.3 Elastic-Plastic Fracture Mechanics Models

In an attempt to better account for local plasticity effects, El Haddad et al. (1980) extended EPFM concepts to short crack growth from notches by making a modification to a $J$-integral solution. In order to do this, they first used an effective crack length parameter to correct for short crack effects, as described previously. This parameter, which is a constant for a given material and material condition (Equation (2.30)), is included in both the elastic part of the $J$-integral, by inclusion into the SIF solution, and in the plastic part of the $J$-integral solution. They also included a modification of the plastic $J$-integral estimate for the exponential hardening case of tension loaded cracked members. The final form of the $J$-integral expression is given below:

$$
\Delta J = 2\pi F^2 a \left[ \frac{f(n)}{n+1} \left\{ \frac{\Delta \sigma \Delta \varepsilon E}{E} \right\} - \left\{ \frac{2f(n)}{n+1} - 1 \right\} \frac{\Delta \sigma^2}{2E} \right] \quad (2.74)
$$

where $f(n)$ is a function of the material strain hardening exponent, $F$ is an elastic geometry factor that accounts for varying component and notch geometries, $E$ is Young’s modulus, and $\Delta \sigma$ and $\Delta \varepsilon$ are local stress and strain ranges, respectively. These local stresses and strains can be derived via Neuber’s rule or finite element analysis. This solution, when used to plot crack growth rate data in terms of an equivalent SIF range, $(\Delta K = \sqrt{E\Delta J})$, for experimental crack growth data obtained using a variety of notch geometries, was shown to provide excellent correlation to long crack data.
2.8.7.4 Notched Crack Growth under Multiaxial Loading

The models discussed to this point have all been for the case of cracks growing from notches under uniaxial constant amplitude loading conditions. Although useful, these models cannot appropriately address issues brought about by the presence of multiaxial stress states. Since multiaxial loadings are very common in application, it is desirable to be able to predict the fatigue crack growth behavior from notches under these conditions as well. The following notched crack growth model, proposed by Hertel and Vormwald (2011), is based on the J-integral and is used to describe crack growth from notches under multiaxial variable amplitude loading conditions based on a critical plane type concept. Although the model is fairly complex, and contains many expressions for quantifying different parameters, only some of the key elements will be highlighted herein.

To begin, in order to account for the presence of multiple crack extension modes, a growth law was first established to describe the effect of each mode on the resulting crack growth rate. The following expression was used by the authors to describe crack growth contributions:

\[
\frac{da}{dN} = C \Delta J^{m}_{I, eff} + C \Delta J^{m}_{II, eff} + C \Delta J^{m}_{III, eff}
\]

(2.75)

\( C \) and \( m \) in the above equation are typical Paris type growth constants, while the cyclic J-integral terms are used to describe the crack driving forces for each cracking mode with respect to various candidate crack growth planes. It is clear from this expression that each cracking mode is considered to contribute independently to the overall crack growth rate. Had a synergistic effect between the modes been assumed, a more complex mixed-mode
crack growth model would have been required. An approximation for $C$, based on the elastic stiffness and crack growth slope of the material, was also given as follows:

$$C = 10^{-5} \left( \frac{5 \times 10^5}{E} \right)^n$$  \hspace{1cm} (2.76)

The core of this model comes in the expression of the crack driving force parameters. Crack driving force for each mode of crack opening are given as follows:

$$\Delta J_{I,\text{eff}} = 2\pi Y_I^2 \Delta W_{x,\text{eff}} \cdot l$$  \hspace{1cm} (2.77a)

$$\Delta J_{II,\text{eff}} = \frac{\pi}{1 + \nu} Y_{II}^2 \Delta W_{xz} U_{\text{eff}} \cdot l$$  \hspace{1cm} (2.77b)

$$\Delta J_{III,\text{eff}} = \pi Y_{III}^2 \Delta W_{xy} U_{\text{eff}} \cdot l$$  \hspace{1cm} (2.77c)

where $\Delta W_{x,\text{eff}}$ is the part of the elastic-plastic strain energy density when the crack is fully open, and $\Delta W_{xy}$ and $\Delta W_{xz}$ are the parts of strain energy density composed of shear stresses and strains in the crack plane corresponding to “case A” cracks and “case B” cracks, respectively. The $U_{\text{eff}}$ parameter, which accounts for friction and roughness induced closure effects in shear-mode crack growth, ranges between 0 and 1, and is set equal to the ratio of crack tip sliding displacements for a mode II crack with and without friction stresses on the crack faces. The $Y$ values are geometry factors corresponding to each loading mode, analogous to those used in SIF solutions to account for various component and crack geometries.

The difficulty in computing these expressions stems from the need to derive complex stress-strain histories, involving multiaxial loading and local plasticity effects, which are required to compute the strain energy density components for crack driving force. This was accomplished by Hertel and Vormwald through the application of advanced cyclic plasticity modeling techniques and a notch stress-strain estimation.
model. Several other parameters are given in the paper as well in order to address issues such as crack opening levels, crack growth thresholds, load sequence effects, and statistical size effects, but the expressions are too numerous to be listed here.

In order to evaluate the prediction capabilities of this model, the authors computed theoretical fatigue lives from an initial microstructural scale crack to a crack length of 0.5 mm. Initial crack lengths were computed from the constant amplitude uniaxial strain-life curve by backward integration of the crack growth law. The resulting crack growth lives were then compared to experimental data, as well as to predictions based on the SWT multiaxial fatigue damage parameter. The short crack model was found to produce predictions in good agreement with the experimental results in terms of both fatigue life and failure plane (maximum crack growth plane). Moreover, the short crack model was found to agree better with experimental data than the SWT parameter, especially for variable amplitude loadings. However, it was noted that results for 90° out-of-phase loading conditions indicated the presence of more complex damage mechanisms, including the time dependent interaction between different loading modes, which are not fully understood and require further investigation.

2.8.8 Crack Growth under Variable Amplitude Loading

One of the key requirements for the application of fracture mechanics concepts in a fatigue crack growth analysis is that conditions of similitude should be retained. Similitude implies that for a particular value of driving force parameter (e.g. J-Integral, SIF, CTOD, etc.), the state of stress surrounding the tip of a crack is uniquely described by the value of that parameter. However, for variable amplitude loading conditions, load history dependence may alter the local stress state at a crack tip, allowing the possibility
for multiple crack growth rates to occur at the same nominal driving force value. Because of this, the amount of crack growth experienced in a variable amplitude loading history cannot be accurately assessed by simply summing the nominal crack growth increment for each applied cycle.

Load history effects on variable amplitude fatigue crack growth are analogous to those for fatigue damage calculation in a crack initiation analysis, especially when considering crack initiation from a notch. One of the biggest contributors to load history dependence in a crack growth analysis is the development and variation of plastic deformation surrounding the crack tip under varying load magnitudes. This is especially notable for cases in which mode I overloads or underloads occur in a loading sequence. Following a mode I tensile overload cycle, applied in an otherwise constant amplitude loading history, a deceleration in crack growth rate is typically observed for a period of time before growth rates eventually return to levels experienced prior to the overload. The magnitude of this “retardation” effect has been shown to directly correspond to the magnitude of the overload. Crack growth retardation has been attributed to a number of mechanisms including: crack blunting, an increase in compressive residual stress in front of the crack tip, increased plasticity induced crack closure due to an increase in residual deformation behind the crack tip, crack deflection (i.e. increased roughness induced closure), and strain hardening effects. Although all of these mechanisms may contribute in some degree to the retardation effects, residual stresses and changes in closure levels tend to be the favored explanations (Anderson, 2005; Geary, 1992).

Opposite to the effect of overloads, underloads (either large compressive loads or occasional tensile loads well below previous minimum levels) have been shown to result
in periods of accelerated crack growth (Ohrloff et al., 1988; Stephens et al., 1976). Acceleration effects have been attributed to tensile residual stresses resulting from the resharpening of a blunted crack following a compressive load, as well as to the destruction of crack face asperities and the resulting decrease in roughness induced closure effects. Ohrloff et al. (1988) showed that the frequency of overloads, underloads, or a combination of both can have a significant effect on the magnitude of the resulting acceleration and/or retardation of subsequent crack growth for two aluminum alloys.

Early models to account for load history effects in variable amplitude crack growth include those proposed by Wheeler (1972) and by Willenborg et al. (1971). Both of these models are based on the assumption that variations in crack growth rate are a direct result of residual stresses developed in front of the crack. The Wheeler model uses a retardation factor to modify the nominally computed LEFM crack growth rate as a crack grows subsequent to a tensile overload. The resulting retardation corrected growth rate can be expressed as follows:

$$\frac{da}{dN}_R = \frac{da}{dN} \left( \frac{\Delta a + r_{y(c)}}{r_{y(o)}} \right)^\gamma$$

(2.78)

where $\Delta a$ is the amount of crack growth following the overload, $\gamma$ is an empirically adjustable shaping parameter (taken as the value that fits best to experimental data for a given material, stress level, crack shape, and loading spectrum), $da/dN$ is growth rate computed from the nominally applied SIF, and the current and overload plastic zone sizes are calculated according to Equations (2.79) and (2.80), respectively:

$$r_{y(c)} = \frac{1}{\beta \pi} \left( \frac{K_{max}}{\sigma_y} \right)^2$$

(2.79)
\[ r_{y(o)} = \frac{1}{\beta \pi} \left( \frac{K_o}{\sigma_y} \right)^2 \] (2.80)

In these equations, \( \beta \) is equal to 2 for plane stress and 6 for plane strain conditions, \( K_o \) is the peak SIF during an overload, and \( K_{max} \) is the maximum SIF in the current cycle. This model assumes that retardation effects disappear after the boundary of the current crack tip plastic zone reaches the boundary of the plastic zone caused by a tensile overload.

The Willenborg model, on the other hand, computes a residual stress SIF which introduces mean stress-like effects in crack growth rate calculation. This can be expressed as follows:

\[
K_r = \begin{cases} 
K_o \left( 1 - \frac{\Delta a}{r_{y(o)}} \right)^{0.5} - K_{max} & \text{for } \Delta a \leq r_{y(o)} \\
0 & \text{for } \Delta a > r_{y(o)}
\end{cases}
\] (2.81)

where parameters are defined the same as in the Wheeler model. This \( K_r \) value is then used to compute an effective \( R \) ratio as follows:

\[
R_{eff} = \frac{K_{min} - K_r}{K_{max} - K_r}
\] (2.82)

This \( R_{eff} \) can then be used in a standard crack growth model that accounts for load ratio dependence, such as the Walker model, to compute the effective growth rate. This model assumes that retardation effects disappear after the crack has grown through the plastic zone caused by a tensile overload. For situations involving variable amplitude loading, as opposed to constant amplitude with occasional overloads, both the Wheeler and Willenborg models still apply, but must be implemented on a cycle-by-cycle basis. However, neither model is able to account for crack growth acceleration effects due to underload events.
More recently, the idea that residual stress distributions surrounding the crack tip are responsible for load sequence effects on crack growth rate has been incorporated into the variable amplitude fatigue crack growth software UniGrow (Mikheevskiy et al., 2010). The UniGrow crack growth model is based on the assumption that a real material can be represented by a set of elementary particles or material blocks with a finite dimension, $\rho^*$, which can either be determined as a function of material fatigue limit stress and threshold SIF range, or through fitting to experimental constant amplitude crack growth rate data. By considering a blunt crack tip with a radius of $\rho^*$, local elastic stress and strain values can be determined from the applied SIF using the Creager-Paris solution (1967). Then, the actual stress-strain response of the material near the crack tip, including residual stresses, can be determined using a cyclic stress-strain relation (e.g. Ramberg-Osgood) in conjunction with typical notch analysis techniques (e.g. Neuber’s rule). By using the material strain-life curve and SWT fatigue damage parameter, the instantaneous crack growth rate can be calculated using the following equation:

$$\frac{da}{dN} = \frac{\rho^*}{N_f}$$

(2.83)

where $N_f$ is the number of cycles to failure computed from the SWT damage parameter at the current crack tip stress-strain state. In order to account for variations in local stress-strain response due to residual stress distributions from all previously applied loading cycles, a set of “memory rules” was established based on experimental observations for fatigue crack growth under variable amplitude loading (Mikheevskiy and Glinka, 2009).

By expressing local stress-strain distributions in terms of SIF, and equating crack growth rates to those obtained through Equation (2.83), the following two-parameter
driving force equation can be used to establish a link between fatigue and fracture mechanics concepts (Noroozi et al., 2007):

\[
\frac{da}{dN} = C[(K_{max} + K_r)^p(\Delta K + K_r)^{1-p}]^m
\]  

(2.84)

where \(K_{max}\) and \(\Delta K\) are the maximum value and range of applied SIF, respectively, \(K_r\) (generally a negative value) is the residual SIF obtained through integration of the residual stress distribution surrounding the crack tip, and \(C, p,\) and \(m\) are fatigue crack growth constant, driving force constant, and fatigue crack growth exponent, respectively. \(C, p,\) and \(m\) can be calculated from material deformation and fatigue properties as follows:

\[
C = 2\rho^* \left[ \frac{1}{2\sigma_f^{1.2}} \left( \frac{\psi_{y,1}}{E^{n'(\sqrt{2\pi\rho^*})}} \right)^{3n'+1} \right]^{1/n'+1} \left[ \frac{1}{2\sigma_f^{1.2}} \left( \frac{\psi_{y,1}}{E^{n'(\sqrt{2\pi\rho^*})}} \right)^{3n'+1} \right]^{-1/2b}
\]  

(2.85)

\[
p = \frac{n'}{n'+1}
\]  

(2.86)

\[
m = -\frac{1}{b}
\]  

(2.87)

where \(\psi_{y,1} = 1.633\). Alternatively, if crack growth rate data are available, these constants can be fit directly to the two-parameter driving force expression using linear regression techniques.

As an alternative to residual SIF and two-parameter approaches, other crack growth models have focused on relating variations in crack growth rate to changing levels of crack closure behind the crack tip, an idea originally proposed by Elber (1970). Evidence supporting closure related growth rate effects includes observations of a delayed retardation effect, where the crack growth rate immediately following an
overload accelerates briefly before falling below levels experienced prior to the overload (Venkateswara Rao and Ritchie, 1988; Ward-Close et al., 1989). It was postulated that this initial acceleration was the result of suppression of near-tip closure due to increased crack opening displacement caused by crack tip blunting after an overload. Then, once the crack begins to grow into the overload induced residual stress field, near-tip closure increases again, resulting in subsequent crack growth retardation. However, there is still much debate on whether crack closure-based or two-parameter models are more appropriate for the consideration of load sequence and/or mean stress effects on fatigue crack growth rate. Some researchers (e.g. (Louat et al., 1993; Vasudevan et al., 1994)) argue that a lack of load ratio effects on threshold SIF levels in inert environments discredits the role of plasticity induced crack closure in the crack growth process.

To consider plasticity induced closure effects on crack growth, Newman (1982, 1981) proposed a model based on a modification of the Dugdale strip-yield model. The Newman model divides the crack tip plastic zone into discreet regions which become broken as the crack advances. Once a segment is broken, it becomes part of the crack wake, and assumes a residual deformation equal to the maximum amount of plastic deformation it experienced while intact. The distribution of residual displacements for each element in the crack wake is then used to compute crack contact stresses and the far-field opening stress level for the current crack configuration. An effective SIF range can then be computed, using the difference between the maximum applied stress and current opening stress, from which crack growth rate can be determined. As a crack grows, residual displacements in the crack wake elements are allowed to change if compressive yielding occurs due to contact stresses at the minimum stress level in a cycle. Load
interaction and load history effects are accounted for in this model through the influence of the residual displacement distribution on the resulting crack opening stress levels.

An advantage of this model is that the plastic-zone size and crack-surface displacements can be calculated from the superposition of two elastic problems. These problems correspond to a crack in a finite-width plate subjected to remote uniform stress, and to a uniform stress applied over a segment of the crack surface. To account for the effects of stress state on plastic-zone size, a constraint factor was also introduced to modify the effective tensile flow stress (average between yield strength and ultimate tensile strength) in order to more properly reflect material yielding under plane stress and plane strain conditions. This model serves as the basis for the fatigue crack growth analysis program, FASTRAN (Newman Jr., 1992).

To this point, variable amplitude loading effects on crack growth rate have only been discussed with respect to mode I crack extension under uniaxial loading. Although important, especially in situations involving crack growth from a notch, multiaxial stress states present some additional complexities in a variable amplitude crack growth analysis. In addition to the possibility for mixed-mode crack growth, stress state and load sequence effects can alter the crack growth rate even for mode I cracks growing under multiaxial nominal loading conditions.

For example, Gladskyi and Fatemi (2014) studied axial and torsion load sequence effects on mode I crack growth in thin-walled tubular specimens of low-carbon steel containing a circular notch. They observed that for cracks growing in mode I under pure torsion and pure axial loadings, cracks in pure torsion tests grew faster despite the same maximum principal stress range on the crack growth plane. Additionally, the insertion of
blocks of pure torsion cycles into an otherwise uniaxial loading history was shown to increase crack growth rate, while the insertion of uniaxial cycles into a pure torsion loading history was shown to decrease crack growth rate. Fatemi et al. (2014) performed similar tests on notched tubes of 7075-T6 aluminum alloy, from which they observed the same crack growth trends. Most of these effects were explained in terms of the stress state at the crack tip. For mode I cracks growing under nominal torsion loading, it was speculated that the presence of T-stress at the crack tip increased crack driving force for a given value of nominal SIF. As such, crack growth rate correlations for the various loading histories were found to improve when considering the effects of T-stress using Equation (2.62).
<table>
<thead>
<tr>
<th>Proposed modification</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i^{(i)} dp_i^{(i)} = \left( a_i^{(i)} : \frac{\partial f}{\partial \sigma} \right) \frac{\partial f}{\partial \sigma} dp_i$</td>
<td>Burlet-Cailletaud 1986</td>
</tr>
<tr>
<td>$dp_i^{(i)} = dp$ for $i=1,2,3$</td>
<td>Chaboche 1991, 1994</td>
</tr>
<tr>
<td>$dp_i^{(4)} = \left( 1 - \frac{\bar{\alpha}_i}{f(\alpha_i)} \right) dp$ for $i=4$, $f(\alpha_i) = \sqrt{\frac{1}{2} a_i^{(i)} : a_i^{(i)}}$</td>
<td>Ohno-Wang 1993</td>
</tr>
<tr>
<td>$dp_i^{(5)} = H(f_i) \left( d \varepsilon_p : a_i^{(i)} \right)$, where $\bar{\alpha}_i = \sqrt{\frac{1}{2} a_i^{(i)} : a_i^{(i)}}$, $f_i = \bar{\alpha}_i^2 - \left( \frac{c_i}{y_i} \right)^2$ and $H(f_i)$ denotes Heaviside step function.</td>
<td>Ohno-Wang 1993</td>
</tr>
<tr>
<td>$dp_i^{(6)} = \left( \frac{\bar{a}_i}{C_i/y_i} \right)^m \left( d \varepsilon_p : a_i^{(i)} \right)$</td>
<td>McDowell 1995</td>
</tr>
<tr>
<td>$dp_i^{(7)} = \left( \frac{\bar{a}<em>i}{C_i/y_i} \right)^m dp_i$, $m_i = A</em>{\alpha} \left( 2 - \frac{4 \bar{\varepsilon}<em>p : \varepsilon_i}{\bar{\varepsilon}<em>i} \right)$, $A</em>{\alpha} = \mu \left( 1 + a</em>{\alpha} e^{b_{\alpha} \beta_i} \right)$</td>
<td>Jiang-Sehitoglu 1996</td>
</tr>
<tr>
<td>$dp_i^{(8)} = \left[ \mu_i + H(f_i) \left( \frac{\partial f}{\partial \sigma} : \frac{a_i^{(i)}}{C_i/y_i} - \mu_i \right) \right] dp_i$</td>
<td>Abdel-Karim / Ohno 2000</td>
</tr>
<tr>
<td>$a_i^{(i)} dp_i^{(i)} = \left[ \delta \cdot a_i^{(i)} + (1-\delta) \left( a_i^{(i)} : \frac{\partial f}{\partial \sigma} \right) \frac{\partial f}{\partial \sigma} \right] dp_i$, (pro i=1,2,3)</td>
<td>Bari a Hassan 2002</td>
</tr>
<tr>
<td>$a_i^{(i)} dp_i^{(i)} = \left[ \delta \cdot a_i^{(i)} + (1-\delta) \left( a_i^{(i)} : \frac{\partial f}{\partial \sigma} \right) \frac{\partial f}{\partial \sigma} \right] \left( 1 - \frac{\bar{\alpha}_i}{f(\alpha_i)} \right) dp_i$, (pro i=4)</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.1 (Continued) Summary of proposed modifications to Armstrong-Frederick type kinematic hardening rules (Halama et al., 2012).

<table>
<thead>
<tr>
<th>Proposed modification</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^{(i)} dp^{(i)} = \left( \frac{\bar{a}_i}{C_i/y_i} \right)^m \left( d \delta^* \cdot a^{(i)} + (1-\delta^*) \left( a^{(i)} : \frac{\partial f}{\partial \sigma} \frac{\partial f}{\partial \sigma} \right) \right) \left( d \varepsilon_p : \frac{a^{(i)}}{\bar{a}_i} \right)$</td>
<td>Chen-Jiao 2004</td>
</tr>
<tr>
<td>$\mu = \frac{\mu_0}{(1 + a \Phi)}$, $\Phi = 1 - \sqrt{\frac{s}{\bar{s}}}$</td>
<td>Kang-Gao-Yang 2004</td>
</tr>
<tr>
<td>$d \varepsilon_p^* = H(f_i) \left( d \varepsilon_p : \frac{a^{(i)}}{\bar{a}_i} \right) a^{(i)}$</td>
<td>Chen et al. 2005</td>
</tr>
<tr>
<td>$a^{(i)} dp^{(i)} = H(f_i) \left( d \varepsilon_p : \frac{a^{(i)}}{\bar{a}_i} \right) a^{(i)} + \xi_i (\bar{a}_i)^{m-1} a^{(i)}$</td>
<td>Yaguchi - Takahashi 2005</td>
</tr>
<tr>
<td>$d \eta = \eta_1 + d \eta_2$, $d \eta_1 = \omega_1 (\eta_1 - \eta_1) dp$, $d \eta_2 = \omega_2 (\eta_2 - \eta_2) dp$, $X = X_0 + (X_0 - X_\infty) e^{-\alpha_x p}$</td>
<td>Halama 2007</td>
</tr>
</tbody>
</table>
Figure 2.1  Flowchart outlining the general steps for estimating the multiaxial variable amplitude fatigue life of notched components based on a variety of analysis procedures.
Figure 2.2  Illustrative comparison of proportional (in-phase) and 90° out-of-phase non-proportional axial-torsion loading (a) in terms of (b) stress path, (c) stress versus time history, and (d) Mohr’s circle at various times in each loading cycle (Stephens et al., 2000).
Figure 2.3  Schematic illustration showing the evolution of the pseudo stress approach in terms of elastic (pseudo stress—pseudo strain), structural (pseudo stress—local strain), and material (local stress—local strain) curves. Numbered arrows represent cyclic plasticity modeling processes. Adapted from (“eFatigue,” 2014).

Figure 2.4  Schematic illustration of the Theory of Critical Distances formulation of (a) the Line Method, (b) the Point Method, and (c) the corresponding reference frame (Susmel, 2008).
Figure 2.5  Procedure for Bannantine-Socie multiaxial cycle counting method. Adapted from (Bannantine and Socie, 1991b; Socie and Marquis, 2000).
Figure 2.6  Schematic illustration of Wang-Brown multiaxial cycle counting method. Adapted from (Wang and Brown, 1995).
Figure 2.7  Schematic illustration of the three fundamental crack extension modes.

Figure 2.8  Schematic illustration of the three crack growth regions.
Figure 2.9  Schematic illustration of the Kitagawa-Takahashi diagram for small crack growth (Stephens et al., 2000).

Figure 2.10  Schematic illustration of a crack growing through a notch affected zone (Stephens et al., 2000).
Figure 2.11  Typical crack growth rate trends for cracks growing from a notch (Stephens et al., 2000).
Chapter 3

Experimental Program

This chapter outlines the experimental program that was implemented in the current study. The experimental program was wide-ranging and designed to supplement various aspects of the computational and analytical work presented in subsequent chapters. Sections 3.1, 3.2, and 3.3 give details on material selection, specimen design and fabrication, and testing equipment, respectively. Then, Section 3.4 outlines the techniques used to monitor the growth of fatigue cracks, while Section 3.5 concludes the chapter with details on the different types of tests performed throughout the testing program.

3.1 Material Selection

The original material of choice for this study was aluminum alloy 2324-T39, because of its use in the construction of newer aircraft structures. The 2324 alloy is a higher strength, higher purity controlled version of the more traditional 2024 alloy. It was developed by Alcoa for tension-dominated, fatigue and fracture critical plate applications. At the present, it is successfully being used on lower wing skin and center wing box components of transport aircraft (Alcoa Inc., n.d.). However, due to its limited
production, 2324-T39 proved to be too difficult to obtain in the proper dimensions and form required for specimen fabrication. Therefore, the material selected for all tests performed was aluminum alloy 2024-T3.

Aluminum alloy 2024-T3 has been produced and used extensively in the aerospace industry since the early 1930’s and is readily available in a wide variety of shapes and sizes. It was chosen for this study due to its common usage and its similarity to the 2324-T39 alloy. Both alloys have the same nominal chemical composition, given in Table 3.1, with the exception of the lower allowable limits of silicon and iron impurities in the 2324 alloy. Because the objectives of this research were meant to be general, and applicable to a wide range of materials, it did not present any limitations when changing the testing material. Additionally, because of the larger availability of studies concerning the 2024-T3 alloy, this change facilitated easier comparisons of test data with supplemental data from literature.

3.2 Specimen Design and Fabrication

Two different types of specimens were fabricated for use in this study. Most tests were performed on specimens of a thin-walled tubular geometry, while some additional tests were performed using specimens of a thin plate geometry. The following sections give more specific details on the design and preparation of each type of specimen.

3.2.1 Tubular Specimens

Tubular test specimens were designed in accordance with the guidelines set forth in ASTM Standard E2207 (2009). Factors considered in the design process included testing machine size and load capacities, available raw material sizes, and the uniformity
of stresses in the specimen gage section, as determined through finite element analysis. The design chosen features a 30 mm long gage section with a 29 mm outside diameter and 25.4 mm inside diameter, resulting in a wall thickness of 1.8 mm. All specimens were machined from drawn tubing with nominal dimensions of 34.9 mm (1.375”) outside diameter and 25.4 mm (1”) inside diameter. Two basic variations of the specimen were produced: one with a smooth/un-notched gage section, and one with a 3.2 mm diameter circular transverse hole drilled through the thickness on one side of the specimen. The tubular specimen geometry and dimensions are shown in Figure 3.1.

The hole size for the notched specimens was carefully selected based on a balance between a number of factors. Chief among these was the desire to maintain a predominately plane stress condition surrounding the hole, so that the stress state at the notch would be similar to that found in the thin sheets used for the production of aircraft wing skins. This was verified using FEA for the axial loading case. Other considerations included having a hole big enough to produce a notch affected zone of considerable size for crack growth analysis, but small enough to avoid significant curvature effects from the specimen’s tubular geometry. The 3.2 mm diameter hole was found to be a good compromise between these factors. The maximum local stress buildup in the specimen thickness direction was found to be around 6% of the maximum stress in the loading direction, thus confirming a predominately plane stress condition at the edge of the hole.

A total of 32 measurements were performed on each tubular specimen, to a precision of at least 0.01 mm, in order to record gage section diameters and to verify that specimens were within the allowable tolerances for taper, trueness, and concentricity. Additionally, all tubular specimens were fully polished, inside and out, to a near mirror
finish prior to testing. This was done to eliminate any adverse effects from machining marks on test results. External polishing was performed in the specimen longitudinal direction using an automated polishing machine and progressively finer lapping films of grades 30, 12, and 3 micron. Final polishing was then performed manually using a liquid rubbing compound and cotton polishing cloth. Internal polishing was performed in a similar manner, but was instead done manually using a pneumatic die grinder and various polishing wheels. Lapping films of 30, 12, and 3 micron were used first, followed by 0.3 and 0.05 micron micro-polishing compounds. For notched specimens, holes were produced by a drilling and reaming operation to avoid the presence of deep machining marks. Any burrs were subsequently removed by light polishing.

At the beginning of the experimental program, a number of initial fatigue tests performed using un-notched specimens revealed that cracks had a tendency to form on the inner surface of the specimens, regardless of the type of loading applied. Although the exact reason is unclear, it was speculated that this was due to variations in the material’s near-surface microstructure due to the tube drawing process. Therefore, since crack initiation on the outside surface of the specimens was necessary to allow for crack growth monitoring, a 26 mm reamer was run through the inside diameter of only the un-notched specimens. This produced a machined surface similar to that on the outside of the specimen and effectively removed any surface irregularities left over from the tube forming process. Specimen bores were then honed and polished following the previously outlined procedure. The reaming process was found not to have a large effect on fatigue life, but was successful in promoting crack initiation and growth on the outer surface of the specimens.
Different variations of the tubular specimen geometry are subsequently denoted using the following prefixes in each specimen number: T for the standard specimen geometry (25.4 mm inside diameter), TM for modified tubular specimens (reamed to 26 mm inside diameter), and TR for reduced outside diameter specimens (subsequently explained). In the event that a suffix is also included following the specimen number, as was done when discussing some of the experimental results and analysis presented in the following chapters, suffixes were used to indicate the specimen condition (i.e. smooth/un-notched or notched) and the type of applied loading. The meaning of each suffix is given in the List of Abbreviations included at the beginning of this dissertation.

3.2.2 Plate Specimens

In addition to the tubular test specimens, some plate specimens were also produced for use in the testing program. Plate specimens featured a 40 mm long gage section with a 40 mm width and a uniform thickness of 3.175 mm. All specimens were machined in the LT orientation from rolled plate with a nominal thickness of 3.175 mm (0.125”), and included a 5.5 mm diameter circular transverse hole through the center of the gage section. The plate specimen geometry and dimensions are shown in Figure 3.2.

Similar to the tubular test specimens, the hole size for plate specimens was selected with the primary goal of ensuring a local plane stress condition at the hole. The 5.5 mm diameter chosen results in a hole diameter to specimen thickness ratio that is approximately equal to that for the notched tubular specimens. Again, the maximum local stress build up in the specimen thickness direction was found, through FEA, to be just less than 6% of the maximum local stress in the loading direction.
For plate specimens, measurements were taken in a total of 12 locations, to a precision of at least 0.01 mm, to record gage section dimensions and to verify the symmetry and flatness of each specimen. The gage sections were then manually polished using 30, 12, and 3 micron lapping films, followed by a final polishing with liquid rubbing compound. Holes in the plate specimens were produced by machining, and any burrs were subsequently removed by light polishing.

3.3 Testing Equipment

All tubular specimen tests were carried out in a closed loop servo-hydraulic axial-torsion load frame with a dynamic rating of 100 kN axial load and 1 kN-m torsional load (shown in Figure 3.3(a)). Load train alignment was verified in accordance with ASTM Standard E1012 (2009) to be within a maximum of 5% bending at 1000 microstrain prior to the beginning of, and periodically throughout, testing. However, typical values at alignment checks were under 4% bending and alignment adjustments were performed to within 2%. Alignment was measured using a tubular test specimen equipped with two arrays of four strain gages (eight gages total) placed circumferentially around its upper and lower gage section. Specimens were held using a hydraulically actuated collet style gripping system. To prevent material deformation in the grip section under high clamping forces, steel plugs were inserted into the ends of each specimen prior to gripping.

For tubular specimen tests in which strain response was recorded, special unreamed specimens (25.4 mm inside diameter) with a slightly reduced outside diameter (27.94 mm) were produced in order to accommodate an Epsilon Technology Corporation Model 3550 axial-torsion extensometer. This extensometer, shown in Figure 3.4, features
a gage length of 25.4 mm (1”) and is capable of measuring ±10% axial strain and ±2.5° of shear strain angle. In order to prevent scratching of the polished surface, three layers of clear tape were applied to areas where the extensometer contacted the specimen.

All plate specimen tests were carried out in another closed loop servo-hydraulic load frame with a dynamic rating of 50 kN axial load (shown in Figure 3.3(b)). Plate specimens were held using a hydraulically actuated gripping system equipped with serrated, wedge-type jaws. Different from the tubular specimen tests, strain gages were attached to each plate specimen to verify that load train alignment was within a maximum of 5% bending at 1000 microstrain for each test performed.

3.4 Crack Monitoring Techniques

Crack initiation and growth were monitored via cellulose acetate surface replication for the un-notched specimen fatigue tests performed in this study. To carry out the periodic replications, tests were first paused and the surface of the specimen was cleaned using reagent grade acetone. The replication film, cut to the proper width and long enough to wrap slightly more than halfway around the gage section of the specimen, was then properly aligned and held against the surface of the specimen. Finally, acetone was applied, via a syringe and hypodermic needle, between the film and the specimen. The capillary action of the liquid acetone then drew the film tightly around the specimen, while at the same time softening the acetate and conforming it to the topography of the specimen surface. After a few minutes, when the acetate film was dry, a permanent marker was used to place a reference mark corresponding to a specific location on the specimen surface. The film was then removed and placed between two glass microscope
slides. This process was performed twice at each replication interval, once for each side of the specimen, to ensure that the entire surface of the gage section was replicated.

The goal was to have approximately 15 surface replicas taken during the crack growth life of each specimen. This was accomplished by initially performing surface replications at intervals corresponding to approximately 10% of the specimen’s total expected fatigue life. Then, these replicas were periodically examined throughout the course of each test to search for small fatigue cracks. If a crack was found, its growth was then monitored, and the cycle interval between replicas was adjusted accordingly, to provide the appropriate number of replications. Crack lengths were measured directly from the surface replicas using an eyepiece scale affixed to a Nikon Labophot–2 optical microscope.

For notched specimen tests of both tubular and plate specimens, on the other hand, the crack initiation location (at the notch) was known prior to testing. Therefore, a Firefly GT820 2.0 megapixel digital microscope camera was used instead to monitor crack growth. This camera is capable of 10-230x optical zoom levels and was fitted with an adjustable polarizing light filter. A picture of the camera setup is shown in Figure 3.5 during a plate specimen test. Images were captured at periodic pauses in the cyclic loading, during which a static tensile load of 5 kN was applied to the specimen. The static load was used in order to help open cracks and make them more visible in the captured images, but was well below a level which would have resulted in any fatigue damage due to its application. Crack lengths were then measured using digital image analysis software. Finally, for the tubular specimen tests, measured crack lengths were input into a curvature correction program to account for the 2D image’s inability to capture the 3D
nature of the crack path along the specimen’s curved surface. This routine, detailed in Appendix A, was verified to be accurate within 2.5% for cracks up to 15 mm in length by comparing the curvature corrected crack length outputs to crack lengths measured directly from cellulose acetate surface replications for a sample notched specimen.

Due to the relative ease of the optical crack length measurement technique, as compared to the surface replication technique, crack length measurements were taken more frequently for these tests. Typically, between 40 and 50 crack length measurements were recorded for each notched specimen fatigue test that was performed.

3.5 Testing Program

Several different types of material tests were performed in order to supplement the analytical work presented in this study. However, they can be generally grouped into three major categories: deformation behavior tests, constant amplitude fatigue tests, and variable amplitude fatigue tests. The following sections will provide an overview of the applied loading conditions and the information gained from each type of test. Table 3.2 has also been compiled to provide a more concise summary of the objectives of the testing program. Experimental results are then presented in subsequent chapters, along with the corresponding analyses for which the tests were designed to supplement.

3.5.1 Deformation Behavior Testing

One of the main goals of the deformation behavior tests was to determine the relevant material properties for the testing material which would be required to perform all subsequent material deformation and fatigue life calculations. Properties were
determined experimentally, as compared to using values from literature, to eliminate a possible source of error in any calculations that were based on these properties.

The most basic deformation test performed was a monotonic tension test. From this test, mechanical properties such as modulus of elasticity, $E$, yield strength, $\sigma_y$, ultimate strength, $\sigma_u$, and percent elongation at failure were determined. Additionally, cyclic deformation tests were performed under axial, torsion, and in-phase and 90° out-of-phase axial-torsion loadings. These tests were carried out in the form of incremental step tests and were necessary to determine cyclic stress-strain relations under different loading conditions. Additionally, they allow for the evaluation of various equivalent stress-strain criteria in terms of their ability to correlate deformation data for different stress states. Finally, the in-phase and 90° out-of-phase tests were used to quantify the amount of non-proportional hardening that takes place within the testing material.

Aside from these more basic tests, used for determining material properties and constitutive relations, some additional deformation tests were performed using more complex loading conditions. These tests utilized constant amplitude block loading histories made up of different sequences of multiaxial strain paths (each having the same equivalent strain amplitude), as well as a short multiaxial variable amplitude simulated service loading history. The goal for these tests was to obtain experimental data for transient and stabilized material stress-strain response under dynamic loading conditions. These data were then used to evaluate the effectiveness of cyclic plasticity modeling techniques in predicting material deformation behavior under general multiaxial loadings.
3.5.2 Constant Amplitude Fatigue Testing

Although understanding the fatigue failure process under multiaxial variable amplitude loading conditions was the core objective of this study, a variety of constant amplitude fatigue tests were also performed for a number of reasons. Most of these tests were conducted in load-control and included un-notched and notched specimens subjected to axial, torsion, and combined axial-torsion loadings. Additionally, a limited number of strain-controlled tests were also performed using un-notched specimens under pure axial and pure torsion loadings. All constant amplitude fatigue tests utilized fully-reversed ($R = -1$) loading conditions and were performed in accordance with applicable ASTM testing standards whenever possible.

The main purpose of the constant amplitude un-notched specimen fatigue tests, performed under pure axial and pure torsion loadings, was to generate the baseline fatigue life curves, both stress-life and strain-life, from which all subsequent fatigue life predictions would be calculated. However, un-notched specimen tests were also performed under in-phase and 90° out-of-phase axial-torsion loading conditions, as well as with a limited number of discriminating load paths (e.g. torsion with static axial stress, triangular load paths, etc.). These tests were performed in order to study the effect of more complex loading component interactions on the resulting fatigue life. Then, all of the constant amplitude un-notched specimen test data combined allowed for the evaluation of multiaxial fatigue damage parameters in situations where the presence of multiaxial stress states and/or varying degrees of loading non-proportionality can impact fatigue damage calculations. If a damage parameter cannot successfully correlate fatigue
life data under simple constant amplitude loading paths, then it cannot be expected to do so under the more complex variable amplitude loading conditions central to this study.

While the nominal loading of engineering components and structures is typically load-controlled, the local deformation at a notch, in many cases, can be dictated by local strain distributions. This, combined with the fact that many fatigue damage parameters rely on strain quantities as the key driving parameters, prompted a limited number of constant amplitude fatigue tests to be performed in strain-control under pure axial and pure torsion loadings. These tests were performed at higher load levels, similar to those used in some of the load-controlled tests, and allowed for the evaluation of any differences in fatigue damage between the two control modes. The stable stress-strain response obtained from these tests was also used to verify the response estimated from the results of cyclic deformation tests.

In addition to the un-notched specimen tests, constant amplitude fatigue tests were performed in nominal load-control under similar loading conditions for notched specimens as well. With respect to crack initiation, the primary goal of these tests was to study notch effects on fatigue life and damage calculation. As such, the experimental data from these tests allowed for the evaluation of notch stress-strain estimation models, stress gradient models, and local life prediction techniques under relatively simple uniaxial and multiaxial loading conditions.

In addition to crack initiation, crack growth was also monitored for each of the constant amplitude fatigue tests performed. As a result, these tests, for both un-notched and notched specimens, were also useful for gaining insight into the cracking mechanisms at work within the testing material. By observing cracking orientations, and
comparing them with planes of maximum shear and normal stress/strain for each loading condition, it made it possible to determine the stress and/or strain component(s) primarily responsible for the initiation and growth of fatigue cracks.

For un-notched specimens, once cracks nucleated, they were found to propagate in a shear- and/or mixed-mode manner for all, or only a portion, of their growth life. Therefore, crack growth data from these tests were useful for evaluating mixed-mode crack growth models in terms of both growth direction and rate. For notched specimen tests, on the other hand, crack growth had the tendency to be mode I dominated, regardless of the nominally applied loading conditions. However, mode I cracks growing under multiaxial stress states often behave differently than similar cracks growing under uniaxial loading conditions. Therefore, the notched specimen tests allowed for the effects of multiaxial stress states, caused by the presence of T-stress and/or loading non-proportionality, to be studied with respect to any differences in the observed mode I crack growth behavior. Additionally, because some growth data were recorded when cracks were still in the notch affected zone, initial crack geometry assumptions and stress concentration effects could also be evaluated with respect to their impact on growth rate correlations.

Given the discrepancy observed throughout the testing program between the preferred crack growth direction/mode in un-notched and notched specimens, some specialized tests were also performed in order to investigate this issue. The difference was thought to be the result of different crack growth mechanisms at work in each type of specimen. As discussed in Section 2.8.5, un-notched specimen crack orientation was thought to be influenced by the coalescence of microcrack networks that develop
throughout the uniformly stressed gage section of the specimen. Therefore, in an attempt to verify or disprove this theory, a small precrack type notch was machined into a limited number of un-notched tubular specimens. The pre-cracked specimens were then tested, under some of the same loading conditions used for regular un-notched specimen tests, to observe the resulting crack path. Since the pre-cracked specimens contained a single crack-like stress concentration at the beginning of the test, crack networks and coalescence processes would not have had time to develop during the course of these tests. Therefore, any effects from crack coalescence could be inferred through comparisons between crack paths in the un-notched and pre-cracked specimens.

Finally, due to the possibility of surface curvature effects on crack growth in the tubular specimen tests, the testing program was supplemented by the addition of a limited number of constant amplitude notched specimen fatigue tests performed using the flat plate specimen geometry. The plate specimens allowed for a more direct comparison of uniaxial crack growth rates to data reported in literature, while a comparison to the tubular specimen growth rates allowed for the evaluation of any curvature effects. These specimens also provided another geometry for which fatigue crack initiation predictions could be compared to experimental results.

3.5.3 Variable Amplitude Fatigue Testing

Given that the primary objective of this study was to better understand fatigue failure mechanisms under multiaxial variable amplitude loading conditions, the testing program was concluded with a variety of variable amplitude fatigue tests. Although there was essentially only one loading history evaluated, the history was applied in several different forms. This simulated service loading history consisted of normal and shear
stress variations derived from recorded flight test data for the lower wing skin area of a large military patrol aircraft (tension-dominated). Fatigue tests were performed for each channel of the loading history independently (i.e. axial stress only and shear stress only), along with tests evaluating the combined effect of both channels. Additionally, an editing procedure was used to remove small loading cycles from the history without eliminating a significant amount of fatigue damage. This, in turn, allowed for reduced testing and analysis time, while at the same time enabling the investigation of small cycle effects on both crack initiation and growth.

In order to remove the complication that notch effects add to a fatigue life analysis, variable amplitude fatigue tests were first performed under axial, torsion, and combined axial-torsion loadings using un-notched tubular specimens. These tests allowed for a more direct comparison between the constant and variable amplitude fatigue test results. In turn, this provided a better means for evaluating analysis procedures specific to variable amplitude loading, such as cycle counting methods and damage summation rules. Additionally, these tests were useful in studying the effects of transient material constitutive behavior and crack initiation definition on variable amplitude life predictions.

Finally, variable amplitude fatigue tests were performed on notched tubular specimens under axial, torsion, and combined axial-torsion loading conditions in order to study the synergistic effect of all of the previously discussed factors. For example, the effects of multiaxial loadings and/or notch effects on both crack initiation and growth were evaluated through constant amplitude fatigue testing of un-notched and notched specimens. Then, the variable amplitude tests performed using un-notched specimens
made it possible to study the effect of variable amplitude loadings on fatigue damage accumulation under both uniaxial and multiaxial loading conditions. These final tests, performed using full and edited versions of the simulated service loading history, reflect a majority of fatigue loading situations encountered in industry and allowed for the evaluation of state-of-the-art fatigue life analysis techniques under such conditions. Not only were fatigue crack initiation predictions evaluated based on the results of these tests, but crack growth predictions computed using both the UniGrow and FASTRAN models were assessed as well.

3.5.4 Notes on Material Anisotropy Effects

Since the specimens tested in this study were machined from drawn tubing and rolled plate stock, it is likely that some degree of anisotropy exists in the material microstructure for these specimens. However, experimental investigations into any effects from this anisotropy were not performed. For the tubular specimens, the primary specimens used for fatigue testing, such investigations were not feasible due to the curved geometry that exists in the direction transverse to the axis of the tube. Although testing in the transverse direction would have been possible for the plate specimens, the degree of anisotropy is likely different between the rolled plate material and the drawn tubes. Therefore, since only a limited number of fatigue tests were performed using the notched plate specimens, all tests were performed in the longitudinal direction.

Although anisotropy effects were not studied experimentally, some results from literature were available for similar materials to help assess any influence that material anisotropy may have had on the deformation, fatigue, and/or crack growth behavior observed in this study. For example, Lin and Nayeb-Hashemi (1993) studied the effects
of anisotropy on the deformation and fatigue behavior of 6061-T6 aluminum alloy. They utilized solid cylindrical specimens machined from cold rolled plate in both the rolling direction (longitudinal), and in the direction transverse to the rolling direction. Specimens from each orientation were then tested in strain-control under fully-reversed axial, torsion, and combined axial-torsion loading conditions. While transverse specimens generally showed slightly higher stress response than longitudinal specimens under monotonic tension and cyclic axial loading, the opposite was true for torsion loading. Under monotonic torsion loading, longitudinal specimens showed a higher stress response, by around 15%, while a considerably smaller difference in stress response was observed under cyclic torsion loading.

In terms of impact on fatigue resistance (tests were performed in the range of $10^2$ to $10^4$ cycles), lives for longitudinal specimens, as calculated from best fit lines, were consistently around a factor of 1.5 longer than those for transverse specimens at the same axial strain amplitude. For pure torsion tests, however, the effect was opposite. At the same shear strain amplitude, lives for transverse specimens ranged from a factor of 1.6 to 2.3 times longer than those for longitudinal specimens, with an average factor of 1.9 difference. Given the opposite effects of material anisotropy on fatigue resistance under axial and torsion loading, it is not surprising that when tests were performed under combined axial-torsion loading, differences in fatigue life between the two specimen orientations were even smaller. For tests performed at strain ratios of $\lambda = \gamma_a/\varepsilon_a = 0.8$ and 2.0, lives were longer for longitudinal specimens by average factors of around 1.4 and 1.2, respectively.
Experimental fatigue data were also available for 7075-T651 aluminum alloy specimens machined from cold rolled plate in both the longitudinal and transverse directions (Zhao and Jiang, 2008). These tests, performed in strain-control under fully-reversed uniaxial loading conditions, again showed similar fatigue lives for flat plate specimens machined in both orientations. At shorter lives (around $10^2$ to $10^4$ cycles), best fit lines show the tendency for slightly longer fatigue lives in the longitudinal orientation, while transverse specimens generally exhibited longer lives as cycles to failure increased. However, the difference at around $10^6$ cycles was less than a factor of 1.6, and decreased at shorter lives. Additional tests were also performed under the same type of loading conditions using solid cylindrical specimens machined in the longitudinal direction from cold finished bar stock. These tests allow for the evaluation of any effects due to microstructural variations resulting from the different processing techniques used to produce each form of the raw material. Although fatigue lives were consistently longer for the cylindrical specimens than for either plate specimen orientation, the difference was less than a factor of 2 in all life regimes considered (up to around $10^6$ cycles).

From these results, it is clear that although microstructural anisotropy can influence material deformation behavior and fatigue resistance, the effect is generally small for some common aluminum alloys. In nearly all cases, the average difference in fatigue life for specimens tested in different material orientations was less than a factor of 2. This is on the same order as the experimental scatter typically observed in duplicate tests as a result of inherent material variability. Additionally, anisotropy effects were shown to have an opposite influence on material behavior under axial and torsion loading.
conditions. As a result, these differences partially cancel and result in an even smaller influence from anisotropy under combined axial-torsion loading conditions.

Furthermore, for aluminum alloys in particular, the effects of anisotropy on fatigue behavior are more closely related to the different distributions and orientations of constituent particles within the material, rather than to variations in mechanical properties brought about by these differences. This is because stress/strain concentration effects, particle/matrix debonding, and/or brittle cracking at constituent particles can all provide favorable sites for fatigue crack initiation. Crack growth, on the other hand, is primarily controlled by local driving forces at the crack tip, which are not so much influenced by the presence of these particles. As a result, material anisotropy should have an even smaller impact on crack growth behavior. Taking all of these factors into consideration, it is reasonable to conclude that any effects from material anisotropy should not have a significant impact on the experimental or analytical results presented in this study. As such, further investigations into material anisotropy were not performed.
Table 3.1 Nominal chemical composition (in weight percent) of 2024 and 2324 aluminum alloys (Alcoa Inc., n.d.). Value is maximum limit if no range is given.

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Si</th>
<th>Fe</th>
<th>Cu</th>
<th>Mn</th>
<th>Mg</th>
<th>Cr</th>
<th>Zn</th>
<th>Ti</th>
<th>Others, each</th>
<th>Others, total</th>
<th>Al</th>
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<td>2024</td>
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<td>0.50</td>
<td>3.8-</td>
<td>0.90</td>
<td>1.2-</td>
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<td>0.15</td>
<td>0.05</td>
<td>0.15</td>
<td>Remainder</td>
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<tr>
<td>2324</td>
<td>0.10</td>
<td>0.12</td>
<td>3.8-</td>
<td>0.90</td>
<td>1.2-</td>
<td>0.10</td>
<td>0.25</td>
<td>0.15</td>
<td>0.05</td>
<td>0.15</td>
<td>Remainder</td>
</tr>
</tbody>
</table>

Table 3.2 Summary of objectives for tests included in the experimental program.

<table>
<thead>
<tr>
<th>Specimen Type</th>
<th>Un-Notched</th>
<th>Notched</th>
<th>Pre-Crack</th>
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<td><strong>Deformation</strong></td>
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<tr>
<td>Obtain:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>• Monotonic and Cyclic Material Properties</td>
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<tr>
<td>• Non-proportional Hardening</td>
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<td></td>
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</tr>
<tr>
<td>• Stable and Transient Constitutive Behavior</td>
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<td></td>
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</tr>
<tr>
<td>Evaluate:</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>• Yield Criteria</td>
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<td></td>
<td></td>
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<tr>
<td>• Cyclic Plasticity Models</td>
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<tr>
<td><strong>Test/Loading Type</strong></td>
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</tr>
<tr>
<td>Obtain:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>• S-N Fatigue Properties</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>• ε-N Fatigue Properties</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Evaluate:</td>
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<td></td>
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<tr>
<td>• Multiaxial Fatigue Damage Parameters</td>
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<td>• Crack Path/Branching Behavior</td>
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<td>• Mode I Crack Growth under Multiaxial Stress States</td>
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<td><strong>Variable Amp. Fatigue</strong></td>
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<td></td>
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<tr>
<td>Evaluate:</td>
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</tr>
<tr>
<td>• Cycle Counting Methods</td>
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<tr>
<td>• Effect of Transient Deformation Behavior on Life Predictions</td>
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<tr>
<td>• Effect of Crack Initiation Definition on Life Predictions</td>
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<td></td>
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</tr>
<tr>
<td><strong>Evaluate:</strong></td>
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<tr>
<td>• Synergistic Effects of Multiaxial Variable Amplitude Loading on Crack Initiation and Growth from Stress Concentrations</td>
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<td>• Predictions from State-of-the-Art Crack Growth Models (i.e. UniGrow and FASTRAN)</td>
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<tr>
<td><strong>Evaluate:</strong></td>
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<td></td>
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<tr>
<td>• Crack Coalescence Effects on Crack Path</td>
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</tr>
<tr>
<td><strong>Evaluate:</strong></td>
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<td>N/A</td>
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</table>
Figure 3.1  Thin-walled tubular specimen geometry and dimensions. All dimensions are given in mm.

Figure 3.2  Notched plate specimen geometry and dimensions (uniform 3.175 mm nominal thickness). All dimensions are given in mm.
Figure 3.3  Instron closed-loop servo-hydraulic load frames used for fatigue testing of (a) thin-wall tubular specimens and (b) notched plate specimens.

Figure 3.4  Epsilon Technology Corporation Model 3550 axial-torsion extensometer.
Figure 3.5  Digital microscope camera setup used to monitor crack growth in notched specimen fatigue tests.
Before fatigue testing was completed, a number of material deformation tests, outlined in Section 3.5.1, were also performed. The primary goal of these tests was the determination of all relevant material properties for the 2024-T3 aluminum alloy tested in this study. These include basic monotonic properties, as well as material constitutive equations relating elastic-plastic stresses and strains under a variety of cyclic loading conditions. In addition to constant amplitude incremental step tests, additional cyclic deformation tests were also performed using more specialized loading histories. Experimental data from these tests were then used to evaluate the plasticity modeling techniques used to estimate stress-strain response for the more complex fatigue life analyses performed in this study.

In this chapter, Sections 4.1 to 4.4 explain in detail the types of deformation tests performed, the testing procedures used, the experimental results, and the information gained from each test. Then, Section 4.5 describes the cyclic plasticity model used to perform stress-strain analyses in this study, while Section 4.6 compares stress-strain estimations derived from the model to experimental results generated under a variety of loading conditions. The work presented in Sections 4.5 and 4.6 was also published in
(Gates and Fatemi, 2016a, 2016b). Finally, Sections 4.7 and 4.8 extend the same plasticity modeling concepts, together with local stress analysis results and notch stress-strain estimation rules, to predict local deformation behavior in notched specimens. Local stress-strain estimations were then compared to solutions derived from nonlinear FEA, both at the notch root and at short distances away from the notch root, in order to evaluate their accuracy under various loading conditions. Results from these latter two sections were also published in (Gates and Fatemi, 2016c, 2014).

### 4.1 Monotonic Tension Test

As stated previously, a monotonic tension test was performed in order to derive the basic mechanical properties necessary for performing subsequent fatigue life analyses. Again, properties were derived experimentally, as opposed to using values from literature, to eliminate one possible source of error from the fatigue life predictions that are based on these values. Properties established by this test include modulus of elasticity, $E$, yield strength, $\sigma_y$, ultimate strength, $\sigma_u$, and percent elongation at failure, $\%EL$.

Only a single monotonic tension test was performed. Load was applied in displacement-control, at a rate of 0.15 mm/min, and strain was recorded until the extensometer reached its limit of 10% strain. After this, the extensometer was removed from the specimen, and testing was continued in displacement-control, at a rate of 0.30 mm/min, until failure occurred.

The results of the tension test are shown in Figure 4.1. From these data, it was determined that the 2024-T3 aluminum alloy used in this study had a modulus of
elasticity of 73.7 GPa, a yield stress (defined with a 0.2% strain offset) of 330 MPa, an ultimate strength of 495 MPa (engineering stress), and experienced 19.5% elongation at failure. A Ramberg-Osgood type fit of the monotonic stress-strain curve yields a monotonic strength coefficient, $K$, and monotonic strain hardening exponent, $n$, of 470 MPa and 0.055, respectively. This fit produces a fairly good representation of the actual stress-strain behavior up to around 2.5% strain. Above this level, however, the fit begins to significantly overpredict the actual strain/underpredict the actual stress values. A summary of all mechanical properties determined from the monotonic tension test can be found in Table 4.1.

## 4.2 Cyclic Incremental Step Tests

Once basic tensile properties were determined, constant amplitude cyclic deformation tests were performed next in order to generate data on stabilized stress-strain response under various loading conditions relevant to the fatigue testing program. These included uniaxial, pure torsion, in-phase axial-torsion, and 90° out-of-phase axial-torsion tests.

Cyclic deformation tests were performed in load-control in the form of fully-reversed ($R = -1$) incremental step tests. Starting from purely elastic loading, load levels were gradually increased, and the specimen was cycled at any given level until stable material response was achieved. Stability was indicated by negligible changes in rotation/displacement amplitude from one recorded cycle to the next. Testing was performed using sinusoidal waveforms at frequencies ranging from 0.05-1.0 Hz, and strain response was recorded every 10 cycles. Due to substantial cyclic hardening,
however, initial cycles at higher load levels for the 90° out-of-phase test resulted in large plastic deformations which exceeded the limits of the extensometer. Therefore, an additional 90° out-of-phase test was performed in strain-control so that stress-strain data could be obtained at the higher loading levels without excessive deformation and/or premature failure of the specimen due to fatigue.

The first incremental step test performed was for uniaxial loading conditions. A total of ten stress amplitude levels, ranging from 50 MPa to 450 MPa, were applied. From the resulting stabilized stress-strain data, it was first determined that the cyclic modulus of elasticity was equal to the modulus of elasticity under monotonic tension loading, i.e. $E' = E = 73.7$ GPa. Then, in determining the constants for the Ramberg-Osgood type power law relation (Equation (2.4)) describing the elastic-plastic portion of the cyclic stress-strain curve, the results from this step test were used in combination with the results from a limited number of strain-controlled fully-reversed fatigue tests. In all, the highest four stress-strain values from the step test, and four mid-life stress-strain values from constant amplitude fatigue tests, were used to determine a cyclic axial strength coefficient, $K'$, and cyclic axial strain hardening exponent, $n'$, of 677.0 MPa and 0.070, respectively. The hysteresis loops corresponding to each set of stabilized stress-strain values used in the fitting of the cyclic stress-strain curve are shown in Figure 4.2. All data had corresponding plastic strain amplitudes of at least 0.01% and did not exceed the ultimate strength of the material. ASTM Standard E646 (2009) was used as a guideline for the fitting procedure whenever applicable. All cyclically stable axial stress-strain data, along with the corresponding best fit curve, are presented with results from other loading
conditions at the end of this section. In addition, all derived material constants are included in Table 4.1.

Next, a similar incremental step test was performed under pure torsion loading. For this test, however, because the level of plastic deformation for each load level (and thus the maximum shear stress at the specimen outer surface) was unknown prior to testing, the nominal shear stress was defined as the average shear stress value, occurring at the mid-thickness of the specimen, based on the following equilibrium equation:

$$\tau_{mid} = \frac{T}{2\pi r_m^2 t}$$  \hspace{1cm} (4.1)

where $T$ is the applied torque, $r_m$ is the mean radius of the specimen gage section, and $t$ is the specimen wall thickness. Therefore, when computing stabilized stress-strain values from the test results, shear strain (measured by the extensometer at the specimen’s outer surface) needed to be adjusted to represent mid-thickness values as well. Since the distribution of shear strain is linear across the thickness of the specimen, regardless of the amount of plasticity present, this was easily done by multiplying the outer surface strain values by the ratio of mean specimen radius to outer specimen radius.

A total of eight shear stress amplitude levels, ranging from 50 MPa to 265 MPa, were applied in this test. After obtaining the full set of stabilized mid-thickness stress-strain values, it was determined that the cyclic shear modulus, $G'$, for the test material was equal to 27.4 GPa, which is nearly identical to values calculated during monotonic modulus checks performed prior to testing. Similar to the procedure used for determining the cyclic axial stress-strain curve, a limited number of data from fully-reversed strain-controlled constant amplitude torsion fatigue tests were used, in combination with the step test results, to compute the constants describing the cyclic shear stress-strain curve.
In all, the highest four stress-strain values from the step test and two mid-life stress-strain values from fatigue tests were used to determine a cyclic shear strength coefficient, $K_{o'}$, and cyclic shear strain hardening exponent, $n_{o'}$, of 382.3 MPa and 0.084 respectively. Corresponding stable cycle shear hysteresis loops are shown in Figure 4.3 for each loading level considered. All data used for curve fitting were at plastic strain amplitudes of at least 0.02%.

For the in-phase axial-torsion incremental step test, a nominal stress amplitude ratio, $\lambda$, of $S_T/S_A = \sqrt{3}/3 = 0.577$ was used so that nominal shear and axial stress components would have equal contributions based on von Mises equivalent stress. Nominal shear stress was again defined using Equation (4.1), and shear strains were converted to mid-thickness values following the test. A total of seven load levels, with von Mises equivalent stress amplitudes ranging from 100 MPa to 450 MPa, were applied during the test. After computing von Mises equivalent stress and strain values from stabilized axial and shear stress-strain amplitudes, a cyclic in-phase stress-strain curve was fit to results from the three highest applied loading levels. The axial and shear stress-strain hysteresis loops corresponding to these levels are shown in Figure 4.4. The cyclic in-phase strength coefficient, $K_{IP'}$, and cyclic in-phase strain hardening exponent, $n_{IP'}$, were calculated to be 688.6 MPa and 0.081, respectively. All equivalent plastic strain amplitudes for the highest three loading levels were above 0.02%, and no fatigue test data were used in the fitting procedure.

Finally, two 90° out-of-phase (OP) axial-torsion incremental step tests were performed in order to obtain information on the level of non-proportional hardening experienced by the 2024-T3 test material. The same nominal shear to axial stress
amplitude ratio was used for these tests as was used in the in-phase axial-torsion step test. The nominal shear stress was defined consistent to the other tests as well. In the first 90° OP test, performed in load-control, a total of five load levels, with von Mises equivalent stress amplitudes ranging from 100 MPa to 350 MPa, were applied. All stabilized stress-strain values from this test corresponded to linear elastic material behavior. The test had to be stopped prematurely, however, due to excessive initial plastic deformation experienced at higher stress levels.

The second test, on the other hand, was performed in strain-control in order to overcome the problem of excessive initial deformation. This test evaluated three different load levels with stabilized von Mises stress amplitudes ranging from 360 MPa to 525 MPa. The stable cycle axial and shear hysteresis loops for this test are shown in Figure 4.5. Additionally, the stable cycle strain (controlled) and stress paths corresponding to these hysteresis loops are shown in Figure 4.6. Shear values are presented in Figure 4.6 in terms of von Mises equivalent stress/strain at the specimen mid-thickness. This way, the circular nature of the loading path (in terms of von Mises stress/strain) becomes evident. Each loading level applied had a plastic strain amplitude of at least 0.02%, and all three levels were used to calculate a cyclic 90° OP strength coefficient, $K_{NP'}$, of 983.0 and a cyclic 90° OP strain hardening exponent, $n_{NP'}$, of 0.107.

Since cyclic stress-strain behavior under 90° OP loading conditions is often not available for a given material, methods were discussed in Section 2.3.2 by which the non-proportional cyclic hardening coefficient, $\alpha_{NP}$, can be estimated based on more common material properties. While Equation (2.19) was shown in (Shamsaei and Fatemi, 2010) to produce excellent estimations of $\alpha_{NP}$ for several different materials, no aluminum alloys
were included in these comparisons. Therefore, this approach was applied to the 2024-T3 aluminum alloy tested in this study in order to evaluate its effectiveness for such materials.

Based on the previously described cyclic in-phase and 90° OP stress-strain curves generated for the testing material, a non-proportional cyclic hardening coefficient of 0.19 was calculated at 1% equivalent strain amplitude. In comparison, by substituting values of $K = 470.0 \text{ MPa}$, $n = 0.055$, $K_{IP}' = 688.6 \text{ MPa}$, and $n_{IP}' = 0.081$ into Equation (2.19), a value of $\alpha_{NP} = 0.22$ was calculated at the same equivalent strain amplitude. Comparisons of $\alpha_{NP}$ at other strain levels are similar and include: 0.22 (predicted) versus 0.16 (experimental) at 0.8% equivalent strain amplitude, and 0.24 (predicted) versus 0.25 (experimental) at 1.8% equivalent strain amplitude. Resulting in a maximum of 6% difference (at 0.8% strain) between experimental and predicted stress amplitude under 90° OP loading conditions, these estimations are very satisfactory.

In comparison, a value of $\alpha_{NP} = 0.14$ was calculated from Equation (2.17), independent of strain level, using only the material yield strength and ultimate strength as input. This latter estimation is fairly accurate at the 0.8% equivalent strain amplitude, but becomes increasingly worse as the strain level exceeds the limit ($\Delta\varepsilon_{eq}/2 < 1\%$) suggested for its applicability. Therefore, in the event that non-proportional hardening levels should need to be estimated, the strain level dependence of Equation (2.19) makes it a more advantageous approach, so long as appropriate deformation properties are known.

From the results of the various deformation tests discussed thus far, basic tensile properties, as well as cyclic stress-strain behavior under different loading conditions, were effectively determined for the 2024-T3 aluminum alloy tested in this study. A
summary of all material properties obtained from these tests can be found in Table 4.1. Additionally, monotonic and stabilized cyclic stress-strain data, along with corresponding best fit lines, are presented in Figure 4.7 for all loading conditions investigated.

By comparing the stress-strain curves in this figure, which are plotted in terms of von Mises equivalent stress and strain, a few conclusions can be made. First of all, it is evident from the difference between the cyclic axial and monotonic tension curves that there is a significant amount of cyclic hardening that occurs when this material is subjected to alternating stresses. Similarly, there is also a significant amount of additional non-proportional hardening that occurs when axial and shear stresses are applied out-of-phase with respect to one another. This is important to note because these changes in material constitutive behavior can play a significant role in the fatigue resistance of the material. Finally, by comparing the cyclic axial, torsion, and in-phase axial-torsion stress-strain curves, it can be concluded that the von Mises equivalent stress and strain criteria are able to correlate the deformation behavior for different loading conditions reasonably well for this material. This is an important observation with respect to subsequent discussions on cyclic plasticity modeling.

4.3 Constant Amplitude Block Loading Histories

In addition to determining basic material properties and constitutive relations, it was also desirable to perform cyclic deformation testing under more complex loading conditions in order to provide a means to evaluate subsequently discussed elastic-plastic stress-strain modeling techniques in such cases. For this reason, two different
deformation tests were performed using constant amplitude block loading histories containing different sequences of fully-reversed multiaxial loading paths.

Each test was performed using loading blocks made up cycles of the same five loading paths. Each loading path was applied as a sinusoidal waveform and resulted in the same von Mises equivalent strain amplitude of 0.7%. The five different loading paths, in order of occurrence, correspond to pure axial, in-phase axial-torsion, 180° out-of-phase axial-torsion (proportional), pure torsion, and 90° out-of-phase axial-torsion loading. Although in both block loading histories the five different loading paths occur in this same order, the difference between the two tests is in the number of consecutive cycles of applied for each path. Block B consists of one only cycle of each path (5 cycles per block), while the Block A consists of 20 consecutive cycles of each loading path (100 cycles per block). Figure 4.8 depicts the constant amplitude block loading histories in terms of both applied strain paths and loading sequence.

Loading blocks were constructed in this manner in order to allow for the evaluation of material response under varying degrees of cross hardening, resulting from sudden changes in proportional loading direction, as well as non-proportional hardening at a nearly constant level of equivalent plastic strain. The consecutive application of 20 cycles of each loading path in Block A also allows for the evaluation of progressive material recovery/softening following the non-proportional loading events. Both transient and stabilized material response were of interest in these tests.

Each block loading deformation test was performed in strain-control, at a cyclic frequency of 0.25 Hz, and stress response was continuously recorded. Tests were stopped once stabilized material response was achieved, as indicated by stable maximum load
values at a particular reference cycle. This corresponded to 500 applied cycles for each test, or 5 repetitions of Block A and 100 repetitions of Block B. Results of these tests are presented in Section 4.6.2 along with comparisons to predicted material response.

4.4 Variable Amplitude Service Loading History

Since this study is concerned with fatigue behavior under multiaxial variable amplitude loading conditions, a material deformation test utilizing a simulated variable amplitude service loading history was also performed. Being able to evaluate elastic-plastic stress-strain predictions for such conditions, based on the results of this test, is useful information when judging the accuracy of fatigue life analyses performed under similar loading conditions.

As a result, the loading history constructed for this test was somewhat based on the type of loading events applied in the variable amplitude fatigue tests analyzed later in this study. However, additional cycles were also added in order to reflect a wider variety of stress states that might be encountered in a more general service loading history. In all, the loading history contains approximately 60 reversals which reflect axial-dominated, shear-dominated, in-phase, out-of-phase, and asynchronous multiaxial loading conditions. Mean stresses and periodic overloads were also included for some cycles. Figure 4.9 depicts the variable amplitude loading history in terms of both the applied strain path and strain versus time history.

The variable amplitude deformation test was performed in strain-control, using triangular waveforms, with maximum applied strain values of 0.7% for the axial channel and 1.085% for the shear channel. These maximum strain values are equal based on von
Mises equivalent strain. The loading was applied at an equivalent cyclic frequency of 0.25 Hz and stress response was continuously recorded. Testing was stopped when stable material response was indicated by negligible changes in maximum load values at a particular reference point in the history. A total of 30 loading blocks were applied during testing. Once again, the results of this test are presented in Section 4.6.3 along with comparisons to predicted material response.

4.5 Cyclic Plasticity Model

Given the importance of accurate stress-strain estimations to subsequent fatigue life predictions, combined with the inadequacies of empirical hardening corrections (as discussed in the literature review), a robust cyclic plasticity model, able to capture all of the material deformation phenomena discussed in the previous sections, was essential to this study. More advanced cyclic plasticity modeling formulations, however, while capable of capturing material behavior under such complex loading conditions, often require the determination of a large number of material constants from experimental data. Since this study is concerned with plasticity modeling for application to fatigue life analysis, where usually only basic information on material deformation behavior is available, an emphasis was placed on developing a more simplified plasticity model. This way, all necessary material constants could be determined from a relatively limited amount of experimental data.

4.5.1 Plasticity Model Formulation

The basis for the plasticity model evaluated in this study was the Armstrong-Frederick-Chaboche style single surface nonlinear kinematic hardening model discussed
in Section 2.3.2. More specifically, a slightly modified version of the hardening rule proposed in (Jiang and Sehitoglu, 1996a; Ohno and Wang, 1994) was chosen due to its ability to accurately model stress-strain behavior under a variety of loading conditions, as demonstrated in (Jiang and Kurath, 1997b, 1996; Jiang and Sehitoglu, 1996b; Zhang and Jiang, 2008). A von Mises yield function was used to describe yielding conditions as deformation test results demonstrated good correlation between uniaxial and multiaxial stress-strain behavior using this criterion. Additionally, to account for the effects of non-proportionally varying stresses, Tanaka’s non-proportionality parameter (Equations (2.20) and (2.21)) was implemented into the model.

The hardening rule assumes the series expansion of backstress components, as described by Equation (2.14), but with the backstress evolution taking the following incremental form:

$$d\tilde{\alpha}^{(i)} = c^{(i)}r^{(i)} \left[ \tilde{n} - \left( \frac{\|\tilde{\alpha}^{(i)}\|}{r^{(i)}} \right)^{\chi^{(i)+1}} \tilde{L}^{(i)} \right] dp + \tilde{\alpha}^{(i)} \frac{r^{(i)}}{r^{(i)}} dr^{(i)} \quad (i = 1, 2, ..., M) \quad (4.2)$$

where $\tilde{n}$ is the unit exterior normal to the yield surface, defined by Equation (2.8), equivalent plastic strain increment, $dp$, is defined by Equation (2.12), $c^{(i)}$ and $r^{(i)}$ are sets of non-negative, single value scalars related to the material constitutive behavior, $\chi^{(i)}$ are a set of non-negative scalar values associated with material ratcheting rate, and $\tilde{L}^{(i)}$ is defined as follows:

$$\tilde{L}^{(i)} = \frac{\tilde{\alpha}^{(i)}}{\sqrt{\tilde{\alpha}^{(i)} \cdot \tilde{\alpha}^{(i)}}} \quad (i = 1, 2, ..., M) \quad (4.3)$$

The last term in Equation (4.2) was not included in the original hardening rule, but was added, per (Döring et al., 2003; Jiang and Zhang, 2008), in order to prevent backstress terms from exceeding their bounds defined by transient kinematic hardening variables, $r^{(i)}$. 

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(subsequently explained). This term was not needed in the original formulation because \( r^{(i)} \) were considered to be constants. The resulting plastic modulus, \( h \), corresponding to this hardening rule can then be calculated using the following expression:

\[
h = \sum_{i=1}^{M} \left( c^{(i)} r^{(i)} \left[ 1 - \left( \frac{\| \alpha^{(i)} \|}{r^{(i)}} \right)^{\chi^{(i)}+1} \right] L^{(i)} : \hat{n} + \frac{\alpha^{(i)} : \hat{n} \, dr^{(i)}}{dp} \right) + \sqrt{2} \frac{dk}{dp} \tag{4.4}
\]

where \( k \) is the material yield stress under pure shear loading.

In order to account for transient material constitutive behavior due to cyclic and/or non-proportional hardening, the parameters, \( r^{(i)} \), in the hardening law were assumed, as a starting point, to vary according to the formulation given in (Zhang and Jiang, 2008):

\[
\text{d}r^{(i)} = b^{(i)} \left[ 1 + m(q) A \right] \left[ R_T^{(i)} (q) - r^{(i)} \right] \text{d}p \quad (i = 1, 2, ..., M) \tag{4.5}
\]

where \( b^{(i)} \) are constants related to the rate of cyclic hardening/softening, \( A \) is Tanaka’s parameter (Equation (2.20)), and \( m(q) \) is a function of strain memory size, \( q \), which describes changes in hardening rate due to loading non-proportionality.

Initial values of \( r^{(i)} \) correspond to values derived from material constitutive behavior under monotonic tension loading, and \( R_T^{(i)} \) are the target values for \( r^{(i)} \) as determined from the following expression:

\[
R_T^{(i)} = \sqrt{2} A R_N^{(i)} + (1 - \sqrt{2} A) R_P^{(i)} \quad (i = 1, 2, ..., M) \tag{4.6}
\]

where \( R_P^{(i)} \) and \( R_N^{(i)} \) are generally functions of strain memory size, \( q \), and are determined from stabilized stress-strain behavior under proportional and 90° out-of-phase non-proportional loading conditions, respectively. The \( \sqrt{2} \) factor is included as the value of \( A \) is approximately equal to 1/\( \sqrt{2} \) for 90° out-of-phase loading conditions. This formulation assumes that cyclic hardening phenomena are purely kinematic in nature. No isotropic
behavior was considered as it was shown by Jiang and Zhang (2008) that hardening is generally not isotropic in nature and cannot be effectively modeled by a change in yield stress.

Cyclic plasticity models, especially those considering transient material behavior, are notorious for requiring a large number of material constants in order to properly model material behavior. Often times, these constants must be derived from specialized deformation tests and/or fit to large amounts of experimental data. However, when using these models in a fatigue life analysis, usually only basic information on material deformation behavior is available. For this reason, an emphasis was placed on using a simplified plasticity model in this study, so that it could be implemented using only a minimal amount of material parameters as input.

One of the major simplifying assumptions made in this study when applying the aforementioned formulations was that the test material does not deviate significantly from a Masing-type behavior. In other words, the shapes of the loading branches of stress-strain hysteresis loops at different strain levels are assumed to be identical, and can be represented by the stabilized cyclic stress-strain curve. The Masing assumption is advantageous because it eliminates the strain level dependence on material constitutive behavior. Therefore, any parameter which is a function of \( q \) in Equations (4.5) and (4.6) can be reduced to a constant. As a result, Equation (4.5) can be expressed as follows:

\[
\begin{align*}
    dr^{(i)} &= b^{(i)} [1 + mA] \left[R_T^{(i)} - r^{(i)} \right] dp \\
    &\quad (i = 1, 2, ..., M)
\end{align*}
\]  

(4.7)

Assuming strain level independent material response has the added benefit of eliminating the need for tracking a strain memory surface in the plasticity modeling process. This, in turn, eliminates the need to calculate any associated material constants. The Masing
assumption was considered to be reasonable in this study because strain level appears to have a relatively small effect on stabilized stress response for the 2024-T3 aluminum alloy investigated. This is evidenced by the similarity in the loading branches of the superimposed axial hysteresis loops in Figure 4.10.

While developing the current plasticity model, it was noted that because the term \([R_f(i)-r(i)]\) always depends on the value of \(r(i) = r(i) + dr(i)\) computed from the previous iteration, implementing Equation (4.7) could lead to a load increment dependence of \(dr(i)\). Essentially, for the same amount of equivalent plastic strain, \(\Delta p\), the total change in \(r(i)\) will depend on the number of increments chosen to model the loading process. Although this dependence can be rendered negligible through the use of appropriate numerical integration schemes and sufficiently small load increments, it is still a potential source for inconsistencies. Not only can this lead to multiple output solutions for the same loading process, but it makes the computation of constants \(b(i)\) difficult as well. Therefore, the equation was modified to take the following form:

\[
dr(i) = b(i)[1 + mA] \frac{R_f(i) - r(i)}{R_f(i) - r(i)} dp \quad (i = 1,2,...,M)
\] (4.8)

In this newly proposed formulation, the potential for load increment dependence was eliminated by normalizing the quantity \([R_f(i)-r(i)]\) by its absolute value. This quantity now only produces a value of ±1 which ensures that \(r(i)\) will stabilize at its target value, \(R_f(i)\).

One problem evident with both Equations (4.7) and (4.8), however, is that all regions of the stress-strain curve (i.e. each component \(r(i)\)) will experience hardening/softening regardless of the magnitude of the applied plastic strain. In the absence of strain level dependent deformation data, this can lead to situations where load
level dependent material behavior may be predicted incorrectly. For example, if a material is cycled for a prolonged period of time at a low plastic strain amplitude before eventually increasing to a higher plastic strain level, one would expect a period of transient material response before stability is reached at the higher load level. However, if sufficient cycles were applied at the lower loading level, Equations (4.7) and (4.8) would result in complete stabilization of the stress-strain curve and predict no transient behavior subsequent to the increase in applied loading level.

In order to avoid this problem when using the Masing assumption, Equation (4.8) was modified so that it only becomes active once the equivalent plastic strain level is sufficient to affect each particular value of $r^{(i)}$. Since Equation (4.12) will subsequently show that $r^{(i)}$ is a function of the slopes of the linearized segments of the stress-strain curve from both $(i-1)$ to $(i)$ and $(i)$ to $(i+1)$, choosing a plastic strain level at which to activate Equation (4.8) is a non-trivial problem. Since this issue is primarily related to the numerical implementation of the model, it was assumed, mostly through trial and error, that when the accumulated equivalent plastic strain since the last reversal, $p^*$, reaches a level that is around halfway between the plastic strain ranges corresponding to points $(i-1)$ and $(i)$ on the material stress-strain curve, hardening should begin to occur for backstress component $(i+1)$. This way, it was found that the error in stabilized material response for any given loading level, due to the only partially stabilized stress-strain curve, was minimized.

By manipulating Equation (4.10) and noting that $Δp = \sqrt{3/2} Δ\varepsilon^p$, this can be mathematically approximated as follows:
\[ \text{dr}^{(i)} = \left( \left( 1 + \frac{c^{(i-1)}}{c^{(i)}} \right) p^* - \frac{4}{c^{(i-1)}} b^{(i)} [1 + mA] \right) \frac{R_{RT}^{(i)} - r^{(i)}}{R_{RT}^{(i)} - r^{(i)}} \, dp \] (4.9)

where \( p^* = p + dp \), and the value of \( p^* \) is reset to zero whenever a loading reversal is detected in the dominant loading component. It should be noted, however, that when using this formulation, unless each component of \( r^{(i)} \) reaches its stabilized value, there will always be some amount of error in steady-state stress-strain predictions. This is because the plastic modulus, Equation (4.4), is a function of a summation dependent on \( r^{(i)} \) values for backstress components \( i \) through \( M \). Therefore, since components tending toward \( M \) will not stabilize at lower plastic strain levels, the correct plastic modulus will not be computed. The resulting error in stress-strain predictions was generally found to be small, however, and further decreased as \( M \) was increased and/or the difference in plastic strain range between points \( (i) \) and \( (i-1) \) decreased. Additionally, for materials where changes in constitutive behavior are small, i.e. exhibit little cyclic and/or non-proportional hardening, this effect should be minimal. Since increasing \( M \) comes at the cost of an increased number of calculations for each loading increment, a good balance must be found between accuracy and computation time.

### 4.5.2 Calculation of Material Constants

With the formulation of the transient hardening parameters complete, this leaves the following material constants to be computed from experimental data: \( c^{(i)} \), \( r_0^{(i)} \), \( R_F^{(i)} \), \( R_N^{(i)} \), \( b^{(i)} \), \( m \), \( \chi^{(i)} \), and \( C_c \) (from Tanaka’s parameter). The procedure for determining \( r^{(i)} \) from a uniaxial or equivalent stress-strain curve by pre-selecting values of \( c^{(i)} \) is described in (Jiang and Sehitoglu, 1996b; Zhang and Jiang, 2008). The first step is to express the stress-strain curve in terms of stress range and plastic strain range, and to select \( M \).
number of points between the yield stress and a user defined maximum value of plastic strain range. Typically, the points where stress-strain values are chosen are based on an equal increment of plastic strain range. However, in this study, it is proposed that a logarithmically increasing increment of plastic strain range should be chosen instead. This way, material stress-strain behavior can be more accurately modeled in the region immediately following yielding, where changes in plastic modulus are the greatest.

At the onset of yielding, \( \Delta \sigma^{(0)} = 2\sigma_y \) and \( \Delta \varepsilon^{p(0)} = 0 \). By simplifying the hardening rule for the one-dimensional uniaxial loading case and solving the resulting differential equation (Jiang and Sehitoglu, 1996b), values for \( c^{(i)} \) can eventually be determined from the following relation:

\[
c^{(i)} = \sqrt{\frac{2}{3}} \frac{2}{\Delta \varepsilon^{p(i)}} \quad (i = 1,2, ..., M) \tag{4.10}
\]

The slope of the stress-strain curve between successive points can also be calculated as:

\[
H^{(i)} = \frac{\Delta \sigma^{(i)} - \Delta \sigma^{(i-1)}}{\Delta \varepsilon^{p(i)} - \Delta \varepsilon^{p(i-1)}} \quad (i = 1,2, ..., M) \tag{4.11}
\]

Finally, values for \( r^{(i)} \) can be computed using the following equation:

\[
r^{(i)} = \frac{2}{3} \frac{H^{(i)} - H^{(i+1)}}{c^{(i)}} \quad (i = 1,2, ..., M) \tag{4.12}
\]

where \( H^{(M+1)} = 0 \).

This procedure is performed three times using the same values of \( c^{(i)} \): once with the material monotonic stress-strain curve to determine \( r_o^{(i)} \) values, another time with the stabilized cyclic in-phase (or uniaxial) equivalent stress-strain curve to determine \( R_o^{(i)} \), and one final time with the stabilized cyclic 90° out-of-phase equivalent stress-strain curve to determine \( R_N^{(i)} \). If a 90° out-of-phase stress-strain curve is not available, it can be
approximated from monotonic and cyclic deformation properties by applying Equations (2.18) and (2.19) over an appropriate range of strain values.

Constants describing transient hardening rates, $b^{(i)}$, are determined from experimental data for tests performed under uniaxial constant amplitude loading conditions. For uniaxial loadings, the non-proportionality parameter, $A$, is equal to zero, and during the transition from monotonic to cyclically stable material behavior, the quantity $[R_{T}^{(i)}-r^{(i)}]$ always has the same sign. Therefore, Equation (4.8) can be reduced and rearranged to produce the following relation:

$$b^{(i)} = \frac{dr^{(i)}}{dp} \quad (i = 1,2, ..., M)$$

(4.13)

From here, it is evident that the amount of hardening in a material is assumed to be a function of equivalent plastic strain alone. Therefore, by knowing the total change in plastic strain during a period of transient material behavior, the constants $b^{(i)}$ can be determined in a fairly straightforward manner.

Due to the Masing assumption, experimental data from only a single fully-reversed uniaxial constant amplitude fatigue test are needed to do this. The only information needed from the test is the constant applied strain level, or stress level (if load-controlled), and the number of cycles required to reach a cyclically stable material response, $N_s$. Knowing the applied load level allows the amount of plastic strain in the first applied cycle to be calculated (from the monotonic stress-strain curve) along with the amount of plastic strain in a cycle experiencing stable material behavior (from the cyclic stress-strain curve). The number of cycles required to transition from one plastic strain level to the next can then be used, along with an assumed relationship between
plastic strain amplitude versus cycle, to calculate the total amount of accumulated plastic strain required to reach stable material behavior.

In the current study, it was found that plastic strain amplitude varied with the number of applied cycles according to a logarithmic relationship:

\[
\varepsilon_p^a = \varepsilon_{a,o}^p - \frac{\log N}{\log N_s} \Delta \varepsilon_p^a
\]  

(4.14)

where \( \Delta \varepsilon_p^a \) is the difference in plastic strain amplitude between the first cycle and stable cycle and \( \varepsilon_{a,o}^p \) is the plastic strain amplitude in the first cycle. Figure 4.11 shows good agreement between Equation (4.14) and the experimentally observed variation in plastic strain amplitude. Using this relationship, transient plastic strain, \( \varepsilon_t^p \), over each cycle can be summed with respect to each backstress component. Because of the plastic strain level condition expressed in Equation (4.9), however, only plastic strain incurred above the level which begins to influence each backstress component (i.e. \( \varepsilon_t^{p(i)} = \varepsilon_p^a - \sqrt{2/3} / c^{(l-1)} \)) are included in the summation. This can be expressed as follows:

\[
\Sigma \varepsilon_t^{p(i)} = 4 \sum_{N=1}^{N_s} \left[ \varepsilon_{a,o}^p - \frac{\log N}{\log N_s} \Delta \varepsilon_p^a - \sqrt{2/3} / c^{(l-1)} \right]
\]  

(4.15)

The factor of 4 comes from converting amplitude to range and accounting for both the loading and unloading portions of a cycle. If it is found that a logarithmic relationship does not adequately describe the variation of plastic strain with cycle, actual experimental values may also be used as an alternative.

Once the values of \( \Sigma \varepsilon_t^{p(i)} \) are known, the hardening rate constants can be determined as follows:

\[
b^{(i)} = \frac{R^{(i)}_p - r^{(i)}_o}{\sqrt{3/2} \Sigma \varepsilon_t^{p(i)}} \quad (i = 1, 2, ..., M)
\]  

(4.16)
where the factor of $\sqrt{3/2}$ is necessary to convert from uniaxial plastic strain to equivalent plastic strain. In order for Equations (4.15) and (4.16) to produce accurate values of $b^{(i)}$, however, the plastic strain range, $\Delta \varepsilon_p^{(i)}$, associated with each backstress component should be exceeded in each cycle of the constant amplitude loading. In other words, Equation (4.17) should be satisfied for each component, $(i)$, so that hardening rates are not calculated for backstress components which may not have been fully saturated at the loading level applied in the test used for determining $b^{(i)}$:

$$\varepsilon_{a,s}^P \geq \sqrt{2/3} / c^{(i)}$$

(4.17)

where $\varepsilon_{a,s}^P$ is the stable cycle plastic strain amplitude. Therefore, when performing these calculations, it is desirable to use experimental data corresponding to a plastic strain amplitude greater than the largest level of plastic deformation expected to be modeled. If hardening rate constants need to be calculated for components $(i)$, which do not satisfy Equation (4.17), it was found that a reasonable approximation method is to assume that $\Sigma \varepsilon_t^{p(i)} = \Sigma \varepsilon_t^{p(i-1)}/2$. Equation (4.16) can then be used to approximate corresponding $b^{(i)}$ values.

In order to calculate the constant, $m$, which describes an increase in hardening rate due to non-proportional loading, the same procedure can be performed using constant amplitude 90° out-of-phase experimental data. In this case, reduction and simplification of Equation (4.8) leads to the following expression for $m$:

$$m = \frac{R_N^{(i)} - r_o^{(i)}}{b^{(i)} \Sigma p_t^{(i)} A} - \frac{1}{A}$$

(4.18)

The value for Tanaka’s non-proportionality factor, $A$, under 90° out-of-phase loading conditions is approximately equal to $1/\sqrt{2}$, or 0.707. A different value of $m$ may be
calculated corresponding to each backstress component, or the values of $m$ for one or more backstress components may be averaged to determine a constant value of $m$. The latter option was used in this study. Note that equivalent plastic strain amplitude must be used to compute $\Sigma p_i$ since the loading is no longer uniaxial.

Constants $\chi(i)$ control the predicted ratcheting or stress relaxation rate under unbalanced loading conditions. In practice, these values are determined by trial-and-error comparisons to experimental results. However, if ratcheting is not expected to be significant, as in the case of this study, these values have little influence on the model output. For this reason, all values of $\chi(i)$ were assumed equal to 5 in this study, as is the case in (“eFatigue,” 2014).

The final material constant left to be calculated is $C_c$ for Tanaka’s parameter. This parameter relates to the rate at which changes in material microstructure occur as a result of the introduction or cessation of non-proportional loading. If the level of non-proportionality does not significantly vary throughout the course of a given loading history, the value of $C_c$ will not have a large impact on the predicted material response. However, when the degree of non-proportionality is frequently changing, this parameter will have a significant effect on the predicted transient response. A small value of $C_c$ predicts significant cross-hardening effects and produces a gradual increase to the maximum hardness level followed by gradual cyclic softening. A large value, on the other hand, predicts a lesser effect of cross-hardening and the maximum hardening level is reached more quickly following the change in loading path (Zhang and Jiang, 2008).

The value of $C_c$ can be determined from comparisons to simple cross-hardening tests, where the loading is abruptly switched from one proportional loading path to
another. It should be noted, however, that the actual rate of hardening reflected in model predictions is a function of constants $b^{(i)}$, $m$, and $C_c$. As such, the values for these constants should be determined in this order so that their dependence on one another is properly reflected. For the 2024-T3 aluminum alloy used in this study, an approximate $C_c$ value of 200 produced the best agreement with experimentally observed cross-hardening trends based on the results of the Block A constant amplitude deformation test (shown in Figure 4.8).

4.5.3 Model Implementation

When implementing this plasticity model in the current study, the number of backstress components, $M$, was chosen to be 6 based on a balance between accuracy and computation time. In a study on the effect of various input parameters on the accuracy of output solutions for the Armstrong-Frederick-Chaboche style plasticity model, Jiang and Kurath (1996) found that predictions converged to experimental values after approximately 5-10 backstress terms. The logarithmic strain range increment chosen in the current study to calculate $c^{(j)}$ values further improves this accuracy for any given value of $M$.

The yield stress used in the plasticity model was defined based on the monotonic stress-strain curve and a $0.01\%$ strain offset. This ensured accurate modeling of stress-strain response/hysteresis loop shape even for small values of plastic strain. Again, the effect of yield stress value on predicted stress-strain response was evaluated in (Jiang and Kurath, 1996). Additionally, the maximum plastic strain range for the model was selected as 5% for all loading histories investigated. This ensured that the model would predict
material constitutive behavior in a consistent manner for all plastic strain levels relevant to the tests performed.

The plasticity model was setup to automatically calculate the base of the logarithmic increment used to calculate the values \(c^{(i)}\) so that the plastic strain range corresponding to \(i = 1\) was equal to approximately three times the strain range offset used to calculate yield stress. Constants \(r_o^{(i)}, R_p^{(i)}, \) and \(R_n^{(i)}\) were then calculated based on the experimentally determined von Mises equivalent stress-strain curves for monotonic tension, cyclic in-phase axial-torsion, and cyclic 90° out-of-phase axial-torsion loading conditions, respectively. A comparison, which shows excellent agreement between the experimental cyclic in-phase stress-strain curve and its piecewise linear representation in the plasticity model, is presented in Figure 4.12. A complete list of the material constants generated for the plasticity modeling analyses is also provided in Table 4.2.

Additionally, in order to improve the computational efficiency of the incremental cyclic plasticity modeling process, a dynamic load discretization algorithm was implemented in order to determine the stress or strain increment used for each computation step. Assuming a constant number of loading increments per reversal, or a maximum stress/strain increase per increment, would result in both excessive computation time and output data/file size. This is especially true if a long variable amplitude loading history is being analyzed where the majority of cycles exhibit linear elastic behavior. However, whether or not a loading process results in plastic deformation behavior cannot be determined \textit{a priori} because it is dependent on the previous deformation history and the current state of hardening. Therefore, the load discretization
routine was integrated directly into the cyclic plasticity algorithm, so that it could be made dependent on the yield function calculation.

The discretization logic proceeds as follows: At the beginning of an analysis, a maximum stress or strain increase to be used in the modeling of plastic deformation processes is specified. This increment is then scaled initially by a factor of 15 (arbitrarily chosen within reasonable bounds) and a corresponding “coarse” load increment is added to the initial stress or strain state. The maximum loading increment is determined based on the loading component with the largest difference between two consecutive points in the input history. If the resulting loading state exceeds the next point in the original input history, the trial state is changed to match that from the input history. The value of the yield function is then computed. If no plasticity is indicated, no values for the current trial state are stored (unless they correspond to values from the input history) and another loading increment is added to generate a new trial state. This process is repeated until plastic deformation is indicated by the yield function. When plasticity is finally detected, the loading state is reverted back to the previous all elastic trial state and the values are stored in the output history. The load increment is then changed back to its original maximum increase value, and plasticity calculations are performed (and output values are stored) for each elastic-plastic load increment analyzed. Once the loading returns to an unloading/elastic process, the maximum load increment is switched back to the coarse increment value, and the process repeats until the entire load history has been analyzed.

Although this discretization routine adds some additional calculation steps compared to an algorithm based on a constant loading increment, in most cases, the small increase in computation time is more than overcome by the reduction in trial loading
states analyzed using the dynamic routine. Additionally, the substantially smaller output files generated using this technique greatly reduce computation time for subsequent analyses such as stress/strain transformations and cycle counting. For the plasticity analyses performed in this study, maximum increments of stress and strain were specified to be on the order of 0.5 MPa and 0.001%, respectively, varying slightly depending on the extent of plastic deformation in each particular analysis.

4.6 Stress-Strain Prediction versus Experiment

In order to evaluate the accuracy of the plasticity model presented in the previous section under a variety of loading conditions, stress-strain outputs computed from the model were compared to experimental data for the deformation tests discussed in Sections 4.2 to 4.4. The following sections present the results of these comparisons for the incremental step tests, constant amplitude block loading histories, and variable amplitude service loading history, respectively.

4.6.1 Cyclic Incremental Step Test Results

The first comparisons made were for the load-controlled incremental step tests. The applied loading histories, in terms of number of cycles at each stress level, were used as input to the plasticity model for each of the uniaxial, pure torsion, and in-phase axial-torsion tests. The predicted strain output histories, in terms of strain amplitude versus cycle, were then used as a basis for comparison with the experimental results. Figures 4.13(a) and 4.13(b) show the predicted versus experimental strain amplitude history for the uniaxial and pure torsion tests, respectively, while Figure 4.13(c) shows the same comparison, in terms of von Mises equivalent strain amplitude, for the in-phase axial-
torsion test. As evidenced by these figures, the strain output predicted by the plasticity model agrees well, both qualitatively and quantitatively, with the experimental data from each test. This is despite the complexity of the observed transient material response. For example, the model is able to correctly predict an initial strain amplitude for some loading levels which is lower than the initial strain amplitude from the previous load level.

In terms of quantitative analysis, the error in predicted strain amplitude for the first cycle of each load level shown in Figure 4.13(a) for the uniaxial step test, in order or increasing load level, was $-1.9\%$, $-5.4\%$, $-2.6\%$ and $4.2\%$. Similarly, for the first cycle of each pure torsion loading level shown in Figure 4.13(b), errors were $-6.4\%$, $-10.2\%$, $7.0\%$, and $-8.9\%$. Finally, for the in-phase axial-torsion step test, errors in equivalent first cycle strain amplitude for each step shown in Figure 4.13(c), again in order of increasing load level, were $0.4\%$, $19.6\%$, and $11.7\%$. Error for subsequent cycles at any given loading level generally decreased from these values in all cases.

It should be noted that some of the error in these strain amplitude predictions, including steady-state values, is likely due to a combination of the quality of fit of the cyclic stress-strain curve and/or premature or incomplete material hardening for a particular backstress component $(i)$, as dictated by Equation (4.9). Effects of the latter are more prevalent at higher loading levels due to a larger difference in the plastic strain range at which each successive backstress component becomes saturated. As such, solution quality is also sensitive to where the plastic strain range for a particular loading level falls with respect to plastic strain ranges, $\Delta \varepsilon_p^{(i)}$, associated with each backstress component. Additionally, although closely monitored, testing machine control could have
adversely affected the experimentally measured strain amplitudes (through improper load application), especially in the first cycle of applied loading at any given level. Regardless of these facts, however, strain amplitude predictions were still very reasonable for all loading conditions considered.

### 4.6.2 Constant Amplitude Block Loading Test Results

After analyzing the results from the incremental step tests, a similar comparison was made between experimental and predicted stress response for the strain-controlled constant amplitude multiaxial block loading histories described in Section 4.3. To avoid any discrepancies due to the definition of equivalent stresses and strains under non-proportional loadings (as discussed in (Jiang and Kurath, 1997b)), quantitative comparisons between predictions and experimental results were based on separate consideration of axial and shear hysteresis loops.

In order to provide a baseline for evaluating predictions, stress-strain analysis for the steady-state loading block of each constant amplitude block loading test was performed twice: once without the inclusion of Tanaka’s parameter and once with. As such, the first comparisons made were aimed at gaging the improvement in stress predictions obtained through just the consideration of non-proportional hardening effects. It should be noted that in the context of these analyses, “steady-state”, or “stable,” are terms used to describe the condition when transient material behavior is only present due to changes in the degree of non-proportionality.

For Block A, the predictions for axial and shear stress amplitudes of the 2nd and 18th cycle of each loading path were compared to the experimental results from the final “stabilized” loading block (10 cycles compared). Without the consideration of non-
proportional hardening effects, the error in predicted stress amplitudes ranged from −10.2% to −3.6%, with an average error of −6.6%. Although predicted stress paths were qualitatively very similar, accounting for transient non-proportional hardening behavior improved these stress amplitude predictions to within a range of −4.1% to 2.2% error, with an average error of −1.7%.

Similarly, for Block B, predicted stress amplitudes for each strain path were compared to experimental results from the stabilized loading block (5 cycles compared). Again, predictions improved from an error range of −12.2% to −1.9%, with an average error of −6.1%, without the consideration of non-proportional hardening effects, to a range of −5.0% to 1.5%, with an average error of −1.4%, when accounting for non-proportional hardening. Summaries of all predicted versus experimental stress amplitudes are given in Tables 4.3 and 4.4 for the Block A and Block B tests, respectively. Additionally, the predicted steady-state stress path and hysteresis loops, presented in Figure 4.14 for the Block B deformation test, show good qualitative agreement between the model output (considering non-proportional hardening) and experimental results. Similar agreement between the experimental and predicted stress path was also observed for the Block A test, but is not shown for brevity.

Additionally, because of the 20 consecutive applications of each load path in Block A, the ability of the transient plasticity model formulation to predict a gradual softening, or recovery, of the material following a decrease in non-proportionality can be evaluated as well. Figure 4.15 shows a comparison between experimental and predicted equivalent stress amplitude versus cycle for the final applied loading block (block 5) in the Block A deformation test. For the first four applied strain paths, it can be seen that the
model generally predicts an initial increase, followed by a gradual decrease, in stress amplitude whenever the loading path is changed. This reflects cross-hardening of the material, due to a sudden change in the proportional loading direction, followed by a gradual softening during the subsequent proportional loading cycles. For the final 90° out-of-phase loading cycles, the degree of non-proportionality is constant throughout and no change in the equivalent stress amplitude is predicted. All of the trends and stress amplitude values predicted by the model agree very well with the experimental data. In comparison, stress amplitude predictions are nearly constant over all cycles of any given loading path when no transient non-proportional hardening is considered. This can be observed by comparing the predicted stress values, reported in Table 4.3, between the 2nd and 18th cycle of each loading path.

After establishing the capability of the plasticity model to reflect changes in material behavior due to varying degrees of non-proportionality, its ability to model transient material behavior due to initial cyclic hardening was also evaluated under the same loading conditions. The experimental results from the Block B deformation test were used for these analyses. Predicted axial and shear stress amplitudes for each strain path were compared separately with their corresponding experimentally measured values for applied loading blocks 2, 4, 8, 16, 32, 50, and 100. As such, comparisons were made based on a total of 35 cycles, or a combination of 56 axial and shear stress amplitudes. Experimental stress amplitudes, predicted stress amplitudes, and the corresponding error percentages for each of these comparisons are summarized in Table 4.4.

The maximum error in any single stress amplitude prediction was −13.7% for the shear stress response in the 180° out-of-phase cycle during the second block of applied
loading. However, the average error for all stress amplitudes in this same block was only −4.8%. When considering all loading blocks analyzed, the error in predicted axial stress amplitudes ranged from −6.9% to 2.6%, with an average of −0.9%, while the error in shear stress amplitude predictions ranged from −13.7% to 5.5%, with an average of −1.4%. This results in an overall average error of −1.2% for all predictions. Figure 4.16 shows a comparison of the predicted and experimental axial and shear stress amplitudes, versus the number of applied loading blocks, for each cycle considered in the analysis. From these results, it is evident that the model is quite capable of predicting transient material deformation behavior, even under complex loading conditions, within a reasonable margin of error.

4.6.3 Variable Amplitude Service Loading Test Results

The final step in evaluating the current plasticity model was to compare predicted stress response to experimental data obtained from the variable amplitude simulated service loading deformation test described in Section 4.4. Again, comparisons were made separately for axial and shear stress amplitudes to avoid any discrepancies due to the definition of equivalent stress. Unlike the constant amplitude block loading tests, however, the definition of a cycle is not always clear for this test. Therefore, comparisons were made based on the minimum and maximum stress values corresponding to five different reversals in the loading history. These reversals correspond to the stress response following the maximum and minimum axial loading events (R1 and R2), the maximum shear loading event (R3), and for in-phase and 90° out-of-phase loading conditions (R4 and R5). The starting point for each of these reversals is indicated in Figure 4.9(b) by a vertical dashed line and a corresponding number in parentheses. For
R5, the shear reversal is analyzed starting at the first peak following the dashed line. Then, to calculate stress ranges, stress values at each starting point were compared to values at the next peak or valley for each loading channel. A total of 10 reversals were analyzed per block (5 axial and 5 shear).

Figure 4.17 shows a comparison between experimental and predicted axial and shear stress amplitudes (half of the range) versus the number of applied loading blocks in the variable amplitude deformation test. Results from a total of five blocks were used for the comparisons: blocks 2, 4, 10, 20, and 30. It can be seen from this figure that the predictions agree relatively well with the experimental results for each reversal analyzed. The maximum error for any stress amplitude prediction was 25.2% for the shear stress reversal at R1 in block 10. However, the range of prediction errors for all reversals in this same loading block was −5.3% to 25.2%, with an average error of 1.1%. This can be compared to stable block (block 30) stress amplitude prediction errors which ranged from −5.8% to 16.0%, with an average error of 0.1%. Again, the maximum error in the stable block corresponds to the shear stress reversal at R1. When considering all loading blocks and all reversals analyzed, the error in axial stress amplitude predictions ranged from −4.4% to 0.4%, with an average error of −1.3%, while error in shear stress amplitudes ranged from −12.0% to 25.2%, with an average error of 1.5%. This results in an overall average error of 0.1% for all predictions.

A comparison showing good agreement between experimental and predicted stress paths for both block 2 and block 30 of the variable amplitude deformation test is shown in Figure 4.18. Additionally, experimental and predicted axial and shear stress-strain hysteresis loops for block 30 are shown in Figure 4.19. Hysteresis loop plots not
only provide another means for evaluating model predictions, but also allow for a quick assessment of the degree of plasticity present in the loading block.

For the variable amplitude deformation test, comparisons were also made between experimental and predicted mean stress values for each reversal. Unlike the fully-reversed constant amplitude deformation tests previously discussed (where mean stresses were typically less than 5% of the stress amplitude), mean stress values can be significant in a variable amplitude loading history. Since mean stress is often an important parameter in fatigue damage calculations, it was important to evaluate the ability of the current plasticity model to accurately represent variations in these values as well. However, comparisons are difficult to make based on typical percentage error calculations because small mean stress values will produce a large percentage error even for small differences in actual values. For this reason, error percentages for mean stress were calculated, in this study, as the difference between the predicted and experimental value, normalized by the experimental stress amplitude for the same reversal. This formulation gives a more accurate representation of error in terms of impact on fatigue damage calculations.

The resulting errors in axial mean stress predictions, averaged for all loading blocks considered, were 3.7%, −10.0%, 8.3%, 0.0%, and −0.8% for reversals R1 through R5, respectively. Similarly, errors in mean shear stress predictions, averaged for all loading blocks considered, were 44.1%, −11.0%, −0.4%, 22.3%, and 23.3% for reversals R1 through R5, respectively. Error values were similar from block to block for any given reversal. Figure 4.20 shows a graphical comparison between the experimental and predicted mean stresses, for each axial and shear stress reversal, as the number of applied loading blocks was increased. In general, axial mean stress predictions had the tendency
to be more accurate than those for mean shear stress, while mean shear stress predictions were more accurate when shear was the dominant loading mode. A complete summary of experimental and predicted stress amplitudes and mean stresses, along with the percentage error values for each comparison, is presented in Table 4.5.

4.6.4 Discussion on Stress-Strain Prediction Results

From the results of the steady-state stress predictions for the constant amplitude block loading tests, it is clear that even for materials displaying moderate levels of non-proportional hardening, and for loading histories composed primarily of proportional loading paths, the consideration of non-proportional hardening is an important aspect in the derivation of accurate stress-strain response. Additionally, it was found that the simplified transient hardening formulation proposed in this study, along with the new calculation methods for determining the related material constants, were able to accurately reflect the effects of non-proportionality on material response, both qualitatively and quantitatively, when coupled with Tanaka’s parameter.

For each type of deformation test performed, the simplified plasticity model formulation was shown to predict the experimentally observed stress-strain behavior within a reasonable amount of error. This is true despite the wide variety of loading conditions investigated. Axial-dominated, shear-dominated, in-phase, out-of-phase, and asynchronous multiaxial loading conditions, along with the inclusion of mean stresses, overload effects, and varying degrees of cross hardening and non-proportional hardening, were all represented. Additionally, the constant amplitude, variable amplitude, and step loading histories all produce significantly different transient hardening behavior due to their distinct differences in loading sequence. Still, the maximum error in any stress or
strain amplitude prediction was just over 25%, and average errors for any given loading history were less than 5%. The combined average error for all stress amplitude predictions considered in the three block loading deformation tests (61 axial and 61 shear) was −0.7%, with a standard deviation of 5.2%.

In general, it was also shown that predictions for axial stress amplitudes, as well as axial mean stresses, were consistently better than those for shear stress. Despite the fact that errors in some predicted mean shear stresses for the variable amplitude deformation test were as large as 58%, mean shear stress typically has a minimal effect on the resulting fatigue damage. Additionally, mean shear stress predictions were shown to be more accurate when shear was the dominant loading mode. Axial mean stresses on the other hand, which can significantly impact fatigue damage, were predicted within 15% error in all cases.

While there is still room for improvement, there will always be a finite amount of error that exists in stress-strain predictions due to modeling assumptions, numerical solving techniques, and inherent material variability. Taking this into consideration, it can be concluded that the current plasticity model, with its simplified material assumptions and revised transient hardening formulation, is capable of predicting material constitutive behavior within a fairly high overall level of accuracy, even under arbitrary multiaxial loading conditions. While this was shown for the 2024-T3 aluminum alloy tested in this study, similar results would be expected for other materials as well, provided they do not exhibit a significant degree of non-Masing deformation behavior.
4.7 Notch Stress Analysis

Unlike the uniformly stressed un-notched specimens discussed in the previous sections, notched components produce localized regions of elevated stress that serve as likely sites for fatigue cracks to initiate. Therefore, being able to accurately characterize and model material deformation behavior in these regions is an essential part of a notched fatigue life analysis. However, the local deformation behavior will change differently depending on the specific notch geometry of interest. For example, in addition to the stress concentration effect, changes in local stress state can occur due to notch boundary conditions, and the severity of stress gradients moving away from the notch can vary depending on the notch root radius. As a result, detailed stress analyses are often required prior to fatigue damage calculation in order to properly quantify these effects.

The main goal of the stress analyses presented in this section was to obtain information on local stress distributions surrounding the hole in the notched tubular specimens tested in this study. As mentioned, this information was necessary in order to derive the local stress-strain histories required for performing subsequent fatigue life calculations. To do this, linear elastic finite element analyses were performed first in order to obtain relevant stress concentration factors and to study local stress states and stress gradients under a variety of loading conditions. Additionally, some nonlinear elastic-plastic analyses were also performed in order to evaluate the effects of plasticity on local deformation behavior. The nonlinear analysis results are not presented until the following section, however, where they are compared to results from notch stress-strain estimation models.
In the following stress analyses, Abaqus/CAE 6.14-5 commercial finite element software was used to perform all stress-strain calculations. A model of the entire test specimen was used in the analyses, over which a free mesh was generated using 10 node quadratic tetrahedral elements (C3D10). The mesh was refined in the critical notch root region down to a minimum node spacing of 0.05 mm. The resulting mesh, shown in Figure 4.21, consists of 25 elements across the specimen thickness and 140 elements around the circumference of the hole. The experimentally determined modulus of elasticity and Poisson’s ratio were input into the software to define the linear elastic isotropic material behavior.

To clarify some of the terminology used in the following discussions, when referring to an angular location around the perimeter of the hole, the 0° location refers to any point on a plane extending radially outward from the center of the hole in a direction rightward (when viewing the hole from outside of the specimen) and perpendicular to the axis of the specimen. Positive rotation is then measured in the counter clock-wise direction. When referring to local stress components, the y-direction is parallel to the axis of the specimen (the direction of applied axial loads) and the x-direction coincides with the 0° plane described previously, but with the added condition of being tangent to the curvature of the specimen at the center of the hole. This leaves the z-direction corresponding to the specimen thickness direction, along the axis of the hole.

The first analyses performed were for linear elastic material behavior under unit axial and unit shear loads. Unit loads were defined in terms of gross cross section nominal stresses. Stress distributions were then evaluated for three paths in the vicinity of the notch root: circumferentially around the outer edge of the hole, along the depth of the
hole at the location of maximum stresses (von Mises, maximum principal, and maximum shear stress), and radially away from the hole at the location of maximum stresses. Figure 4.22 shows the distribution of von Mises, maximum principal, and maximum shear stresses around the circumference of the hole, at the outer surface of the specimen, for both pure axial and pure torsion loadings. Since nominal loadings correspond to the unit stress case, output values show the stress concentration effect of the hole on each stress quantity. It is evident from these plots that the location of maximum stress for all equivalent stresses considered is at $0^\circ$ for axial loading and at $45^\circ$ for torsion loading. After determining these locations, stress distributions along the depth of the hole and radially away from the hole could then be evaluated where stresses were highest.

Figure 4.23 shows the distribution of local elastic stress components along the depth of the hole for pure axial and pure torsion loadings. As stated previously, the distributions are given at the location of maximum stresses around the circumference of the hole. For both loading conditions, the maximum value of all stress components is found near the specimen’s mid-thickness region. This is in part due to increased constraint imposed on the mid-thickness area of the hole by the surrounding elastic material. This constraint, not present at the specimen inner and outer surfaces, restricts the deformation of the mid-thickness material and, in turn, causes a buildup of stress in the specimen thickness direction. Although the hole size and wall thickness of the notched specimens used in this study were carefully chosen to maintain a predominately plane stress state at the notch root, inevitably there is still a small amount of transverse stress, $\sigma_z$, present. In this particular case, mid-thickness von Mises stresses are around 7% higher than the highest surface stresses for axial loading, and around 6% higher for
torsion loading. The implications of this variation with respect to fatigue damage calculation are subsequently discussed in Section 5.3.2.

From these results, changes in stress concentration across the thickness of the specimen can also be evaluated for each applied loading. For multiaxial stress states, however, the value of stress concentration factor can vary significantly depending on its definition. When computing stress concentration factors based on the ratio of the maximum von Mises equivalent stress at the notch to the nominally applied (axial or shear) stress, values of 3.14 and 4.09 were derived from FEA results at the specimen’s mid-thickness location for pure axial and pure torsion loading, respectively. Alternatively, substituting local von Mises stress for maximum principal stress at the notch root yields values of 3.22 for pure axial and 4.27 for pure torsion loading. This can be compared with theoretical stress concentration factors of 3.21 and 3.89 for axial and torsion loading, respectively. Theoretical concentration factors were calculated using the stress concentration finder available on the website eFatigue (“eFatigue,” 2014). Although there are differences between the theoretical and FEA values, this could be the result of multiple factors such as the assumption of plane stress or plane strain conditions and the local stress quantity used to define stress concentration factor. In subsequent analyses presented in this study, $K_t$ values always refer to those derived through linear elastic FEA.

Finally, normalized stress distributions moving radially away from the hole on the specimen outer surface are presented in Figure 4.24. Again, values are taken starting from the 0° location for axial loading, and from the 45° location for torsion loading. As can be seen from both plots in the figure, von Mises stress is equal to maximum principal stress
at the edge of the hole, indicating a uniaxial stress state around the perimeter of the hole. This condition is expected due to the prevailing free surface boundary conditions, which allow a normal stress, but no shear stresses, on planes perpendicular to the curvature of the hole. However, as the distance from the hole is increased, the stress state quickly becomes multiaxial. This can be seen in the axial loading plot, Figure 4.24(a), as the variation of stress in the $x$-direction, perpendicular to the applied loading. This stress component reaches its maximum at a distance from the edge of the hole of around 43% of its radius before returning to zero as the notch effect diminishes. For the torsion case, the increasing degree of multiaxiality is evidenced by the increasing difference between von Mises equivalent stress and maximum principal stress. This difference continues to increase away from the hole until the notch effect has vanished and the stress state is that of pure torsion (biaxial). It should be noted that at the mid-thickness location of the specimen, stresses may be multiaxial even at the free surface of the hole. This is due to the presence of transverse stress, in the specimen’s thickness direction, as a result of the notch constraint discussed previously.

These changes in multiaxiality/stress state can be an important consideration when accounting for notch effects in a fatigue life analysis. For example, an analysis performed using Neuber’s rule, which is based on maximum stress and strain values at the edge of the hole, would consider a uniaxial stress state regardless of the nominally applied loading conditions. This is true even if fatigue notch factor, $K_f$, is used to account for stress gradient effects. An approach such as the Theory of Critical Distances, however, considers stress gradient effects by evaluating stress-strain values a short distance away from the edge of the hole. Therefore, such an approach could be better able
to reflect changes in fatigue damage due to variations in local stress state. This can be especially significant in cases where non-proportional loading is present at the notch, and will be discussed in more detail in the subsequent fatigue life analysis chapters.

In the case of combined axial-torsion loading, local elastic stress components are often computed through the superposition of each nominal stress component multiplied by its respective stress concentration factor, i.e. Equation (2.3). It is important to note that in some cases, such as a sharp notch, the location of maximum local stress due to axial loading may coincide, or very nearly coincide, with that due to shear loading. However, for a blunt notch or hole, such as the one considered in this study, these locations will vary around the notch root. Therefore, the location of maximum local stress will continually change with a changing ratio of applied axial to torsion loading. It is for this reason, that when using the principle of superposition to derive local elastic stresses, care must be taken to ensure that the stress concentration factors used for each nominal loading component correspond to the same particular location being analyzed.

Figure 4.25 shows the von Mises stress distribution, location of maximum stress, and equivalent stress concentration factor on the specimen outer surface for different ratios of nominal axial to shear stress. As expected, as the axial stress is increased with respect to shear stress, the location of maximum stress tends towards $0^\circ$, and as shear stress is increased relative to axial stress, this location tends towards $45^\circ$. Also, as the nominal stress ratio moves from axial-dominated to shear-dominated, the equivalent stress concentration factor reduces to a minimum for pure torsion loading. Note that in order to maintain consistency in the definition of nominal stress for combined loading,
stress concentration factor is defined here as local von Mises stress divided by nominal von Mises stress, whereas in the pure torsion case, nominal shear stress was used.

To avoid the need for FE analysis at every desired combination of loads, a polynomial equation can be fit to the stress concentration variation around the hole for each individual load case and stress component. Then, for any given combination of axial and shear loading, the local stress components around the hole can be calculated as a function of the angular location as follows:

\[
\sigma_{ij} = S_A a_{ij}(\theta) + S_T t_{ij}(\theta)
\]  

(4.19)

where \(a_{ij}(\theta)\) and \(t_{ij}(\theta)\) are functions describing the elastic stress concentration variation for stress component, \(\sigma_{ij}\), due to axial and shear loading, respectively, \(S_A\) and \(S_T\) are nominal axial and shear stress, respectively, and \(\theta\) is the angular location around the hole. Local elastic equivalent stress can then be calculated at each angular location using the resulting stress components and an appropriate equivalent stress criterion.

However, as shown previously for the geometry used in this study, there is always a uniaxial stress state which exists at the edge of the hole in a direction tangential to its curvature. Therefore, in this case, only one function for each nominal loading, \(a(\theta)\) and \(t(\theta)\), is needed to directly describe the variation of equivalent stress around the hole. On the outer surface of the tubular test specimens used in this study, these functions can be expressed using the following third degree polynomials:

\[
a(\theta) = 1.24(10^{-5})\theta^3 - 1.56(10^{-3})\theta^2 + 1.53(10^{-3})\theta + 2.93
\]  

(4.20)

\[
t(\theta) = -1.82(10^{-5})\theta^3 - 4.27(10^{-4})\theta^2 + 1.31(10^{-1})\theta + 3.79(10^{-2})
\]  

(4.21)

where \(\theta\) is in degrees. Equation (4.19) can then be maximized to obtain the value and location of maximum local stress for a given nominal loading condition.
In some cases, analytical expressions based on theory of elasticity may also be available for such computations. It should also be noted that for more complex component geometries, where nominal stress is not easily defined, the same type of calculation procedure may be carried out using applied load values and stress proportionality factors in the place of nominal stresses and stress concentration factors.

For a constant amplitude fatigue life analysis, using an equation such as Equation (4.19) is an effective approach to determine the maximum stress location, and thus the likely failure location, for any combination of applied loadings. For non-proportional nominal loadings, the equation may need to be evaluated at several different points in the loading cycle in order to determine the location of the absolute maximum stresses occurring over the entire cycle. Then, since all loading cycles are the same, plasticity corrections can be performed and local stress-strain values at this critical location can be used to compute a corresponding fatigue life estimate. In a non-proportional variable amplitude fatigue life analysis, however, the ratio of nominally applied loads, and consequently the location of maximum local stress, can be continuously changing throughout the loading history. This, combined with the fact that the magnitude of the stress concentration effect is also changing with the nominal load ratio, makes determining a failure location prior to performing such an analysis very difficult. For this reason, it may often be necessary to perform variable amplitude fatigue life analyses at multiple locations in the vicinity of a notch root in order to determine which location will experience the maximum amount of fatigue damage.
4.8 Notch Stress-Strain Estimation

In order to derive accurate stress-strain histories for critical, plastically deformed, locations within a notched component, local stresses and strains must often be estimated from a nominally applied loading history. Although this may be rather straightforward for cases involving simple geometries and uniaxial loadings, it can become quite complicated as geometries and loading conditions become more complex. Nonlinear finite element analysis is a very useful tool in this respect, but for complex loading conditions and/or long loading histories, it can be computationally very expensive to rely solely on this method to obtain local elastic–plastic stress–strain histories. Therefore, notch root stress–strain estimation rules, coupled with elastic stress concentration factor solutions and the nominally applied loading history, are often used in practice to efficiently derive local stresses and strains at the fatigue critical locations within a component.

4.8.1 2024-T3 Notched Tubular Specimen

In this section, local notch root stresses and strains are first estimated using Neuber’s rule for the 2024-T3 notched tubular specimen tested in this study. Several different levels of pure axial, pure torsion, and combined axial-torsion loading conditions were considered. For all analyses, the generalized form of Neuber’s rule, as presented by Hoffmann and Seeger (1989), was used in the form of Equation (2.24). Effective stress concentration factors, $K_{tq}$, used in Neuber’s rule calculations were those obtained from linear elastic FEA based on gross cross section nominal stresses and local von Mises stresses. Therefore, the resulting notch root stress and strain quantities, $\sigma_q$ and $\varepsilon_q$ respectively, are also equivalent quantities based on von Mises stress.
In order to evaluate the accuracy of these notch plasticity corrections, local stress-strain predictions were compared to solutions derived using nonlinear FEA. Nonlinear analyses were performed using Abaqus/CAE 6.14-5 commercial finite element software using the same model and mesh described in Section 4.7 for linear elastic analyses. Elastic–plastic simulations were conducted in a quasi-static fashion by applying the desired nominal load amplitude and evaluating the resulting stress-strain solutions after a single loading from zero to maximum. The cyclic axial stress–strain curve was input into the software to define the material constitutive behavior, and due to the monotonic nature of the analyses, isotropic hardening behavior was assumed.

All FEA solutions and Neuber’s rule predictions are based on stresses and strains occurring at the specimen’s outer surface. It should be noted that in the following analyses, local equivalent stresses and strains from Neuber’s rule are being compared directly to those derived from FEA. Therefore, the additional relations of the generalized form of Neuber’s rule, which relate principal notch stresses and strains to the equivalent quantities, are not needed. Additionally, in the case of a circular hole under true plane stress conditions, as previously shown, the stress state at the notch surface is uniaxial for all loading cases. Therefore, the equivalent stress would be equal to the only non-zero principal stress. However, this is not always the case and in general, the additional relations should be considered.

Figure 4.26 shows the comparison of Neuber’s rule predictions for local von Mises equivalent stress and strain versus nominal stress, against solutions generated from nonlinear FEA, for pure axial \((\lambda = 0)\), pure torsion \((\lambda = \infty)\), and combined axial-torsion \((\lambda = 0.62)\) loading conditions. In all cases, prediction trends are similar, and all local stress
and strain estimations are in good agreement with the FEA solutions. Neuber’s rule does, however, have the tendency to slightly over predict notch root stresses and strains for the geometry of interest. This is expected, however, since Neuber’s rule was derived for plane stress conditions and typically serves as an upper bound for stress-strain estimations (Zeng and Fatemi, 2001). The presence of notch constraint, even to a small extent, acts to reduce equivalent stress and strain values compared to those which would be expected under truly plane stress conditions. Plots of elastic–plastic stress and strain concentration factors calculated from Neuber’s rule predictions are compared to those derived from FEA results in Figure 4.27 for several pure axial and pure torsion nominal loading levels. These plots show the decrease in stress concentration, and corresponding increase in strain concentration, as the amount of plasticity present at the notch root increases.

For all loading conditions analyzed, Neuber’s rule estimations for local equivalent stress are plotted against nonlinear FEA solutions in Figure 4.28(a). Similarly, all local equivalent strain estimations are shown Figure 4.28(b). These plots help to give an overall indication of the amount of error present in the local stress-strain predictions. All local stress estimations were found to be within ±5% error, while the less accurate local strain estimates fell within an error margin of ±25%. It is important to note, however, that the higher stress levels considered in these analyses are much higher than what would exist in typical fatigue loading applications. As will be seen in subsequent chapters, the highest notch root equivalent strain amplitude encountered in the experiments performed in this study was around 0.5%. At the lower, more practical loading levels, Neuber’s rule stress and strain estimations are in very good agreement with the results from FEA.
Therefore, it would be reasonable to expect equivalent stress-strain predictions within ±5% error when performing a typical fatigue life analysis.

Despite the popularity of Neuber’s rule for local stress-strain prediction, and the good agreement it demonstrates with elastic-plastic FEA solutions in this study, it is not without its shortcomings. As discussed in Section 2.3.3, because of their dependence on Hencky’s flow rule, notch estimation rules which correct for plasticity based on an equivalent stress range generally cannot be directly applied to non-proportionally varying stress histories without the use of plasticity modeling techniques. Additionally, changes in material constitutive behavior due to cyclic and/or non-proportional hardening cannot be accounted for outside of a cyclic plasticity model framework. Since these phenomena are of particular interest in this study, an alternative approach to local stress-strain estimation was also evaluated. The particular model considered was the pseudo stress approach proposed by Köttgen et al. (1995), as discussed in Section 2.3.3.

Because notch effects are considered through the introduction of a “structural yield surface,” which is defined based on a local elastic stress versus local elastic-plastic strain curve, this model can be used in conjunction with any desired cyclic plasticity model. For this study, the plasticity model formulation outlined in Section 4.5 was chosen. In order to evaluate the accuracy of the pseudo stress-based plasticity corrections, local stress-strain predictions were performed for various loading conditions and compared to the previously presented nonlinear FEA solutions.

Before local stress-strain response could be computed, the structural stress-strain curve first needed to be generated. Although this can be done in a number of ways, uniaxial Neuber’s rule was chosen in this study due to its simplicity and popularity. The
first step in generating the structural stress-strain curve is to generate a set of linear elastic/pseudo stress-strain values which encompasses the range of equivalent elastic notch root stresses and strains expected in practical loading situations. Then, Neuber’s rule, along with an appropriate material constitutive relation, is used to generate the elastic-plastic stress-strain values corresponding to each pseudo stress-strain state. Once this is done, pseudo stress can be plotted against elastic-plastic strain and a corresponding Ramberg-Osgood style relation can be fit to the resulting structural stress-strain curve.

The linear elastic, cyclic elastic-plastic, and elastic stress-true strain deformation curves are shown in Figure 4.29, along with the structural stress-strain best fit curve, for the 2024-T3 aluminum alloy used in this study. The deformation curve parameters corresponding to the structural stress-strain curve fit were found to be $K^* = 3400$ MPa and $n^* = 0.290$. The cyclic stress-strain relation used in generating this curve (used in subsequent fatigue life calculations) is that derived from the in-phase axial-torsion incremental step test ($K_{IP}' = 688.6$, $n_{IP}' = 0.081$). For the current comparisons with FEA stress-strain solutions, however, an older cyclic stress-strain fit, corresponding to uniaxial incremental step test results, was used in order to be consistent with the material properties input into the finite element model.

Prior to performing plasticity corrections, local pseudo stress histories were first constructed by multiplying the elastic stress concentration factor for each local stress component, as determined from linear elastic FEA, by the nominally applied constant amplitude loading history. For combined axial-torsion loadings, linear superposition, i.e. Equation (2.3), was used to combine local stress contributions from each applied loading. Plasticity calculations were then performed by inputting the local pseudo stress histories,
along with the material structural stress-strain curve, into the plasticity model. Once the resulting elastic-plastic strain output was calculated, it was input back into the plasticity model, along with the material cyclic stress-strain curve, to compute the corresponding elastic-plastic notch stresses. When performing local stress-strain predictions for the purpose of comparison to FEA solutions, only steady-state stress-strain values were desired. Therefore, the cyclic plasticity model used for these calculations did not consider transient material behavior.

Because the pseudo stress approach is subsequently used, in conjunction with Theory of Critical Distances concepts, to consider stress gradient effects on fatigue damage, stress-strain predictions were not only performed at the notch root, but also at distances of 0.210 mm and 0.500 mm radially outward from the edge of the notch from the location experiencing the largest stress concentration for each loading condition. This corresponds to the 0°, 45°, and 23° locations for axial, torsion, and in-phase axial-torsion loading conditions, respectively. Stress analysis paths for each loading condition are shown in subsequent results figures. Since the stress state away from the edge of the hole is multiaxial, evaluating the ability of this approach to accurately predict equivalent, as well as individual, stress and strain components is important when judging the overall quality of subsequent fatigue damage calculations based on these values. Again, all FEA solutions and pseudo stress-based plasticity corrections are based on stresses and strains occurring at the outer surface of the specimen.

Similar to the Neuber’s rule analyses, the first comparisons made for the pseudo stress-based predictions were aimed at evaluating its ability to accurately predict local equivalent stress and strain at the notch root. Results of these predictions are shown,
along with those from Neuber’s rule, in Figure 4.26. Similar to Neuber’s rule predictions, the pseudo stress approach has the tendency to slightly overestimate notch root stresses and strains, although to a lesser extent. However, since Neuber’s rule was used to derive the structural stress-strain curved used in the pseudo stress approach, similar equivalent stress and strain predictions are expected. All von Mises stress values at the notch root were predicted within 2% error, regardless of the nominal stress state or loading level, when using the pseudo stress approach. A summary of all predicted and FEA derived local stress-strain solutions at the notch root, along with the percentage error for each prediction, can be found in Table 4.6.

Additionally, results for local equivalent stress predictions moving away from the hole are shown, along with nonlinear FEA solutions, in Figure 4.30. Individual plots within the figure represent different nominal loading conditions (i.e. axial, torsion, and in-phase axial-torsion). Each plot contains traces for the FEA derived variation of von Mises equivalent stress starting at the maximum stress location at the notch root and moving radially away from the notch along a path normal to its curvature. Each trace corresponds to a different nominal loading level, and corresponding pseudo stress-based predictions are represented by solid markers. The stress analysis paths for each loading condition are only shown in Figure 4.30, but are the same for all subsequent stress-strain comparisons.

Unlike maximum stress predictions at the notch root, equivalent stress has the tendency to be underpredicted as the distance from the notch is increased. Although the maximum error in any von Mises stress prediction is only 6.3%, the underprediction of stress values was not expected, as Neuber’s rule is generally considered to provide an
upper bound for local stress-strain estimations. Possible reasons for this behavior are discussed, along with other results, in Section 4.8.3. Again, a summary of all predicted and FEA derived local stress-strain solutions at distances of 0.210 mm and 0.500 mm from the notch root, along with the percentage error for each prediction, can be found in Tables 4.7 and 4.8, respectively.

Next, comparisons were made between pseudo stress-based plasticity corrections and nonlinear FEA results for maximum principal stress and strain. Similar to equivalent stress/strain, given the popularity of these quantities in fatigue damage analysis, such comparisons are useful when assessing the impact of local stress-strain estimation on subsequent life predictions. These comparisons are shown in Figures 4.31 and 4.32 for maximum principal stress and strain, respectively. Figures are organized by loading condition and loading level according to the same logic as the previous figure for von Mises stress distributions. From these figures, it can be seen that maximum principal stress and strain predictions follow the same trends for each respective loading case. While these stress/strain predictions tend to be fairly accurate for the lower applied loading levels, they increasingly deviate from the FEA solutions as the loading level is increased and/or the distance from the notch root is decreased.

For the higher loading levels analyzed, maximum principal stresses are significantly overpredicted at the notch root, resulting in prediction errors of up to 20.3%. This is somewhat surprising for the notched tubular specimen because the boundary conditions of the circular hole, combined with plane stress conditions across the thickness of the specimen, result in a uniaxial stress state at the notch root regardless of the nominal stress state. Therefore, the only non-zero principal stress should be equal to the von Mises
equivalent stress in all cases (as is the case for the FEA solutions). However, there is a significant difference between the two stress quantities which results from the pseudo stress-based plasticity corrections. Again, possible reasons for this behavior are offered in the following discussion section (Section 4.8.3).

As the distance from the notch root is increased, however, principal stress predictions tend to cross over the FEA distributions, and all stresses are underpredicted at a distance of 0.5 mm from the notch root. The maximum error in principal stress predictions at a distance of 0.5 mm from the notch root is 5.8%. Trends for maximum principal strain predictions are similar, in all cases, to those for maximum principal stress. This is expected, however, since local stress histories are derived from the local strain histories. Maximum principal strain prediction errors peak at 15.4%.

Finally, due to its use in shear-based fatigue damage parameters, comparisons were made between predicted and nonlinear FEA derived distributions of maximum shear strain in Figure 4.33. From this figure, it can be seen that the accuracy of maximum shear strain predictions are qualitatively and quantitatively (in terms of error) very similar to maximum principal strain predictions for all and loading conditions considered. This is expected, however, since maximum shear strain is directly computed from principal strain quantities. The maximum error in maximum shear strain predictions is 13.1%.

4.8.2 AISI 1141 Notched Shaft Specimens

In order to study the generality of the findings presented in the previous section, similar comparisons between pseudo stress-based local stress-strain estimations and nonlinear FEA results were performed for data found in literature for a different material and specimen geometry. The second material investigated was AISI 1141 medium carbon
steel, treated with vanadium-microalloy, and tested in an as-forged condition (Fang, 2001). The material was first hot rolled into 76 mm diameter bars before being hot forged into 25 mm thick pancakes, cooled, and finally machined in the longitudinal orientation into two different notched cylindrical specimen geometries. Mechanical properties for the material, summarized in Table 4.9, include a yield strength (0.2% offset) of 524 MPa, ultimate tensile strength of 875 MPa, modulus of elasticity of 200 GPa, and Brinell hardness of 262 BHN.

The two different specimen geometries investigated for this material were that of a stepped shaft and a grooved shaft. The stepped shaft features a 12.7 mm minor diameter, 15.9 mm major diameter, and notch root radius of 0.40 mm at the center of a 38.1 mm long gage section. The second geometry, a grooved shaft, features a 12.7 mm minor diameter, 14.6 mm major diameter, and a notch root radius and opening angle of 0.79 mm and 60°, respectively, at the center of a 38.1 mm gage section. Complete geometry and dimensions are shown in Figures 4.34(a) and 4.34(b) for the stepped and grooved shaft specimens, respectively.

Nominal loading conditions analyzed for the notched shaft specimens included constant amplitude pure torsion and in-phase axial-torsion loadings. Since local stress-strain solutions were not available in the literature for these specimens, Abaqus/CAE 6.14-5 was used to generate these results using the same nonlinear modeling techniques outlined in the previous section. However, due to the much smaller notch root radii of the shaft specimens, a submodeling technique was used to generate accurate stress-strain response in the specimens’ critical regions. A more refined mesh was generated on the submodel using eight node linear hexahedral brick elements (C3D8R), down to a
minimum node spacing of 0.005 mm. Again, stress-strain predictions were evaluated at 
the notch root, as well as at distances of 0.2 mm and 0.5 mm away from the notch root. 
Stress analysis paths for each loading condition, including their orientation with respect 
to the notch bisector, are shown in subsequent results figures.

The procedures for estimating local stress-strain response for the AISI 1141 
notched shaft specimens were identical to those used in the previous section for the 2024-
T3 tubular specimen. Again, before local plasticity corrections could be performed using 
the pseudo stress approach, the structural stress-strain curve was first generated based on 
uniaxial Neuber’s rule. The cyclic stress-strain relation \( K' = 1205 \text{ MPa}, n' = 0.122 \) used 
in generating this curve was fit to stabilized strain-controlled uniaxial fatigue data (Yang 
and Fatemi, 1995). The deformation curve parameters corresponding to the structural 
stress-strain curve fit were found to be \( K^* = 7200 \text{ MPa} \) and \( n^* = 0.335 \). The linear elastic, 
cyclic elastic-plastic, and elastic stress-true strain deformation curves are shown in Figure 
4.35, along with the structural stress-strain best fit curve.

Similar to the tubular specimen analyses, the first stress-strain comparisons made 
for the AISI 1141 notched shaft specimens were aimed at evaluating the ability of the 
pseudo stress approach to accurately estimate local equivalent stress. Results of these 
comparisons are shown in Figure 4.36 for both specimens. Individual plots within the 
figure represent different nominal loading conditions and/or specimen geometries. The 
stress analysis paths for each loading condition are only shown in Figure 4.36, but are the 
same for all subsequent stress-strain comparisons.

Overall, equivalent stress predictions for the notched shaft specimens were found 
to be slightly less accurate than those derived for the tubular specimen. For both shaft
specimens, all local von Mises stress values at the notch root were predicted within 5.5% error, but tend to be underpredicted for the higher loading levels of each loading condition. Moving away from the notch root, the maximum error in any equivalent stress prediction is 12.2%, corresponding to the highest pure torsion loading level of the grooved shaft specimen at a distance of 0.5 mm from the notch root. Underpredicted stress values are especially surprising for the shaft specimens, as a local plane strain condition exists at the notch for both specimens. Under these conditions, Neuber’s rule typically produces conservative stress-strain estimates and the ESED rule is generally considered to be more accurate. However, in this case, implementing the ESED rule in the derivation of the structural stress-strain curve would result in even lower stress-strain predictions, and larger errors, than those based on Neuber’s rule.

In general, error in von Mises stress predictions is fairly consistent between the different specimen geometries and loading conditions when compared at the same local equivalent stress magnitude. This is despite the fact that the in-phase axial-torsion nominal loading condition produces a multiaxial local stress state for both notched shaft specimens, while pure torsion nominal loading results in a pure shear local stress state. Similar to the tubular specimen results, a summary of all predicted and FEA derived local stress-strain solutions at all distances from the notch root, along with the percentage error for each prediction, can be found in Tables 4.6 to 4.8.

Next, comparisons were made between pseudo stress-based plasticity corrections and nonlinear FEA results for maximum principal stress and strain. Again, given the popularity of these quantities in fatigue damage analysis, such comparisons are useful when assessing the impact of local stress-strain estimation on life predictions. For
maximum principal stress, these comparisons are shown in Figure 4.37 for both AISI 1141 notched shaft geometries. Similarly, maximum principal strain comparisons are shown in Figure 4.38. While these stress/strain predictions tend to be fairly accurate for the lower applied loading levels, they increasingly deviate from the FEA solutions as the loading level is increased and/or the distance from the notch root is decreased.

Observations for in-phase axial-torsion loading of both AISI 1141 shaft specimen geometries (Figures 4.37(b) and 4.37(d)) are similar to those for all nominal loading conditions of the 2024-T3 tubular specimen (Figure 4.31). For the higher loading levels, maximum principal stresses are significantly overpredicted at the notch root, resulting in errors of up to 16.3% for the shaft specimens. As the distance from the notch root is increased, however, principal stress predictions for both the 2024-T3 specimen and in-phase axial-torsion loading of the shaft specimens tend to cross over the FEA distributions, and all stresses are underpredicted at a distance of 0.5 mm from the notch root. The maximum error in principal stress predictions at 0.5 mm from the notch root is 9.7% for in-phase axial-torsion loading of the AISI 1141 shaft specimens.

For pure shear loading of the notched shaft specimens, however, maximum principal stress predictions do not follow the same trends observed for the other loading nominal conditions. In these cases, maximum principal stresses are consistently underpredicted, regardless of the distance from the notch root. Although maximum principal stress values are much lower under pure torsion nominal loading of these specimens, the maximum prediction error is still 12.2%. Again, possible reasons for this behavior are offered in the following discussion section.
As with the tubular specimen, trends for maximum principal strain predictions are similar, in all cases, to those for maximum principal stress. While predictions errors are very similar between stress and strain distributions for the 2024-T3 tubular specimen, maximum principal strain predictions tend to be more non-conservative than their corresponding stress predictions for the shaft specimens. Maximum prediction errors are fairly consistent between both notched shaft geometries at around 46% for pure torsion loading and 25% for axial-torsion loading.

Finally, comparisons between predicted and nonlinear FEA derived distributions of maximum shear strain are shown in Figure 4.39 for the AISI 1141 notched shaft specimens. By studying these figures, it can be seen that the accuracy of maximum shear strain predictions are qualitatively and quantitatively (in terms of error) very similar to maximum principal strain predictions for all specimens and loading conditions considered. Again, this is expected since maximum shear strain is directly computed from principal strain quantities. The maximum error in maximum shear strain predictions is 46%, for either notched shaft specimen geometry.

4.8.3 Discussion on Notch Stress-Strain Estimation Results

For any given plot presented in the previous two results sections, the shape of the pseudo stress-based stress and strain distributions are very similar between the different loading levels investigated. The shape of the FEA derived stress distributions, however, is observed to change in many cases as the loading level is increased. As a result, it is likely that a significant amount of error in the pseudo stress-based plasticity corrections can be attributed to the redistribution of notch root stresses and strains due to excessive notch root plasticity.
Because the pseudo stress approach is based on linear elastic stress distributions, it is unable to account for the transfer of loading to surrounding material when yielding occurs at the notch root. This would, in turn, lead to the observed over-prediction of principal stresses and strains near the notch root for the more plastically deformed loading states. This means that the minimum and/or mid principal stress/strain values must also be overpredicted in order to limit the equivalent stresses and strains to the levels dictated by the structural stress-strain curve. Then, as the distance from the notch root is increased, the nonlinear stress distribution must eventually produce higher stresses and strains than the corresponding linear distribution in order to satisfy equilibrium conditions across the specimen cross section. This is reflected in the pseudo stress-based predictions as the shift to underpredicted principal stresses and strains as the distance from the notch root is increased.

The exception to this behavior is only observed for pure shear loading of both notched shaft specimens. For this loading condition, maximum principal stresses and strains, along with maximum shear strains, are always underpredicted at the higher loading levels, regardless of the distance from the notch root. In these cases, however, the local stress state is pure shear. Therefore, maximum and minimum principal stress/strain components must always be equal. Because of this, maximum principal stress/strain predictions change proportionally with equivalent stress/strain. Since the values of equivalent stress/strain are limited by the structural stress-strain curve, this does not allow for the over-prediction of maximum principal quantities observed for the other loading conditions.
From the results of these analyses, all significant local stress and strain component predictions were found to fall within a range of ±20% error for the 2024-T3 notched tubular specimen. Although local stress and strain predictions obtained using the pseudo stress approach become increasingly worse as the degree of plasticity at the notch increases, it is worth mentioning that the higher loading levels analyzed are much higher than those which would be expected in typical fatigue loading applications. For this study, the highest nominal stress levels applied in the testing of these specimens was 145 MPa for pure axial, 108 MPa for pure torsion, and 125 MPa axial 72 MPa shear for in-phase axial-torsion loading. At stress levels corresponding to these maximum testing loads, all stress-strain predictions are observed to be within ±7% error. Taking this into consideration, it should be reasonable to expect highly accurate local stress-strain predictions when computing local stress-strain histories for a typical fatigue life analysis.

It should also be noted that although fatigue life analyses are later performed for 90° OP axial-torsion loading of the 2024-T3 specimens, in which case pseudo stress-based plasticity corrections are used to obtain local stress-strain estimations, no nonlinear FEA were performed for this loading condition. This is because, due the nature of the loading, the 90° OP loading cycle can be viewed as a combination of the other three loading conditions investigated. At two points in the cycle, the nominal loading is axial only, at another two points it is torsion only, and in between the loading is combined axial-torsion. Therefore, the accuracy of local plasticity corrections for 90° OP loadings can be indirectly assessed from the comparisons made for the other loading conditions. Although the material stress-strain behavior may change due to non-proportional
hardening, this would only increase the accuracy of the calculations by reducing the amount of plastic deformation predicted.

Unlike the 2024-T3 tubular specimen analyses, all nominal stress levels used in deriving local stress-strain predictions for both notched shaft specimen geometries correspond to actual fatigue testing conditions used for these specimens. While maximum principal and shear strain prediction errors reach up to 46% for these specimen geometries, these errors correspond to loading levels where significant local plasticity is present and fatigue lives are on the order of a few thousand cycles. Additionally, at these higher applied loading levels, maximum equivalent nominal stresses exceed the yield strength of the material, which can lead to errors in local pseudo stress values derived using linear elastic stress concentration factors. Excluding these rather extreme loading cases (corresponding to the highest loading level for each shaft specimen and loading condition), the vast majority of local stress and strain components are predicted within ±15% error. Therefore, reasonably accurate local stress-strain predictions can still be expected for a majority of fatigue loading conditions using the pseudo stress approach.

To summarize, local pseudo stress-based plasticity corrections were evaluated against nonlinear FEA solutions for several different combinations of specimen geometry, loading conditions, and material. On one hand, the 2024-T3 notched tubular specimen contains a blunt notch under plane stress conditions, while on the other, the AISI 1141 notched shaft specimens contain sharp notches under plane strain conditions. Additionally, the uniaxial, pure torsion, and combined axial-torsion loading conditions result in a variety of local stress states at different distances from the notch root for each specimen geometry investigated. Despite these differences, the majority of local stress-
strain predictions were found to be within a reasonable margin of error (±15%) for loading levels relevant to typical fatigue loading conditions. As such, the pseudo stress approach shows much promise for application to the complex multiaxial variable amplitude loading histories which are central to this study. As loading levels increase, however, significant local plasticity is found to result in excessive error in stress-strain predictions, most likely due to the redistribution of elastic-plastic stresses and strains not accounted for in linear pseudo stress distributions.

4.9 Summary and Conclusions

Given the dependence of many fatigue damage parameters on both stress and strain values, being able to produce accurate stress-strain estimations is an essential first step when performing a fatigue life analysis. If not done correctly, errors in this step can continue to propagate and magnify throughout subsequent steps of the analysis. As a result, cyclic plasticity modeling techniques were investigated in this chapter, with and without the incorporation of notch stress-strain estimation rules, in order to evaluate their accuracy when applied to complex multiaxial loading situation involving both un-notched and notched specimens.

Based on the experimental results and analysis presented for material deformation behavior in un-notched specimens, some key findings and conclusions can be summarized as follows:

1) Significant cyclic and non-proportional hardening were observed for the 2024-T3 aluminum alloy tested in this study.
2) von Mises equivalent stress and strain criteria were able to correlate experimental cyclic deformation behavior for axial, torsion, and in-phase axial-torsion loading conditions reasonably well for this material.

3) An Armstrong-Frederick-Chaboche style cyclic plasticity model, with the inclusion of Tanaka’s non-proportionality parameter, was simplified based on the assumption of Masing material behavior and through modification of the transient hardening formulation.

4) All material constants for the plasticity model were determined from the following experimental data: monotonic stress-strain curve, cyclic stress-strain curve, 90° OP equivalent stress-strain curve (can also be estimated from the first two curves mentioned), number of cycles until material stabilization for a single constant amplitude uniaxial and 90° OP fatigue test, and the stress-strain behavior from a single loading history containing cross-hardening events.

5) The maximum error in any stable cycle stress amplitude prediction was just over 16%, but average errors for any given loading history were less than ±2%. The combined average prediction error for all stable cycle stress amplitudes considered (17 axial and 17 shear) was −1.1%, with a standard deviation of 3.7%.

6) The maximum error in any stress amplitude prediction (including transient material behavior) was just over 25%, but average errors for any given loading history were less than ±5%. The combined average error for all stress amplitude predictions considered in the three block loading deformation tests (61 axial and 61 shear) was −0.7%, with a standard deviation of 5.2%.
7) All predictions for axial stress amplitudes (−6.9% max, −1.3% average, and 2.2% standard deviation in error) were consistently better than those for shear stress (25% max, −0.1% average, and 7.0% standard deviation in error).

8) Although some errors in mean shear stress predictions from the variable amplitude deformation test were as large as 58%, with an average error of 16%, mean shear stress typically has a minimal influence on fatigue damage.

9) Axial mean stresses, which can significantly impact fatigue damage calculations, were predicted within ±15% error for all cases (in the variable amplitude deformation test), with an average error of 0.3%.

Similarly, based on the numerical results and analysis presented for local material deformation behavior in notched specimens, some key findings and conclusions can be summarized as follows:

10) The presence of notches in engineering components can influence local deformation behavior by producing a stress concentration effect, altering the local stress state, and/or introducing stress gradients surrounding the notch root. As a result, detailed stress analyses and notch stress-strain estimation rules are often required in order to accurately quantify and predict this behavior under complex loading conditions.

11) For the 2024-T3 tubular specimen geometry used in this study, local stress-strain analysis was first performed using Neuber’s rule. All stabilized local equivalent stress predictions were found to be within ±5% of nonlinear FEA results, while all local equivalent strain predictions fell within an error margin of ±25% under axial, torsion, and combined axial-torsion loading conditions.
12) Despite the popularity of Neuber’s rule and its agreement with nonlinear FEA results for the local stress-strain predictions performed in this study, notch estimation rules which correct for plasticity based on equivalent stress range generally cannot be applied to non-proportionally varying stress histories without the use of plasticity modeling techniques.

13) Using a pseudo stress-based plasticity modeling technique, all stabilized local stress (von Mises and maximum principal) and strain (maximum principal and maximum shear) predictions were found to fall within $\pm 20\%$ error for the 2024-T3 notched tubular specimen geometry. Furthermore, at nominal stress levels representative of the maximum fatigue testing loads applied in this study, all local stress-strain predictions were found to be within $\pm 7\%$ error.

14) While maximum errors in local stress-strain predictions were as large as 46% for the AISI 1141 notched shaft specimen geometries investigated, these errors correspond to loading conditions where significant local plasticity was present and fatigue lives were on the order of a few thousand cycles.

15) Excluding the more extreme loading cases, the majority of local stress-strain predictions derived using the pseudo stress-based plasticity modeling technique were found to be within $\pm 15\%$ error for loading levels relevant to typical fatigue loading applications, regardless of specimen geometry and/or local stress state.

16) At the higher loading levels investigated, significant local plasticity was found to result in excessive errors in local stress-strain predictions. This is likely due to the redistribution of elastic-plastic stresses and strains, which is not accounted for in linear pseudo stress distributions. In these cases, local stress-strain values were
overpredicted at the notch root and eventually underpredicted as the distance from the notch was increased.

- **Looking forward:**

  For each un-notched specimen deformation test performed in this study, the cyclic plasticity model formulation presented in this chapter was shown to predict the experimentally observed stress-strain behavior within a reasonable margin of error. This was true even for complex loading histories where cyclic hardening, varying degrees of non-proportional hardening, and/or variable amplitude multiaxial loading conditions were present. Therefore, it can be concluded that the plasticity model should be capable of predicting material constitutive behavior with a fairly high level of accuracy for even the most complex loading histories. This should be especially true when considering the primarily elastic and axial-dominated variable amplitude fatigue loading histories subsequently analyzed in this study.

  Furthermore, while some stress-strain prediction errors were found to be conservative (overpredictions) and some were non-conservative (underpredictions), average prediction errors had the tendency to be slightly non-conservative, regardless of the loading history and/or degree of hardening in the material. Therefore, any errors in un-notched specimen fatigue life predictions resulting from stress-strain histories derived using this model would also be expected to have a slightly non-conservative tendency.

  Similar to the deformation analysis results for un-notched specimens, the pseudo stress-based plasticity modeling technique was shown to produce fairly accurate local stress-strain predictions for the various combinations of material, specimen/notch geometry, local stress state, and analysis location investigated. This was especially true at
the lower applied loading levels considered. Therefore, these results suggest that this method should also be capable of producing accurate local stress-strain histories for the complex variable amplitude loading conditions applied in the notched specimen fatigue tests performed in this study.

At the notch root, all local stress-strain predictions for the 2024-T3 notched tubular specimen geometry had the tendency to be conservative. While prediction errors increased with loading level, the average error for all stress-strain predictions at the notch root for this specimen was 7.0%, with a standard deviation of 5.2%. Conversely, predictions away from the notch had the tendency to be slightly non-conservative. The average error for all stress-strain predictions at a distance of 0.2 mm from the notch root was −1.9%, with a standard deviation of 3.0%. Therefore, any errors in notched specimen fatigue life predictions resulting from local stress-strain histories derived using this technique would also be expected to have a slightly conservative tendency at the notch root, and a slightly non-conservative tendency away from the notch root. The accuracy of stress-strain predictions both at, and away from, the notch root is an important consideration with respect to the subsequent modeling of stress gradient effects.
Table 4.1 Summary of experimentally determined monotonic and cyclic material properties for 2024-T3 aluminum alloy test material.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Monotonic Properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2% Yield Strength</td>
<td>(\sigma_y)</td>
<td>330</td>
<td>MPa</td>
</tr>
<tr>
<td>Ultimate Tensile Strength</td>
<td>(\sigma_u)</td>
<td>495</td>
<td>MPa</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>(E)</td>
<td>73.7</td>
<td>GPa</td>
</tr>
<tr>
<td>Shear Modulus</td>
<td>(G)</td>
<td>27.4</td>
<td>GPa</td>
</tr>
<tr>
<td>Elastic Poisson’s Ratio</td>
<td>(\nu)</td>
<td>0.343</td>
<td></td>
</tr>
<tr>
<td>Elongation at Fracture</td>
<td>%EL</td>
<td>19.5</td>
<td>%</td>
</tr>
<tr>
<td>Monotonic Strength Coefficient*</td>
<td>(K)</td>
<td>470</td>
<td>MPa</td>
</tr>
<tr>
<td>Monotonic Strain Hardening Exponent*</td>
<td>(n)</td>
<td>0.055</td>
<td></td>
</tr>
<tr>
<td><strong>Cyclic Properties</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0.2% Cyclic Axial Yield Strength</td>
<td>(\sigma_y')</td>
<td>415</td>
<td>MPa</td>
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<tr>
<td>Cyclic Axial Strength Coefficient</td>
<td>(K')</td>
<td>677.0</td>
<td>MPa</td>
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<tr>
<td>Cyclic Axial Strain Hardening Exponent</td>
<td>(n')</td>
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<td></td>
</tr>
<tr>
<td>Cyclic Shear Strength Coefficient</td>
<td>(K_\theta')</td>
<td>382.3</td>
<td>MPa</td>
</tr>
<tr>
<td>Cyclic Shear Strain Hardening Exponent</td>
<td>(n_\theta')</td>
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</tr>
<tr>
<td>Cyclic In-Phase Strength Coefficient</td>
<td>(K_{IP}')</td>
<td>688.6</td>
<td>MPa</td>
</tr>
<tr>
<td>Cyclic In-Phase Strain Hardening Exponent</td>
<td>(n_{IP}')</td>
<td>0.081</td>
<td></td>
</tr>
<tr>
<td>Cyclic 90° OP Strength Coefficient</td>
<td>(K_{NP}')</td>
<td>938.0</td>
<td>MPa</td>
</tr>
<tr>
<td>Cyclic 90° OP Strain Hardening Exponent</td>
<td>(n_{NP}')</td>
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<tr>
<td>Non-Proportional Cyclic Hardening Coefficient</td>
<td>(\alpha_{NP})</td>
<td>0.19</td>
<td></td>
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<tr>
<td>Structural Cyclic Strength Coefficient</td>
<td>(K^*)</td>
<td>3400</td>
<td>MPa</td>
</tr>
<tr>
<td>Structural Cyclic Strain Hardening Exponent</td>
<td>(n^*)</td>
<td>0.290</td>
<td></td>
</tr>
</tbody>
</table>

*Properties valid up to approximately 2.5% strain
Table 4.2  Summary of 2024-T3 aluminum alloy material constants in Armstrong-Frederick-Chaboche style cyclic plasticity model.

<table>
<thead>
<tr>
<th></th>
<th>$E$ (GPa)</th>
<th>$v$</th>
<th>$\sigma_y$ (MPa)</th>
<th>$m$</th>
<th>$C_c$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>73.7</td>
<td>0.343</td>
<td>283</td>
<td>12</td>
<td>200</td>
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</tbody>
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<table>
<thead>
<tr>
<th>(i)</th>
<th>$c^0$</th>
<th>$r_s^0$ (MPa)</th>
<th>$R_d^0$ (MPa)</th>
<th>$R_y^0$ (MPa)</th>
<th>$h^0$</th>
<th>$\chi^0$</th>
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<tbody>
<tr>
<td>1</td>
<td>2749.7</td>
<td>6.819</td>
<td>47.259</td>
<td>84.875</td>
<td>22.761</td>
<td>5.0</td>
</tr>
<tr>
<td>2</td>
<td>874.3</td>
<td>14.330</td>
<td>24.961</td>
<td>38.035</td>
<td>6.309</td>
<td>5.0</td>
</tr>
<tr>
<td>3</td>
<td>355.0</td>
<td>12.405</td>
<td>22.158</td>
<td>34.637</td>
<td>6.553</td>
<td>5.0</td>
</tr>
<tr>
<td>4</td>
<td>156.1</td>
<td>9.760</td>
<td>22.010</td>
<td>35.174</td>
<td>6.309</td>
<td>5.0</td>
</tr>
<tr>
<td>5</td>
<td>70.9</td>
<td>4.295</td>
<td>22.731</td>
<td>37.094</td>
<td>6.309</td>
<td>5.0</td>
</tr>
<tr>
<td>6</td>
<td>32.7</td>
<td>52.371</td>
<td>46.973</td>
<td>79.814</td>
<td>20.237</td>
<td>5.0</td>
</tr>
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</table>

Table 4.3  Summary of experimental vs. predicted steady-state stress amplitudes for Block A constant amplitude deformation test. No NPH refers to predictions which do not consider non-proportional hardening.

<table>
<thead>
<tr>
<th>Path/Cycle</th>
<th>Axial Stress Amp (MPa)</th>
<th>Shear Stress Amp (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exp.</td>
<td>Pred.</td>
</tr>
<tr>
<td>A/2</td>
<td>463.8</td>
<td>439.3</td>
</tr>
<tr>
<td>A/18</td>
<td>459.1</td>
<td>439.9</td>
</tr>
<tr>
<td>IP/2</td>
<td>331.2</td>
<td>311.4</td>
</tr>
<tr>
<td>IP/18</td>
<td>327.4</td>
<td>310.5</td>
</tr>
<tr>
<td>180° OP/2</td>
<td>330.6</td>
<td>311.9</td>
</tr>
<tr>
<td>180° OP/18</td>
<td>326.2</td>
<td>310.7</td>
</tr>
<tr>
<td>T/2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>T/18</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

220
Table 4.4  Summary of experimental vs. predicted stress amplitudes for Block B constant amplitude deformation test (shown in Figure 4.8). No NPH refers to predictions which do not consider non-proportional hardening.

<table>
<thead>
<tr>
<th>Block</th>
<th>Path</th>
<th>Axial Stress Amp (MPa)</th>
<th>Shear Stress Amp (MPa)</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>No NPH</td>
<td>Tanaka</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Exp.</td>
<td>Pred.</td>
</tr>
<tr>
<td>Block 2</td>
<td>A</td>
<td>391.5</td>
<td>390.9</td>
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<tr>
<td></td>
<td>IP</td>
<td>288.1</td>
<td>286.7</td>
</tr>
<tr>
<td></td>
<td>180° OP</td>
<td>285.3</td>
<td>289.9</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>90° OP</td>
<td>405.1</td>
<td>377.0</td>
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<td>Block 4</td>
<td>A</td>
<td>423.2</td>
<td>413.1</td>
</tr>
<tr>
<td></td>
<td>IP</td>
<td>309.5</td>
<td>301.3</td>
</tr>
<tr>
<td></td>
<td>180° OP</td>
<td>302.7</td>
<td>303.3</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>90° OP</td>
<td>427.2</td>
<td>402.0</td>
</tr>
<tr>
<td>Block 8</td>
<td>A</td>
<td>441.7</td>
<td>442.5</td>
</tr>
<tr>
<td></td>
<td>IP</td>
<td>321.2</td>
<td>319.5</td>
</tr>
<tr>
<td></td>
<td>180° OP</td>
<td>313.3</td>
<td>320.1</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>90° OP</td>
<td>442.0</td>
<td>435.9</td>
</tr>
<tr>
<td>Block 16</td>
<td>A</td>
<td>451.4</td>
<td>457.0</td>
</tr>
<tr>
<td></td>
<td>IP</td>
<td>327.5</td>
<td>327.1</td>
</tr>
<tr>
<td></td>
<td>180° OP</td>
<td>319.2</td>
<td>327.6</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>90° OP</td>
<td>450.4</td>
<td>448.6</td>
</tr>
<tr>
<td>Block 32</td>
<td>A</td>
<td>457.6</td>
<td>457.2</td>
</tr>
<tr>
<td></td>
<td>IP</td>
<td>331.6</td>
<td>327.1</td>
</tr>
<tr>
<td></td>
<td>180° OP</td>
<td>322.7</td>
<td>327.8</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>90° OP</td>
<td>456.7</td>
<td>448.7</td>
</tr>
<tr>
<td>Block 50</td>
<td>A</td>
<td>461.0</td>
<td>457.2</td>
</tr>
<tr>
<td></td>
<td>IP</td>
<td>333.4</td>
<td>327.2</td>
</tr>
<tr>
<td></td>
<td>180° OP</td>
<td>324.5</td>
<td>327.8</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>90° OP</td>
<td>460.7</td>
<td>448.7</td>
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<td>Block 100 (stable)</td>
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<td>464.9</td>
<td>444.6</td>
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<tr>
<td></td>
<td>IP</td>
<td>335.9</td>
<td>319.2</td>
</tr>
<tr>
<td></td>
<td>180° OP</td>
<td>327.2</td>
<td>321.1</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>90° OP</td>
<td>464.1</td>
<td>432.5</td>
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Table 4.5  Summary of experimental vs. predicted stress amplitudes and mean stresses for variable amplitude deformation test (shown in Figure 4.9).

<table>
<thead>
<tr>
<th>Block 2</th>
<th>Axial Stress (MPa)</th>
<th>Shear Stress (MPa)</th>
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<tbody>
<tr>
<td>R1</td>
<td>225.6</td>
<td>232.2</td>
</tr>
<tr>
<td>R2</td>
<td>195.4</td>
<td>193.5</td>
</tr>
<tr>
<td>R3</td>
<td>122.6</td>
<td>123.1</td>
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<tr>
<td>R4</td>
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<td>333.5</td>
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<td>R5</td>
<td>368.4</td>
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</thead>
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<tr>
<td>R1</td>
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<td>232.2</td>
</tr>
<tr>
<td>R2</td>
<td>197.3</td>
<td>193.5</td>
</tr>
<tr>
<td>R3</td>
<td>130.3</td>
<td>126.4</td>
</tr>
<tr>
<td>R4</td>
<td>355.7</td>
<td>342.2</td>
</tr>
<tr>
<td>R5</td>
<td>373.7</td>
<td>360.2</td>
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<table>
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<td>R2</td>
<td>199.4</td>
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<td>R5</td>
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<table>
<thead>
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<td>232.2</td>
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<tr>
<td>R2</td>
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<tr>
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<td>359.8</td>
</tr>
<tr>
<td>R5</td>
<td>380.5</td>
<td>372.9</td>
</tr>
</tbody>
</table>

* % Error for mean stress calculated as (Pred. Mean – Exp. Mean)*100 / (Exp. Amp)
Table 4.6 Comparison of pseudo stress-based local stress-strain predictions and FEA solutions at the notch root.

<table>
<thead>
<tr>
<th>Loading Path</th>
<th>Nominal Loading</th>
<th>Local Stresses (MPa)</th>
<th>Local Strains (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_n$ (MPa)</td>
<td>$\tau_n$ (MPa)</td>
<td>$\sigma_l$ FEA</td>
</tr>
<tr>
<td><strong>2024-T3 Notched Tubular Specimen</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial</td>
<td>150</td>
<td>0</td>
<td>407</td>
</tr>
<tr>
<td>Axial</td>
<td>200</td>
<td>0</td>
<td>455</td>
</tr>
<tr>
<td>Axial</td>
<td>250</td>
<td>0</td>
<td>484</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>100</td>
<td>364</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>125</td>
<td>414</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>150</td>
<td>444</td>
</tr>
<tr>
<td>In-Phase</td>
<td>125</td>
<td>72</td>
<td>427</td>
</tr>
<tr>
<td>In-Phase</td>
<td>200</td>
<td>115</td>
<td>493</td>
</tr>
<tr>
<td><strong>AISI 1141 Stepped Shaft Specimen</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>344</td>
<td>696</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>294</td>
<td>625</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>240</td>
<td>553</td>
</tr>
<tr>
<td>In-Phase</td>
<td>260</td>
<td>274</td>
<td>653</td>
</tr>
<tr>
<td>In-Phase</td>
<td>201</td>
<td>214</td>
<td>573</td>
</tr>
<tr>
<td>In-Phase</td>
<td>140</td>
<td>154</td>
<td>488</td>
</tr>
<tr>
<td><strong>AISI 1141 Grooved Shaft Specimen</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>344</td>
<td>694</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>294</td>
<td>620</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>240</td>
<td>543</td>
</tr>
<tr>
<td>In-Phase</td>
<td>260</td>
<td>274</td>
<td>646</td>
</tr>
<tr>
<td>In-Phase</td>
<td>201</td>
<td>214</td>
<td>564</td>
</tr>
<tr>
<td>In-Phase</td>
<td>140</td>
<td>154</td>
<td>475</td>
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</table>
Table 4.7  Comparison of pseudo stress-based local stress-strain predictions and FEA solutions at a distance of approximately 0.200 mm from the notch root.

<table>
<thead>
<tr>
<th>Loading Path</th>
<th>Nominal Loading</th>
<th>Local Stresses (MPa)</th>
<th>Local Strains (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_p$</td>
<td>$\tau_p$</td>
<td>$\sigma_q$ FEA</td>
</tr>
<tr>
<td>2024-T3 Notched Tubular Specimen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial</td>
<td>150</td>
<td>0</td>
<td>326</td>
</tr>
<tr>
<td>Axial</td>
<td>200</td>
<td>0</td>
<td>407</td>
</tr>
<tr>
<td>Axial</td>
<td>250</td>
<td>0</td>
<td>451</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>100</td>
<td>238</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>125</td>
<td>305</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>150</td>
<td>361</td>
</tr>
<tr>
<td>In-Phase</td>
<td>125</td>
<td>72</td>
<td>347</td>
</tr>
<tr>
<td>In-Phase</td>
<td>200</td>
<td>115</td>
<td>461</td>
</tr>
<tr>
<td>AISI 1141 Stepped Shaft Specimen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>344</td>
<td>655</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>294</td>
<td>580</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>240</td>
<td>494</td>
</tr>
<tr>
<td>In-Phase</td>
<td>260</td>
<td>274</td>
<td>590</td>
</tr>
<tr>
<td>In-Phase</td>
<td>201</td>
<td>214</td>
<td>491</td>
</tr>
<tr>
<td>In-Phase</td>
<td>140</td>
<td>154</td>
<td>382</td>
</tr>
<tr>
<td>AISI 1141 Grooved Shaft Specimen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>344</td>
<td>669</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>294</td>
<td>591</td>
</tr>
<tr>
<td>Torsion</td>
<td>0</td>
<td>240</td>
<td>505</td>
</tr>
<tr>
<td>In-Phase</td>
<td>260</td>
<td>274</td>
<td>608</td>
</tr>
<tr>
<td>In-Phase</td>
<td>201</td>
<td>214</td>
<td>512</td>
</tr>
<tr>
<td>In-Phase</td>
<td>140</td>
<td>154</td>
<td>404</td>
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Table 4.8  Comparison of pseudo stress-based local stress-strain predictions and FEA solutions at a distance of 0.500 mm from the notch root.

<table>
<thead>
<tr>
<th>Loading Path</th>
<th>Nominal Loading</th>
<th>Local Stresses (MPa)</th>
<th>Local Strains (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma_0 )</td>
<td>( \tau_0 )</td>
<td>( \sigma_q ) FEA</td>
</tr>
<tr>
<td>2024-T3 Notched Tubular Specimen</td>
<td>Axial</td>
<td>150</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Axial</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Axial</td>
<td>250</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Torsion</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Torsion</td>
<td>0</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td>Torsion</td>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>In-Phase</td>
<td>125</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>In-Phase</td>
<td>200</td>
<td>115</td>
</tr>
<tr>
<td>AISI 1141 Stepped Shaft Specimen</td>
<td>Torsion</td>
<td>0</td>
<td>344</td>
</tr>
<tr>
<td></td>
<td>Torsion</td>
<td>0</td>
<td>294</td>
</tr>
<tr>
<td></td>
<td>Torsion</td>
<td>0</td>
<td>240</td>
</tr>
<tr>
<td></td>
<td>In-Phase</td>
<td>260</td>
<td>274</td>
</tr>
<tr>
<td></td>
<td>In-Phase</td>
<td>201</td>
<td>214</td>
</tr>
<tr>
<td></td>
<td>In-Phase</td>
<td>140</td>
<td>154</td>
</tr>
<tr>
<td>AISI 1141 Grooved Shaft Specimen</td>
<td>Torsion</td>
<td>0</td>
<td>344</td>
</tr>
<tr>
<td></td>
<td>Torsion</td>
<td>0</td>
<td>294</td>
</tr>
<tr>
<td></td>
<td>Torsion</td>
<td>0</td>
<td>240</td>
</tr>
<tr>
<td></td>
<td>In-Phase</td>
<td>260</td>
<td>274</td>
</tr>
<tr>
<td></td>
<td>In-Phase</td>
<td>201</td>
<td>214</td>
</tr>
<tr>
<td></td>
<td>In-Phase</td>
<td>140</td>
<td>154</td>
</tr>
</tbody>
</table>
Table 4.9  Summary of material properties for AISI 1141 steel alloy.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Monotonic Properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2% Yield Strength</td>
<td>$\sigma_y$</td>
<td>524</td>
<td>MPa</td>
</tr>
<tr>
<td>Ultimate Tensile Strength</td>
<td>$\sigma_u$</td>
<td>875</td>
<td>MPa</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>$E$</td>
<td>200.0</td>
<td>GPa</td>
</tr>
<tr>
<td>Shear Modulus</td>
<td>$G$</td>
<td>80.2</td>
<td>GPa</td>
</tr>
<tr>
<td>Elastic Poisson’s Ratio (calculated)</td>
<td>$\nu$</td>
<td>0.290</td>
<td></td>
</tr>
<tr>
<td>Elongation at Fracture</td>
<td>$%EL$</td>
<td>26.1</td>
<td>%</td>
</tr>
<tr>
<td><strong>Cyclic Properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2% Cyclic Axial Yield Strength</td>
<td>$\sigma_y'$</td>
<td>564</td>
<td>MPa</td>
</tr>
<tr>
<td>Cyclic Axial Strength Coefficient</td>
<td>$K'$</td>
<td>1205.0</td>
<td>MPa</td>
</tr>
<tr>
<td>Cyclic Axial Strain Hardening Exponent</td>
<td>$n'$</td>
<td>0.122</td>
<td></td>
</tr>
<tr>
<td>Structural Cyclic Strength Coefficient</td>
<td>$K^*$</td>
<td>7200</td>
<td>MPa</td>
</tr>
<tr>
<td>Structural Cyclic Strain Hardening Exponent</td>
<td>$n^*$</td>
<td>0.335</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.1  Monotonic stress-strain curve for 2024-T3 aluminum alloy.
Figure 4.2  Stable cycle hysteresis loops for uniaxial cyclic loading of 2024-T3 aluminum alloy.

Figure 4.3  Stable cycle hysteresis loops for pure shear cyclic loading of 2024-T3 aluminum alloy.
Figure 4.4  Stable cycle (a) axial and (b) shear hysteresis loops for in-phase axial-torsion cyclic loading of 2024-T3 aluminum alloy.
Figure 4.5  Stable cycle (a) axial and (b) shear hysteresis loops for 90° out-of-phase axial-torsion cyclic loading of 2024-T3 aluminum alloy.
Figure 4.6  Stable cycle axial vs. shear (a) strain (controlled) and (b) stress paths for 90° out-of-phase axial-torsion cyclic deformation test. Shear values are presented in terms of von Mises equivalent stress/strain at the specimen mid-thickness.
Figure 4.7  Monotonic and cyclic stress-strain curves generated through material deformation tests.
Figure 4.8  Constant amplitude block loading histories in terms of (a) strain paths and sequence and (b) load vs. time history (shown for Block B only).
Figure 4.9  Variable amplitude simulated service loading history for deformation testing in terms of (a) strain path and (b) load vs. time history. Vertical dashed lines and numbers in parentheses in (b) indicate the starting points of reversals used for comparisons between predicted and experimental response.
Figure 4.10  Comparison of stabilized cyclic axial hysteresis loops to cyclic stress-strain curve.
Figure 4.11 Comparison of experimental and predicted plastic strain amplitude variation vs. number of applied loading cycles.

Figure 4.12 Comparison of experimental cyclic in-phase axial-torsion stress-strain curve (in range form) to predicted stable stress-strain values from plasticity model.
Figure 4.13 Comparison of experimental and predicted strain amplitude variation vs. number of applied loading cycles for (a) uniaxial, (b) pure torsion, and (c) in-phase axial-torsion incremental step tests.
Figure 4.14 Comparisons of (a) experimental and (b) predicted stress paths, (c) experimental and (d) predicted axial hysteresis loops, and (e) experimental and (f) predicted shear hysteresis loops for block 100 of the Block B constant amplitude deformation test.
Figure 4.15  Comparison of experimental and predicted steady-state stress amplitude vs. cycle for the Block A constant amplitude deformation test.
Figure 4.16  Comparison of experimental and predicted (a) axial stress amplitude and (b) shear stress amplitude vs. number of applied loading blocks for the Block B constant amplitude deformation test.
Figure 4.17  Comparison of experimental and predicted (a) axial stress amplitude and (b) shear stress amplitude vs. number of applied loading blocks for the variable amplitude simulated service loading deformation test.
Figure 4.18 Comparison of (a) experimental and (b) predicted stress paths for loading block 2, and (c) experimental and (d) predicted stress paths for loading block 30 of the variable amplitude service loading deformation test.
Figure 4.19 Comparison of (a) experimental and (b) predicted axial hysteresis loops, and (c) experimental and (d) predicted shear hysteresis loops for loading block 30 of the variable amplitude service loading deformation test.
Figure 4.20  Comparison of experimental and predicted (a) axial mean stress and (b) mean shear stress vs. number of applied loading blocks for the variable amplitude simulated service loading deformation test.
Figure 4.21  Finite element mesh used for modeling of local stress-strain behavior in notched tubular specimens.
Figure 4.22 Elastic FEA results for distribution of von Mises equivalent stress, maximum principal stress, and maximum shear stress around the hole circumference on the specimen outer surface under (a) pure axial loading and (b) pure torsion loading.
Figure 4.23  Elastic FEA results for distribution of non-zero stress components along hole depth at location of maximum stresses for (a) pure axial loading and (b) pure torsion loading.
Figure 4.24  Elastic FEA results for distribution of non-zero stress components radially away from hole edge on specimen outer surface at location of maximum stresses for (a) pure axial loading and (b) pure torsion loading. Distance is normalized by hole radius, $r$. 
Figure 4.25  Elastic FEA results for (a) distribution of von Mises equivalent stress around the hole circumference on specimen outer surface, (b) location of maximum stresses and, (c) equivalent stress concentration factor vs. nominal axial to shear stress ratio. Stresses are normalized by nominally applied von Mises stress values.
Figure 4.26 Neuber’s rule estimations compared to elastic–plastic FEA results for nominal vs. local equivalent stress for (a) axial loading, (c) torsion loading, and (e) combined axial-torsion loading along with nominal stress vs. local equivalent strain for (b) axial loading, (d) torsion loading, and (f) combined axial-torsion loading.
Figure 4.27  Elastic–plastic stress and strain concentration factors versus nominal stress, as computed from Neuber’s rule, compared to FEA results for (a) axial loading and (b) torsion loading.
Figure 4.28 Local von Mises equivalent elastic–plastic (a) stress and (b) strain predictions, as computed from Neuber’s rule, compared to FEA results for axial, torsion, and combined axial-torsion loadings at several nominal stress levels. Error bands of ±5% and ±15% are shown around the perfect prediction line ($x = y$).
Figure 4.29  Pseudo stress approach stress-strain curves and fits for 2024-T3 aluminum alloy.
Figure 4.30 Comparison of von Mises stress solutions from nonlinear FEA and pseudo stress-based plasticity corrections for various nominal stress levels of (a) pure axial, (b) pure torsion, and (c) in-phase axial-torsion loading of notched tubular specimens of 2024-T3 aluminum alloy. The stress analysis path is also shown for each nominal loading condition.
Figure 4.31 Comparison of maximum principal stress solutions from nonlinear FEA and pseudo stress-based plasticity corrections for various nominal stress levels of (a) pure axial, (b) pure torsion, and (c) in-phase axial-torsion loading of notched tubular specimens of 2024-T3 aluminum alloy.
Figure 4.32  Comparison of maximum principal strain solutions from nonlinear FEA and pseudo stress-based plasticity corrections for various nominal stress levels of (a) pure axial, (b) pure torsion, and (c) in-phase axial-torsion loading of notched tubular specimens of 2024-T3 aluminum alloy.
Figure 4.33 Comparison of maximum shear strain solutions from nonlinear FEA and pseudo stress-based plasticity corrections for various nominal stress levels of (a) pure axial, (b) pure torsion, and (c) in-phase axial-torsion loading of notched tubular specimens of 2024-T3 aluminum alloy.
Specimen geometry and dimensions for (a) AISI 1141 steel stepped shaft specimen, and (b) AISI 1141 steel grooved shaft specimen. All dimensions are given in mm.

Pseudo stress approach stress-strain curves and fits for AISI 1141 steel alloy.

Figure 4.34

Figure 4.35
Figure 4.36 Comparison of von Mises stress solutions from nonlinear FEA and pseudo stress-based plasticity corrections for various nominal stress levels of (a) pure torsion and (b) in-phase axial-torsion loading of stepped shaft specimens, and (c) pure torsion and (d) in-phase axial-torsion loading of grooved shaft specimens of AISI 1141 steel. The stress analysis path is also shown for each specimen geometry.
Figure 4.37 Comparison of maximum principal stress solutions from nonlinear FEA and pseudo stress-based plasticity corrections for various nominal stress levels of (a) pure torsion and (b) in-phase axial-torsion loading of stepped shaft specimens, and (c) pure torsion and (d) in-phase axial-torsion loading of grooved shaft specimens of AISI 1141 steel.
Figure 4.38  Comparison of maximum principal strain solutions from nonlinear FEA and pseudo stress-based plasticity corrections for various nominal stress levels of (a) pure torsion and (b) in-phase axial-torsion loading of stepped shaft specimens, and (c) pure torsion and (d) in-phase axial-torsion loading of grooved shaft specimens of AISI 1141 steel.
Figure 4.39  Comparison of maximum shear strain solutions from nonlinear FEA and pseudo stress-based plasticity corrections for various nominal stress levels of (a) pure torsion and (b) in-phase axial-torsion loading of stepped shaft specimens, and (c) pure torsion and (d) in-phase axial-torsion loading of grooved shaft specimens of AISI 1141 steel.
Chapter 5

Constant Amplitude Fatigue Behavior and Life Predictions

As mentioned in Section 3.5.2, constant amplitude fatigue testing, the results from which are presented in Section 5.1, was a large part of this study for a number of reasons. To begin, un-notched specimen fatigue tests were performed under pure axial and pure torsion loading conditions in order to generate the baseline fatigue life curves, as detailed in Section 5.2, from which all subsequent fatigue life predictions were calculated. However, tests were also performed using in-phase, 90° out-of-phase (OP), and a limited number of more specialized axial-torsion load paths. Since relating the variation of stresses and strains to the fatigue damage that occurs within a material is the most fundamental part of any fatigue life analysis, these tests were useful in studying the effects of various stress states and loading component interactions on the resulting fatigue damage. Additionally, all of the un-notched specimen tests combined allowed for the evaluation of various multiaxial fatigue damage parameters in terms of their ability to adequately correlate the experimental data. Results for un-notched specimen life predictions are presented in Section 5.3.1 based on equivalent stress/strain approaches, and in Sections 5.4.1 and 5.4.2 based on several different critical plane damage parameters.
In addition to the un-notched specimen constant amplitude fatigue tests, tests were performed under similar nominal loading conditions for notched specimens as well. With respect to crack initiation, the primary goal of these tests was to study notch effects on fatigue life and damage calculation. As such, the experimental data from these tests allowed for the evaluation of notch stress-strain estimation models, stress gradient models, and local life prediction techniques under relatively simple uniaxial and multiaxial loading conditions. Results for the notched specimen fatigue life analyses are presented in Section 5.3.2 based on equivalent stress/strain approaches, and in Section 5.4.3 based on critical plane approaches. Various aspects of the work presented in this chapter, for both notched and un-notched specimens, were published in several journal papers as well, i.e. (Gates et al., 2014; Gates and Fatemi, 2016c, 2016d, 2015, 2014).

Finally, in order to evaluate any effects from the curvature of the tubular specimen geometry on crack initiation and/or growth in notched specimens, the testing program was supplemented by the addition of a limited number of uniaxial fatigue tests utilizing a notched plate specimen geometry. The details of these tests, including experimental results and comparisons with tubular specimen fatigue life predictions, are presented towards the end of this chapter in Section 5.5.

5.1 Constant Amplitude Fatigue Test Summary

All tubular specimen constant amplitude fatigue tests were performed under fully-reversed ($R = -1$) loading conditions. Additionally, with the exception of a few un-notched specimen axial and torsion tests at shorter lives, all fatigue tests were performed
in load-control. All loads were applied in the form of sinusoidal waveforms at cyclic frequencies ranging from 0.1 – 7.0 Hz.

As mentioned previously, because of the nonlinear distribution of shear stress across the thickness of the specimen under elastic-plastic loading conditions, nominal shear stresses could not be computed using traditional elastic equations. Therefore, nominal shear stresses, \( S_T \), were calculated for all specimens by the following procedure. First, an average shear stress at the mid-thickness of the specimen was calculated based on equilibrium conditions (Equation (4.1)). Once mid-thickness shear stress was computed, this value was converted to mid-thickness shear strain using the Ramberg-Osgood relation for cyclic shear, Equation (2.5), along with the shear deformation properties given in Table 4.1. Since the distribution of shear strain is linear across the specimen thickness regardless of the amount of plasticity present, outer surface shear strain was extrapolated using the ratio of outer specimen radius to mean radius. From this new value of shear strain, outer surface shear stress was then calculated using the same cyclic stress-strain relation. The resulting elastic-plastic outer surface shear stress was then taken to be the nominal shear stress value. For the limited number of strain-controlled tests performed, nominally applied shear strain values at the specimen outer surface were directly converted to nominal shear stresses using the cyclic shear stress-strain relation. When nominal stresses were elastic, this procedure produced nearly identical values (less than 0.5% difference) to those computed based on elastic shear stress assumptions.

For notched specimens, all nominal stresses were computed in the same manner based on gross cross sectional area. For axial-torsion loading conditions where maximum
axial and shear stresses were applied simultaneously (e.g. in-phase), however, this procedure for calculating nominal shear stress is not correct from a theoretical standpoint. This is because the combined effect from both the axial and shear stress components is not accounted for in the consideration of material yielding behavior. Given the fact that nominally applied equivalent stress amplitudes were near or below the material yield strength in all but one of these tests, however, the same procedure was used for these loading conditions with the assumption that any errors in computed shear stress amplitude would be small. It should be noted that the largest error that can exist in these nominal shear stress calculations is equal to the difference between the mid-thickness equilibrium shear stress and the outer surface shear stress calculated based on elastic material assumptions. For a tubular specimen geometry, this is equal to the percentage difference between the mean gage section radius and the outer surface radius, or around 5.5% in the case of this study.

Although crack growth was monitored for both un-notched and notched specimen tests using the techniques outlined in Section 3.4, crack initiation on the inner surface of some un-notched specimens made judging crack initiation life based on crack length difficult. Therefore, for consistency, the definition of crack initiation for all un-notched specimens was considered to be a 3% change in displacement or rotation amplitude when compared to a stable reference cycle. This generally corresponded to final crack lengths of approximately 10–15 mm, with growth from 1 mm to final length occurring fairly rapidly.

For notched specimens, on the other hand, the crack initiation definition was based on the crack transition length found from a Kitagawa–Takahashi diagram
(discussed in Section 2.8.6). This transition represents the length at which fatigue
damage/failure switches from being controlled by the fatigue limit stress to being
controlled by the threshold stress intensity factor for the material. A crack length of 0.2
mm was calculated based on material properties reported in literature. Therefore, the
fatigue life corresponding to the time when the first individual crack growing from the
notch root reached a length of 0.2 mm, as observed on the specimen outer surface, was
considered to be the crack initiation life for notched specimens.

The first constant amplitude fatigue tests performed were for un-notched
specimens subjected to pure axial and pure torsion loading conditions, and were primarily
aimed at establishing the baseline fatigue life curves from which all subsequent fatigue
life predictions would be based. A summary of these tests, including all relevant testing
parameters and the resulting fatigue lives, is given in Table 5.1 for pure axial loading and
in Table 5.2 for pure torsion loading. Notched specimens fatigue tests were also
performed under the same loading conditions, the results of which are included in each
respective summary table.

These tables list, when applicable: Specimen ID, average gage section inside
diameter, \( d_i \), average gage section outside diameter, \( d_o \), testing frequency, applied load
amplitude, \( P_a \), applied mean load, \( P_m \), applied torque amplitude, \( T_a \), applied mean torque,
\( T_m \), nominal axial stress amplitude, \( S_{A,a} \), mean nominal axial stress, \( S_{A,m} \), nominal shear
stress amplitude, \( S_{T,a} \), mean nominal shear stress, \( S_{T,m} \), and fatigue life to crack lengths of
0.2 mm, 1.0 mm, and final failure. Fatigue lives are reported in terms of cycles to failure.
For notched specimens, crack lengths of 0.2 mm and 1.0 mm refer to the overall length of
the longest individual crack, measured from the tip of that crack to the edge of the hole,
while final failure corresponds to a 15 mm tip-to-tip crack length and includes the diameter of the hole. For un-notches specimens, 0.2 mm and 0.5 mm refer to half crack lengths (half of the overall tip-to-tip crack length), while the final failure criterion typically produced tip-to-tip crack lengths of around 10-15 mm.

Also included in these tables are axial strain amplitude and mean strain, $\varepsilon_a$ and $\varepsilon_m$, respectively, and shear strain amplitude and mean, $\gamma_a$ and $\gamma_m$, respectively. For strain-controlled tests, these values correspond to the applied strains at the specimen outer surface, and corresponding stress values were calculated from recorded testing loads at midlife/stable cycles. For load-controlled tests, on the other hand, strain amplitudes were estimated from Ramberg-Osgood cyclic stress-strain relations generated under each respective loading condition.

Next, in order to study the effects of multiaxial stress states and non-proportional loadings on the resulting fatigue damage, a number of combined axial-torsion fatigue tests were also performed under both in-phase and 90° out-of-phase loading conditions. As was the case for the cyclic deformation tests discussed in Section 4.2, a nominal stress ratio, $\lambda$, of $S_T/S_A = \sqrt{3}/3 = 0.577$ was used in these tests so that nominal shear and axial stress contributions would be equal based on von Mises equivalent stress. Relevant testing parameters and resulting fatigue lives for each in-phase and 90° OP test, for both un-notched and notched specimens, are summarized in Tables 5.3 and 5.4, respectively. These tables contain the same information as Tables 5.1 and 5.2, but additionally list the nominally applied von Mises stress amplitude, $S_{vm,a}$, for each test. For these loading conditions, individual shear and axial strain amplitudes listed in the tables were
calculated using the plasticity modeling techniques detailed in Section 4.5 in conjunction with the equivalent cyclic stress-strain curves generated for each respective loading path.

In addition to the un-notched and notched specimen fatigue tests summarized in Tables 5.1 to 5.4, additional information is also given in these tables for tests performed under each loading path using un-notched tubular specimens containing a small machined precrack. Since these tests were aimed at evaluating shear-mode crack growth mechanisms only, they are not discussed in this chapter. Instead, the results of these tests are presented and discussed in Section 7.2 with respect to constant amplitude crack growth in un-notched specimens.

Finally, for un-notched specimens only, more specialized loading paths were also applied, in a limited number of fatigue tests, to study various degrees of loading component interaction on fatigue damage. Loading conditions for these tests included cyclic torsion with static axial stress and triangular load paths (in shear versus axial stress space) with different ratios of nominally applied stress components. All testing parameters and resulting fatigue lives for these tests are summarized in Table 5.5. For these loading conditions, stabilized strain amplitudes were calculated using plasticity modeling techniques, based on the equivalent in-phase cyclic deformation curve, in conjunction with Tanaka’s parameter for estimating non-proportional hardening. Although the cyclic axial stress-strain curve could have been used with the plasticity model instead, the in-phase curve was chosen due to its slightly better overall agreement with the deformation data presented in Figure 4.7 for all stress states considered. Analysis of all constant amplitude fatigue test data, using various multiaxial fatigue damage parameters, is presented in the following sections.
5.2 Baseline Fatigue Life Curves

From the fatigue life data presented in Tables 5.1 and 5.2, fully-reversed stress-life (S-N) curves were first constructed for both uniaxial stress amplitude versus cycles to failure and shear stress amplitude versus cycles to failure. In the least squares curve fitting procedure, applied stress values were treated as the independent variable in order to properly reflect the manner in which the data were generated. Additionally, any data from runout tests were not included in the fits. The resulting fatigue life curves take the following power law form for uniaxial and shear stress-life, respectively:

\[ S_{A,N_f} = A_A (N_f)^b \]  
\[ S_{T,N_f} = A_T (N_f)^{b_0} \]  

where \( S_{A,N_f} \) and \( S_{T,N_f} \) represent the fully-reversed fatigue strength at \( N_f \) cycles under uniaxial and pure shear loading, respectively. The uniaxial stress-life coefficient, \( A_A \), for the testing material, was found to be 1089.2 MPa with a uniaxial fatigue strength exponent, \( b \), of \(-0.133\). Similarly, for pure torsion loading, the shear stress-life coefficient, \( A_T \), was determined to be 416.2 MPa, while the resulting shear fatigue strength exponent, \( b_0 \), was \(-0.078\). These fits, along with the corresponding experimental data, are presented in Figures 5.1 and 5.2 for the uniaxial and pure shear cases, respectively.

In addition to stress-life fatigue curves, uniaxial (\(\varepsilon\)-N) and shear (\(\gamma\)-N) strain-life curves were also fit to the experimental data. In order to do this, it was first necessary to divide the total applied strain amplitudes into elastic and plastic strain components. Again, since the majority of these tests were performed in load-control, where strain response was not measured, applied strain amplitudes were estimated using appropriate
material constitutive relations derived from the cyclic incremental step tests described in Section 4.2. For strain-controlled tests, however, applied strain values were used directly. Elastic strain amplitude was calculated as $\varepsilon_a^e = \sigma_a/E$ for uniaxial loading and as $\gamma_a^e = \tau_a/G$ for shear loading. Then, plastic strain amplitude was calculated as the difference between the total applied strain and the elastic strain amplitude.

Strain-life fatigue curves can then be expressed in terms of elastic and plastic strain contributions, as follows, for uniaxial and shear loadings, respectively:

$$\varepsilon_a = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c$$  \hspace{1cm} (5.3)

$$\gamma_a = \frac{\tau'_f}{G} (2N_f)^{b_o} + \gamma'_f (2N_f)^{c_o}$$  \hspace{1cm} (5.4)

where $\varepsilon_a$ and $\gamma_a$ are the fully-reversed uniaxial and shear strain amplitudes, respectively, corresponding to a fatigue life of $2N_f$ reversals, $\sigma'_f$ and $\tau'_f$ are the uniaxial and shear fatigue strength coefficients, respectively, $b$ and $b_o$ are the uniaxial and shear fatigue strength exponents, respectively, $\varepsilon'_f$ and $\gamma'_f$ are the uniaxial and shear fatigue ductility coefficients, respectively, and $c$ and $c_o$ are the uniaxial and shear fatigue ductility exponents, respectively.

For the axial data, a least squares fit of elastic strain versus reversals to failure for each test performed (excluding data from runout tests) led to values of $\sigma'_f = 1194.2$ MPa and $b = -0.133$. From plastic strain versus reversals to failure data, the remaining constants $\varepsilon'_f = 0.066$ and $c = -0.445$ were determined. Similarly, through curve fitting of the pure torsion fatigue test data, shear strain-life constants of $\tau'_f = 439.3$ MPa, $b_o = -0.078$, $\gamma'_f = 0.834$ and $c_o = -0.705$ were determined. Only plastic strain amplitudes greater than 0.02% were considered in either fit, and applied strain values were treated as
the independent variable in the fitting procedure. The resulting strain-life curves, along with the corresponding experimental data, are presented in Figures 5.3 and 5.4 for uniaxial and pure shear loading conditions, respectively. A summary of the baseline fatigue properties generated, including all uniaxial and shear stress- and strain-life constants, is also given in Table 5.6 for the 2024-T3 testing material.

5.3 Equivalent Stress/Strain-Based Life Predictions

More traditional approaches to multiaxial fatigue damage calculation have focused on extending static yield criteria to cyclic loading situations in order to compute equivalent stress or strain values for use with uniaxial stress- or strain-life fatigue curves. Typically, stress-based approaches are used in the high cycle fatigue regime, where the loading is primarily elastic, while strain-based approaches are better able to account for the effects of plasticity on the resulting fatigue damage.

The oldest and simplest methodology for fatigue life prediction, the stress-life approach, relates the alternating stress within a material to the number of cycles a material can withstand at that stress level before failure occurs. Most often, when applied to multiaxial loading situations, an equivalent stress is first computed using criteria such as von Mises equivalent stress or maximum principal stress. This equivalent stress is then assumed to be equal, in terms of fatigue damage, to a uniaxial stress of the same magnitude. As such, fatigue life can be approximated from a stress-life curve derived under fully-reversed uniaxial loading conditions.

In order to account for other effects such as component size, temperature, surface finish, environment, etc., the slope of the S-N line is typically modified by reducing the
fatigue strength at the assumed endurance limit life by empirically derived correction factors. However, since none of these factors were investigated in the current study, they were not considered in the following analyses. Additionally, the presence of mean stresses must be accounted for in an S-N analysis, through empirical modifications to the equivalent alternating stress value, using one of a number of available models. However, since all of the constant amplitude fatigue tests in this study were performed under fully-reversed loading conditions, no consideration of mean stress was required in this chapter.

As an alternative to equivalent stress-based fatigue life analyses, equivalent strain-based approaches are often used as well because of their ability to better account for plastic deformations sometimes encountered in cyclic loading situations. These approaches are analogous to their stress-based counterparts, but instead rely on strain-based quantities for computing fatigue damage. After an equivalent strain amplitude is computed, based on criteria such as von Mises equivalent strain or maximum principal strain, fatigue life can then be approximated from a typical strain-life curve generated under fully-reversed uniaxial loading conditions. Again, if mean stresses were present, they would need to be considered through one of a number of empirical modifications.

5.3.1 Un-Notched Specimen Life Predictions

Two different equivalent stress criteria were used to evaluate the ability of the S-N approach to correlate the multiaxial fatigue data obtained from constant amplitude un-notched specimen fatigue tests: von Mises equivalent stress and maximum principal stress. Maximum principal stress amplitude was derived from Mohr’s circle equations, while von Mises stress amplitude was calculated from nominally applied axial, $S_{A,a}$, and shear, $S_{T,a}$, stress amplitudes using the following equation:
\[ S_{vm,a} = \sqrt{S_{A,a}^2 + 3S_{T,a}^2} \] (5.5)

For the constant amplitude fatigue data generated in this study, the S-N approach yields the fatigue life correlations shown in Figure 5.5, for maximum principal stress-based predictions, and in Figure 5.6 for von Mises equivalent stress-based predictions.

Multiaxial stress-life correlations based on maximum principal stress (Figure 5.5) show that, with the exception of in-phase axial-torsion data, non-conservative fatigue lives were predicted for all other multiaxial loading conditions. Using this approach, only 23\% and 68\% of life predictions were within factors of ±3 and ±10, respectively, of experimental results. For von Mises S-N predictions (Figure 5.6), correlations were improved over the maximum principal stress criterion, and have the tendency to be mostly conservative (with the exception of 90° OP life predictions). Using this approach, 43\% and 93\% of life predictions were within factors of ±3 and ±10, respectively, of experimental results. It should be noted that life predictions for uniaxial fatigue tests were not included in these percentages, as the prediction curve was fit to these data. Additionally, as a clarification, in the sense that it is used throughout this dissertation, when the “±” sign precedes a factor of difference expression, this means a factor of 3 or 10 greater than, or a factor of 3 or 10 less than the experimental result (as opposed to positive or negative 3 or 10 times the experimental result).

As an alternative to the equivalent stress-based approaches, an equivalent strain-based approach was also evaluated in this study due to the presence of plastic deformation in several tests performed in the low cycle fatigue regime. Since von Mises S-N predictions were shown to be superior to those based on maximum principal stress, only von Mises equivalent strain was considered for multiaxial strain-life predictions.
Again, since the majority of these tests were performed in load-control, where strain response was not measured, equivalent strain amplitudes were estimated from equivalent stress amplitudes (Equation (5.5)) using the material constitutive relation, Equation (2.4), along with the relevant uniaxial deformation properties.

The resulting fatigue life predictions based on the von Mises equivalent strain-life approach are shown in Figure 5.7. As expected, it is apparent from these results that both the von Mises stress- and strain-life approaches produced similar life prediction trends for the different multiaxial loading conditions investigated. Slight improvements in life prediction accuracy, however, were gained by using the strain-based analysis approach. For all un-notched constant amplitude fatigue data generated in this study (excluding uniaxial data), 50% and 90% of the strain-based life predictions were within factors of ±3 and ±10, respectively, of experimental results. A summary of fatigue life predictions for all loading conditions and analysis approaches considered, along with the percentage of predictions within scatter bands of ±3 and ±10, is provided in Table 5.7.

5.3.2 Notched Specimen Life Predictions

Notched specimen fatigue life analyses were performed using von Mises equivalent stress and strain approaches similar to those described in the previous section for un-notched specimens. The primary difference in the notched predictions is the need to determine local elastic-plastic notch root stresses and strains from the nominally applied loading conditions. Additionally, a third approach was used to predict notched specimen fatigue lives based on local elastic equivalent stresses.

When performing a notched specimen fatigue life analysis based on a local stress or strain approach, the location at which stress-strain variations are computed is an
important consideration. Generally, the predicted failure location will correspond to the location at the notch root experiencing the highest stress amplitudes. For proportional loading paths, this location can be determined by multiplying nominally applied stress values by their respective stress concentration variations around the notch. Superposition can then be used to add the local contributions from each nominal stress component and determine which location produces highest combined stresses. From the stress analysis results presented in Section 4.7 for the notched tubular specimen geometry tested in this study, it is easy to establish maximum stress/predicted failure locations of 0° and 180° for uniaxial nominal loadings, ±45° for torsion loading, and ±23° for in-phase axial-torsion loading at the nominal stress ratio applied in testing.

As previously mentioned, however, for non-proportional nominal loadings, the location of maximum stress at a notch can change continuously throughout a cycle. Therefore, if the failure location is unknown prior to performing fatigue analysis, notch root stress variations at several locations may need to be evaluated in order to determine which will experience the maximum equivalent stress amplitude over the course of any given cycle. In this case, the principle of superposition, along with the applied loading conditions, can be used to compute time histories for the variation of each local elastic stress component at each potential failure location. Then, if an equivalent stress/strain approach is to be used for damage calculation, for each time point in the history, a signed elastic/pseudo equivalent stress can be computed. With the time variation of equivalent stress known over the entire cycle, the elastic equivalent stress amplitude for each location can easily be determined. In the case of this study, maximum local stress amplitudes occur at the 0° and 180° locations around the hole for 90° OP non-
proportional loadings at the nominal stress ratio applied in testing. More information on signed equivalent stress histories is given in Chapter 6.

It should be noted that although notched specimen life predictions are also commonly computed based on nominally applied stress levels, through the consideration of appropriate stress concentration factors, these approaches are difficult to apply when non-proportional loading conditions exist. Due to the potential change in both the magnitude and location of maximum local stresses with a change in nominal stress ratio, equivalent stress concentration factors can be difficult to define in such cases. This is especially true under the variable amplitude loading conditions discussed in the next chapter. Therefore, under these conditions, nominal stress approaches based on a modified stress-life fatigue curve can lose their relevance. Local analysis approaches must be applied, instead, in order to properly account for the variation of stresses and strains at the notch root. As such, no nominal stress-based fatigue life analyses were performed in this study.

In addition to the analysis location with respect to the curvature of the notch, the location with respect to the specimen thickness can also have an impact on resulting fatigue life predictions. In this study, two options were considered: either the mid-thickness location of the specimen, or an inner/outer surface location. It was ultimately decided that the stress concentration values used for each respective loading case would reflect those at the highest stress location around the edge of the hole on either the inner or outer surface of the specimen. This corresponds to the specimen outer surface for axial and combined loading conditions, and to the inner surface for pure torsion loadings.

Although it was shown in Section 4.7 that stresses are slightly higher at the specimen mid-thickness, due to notch constraint and the buildup of transverse stress, the
difference is small for the specimens used in the study. Therefore, using surface stress values instead was not expected to have a large impact on predicted fatigue lives. Locations at the specimen surface were chosen based on the idea that fatigue damage mechanisms are less restricted at the two free surfaces present where the notch and specimen surfaces intersect. As a result, performing fatigue life analyses at these locations was thought to better reflect the stresses where actual crack initiation was more likely to occur. Additionally, choosing surface locations helped to facilitate some of the more in depth notch effect studies discussed in subsequent sections.

5.3.2.1 Equivalent Pseudo Stress-Based Approach

Notched specimen fatigue lives were first predicted using a local pseudo stress-based approach. This approach computes fatigue damage directly from the local elastic equivalent stress history, which simplifies the analysis by eliminating the need to perform local plasticity corrections. In the case of this study, the equivalent stress criterion used was von Mises stress. In order to apply this approach, a suitable fatigue life curve must first be obtained from which fatigue damage can be computed. This so-called pseudo stress-life curve was constructed from the material uniaxial stress-life curve (Equation (5.1)) by following the procedures outlined in (Haibach, 2003a; Lee et al., 2012). First, the un-notched specimen endurance limit, 5(10⁶) cycles in this case, is raised by a factor of \(K_t/K_f\), where \(K_t\) is the uniaxial elastic stress concentration factor and \(K_f\) is the fatigue notch factor. Doing so allows for a qualitative consideration of stress gradient effects in the vicinity of the notch root. \(K_f\) can either be determined experimentally, as the ratio between the un-notched and notched specimen endurance limits, or through a suitable approximation formula. For this study, \(K_f\) was estimated using Neuber’s formulation.
(Equation (2.28)) where, for the tubular specimens under consideration, the notch root radius, $r$, is $1.6$ mm, the material characteristic length, $\rho$, is $0.5$ mm, and $K_t = 2.94$ for uniaxial loading (based on gross nominal stress and evaluated at the specimen outer surface).

Once the modified endurance limit has been determined, the remainder of the pseudo stress-life curve is constructed by retaining the same S-N slope factor as the un-notched specimen fatigue curve. Maintaining the same slope helps to account for a decrease in fatigue damage, due to an increasing difference between elastic/pseudo stress and actual stress, as localized plasticity increases and fatigue life decreases. With a ratio of $K_t / K_f$ equal to 1.31, the resulting pseudo stress-life curve for this study takes the same form as Equation (5.1), but with the coefficient changing from $1089$ MPa to $1426$ MPa. A schematic illustration, showing the construction of the pseudo stress-life curve, is presented in Figure 5.8. Knowing the local von Mises equivalent stress amplitude and pseudo stress-life curve, fatigue lives can then be predicted analogous to the stress-life approach used for un-notched specimens.

Notched specimen fatigue lives for all constant amplitude fatigue tests performed in this study were predicted first using this approach. Results from these predictions are shown in Figure 5.9 in terms of experimental versus predicted life. Of all the tests performed, $89\%$ and $100\%$ of fatigue lives were predicted within factors of $\pm 3$ and $\pm 10$, respectively, of experimental results. Additionally, it should be noted that because the local stress state at the analysis location was uniaxial, regardless of the nominal loading conditions, local maximum principal stress-based analyses would have resulted in the same life predictions for the specimen geometry considered. A summary of all notched
specimen fatigue life prediction results, along with the percentage of predictions within scatter bands of ±3 and ±10, is provided in Table 5.8.

5.3.2.2 Equivalent Stress-Based Approach

In order to consider the effects of localized plasticity in fatigue damage calculations, a local stress-based approach, similar in concept to that used for un-notched specimens, was also used to predict notched specimen fatigue lives. In order to derive local elastic-plastic stresses and strains from the theoretical elastic stress amplitudes used in the pseudo stress-based approach, Neuber’s rule was used. Neuber’s rule was shown in Section 4.8.1 to provide accurate stress-strain predictions for the tubular specimens used in this study under a variety of uniaxial and combined loading conditions. In this case, the local von Mises pseudo stress was used, along with a Ramberg-Osgood type cyclic stress-strain relation, Equation (2.4), as input to the following expression of Neuber’s rule:

\[
\frac{(K_{tq} \Delta S_{vm})^2}{E} = \frac{(\Delta \sigma_q)^2}{E} + 2\Delta \sigma_q \left(\frac{\Delta \sigma_q}{2K'}\right)^{1/n'}
\]  

where \(\Delta S_{vm}\) is the nominally applied von Mises equivalent stress range for each loading path, \(K_{tq}\) is the equivalent stress concentration factor (different for each loading path), defined as the ratio of local elastic equivalent stress over nominal equivalent stress, \(K'\) is the material cyclic axial strength coefficient, and \(n'\) is the cyclic axial hardening exponent. Once the local equivalent stress range, \(\Delta \sigma_q\), and corresponding amplitude were determined, damage calculation (from Equation (5.1)) was then performed using the same stress-based procedures outlined for un-notched specimens.

Local equivalent stress-based analyses were performed twice for each notched specimen fatigue test performed: once as just described, using maximum equivalent stress
values at the notch root, and once by substituting $K_{tq}$ for $K_{fq}$ in Equation (5.6) to account for the effects of stress gradients in the resulting fatigue life predictions. Different values of $K_{fq}$ were computed for each loading path based on Equation (2.28).

Local von Mises stress-based fatigue life predictions for the constant amplitude notched specimen fatigue tests are presented in Figures 5.10 and 5.11. The results of the first set of analyses, based on the maximum equivalent stress values at the notch root (as calculated from Neuber’s rule), are shown in Figure 5.10. Similarly, the results of the second set of analyses, evaluated using the fatigue notch factor in conjunction with Neuber’s rule, are shown in Figure 5.11. For the latter case, 89% and 100% of fatigue lives were predicted were within factors of ±3 and ±10, respectively, of experimental results. While life predictions based on maximum local stresses are qualitatively similar, they lack overall accuracy due to their conservative nature. Again, because the local stress state at the analysis location was always uniaxial, local maximum principal stress-based analyses would have resulted in the same life predictions for the specimen geometry considered.

5.3.2.3 Equivalent Strain-Based Approach

In addition to the local stress approach, a local strain-based approach was considered using von Mises equivalent strain values derived from Neuber’s rule. Using the stress ranges obtained from Equation (5.6), local elastic-plastic strain values were subsequently determined for each loading condition based on Equation (2.4). Having computed the local equivalent strain amplitude for each testing condition, fatigue life was then calculated, from Equation (5.3), using identical procedures to those used for the un-notched strain-life analyses. As in the case of the local stress approach, each analysis was
performed twice: once using $K_{tq}$ and maximum equivalent stress values at the notch root, and once using $K_{fq}$ to account for the effects of stress gradients in the resulting fatigue life predictions.

Similar to the other analysis approaches, constant amplitude notched specimen life predictions based on the local von Mises equivalent strain approach are presented in Figures 5.12 and 5.13 with and without the consideration of stress gradient effects, respectively. Again, analyses which use fatigue notch factor to derive local stresses were found to be more accurate than those based on maximum stresses with 89% and 100% of fatigue lives predicted within factors of $\pm 3$ and $\pm 10$ of experimental results, respectively. Again, a summary of notched specimen fatigue life predictions for all loading conditions and analysis approaches considered, along with the percentage of predictions within scatter bands of $\pm 3$ and $\pm 10$, is provided in Table 5.8.

### 5.3.3 Discussion on Equivalent Stress- and Strain-Based Life Predictions

In order to interpret the results of the equivalent stress- and strain-based fatigue life predictions presented in the previous sections, attention was first focused on the notched specimen fatigue tests, as life predictions for these tests tend to show the best agreement with experimental results. For the notched specimens, fatigue life analyses were performed twice: once considering the maximum stress-strain values at the notch root, and once considering stress gradient effects through the use of fatigue notch factor in Neuber’s rule. In both cases, it is easy to see from Figures 5.10 to 5.13 that the von Mises equivalent stress/strain approaches are able to collapse the notched fatigue data into a single band with a relatively small amount of scatter.
When comparing life predictions based on $K_{tq}$ to those based on $K_{fq}$, the primary difference is that predictions based on $K_{tq}$ are shifted towards being more conservative. This is expected, however, due to the presence of stress gradients at the notch root. Because the processes that drive fatigue damage occur within a finite volume of material, the variation of stress and strain components over this volume would be expected to produce less fatigue damage than what would be predicted based on maximum values. Therefore, by reducing the stress and strain values used for fatigue life analysis through the application of fatigue notch factor, the degree of conservatism in life predictions was effectively reduced. Figures 5.11 and 5.13 show that, by using $K_{fq}$ when performing plasticity corrections, 89% of notched fatigue lives were predicted within a factor of ±3 of experimental results, regardless of the nominally applied loading conditions, for both the equivalent stress- and equivalent strain-based approaches.

The local pseudo stress approach evaluated in this study, which is based on the theoretical elastic equivalent stress values at the notch root, also produced excellent life predictions for the notched specimen fatigue tests. In this approach, stress gradient and plasticity effects were considered empirically through the modification of the stress-life curve used to compute fatigue damage. Of the tests performed, Figure 5.9 shows that 89% of fatigue lives were predicted within a factor of ±3 using this approach.

Although it may appear surprising at first, that simple equivalent stress- and strain-based analysis approaches produced relatively accurate fatigue life predictions for the variety of nominal loading conditions investigated, the notch geometry considered in this study produces favorable conditions for these procedures. Because of the free surface boundary conditions of the hole, and a state of plane stress in the specimen thickness
direction, a uniaxial stress state always exists at any location around the perimeter of the notch regardless of the degree of multiaxiality of the nominally applied loading. Therefore, no effects from multiaxial stress states and/or non-proportional hardening need be accounted for in these analyses.

However, evidence of the shortcomings of these approaches begins to emerge as attention is switched to the un-notched specimen constant amplitude fatigue life analyses performed in this study. Although these analyses should be simplified in the absence of a notch, in this case, they become more complex as a result of multiaxial stress state effects (excluding pure axial loading) not present in the notched specimen life predictions.

For the un-notched specimen analyses, both the equivalent stress- and equivalent strain-based approaches tend to bring together fatigue life predictions for axial and in-phase axial-torsion data, while predictions for pure torsion loading tend to be less non-conservative/more conservative. However, the correlation between the different loading conditions becomes increasingly worse at longer life. It is also clear that when significant loading non-proportionality was present, as was the case for the 90° OP axial-torsion tests, both the stress- and strain-based approaches resulted in life predictions which were consistently non-conservative by a factor of around 5-10. Excluding results for uniaxial loading conditions (since life prediction curves were fit to these data), only 43% and 50% of fatigue life predictions fall within a factor of ±3 of the experimental results based on the von Mises equivalent stress-life (Figure 5.5) and strain-life (Figure 5.7) approaches, respectively. Furthermore, using a maximum principal stress criterion with the S-N approach resulted in even worse fatigue life correlations. For these analyses, the majority
of life predictions were found to be non-conservative, and only 23% of predictions were within a factor of ±3 of experimental results.

Although equivalent stress- and equivalent strain-based analysis approaches are simple to implement under a wide range of multiaxial loading conditions, the constant amplitude fatigue life predictions presented in this chapter indicate that they are not capable of properly accounting for fatigue damage under a variety of such loading conditions. As a result, these approaches cannot be expected to provide accurate fatigue life predictions for more complex multiaxial variable amplitude loading conditions, which are the main focus of this study. Additionally, because equivalent stress/strain quantities such as von Mises do not change with respect to coordinate system orientation, they cannot predict a crack initiation plane. Therefore, if subsequent crack growth analyses are to be performed, the initial cracking orientation must be assumed independent of the damage parameter.

5.4 Critical Plane-Based Life Predictions

Given the disadvantages of equivalent stress- and strain- based approaches for multiaxial fatigue damage calculation, more recent damage parameters have made use of the critical plane concept as a more physically meaningful basis for damage calculation. The critical plane concept reflects experimentally observed behavior which shows that cracks tend to initiate and grow on preferred planes within a material. These planes usually correspond to either planes of maximum shear or maximum principal stress/strain, depending on the cracking behavior of the material being analyzed. Given the relative success these approaches have demonstrated in predicting fatigue damage for
a variety of materials and multiaxial loading conditions (as noted in the literature review),
the experimental data generated in this study were also analyzed using some of the more
popular critical plane damage parameters. These include the Smith-Watson-Topper
(SWT) parameter and the Fatemi-Socie (FS) parameter. Since both of these parameters
contain both stress and strain terms in their formulation, they have the added advantage,
over stress- or strain-based approaches, of being able to reflect changes in fatigue damage
brought about by cyclic and/or non-proportional hardening behavior.

5.4.1 Un-Notched Specimen Life Predictions

The Smith-Watson-Topper parameter, Equation (2.39), is based on the
assumption that crack initiation and small crack growth mechanisms occur on planes of
maximum tensile stress/strain. Similarly, cracks are often assumed to initiate via tensile
mechanisms in aluminum alloys due to the stress concentration/notch effect of inclusions
contained within the material. As a result, the SWT parameter was used to predict fatigue
lives for the 2024-T3 constant amplitude fatigue tests performed in this study.

In cases where loading is fully-reversed, however, Equation (2.39) should, but
does not, predict fatigue lives equivalent to those obtained from the uniaxial strain-life
equation on which the predictions are based. This is because the stress amplitude derived
from the elastic portion of the strain-life equation, which is multiplied with the entire \( \varepsilon \cdot N \)
equation to derive the SWT life prediction curve, does not always properly account for
the effects of plasticity on material constitutive behavior (Khosrovaneh et al., 2004). A
more proper form of the SWT life curve equation can be expressed as follows:

\[
\frac{\Delta \varepsilon_1}{2} \sigma_{n,max} = \left[ \frac{\sigma_f'}{E} \left(2N_f\right)^b + \varepsilon_f' \left(2N_f\right)^c \right] \sigma_{a,Nf}
\]  

(5.7)
where the elastic-plastic stress amplitude corresponding to the fully-reversed uniaxial strain amplitude at a given fatigue life, $\sigma_{a,N_f}$, is calculated numerically from the following expression:

$$\frac{\sigma_f'}{E} \left( 2N_f \right)^b + \epsilon_f' \left( 2N_f \right)^c = \frac{\sigma_{a,N_f}}{E} + \left( \frac{\sigma_{a,N_f}}{K'} \right)^{1/n'}$$  \hspace{1cm} (5.8)

Computing fatigue life using this approach requires an iterative solving procedure where $N_f$ is varied until the equality in Equation (5.7) becomes satisfied.

In this study, fatigue life analyses using the SWT parameter were performed twice for each loading case considered: once using Equation (2.39) and once using Equation (5.7). The deviatoric formulation of the parameter, mentioned in Section 2.4, was not considered due to the fully-reversed nature of the tests performed in this study. In the absence of mean stress, deviatoric SWT predictions are expected to be similar to those based on the traditional formulation.

Since the SWT parameter relates both stress and strain terms to fatigue damage, stabilized stress-strain response was estimated for each loading condition, prior to damage calculation, by using the cyclic plasticity model outlined in Section 4.5. Since only steady state stress-strain values are of interest in these constant amplitude life predictions, no transient material behavior was considered in the plasticity modeling process. The cyclic stress-strain curve used as input to the model was that derived under in-phase axial-torsion loading conditions ($K_{IP}' = 688.6$, $n_{IP}' = 0.081$). Non-proportional hardening was accounted for in 90° OP fatigue tests, however, by instead using the 90° OP cyclic deformation curve as input to the plasticity model ($K_{NP}' = 938.0$, $n_{NP}' = 0.107$). Once stresses and strains were computed, stress transformations were then performed in 5° increments to identify the plane experiencing the largest amount of fatigue damage.
With the critical plane identified, multiaxial fatigue life predictions were computed based on the uniaxial strain-life properties in Table 5.6.

The results of the SWT fatigue life predictions based on Equation (2.39) are presented in Figure 5.14. Similarly, the results of predictions based on Equation (5.7) are shown in Figure 5.15. It can be seen from these figures that predictions based on both SWT formulations are qualitatively very similar, but Equation (2.39) (Figure 5.14) results in slightly more accurate predictions. For the life predictions in Figure 5.14, 28% and 73% are within factors of ±3 and ±10, respectively, of experimental lives, while only 20% and 60% are within factors of ±3 and ±10, respectively, for the predictions in Figure 5.15. Axial test data were not included in these percentages, as the prediction curve was fit to these data. From these results, it is clear that the data from different loading conditions are not correlated well using the SWT parameter. In fact, the error in predicted fatigue lives is very similar, both qualitatively and quantitatively, to that observed in Figure 5.6 for maximum principal stress-based predictions. A summary of fatigue life predictions for all loading conditions and analysis approaches considered, along with the percentage of predictions within scatter bands of ±3 and ±10, is provided in Table 5.7.

Although the SWT parameter did not produce adequate fatigue life predictions for the various loading conditions applied in this study, it is meant for materials which fail by a tensile-dominated damage mechanism. Crack replicas from the tests performed in this study, however, indicate that cracks in the 2024-T3 aluminum alloy consistently initiate on planes of maximum shear, regardless of the applied loading conditions. Figure 5.16 illustrates this by showing crack replica images for small cracks initiated under pure torsion, Figure 5.16(a), in-phase axial-torsion, Figure 5.16(b), and 90° OP, Figure
5.16(c), loading conditions. Images are ordered from left to right according to the highest to lowest applied stress amplitude. Specimen numbers followed by an asterisk indicate images which correspond to secondary fatigue cracks, i.e. not the crack that caused final failure. For reference, the maximum shear and principal stress planes for each loading condition are included as well.

For the lower stress levels, although crack branching to maximum principal planes is observed for some specimens (TM4-ST, TM78-ST, TM67-ST, TM37-SC, and TM62-SC), crack nucleation and several hundred microns of small crack growth are still observed to be shear driven processes in these cases. More information and discussion on crack branching is provided in Section 7.2. Cracking orientations are not shown for uniaxial loading conditions because the preferred maximum shear plane is located at a 45° angle in the specimen thickness direction. Therefore, a preference between shear and tensile driven crack initiation would not be observable from the surface of the specimen in this situation.

Given the fact that cracks tend to initiate on maximum shear planes for the material tested in this study, the Fatemi-Socie parameter, which is based on this type of damage mechanism, would be expected to provide improved multiaxial fatigue life correlations compared to those observed for the SWT parameter. Once again, since the FS parameter relates both stress and strain terms to fatigue damage, the same stress-strain estimations generated for the SWT analyses were used to compute fatigue damage. Again, stress transformations were performed in 5° increments, for each applied loading condition, prior to damage calculation. For the analyses performed in this study, the critical plane was taken as the plane experiencing the maximum value of the damage
parameter, not the maximum shear strain plane. This was done to maintain consistency between constant and variable amplitude fatigue life prediction procedures. Due to the nature of variable amplitude loading, damage cannot be summed on the maximum shear plane, because the maximum shear plane can be different for each applied cycle. For the 2024-T3 aluminum alloy used in this study, however, the difference between the maximum shear plane and maximum FS damage plane was typically less than 10° and differences in fatigue life predictions were relatively small.

A value of \( k = 1 \) was used for the FS fatigue life analyses. This value was determined based on comparisons between fatigue damage calculated for uniaxial and pure torsion loading conditions at the same life. However, since the theoretical value for \( k \) can vary with life (subsequently discussed), this value was chosen based on comparisons made near the middle of the fatigue data range (i.e. around \( 10^4 \) to \( 10^5 \) reversals).

Fatigue life predictions using the FS parameter were computed based on both uniaxial and pure shear strain-life properties, given in Table 5.6, using Equations (2.37) and (2.38), respectively. Results based on uniaxial properties are presented in Figure 5.17, while those based on shear fatigue properties are shown in Figure 5.18. Overall, predictions based on uniaxial fatigue properties tend to be more accurate with 88% and 98% falling within factors of \( \pm 3 \) and \( \pm 10 \), respectively, of experimental lives. For predictions based on shear properties, 56% and 100% are within factors of \( \pm 3 \) and \( \pm 10 \), respectively. Similar to previous un-notched specimen life comparisons, axial data were excluded from scatter band percentages for predictions based on uniaxial properties, and pure torsion data were excluded when predictions are based on shear fatigue properties. Results from runout tests were also excluded. Even though the accuracy of life
predictions can vary based on the fatigue properties used to generate them, the correlation of data from different loading conditions is found to be very reasonable using the FS parameter. This indicates that the parameter is able to properly account for the shear-based damage mechanism observed in the testing material.

5.4.2 Modified Fatemi-Socie Parameter

In the previous section, it was shown that the Fatemi-Socie critical plane-based damage parameter was able to correlate all of the constant amplitude fully-reversed 2024-T3 fatigue data generated in this study relatively well. Additionally, as mentioned in Section 2.4, similar results have been reported in literature for several other materials and loading conditions as well. Despite this fact, when analyzing some recent literature data, e.g. (Meyer, 2014; Zhao and Jiang, 2008), it was found that the FS parameter can result in non-conservative life predictions when significant tensile mean stress is present. While increasing the \( k \) value in the parameter can improve mean stress correlations by increasing the influence of the maximum normal stress term, this has a detrimental effect on the correlation of fatigue data generated under other multiaxial loading paths.

Although the constant amplitude fatigue tests in this study were all performed under fully-reversed loading conditions, the variable amplitude tests, discussed in the following chapter, were conducted using a tension-dominated loading spectrum. Therefore, making sure the parameter was able to properly account for mean stress effects was an important aspect of this study.

In addition to mean stress effects, certain conditions have also been noted in literature where the correlation of various multiaxial fatigue data using the FS parameter is less than ideal. For example, when computing the experimental value of \( k \) for the FS
parameter from uniaxial and pure shear fatigue properties, Fatemi and Kurath (1988) (1045 steel and Inconel 718) and Kim and Park (1999) (S45C steel) found that the value of $k$ increased with fatigue life. However, when assuming a constant $k$ value, data were still found to be correlated within a reasonable margin of error. As a result, when reanalyzing their fatigue data and assuming a linear variation of $k$ with $\log(2N_f)$, Kim and Park reported a negligible change in fatigue life predictions for most loading conditions investigated.

It is important to note that there are several factors, in addition to stress state, which can affect the correlation of multiaxial fatigue data. For example, if the criterion used for defining crack initiation does not result in a sufficiently small crack length, differences in crack growth behavior between different stress states can cause disproportionate changes in experimentally measured fatigue lives. This is especially true if a material exhibits a transition in crack growth mechanisms (e.g. shear-mode to tensile-mode) as fatigue life increases. However, there is also the possibility that at least some of the variation in $k$ value observed in the aforementioned studies may relate to changes in fatigue damage due to the interaction of normal and shear stress/strain components.

In a more specific investigation into the effects of normal-shear stress/strain interaction, Wu and Yang (1987) performed multiaxial fatigue tests on tubular specimens of 304 stainless steel using two different strain paths and $\lambda$ ratios of 0.6, 1.5, and 3.0. Although the same maximum and minimum strain values were used, the difference between the paths was the sequence in which each strain component was applied. For path A, the axial strain was first increased from zero to its maximum value, while the shear strain was kept at zero. Then, while holding the axial strain at its maximum value,
the shear strain was increased to its maximum value. Finally, these steps were reversed for unloading. Path B was identical to path A, but with the sequence of applied axial and shear strain reversed. After performing fatigue tests at various loading levels and \( \lambda \) ratios, it was found that the experimental lives for tests performed using path B were always longer than those for tests performed under the same loading conditions using path A. The difference in life was as much as a factor of 2. Although similar damage values would generally be calculated for each path based on most damage parameters (some differences may exist due to changes in stress response), this difference in experimental life demonstrates the effect that time dependent normal-shear stress/strain interactions can have on fatigue damage.

Similarly, Shamsaei (2010) performed a limited number of fatigue tests on tubular specimens of 1050 steel for different constant amplitude loading paths which resulted in the same damage value based on the FS parameter. These paths included cyclic torsion with static tension loading, and cyclic torsion with a pulsating tensile stress applied midway through the shear cycle. Based on the results of these tests, it was found that the experimental fatigue life for a cyclic torsion with static tension test was only around half of the life predicted for in-phase axial-torsion loading at the same damage value. It should be noted, however, that significant cyclic ratcheting (up to 10% tensile strain) was observed in this test, and may have contributed in some degree to the shorter observed life. Conversely, the average life for cyclic torsion with pulsating tension tests, the loading path for which is shown in Figure 5.19 in terms of normalized stress/strain variations, was around a factor of 2.5 times longer than for in-phase loading. Given the difference in fatigue life observed between the two loading paths, combined with their
time dependent differences in normal-shear stress/strain interaction, these results suggest that this is an area where improvements can be made with respect to the quantification of fatigue damage.

Again, despite good correlation of the constant amplitude multiaxial fatigue data generated in this study, given the varying degrees of loading non-proportionality in the variable amplitude axial-torsion fatigue tests performed, making sure the FS parameter was able to properly account for differences in fatigue damage due to normal-shear stress/strain interaction effects was an important aspect of this study. These interactions can occur in many forms including: the interaction between shear and normal stress components on the maximum shear plane (even under uniaxial loading, e.g. mean stress effects), different ratios of applied shear to normal stress amplitude, and time/load path dependent interaction effects between normal and shear stress/strain components. As a result, the following sections were aimed at evaluating the FS parameter using data generated under a variety of mean stress and multiaxial loading paths. Modifications to the parameter were then proposed in an attempt to improve life predictions.

5.4.2.1 Mean Stress Effects on Damage Calculation

The first analyses performed were for fatigue test data reported by Zhao and Jiang (2008) for 7075-T651 aluminum alloy ($\sigma_y = 501$ MPa, $\sigma_u = 561$ MPa). A summary of relevant deformation and fatigue properties for this material, either reported or calculated from reported data, can be found in Table 5.9. Loading conditions included were similar to those used in the current study: uniaxial (131 tests), torsion (17 tests), torsion with static axial stress (9 tests), in-phase (9 tests), and 90° OP axial-torsion (7 tests). Uniaxial fatigue tests were conducted at $R$ ratios ranging from $-\infty$ to 0.7, while tests for all other
loading paths were performed under fully-reversed conditions. While tests were performed in strain-control, $R$ ratios for uniaxial tests were computed based on minimum and maximum stress values reported near the midlife cycle of each fatigue test.

Although, Zhao and Jiang observed mixed cracking behavior for this material, i.e. shear-mode cracking at shorter fatigue lives and tensile cracking in the high cycle regime, the crack orientations reported were measured for macroscopic cracks in the millimeter scale of crack length. However, as will be subsequently shown in Section 7.2 for 2024-T3 aluminum alloy, even if fatigue cracks are observed to branch and grow macroscopically on planes of maximum tensile stress, crack nucleation and a small amount of growth typically still take place on the maximum shear plane(s). Therefore, the 7075-T651 data were analyzed using the shear-based FS damage parameter in order to evaluate resulting fatigue life correlations.

Correlation of the 7075-T651 fatigue test data using the FS parameter ($k = 1$) is shown in Figure 5.20, where the life prediction curve is based on uniaxial strain-life properties. Data for tests with experimental lives less than 50 cycles are excluded from the figure due to the possibility of unstable material behavior when maximum stresses are near the ultimate strength of the material. Additionally, data from runout tests are also excluded. It is clear from this figure that, despite reasonably good correlation between the different multiaxial load paths under fully-reversed conditions, the parameter results in increasingly non-conservative life predictions for uniaxial loading as the $R$ ratio, and thus the tensile mean stress, is increased.

Although increasing the $k$ value in the FS parameter can improve the correlation of mean stress data by increasing the influence of the maximum normal stress term, this
also has a detrimental effect on the correlation of fatigue data generated under other multiaxial loading paths. This is especially true for pure torsion loading where the maximum normal stress on the maximum shear plane is zero. To demonstrate this effect, Figure 5.21 presents fatigue life predictions for the same 7075-T651 test data, but in this case, the FS parameter is calculated using a $k$ value of 4. As expected, although increasing $k$ did have a beneficial on the correlation of uniaxial mean stress data, this moderate improvement came at the expense of the accuracy of life predictions for other multiaxial loading paths. It should also be noted that the accuracy of life predictions based on shear strain-life properties is more sensitive to changes in $k$ value. This is because the $\gamma-N$ life prediction curve (Equation (2.37)) does not change with the $k$ value in the same manner that the uniaxial life prediction curve (Equation (2.38)) does.

Similar to the 7075-T651 data, a trend of non-conservative fatigue life predictions in the presence of tensile mean stress was also observed for ductile cast iron ($\sigma_y = 644$ MPa, $\sigma_u = 1006$ MPa) data reported by Meyer (2014). Again, a summary of relevant deformation and fatigue properties reported for this material is included in Table 5.9. These fatigue data were generated in load-control using un-notched solid cylindrical specimens and uniaxial loading conditions with $R$ ratios ranging from $-7$ to 0.75 (54 tests total). Fatigue life correlations for these tests (excluding runout data) were computed using the Fatemi-Socie parameter, evaluated at $k$ values of 1 and 4, and are shown in Figures 5.22 and 5.23, respectively, based on uniaxial strain-life properties. Again, as the $R$ ratio is increased, the predictions become increasingly more non-conservative. Conversely, life predictions for $R$ ratios less than $-1$ tend to fall on the conservative side. Although increasing the $k$ value significantly improved the correlation of the uniaxial test
data, if results for multiaxial loading conditions had been available, a corresponding worsening in the correlation of these data could also be expected. This would be consistent with observations for the 7075-T651 data.

5.4.2.2 Proposed Modification to the Fatemi-Socie Damage Parameter

Given the results presented in the previous section, modifications to the FS parameter were investigated in an attempt to improve life predictions. Because life predictions were found to be non-conservative in the presence of significant tensile mean stress, it was apparent that the effect of maximum normal stress should be increased when such conditions exist. However, in order to maintain good fatigue life correlations for fully-reversed and multiaxial loading conditions, the influence of the normal stress term should not change in these cases. It was determined that substituting the yield strength in the FS equation for a quantity based on stress amplitude/range could achieve this effect. In addition, it can be observed in Figures 5.17 and 5.20 that the correlation between fully-reversed axial and torsion fatigue data begins to degrade in the high cycle fatigue regime. Therefore, substituting yield strength for a value based on shear stress was thought to be beneficial. This was based on the idea that the ratio of normal stress to shear stress could allow for better consideration of interaction effects between the two types of stresses.

Through some trial and error, it was found that replacing $\sigma_y$ in Equation (2.37) with $G\Delta\gamma$, where $\Delta\gamma$ is the shear strain range on the maximum shear strain plane, resulted in improved fatigue life correlations in the presence of mean stress. This yields the following equation for multiaxial fatigue damage calculation:
\[
\frac{\Delta \gamma_{max}}{2} \left( 1 + k \frac{\sigma_{n, max}}{G \Delta \gamma} \right) = \frac{\tau'}{G} (2N_f)^{b_o} + \gamma' (2N_f)^{c_o}
\] (5.9)

Shear stress range was expressed using \( G \Delta \gamma \) as opposed to \( \Delta \tau \), in order to account for the effect that changes in material constitutive behavior can have on fatigue damage. Although \( G \Delta \gamma \) is equal to \( \Delta \tau \) at longer lives, where deformation is elastic, normalizing \( \sigma_{n, max} \) by a quantity based on strain predicts an increase in damage when cyclic and/or non-proportional hardening occur. Normalizing \( \sigma_{n, max} \) by \( \Delta \tau \), on the other hand, would result in the same damage value in situations where the two stress components change proportionally as a material hardens or softens.

This modified damage parameter maintains all of the advantages and physical interpretations of the FS parameter, as discussed in Section 2.4, without introducing any additional empirical fitting constants. Additionally, from a physical standpoint, the reduction of the normal stress term with increasing shear stress/strain range may reflect the idea that as shear stress/strain increases, larger local shear deformations are able to overcome some of the resistance caused by friction and interlocking between opposing crack faces. The implementation of Equation (5.9) remains simple, requiring no additional information for fatigue damage calculation than the original FS parameter.

Although there is a possibility for unrealistically large damage values to be computed as the shear strain range approaches zero, this mathematical issue can be overcome by imposing appropriate limits on the shear strain range required to produce fatigue damage. For example, in situations where the shear strain range is below its fatigue limit value, the damage parameter can be assumed to take a value of zero. While this issue could also be avoided by multiplying the parameter out and simplifying
\[ \Delta \gamma_{\text{max}}/2 + k \sigma_{n,\text{max}}/(2G) \], this can lead to situations where fatigue damage is predicted in the absence of alternating shear strain.

Similar to the original FS parameter, the right-hand side of Equation (5.9), which relates the value of the damage parameter to fatigue life based on shear strain-life properties, can alternatively be expressed in terms of uniaxial fatigue life properties. To establish the uniaxial fatigue life curve, a Mohr’s circle analysis is first used to establish a relationship between the fully-reversed uniaxial loading, \( \sigma_a \), and stresses \( \sigma_{n,\text{max}} = \sigma_a/2 \) and strains \[ \Delta \gamma_{\text{max}}/2 = \gamma_{a,e} + \gamma_{a,p} = (1+\nu_e)\varepsilon_{a,e} + (1+\nu_p)\varepsilon_{a,p} \] on the maximum shear plane. Subscripts \( a, e, \) and \( p \) denote amplitude, elastic, and plastic quantities, respectively. The damage parameter can then be rewritten in terms of uniaxial loading using these relationships, \[ ((1+\nu_e)\varepsilon_{a,e} + (1+\nu_p)\varepsilon_{a,p})\left[1 + k \sigma_a/(4G(1+\nu_e)\varepsilon_{a,e} + (1+\nu_p)\varepsilon_{a,p})\right]. \] Finally, the uniaxial stress and strain quantities can be expressed in terms of fatigue life properties. After rearranging some terms, the following life prediction curve is derived for the modified FS parameter based on uniaxial strain-life properties:

\[
\frac{\Delta \gamma_{\text{max}}}{2} \left(1 + k \frac{\sigma_{n,\text{max}}}{G\Delta \gamma}\right) = \left[ (1 + \nu_e)\frac{\sigma_f'}{E} (2N_f) + (1 + \nu_p)\varepsilon_f' (2N_f) \right] + k \frac{\sigma_f'}{4G} (2N_f)^b \quad (5.10)
\]

where \( \nu_e \) is the elastic value of Poisson’s ratio, \( \nu_p \) is the plastic value of Poisson’s ratio (usually taken to be 0.5), and all other fatigue properties are taken from the fully-reversed uniaxial strain-life equation.

As mentioned previously, in cases where material deformation is linear elastic, expressing shear stress range as \( G\Delta \gamma \) will result in the same damage value that would be obtained using \( \Delta \tau \). Since the \( \Delta \tau \) version may be simpler to implement in certain
situations, the life prediction curve for this form of the modified parameter, based on uniaxial strain-life properties, is also presented as follows:

\[
\frac{\Delta \gamma_{\text{max}}}{2} \left( 1 + k \frac{\sigma_{n,\text{max}}}{\Delta \tau} \right) = \left[ (1 + \nu_e) \frac{\sigma'_f}{E} (2N_f)^b + (1 + \nu_p) \varepsilon'_f (2N_f)^c \right] \left[ 1 + \frac{k}{2} \right] \]  

(5.11)

By recalculating fatigue damage for the 7075-T651 and ductile cast iron fatigue test data, plotted against the original FS parameter in Figures 5.20 and 5.22, respectively, improvements offered by the modified FS parameter become evident. Figures 5.24 and 5.25 show the results of these calculations for 7075-T651 and ductile cast iron, respectively. Although the \(k\) values for the original and modified versions of the FS parameter are generally not related, a \(k\) value of 1 was also used with Equation (5.10) in all modified parameter damage calculations for both materials. Fatigue life correlations are not only qualitatively improved, with a tighter grouping of test data from all loadings conditions, but the overall accuracy of predictions is also increased. Additionally, comparisons between modified parameter calculations based on the inclusion of \(\Delta \tau\) versus \(G \Delta \gamma\) revealed that differences in life predictions for either version of the parameter were negligible above 1000 reversals, and within a factor of around 1.5 at lives on the order of 50 reversals.

5.4.2.3 Load Path Effects on Damage Calculation

In addition to improved mean stress consideration, multiaxial fatigue life correlations and predicted failure planes under fully-reversed loading conditions remain very similar to the already accurate predictions based on the original FS parameter. In fact, fully-reversed data correlations were even found to improve slightly using the
modified version of the parameter. This is especially evident in the comparison between fully-reversed uniaxial and torsion loading conditions in Figures 5.20 and 5.24.

To help illustrate this point, the theoretical value of constant $k$, which relates fatigue damage under these two loading conditions, can be computed as a function of fatigue life for both versions of the parameter. Equating uniaxial- and shear-based life prediction curves and solving for $k$ results in the following equations for the original FS parameter and the modified parameter, respectively:

$$k = \left[ \frac{\tau_f (2N_f)^{b_0} + \gamma_f (2N_f)^{c_0}}{(1 + v_e) \frac{\sigma_f}{E}(2N_f)^b + (1 + v_p) \epsilon_f (2N_f)^c} - 1 \right] \frac{2\sigma_y}{\sigma_f(2N_f)^b} \tag{5.12}$$

$$k = \left[ \frac{\tau_f (2N_f)^{b_0} + \gamma_f (2N_f)^{c_0}}{(1 + v_e) \frac{\sigma_f}{E}(2N_f)^b + (1 + v_p) \epsilon_f (2N_f)^c} - 1 \right] \frac{4G}{\sigma_f(2N_f)^b} \tag{5.13}$$

For comparison purposes, the theoretical value for $k$ can also be computed for the modified parameter, when substituting $\Delta \tau$ in place of $G\Delta \gamma$, as follows:

$$k = \left[ \frac{\tau_f (2N_f)^{b_0} + \gamma_f (2N_f)^{c_0}}{(1 + v_e) \frac{\sigma_f}{E}(2N_f)^b + (1 + v_p) \epsilon_f (2N_f)^c} - 1 \right] 2 \tag{5.14}$$

While there are other factors in addition to stress state which can affect the correlation of uniaxial and pure shear fatigue data (e.g. crack initiation definition through differences in crack growth behavior), if the damage parameters were able to perfectly account for these differences over the entire range of fatigue lives, the value of $k$ would remain constant.

Plots showing the variation of $k$ with life for several different materials are shown in Figures 5.26(a), 5.26(b), and 5.26(c) based on Equations (5.12), (5.13), and (5.14), respectively. Materials include the 2024-T3 aluminum alloy tested in the current study,
7075-T651 aluminum alloy (Zhao and Jiang, 2008), Pure Titanium and Titanium alloy BT9 (Shamsaei et al., 2010b), AISI 1141 vanadium-microalloyed steel (Fang, 2001), 1045 hot-rolled steel (Fatemi and Kurath, 1988), and Inconel 718 (Koch, 1985).

From these plots, it is evident that the variation in $k$ value is reduced at longer life, using both forms of the modified FS parameter, for the aluminum alloys and AISI 1141 steel alloy. For the other materials, however, the variation in $k$ at longer life does not change significantly depending on which form of the parameter is used. Although there is significantly more variation in $k$ at shorter lives for the $G\Delta\gamma$ version of the modified parameter, it is important to note that for uniaxial loading, as fatigue life decreases, shear strain range increases at a higher rate than normal stress due to the presence of plastic deformation. As a result, the ratio of normal stress to shear strain range reduces with life. Conversely, for the original FS parameter, this ratio increases as life decreases, and is constant with respect to fatigue life for the $\Delta\tau$ version of the modified parameter. Therefore, a larger $k$ value is required for the $G\Delta\gamma$ version of the modified parameter in order to have the same impact on fatigue damage correlations. Additionally, differences in fatigue damage values due to variations in $k$ do not have as large of an effect on fatigue life predictions in the low cycle regime as they do at longer life. As such, it is not surprising that the modified parameter resulted in improved correlation of the 7075-T651 uniaxial and shear fatigue data.

In order to further evaluate the characteristics of the proposed modified FS parameter under multiaxial loading conditions, a limited number of fatigue tests were also performed in this study using specialized load paths meant to differentiate between the two different versions of the parameter. The idea of these tests was to apply different
loading conditions to un-notched specimens which would result in the same damage value based on the original FS parameter, but different damage values based on the modified parameter. Then, differences in experimental lives were compared to predictions to determine which parameter more closely reflects the fatigue damage variation between loading paths. These, so-called, discriminating load path tests were conducted using a triangular shaped load path in shear versus axial stress space. This path, along with its corresponding stress-time history, is shown in Figure 5.27 based on normalized axial and shear stress variations.

In analyzing potential load paths, it was found that by changing the ratio of applied shear to axial stress, relatively large differences in predicted fatigue damage could be obtained between the original and modified FS parameters. Therefore, the same loading path was used in these tests, but with two different ratios of applied shear to axial stress, \( \lambda = 2 \) and \( \lambda = 0.5 \). Stress values were carefully selected so that each path would result in the same fatigue damage value on the maximum damage plane according to the FS parameter. A summary of the tests performed, two at each stress ratio, is included in Table 5.5. Additionally, experimental results from fully-reversed torsion tests with static tensile stress were also available for comparison at the same damage value. This additional loading path allows for the evaluation of damage predictions for cycles containing different load-time interaction between shear and normal stress components as well. Stress levels below yielding were chosen so that plasticity effects would not be a factor in damage calculations. The maximum von Mises equivalent stress was similar for all tests, with values ranging from 283 MPa to 287 MPa.
The results of the discriminating load path tests are shown in Figure 5.28 along with life predictions based on both the FS and modified FS parameters. Results from the triangular load path tests (Tri), at both nominal stress ratios, along with results from torsion with static tension tests (STSA), are included. A $k$ value of 1 and uniaxial fatigue life properties were used with both damage parameters in all analyses. In this figure, gray columns represent the average experimental fatigue life from two duplicate tests, while error bars indicate the individual life of each test. From these results, it is clear that variations in experimental fatigue damage exist between the different loading conditions. While the FS parameter, in its original form, does not account for these differences, the modified form of the parameter reflects the observed differences in fatigue life relatively well for each loading path considered. This supports the idea that including the ratio of normal stress to shear stress in the damage parameter can allow for better consideration of at least some of the interaction effects between the two types of stresses.

Given the improvements in life predictions obtained when using the modified FS parameter to calculate fatigue damage, as evidenced for the wide variety of loading conditions and the various materials presented thus far, one final step was to analyze the remaining fully-reversed fatigue data generated for the 2024-T3 aluminum alloy tested in this study. These data, plotted against the original FS parameter in Figures 5.17 and 5.18, are presented again in Figures 5.29 and 5.30 with damage calculated based on the modified FS parameter. Figure 5.17 and 5.29 show life predictions and correlations based on uniaxial strain-life properties, while Figure 5.8 and 5.30 show predictions and correlations based on shear strain-life properties.
Similar to the two materials from literature, it can be seen from these figures that the modified FS parameter provides somewhat improved fatigue life correlations for the 2024-T3 aluminum alloy as well. This is again evidenced by the slightly tighter grouping of test data from all loadings conditions considered. Overall life prediction accuracy, however, was similar to that obtained using the original form of the FS parameter. When based on uniaxial fatigue properties, and $k = 1$, 88% and 100% of predictions (excluding axial data) were within factors of ±3 and ±10 of experimental results, respectively. For life predictions based on shear properties (excluding pure shear data), 50% and 97% were within factors on ±3 and ±10, respectively.

The slight conservative tendency of these predictions (and the offset between pure shear data and the shear strain-life curve) is due to fact that the life prediction curves are based on damage values on the maximum shear plane. Since fatigue damage was computed on the maximum damage plane in these analyses, damage values are slightly higher and result in shorter life predictions for most loading conditions. Additionally, since the difference between the maximum shear plane and maximum damage plane is slightly larger based on the modified FS parameter, this conservative tendency is more pronounced for life predictions based on this version of the parameter. Again, a summary of 2024-T3 fatigue life predictions for all loading conditions and analysis approaches considered, along with the percentage of predictions within scatter bands of ±3 and ±10, is provided in Table 5.8.

5.4.2.4 Cracking Orientation Comparisons

Aside from life predictions, the discriminating load path tests also allow for the evaluation of damage parameters in terms of predicted failure planes. While maximum
shear and damage planes agree well with experimental cracking observations for the loading paths discussed in the previous section (i.e. torsion, in-phase axial-torsion, 90° OP axial-torsion), the discriminating load path tests provide more unique distributions of damage versus plane orientation which can provide additional insight on damage mechanisms.

The variations of shear strain amplitude, maximum normal stress, FS parameter, and modified FS parameter with plane orientation are plotted in Figures 5.31(a), 5.32(a), and 5.33(a) for the torsion with static tension loading, and for the triangular load paths with nominal stress ratios of $\lambda = 0.5$ and $\lambda = 2$, respectively. As seen in these figures, the triangular path at $\lambda = 2$ and the STSA loading result in two distinct maximum shear planes ($0^\circ$ and $90^\circ$). However, because of the maximum normal stress variation, only one of these planes produces the maximum value of damage parameter on planes oriented at $\pm 7^\circ$, for the FS parameter, and $\pm 11^\circ$ for the modified FS parameter. On the contrary, there are several planes at or near the maximum shear strain amplitude for the triangular path at $\lambda = 0.5$, and the overall variation in shear strain amplitude is much smaller. Due to the normal stress variation, however, still only two maximum damage planes exist for this loading condition. They are at orientations of $\pm 30^\circ$ for the FS parameter, and $\pm 28^\circ$ for the modified FS parameter.

Experimental crack paths for the STSA tests, and for the triangular load path tests with nominal stress ratios of $\lambda = 0.5$ and $\lambda = 2$, are shown in Figures 5.31(b), 5.32(b), and 5.33(b), respectively. For the STSA loading, an overall crack orientation of approximately $0^\circ$ agrees well with predictions for both specimens tested. Although the earlier crack replicas of specimen TM96-STSA show a short section of vertical ($90^\circ$)
growth, this is the result of an inclusion from which the crack initiated. All other regions of crack growth took place in a planar manner on the horizontal 0° plane.

Similarly, for the triangular load path at $\lambda = 0.5$ (Figure 5.32), overall crack orientations were observed to be close to the predicted maximum shear/damage planes, with macroscopic crack orientations of approximately 25° for both specimens. Nucleation and small crack growth under this loading condition, however, were observed on several different planes, with cracks on the order of 1-2 mm showing a preference for the 0° plane. While maximum shear planes exist at orientations of 32°, 58°, 122°, and 148°, all planes experience at least 80% of the maximum shear strain amplitude. Additionally, the applied normal stress results in a damage value near the 0° plane that is around 90% of the maximum damage value predicted for both versions of the FS parameter. Therefore, the discrepancy between experimental and predicted small crack orientations is not alarming, as there is a high probability of failure on the experimentally observed crack planes as well. This is especially true when considering variations in stress-strain behavior that can exist on each plane at the microstructural level due to material inhomogeneity.

For the last loading condition, the triangular path with $\lambda = 2$ (Figure 5.33), the shear strain and damage distributions are similar to those for STSA loading. While maximum shear strain planes exist at orientations of both 0° and 90°, the maximum normal stress distribution results in higher damage predictions near the 0° plane. Unlike the STSA tests, however, cracks were not observed to nucleate and grow on this plane in experiments. Rather, cracks were consistently found, in all growth regimes, on planes at the 90° orientation. This is despite the fact that the predicted fatigue damage on this
plane, using both forms of the FS parameter, is only around 80% of that experienced on the 0° plane. Similar observations were also reported by Fatemi and Socie (1988) for tests performed on 1045 steel using 90° OP sinusoidal waveforms (elliptic stress path) at the same nominal stress ratio.

There are a few key points that can be taken away from these crack orientation comparisons. The first is the observation that, despite the limitations of performing maximum shear plane damage analyses in variable amplitude loading situations, experimentally observed crack orientations tend to be more closely aligned with maximum shear planes as opposed to maximum damage planes. This is evidenced by the STSA tests where, although maximum damage planes deviate 7° to 11° from maximum shear planes (depending on parameter), cracks were observed to be nearly perfectly horizontal (0°). For this loading condition, however, the 0° plane experiences at least 94% of the maximum predicted fatigue damage, thus leading to a high probability of failure on this plane as well.

Additionally, given the relatively small variation in shear strain amplitude versus plane orientation, the variety of crack nucleation planes observed in Figure 5.32 for the triangular path with \( \lambda = 0.5 \) suggests that normal stress may have less influence on shear-mode crack nucleation/growth when cracks are on the order of a few hundred microns. This is consistent with the findings reported in (Socie and Schield, 1984) and, again, seems to agree with small crack growth on maximum shear planes, rather than maximum damage planes, observed for other loading paths. As the cracks grow, however, the effect of the normal stress acting on maximum shear strain planes seems to have the tendency to turn cracks onto the higher damage planes predicted by the FS parameter.
Finally, the difference in cracking orientation observed between the triangular load path tests at $\lambda = 2$ and the STSA tests could suggest a number of things. The most obvious take away from the triangular load path tests is that although failure occurred on a maximum shear plane, unlike the STSA tests, the maximum applied normal stress on the $0^\circ$ plane did not cause a preference for cracks to grow in this direction. The primary differences between these two loading conditions are the smaller applied normal stress, and its fully-reversed cyclic nature, in the triangular load path tests. Therefore, it is possible that normal stress magnitude, via a threshold condition, and/or the inclusion of compressive stresses in a loading cycle could contribute to the fatigue damage, or lack thereof, experienced on any given plane.

For example, the idea behind the maximum normal stress term in the FS parameter is that tensile stresses help to open shear cracks and accelerate their growth. However, if the normal stress is low enough (only 79.5 MPa for the $\lambda = 2$ triangular load path), it may not be sufficient to overcome local crack interaction effects, and the crack may remain closed throughout the entire loading cycle. This could then lead to a situation where cracks on both the $0^\circ$ and $90^\circ$ planes are subjected to very similar driving forces. In such a case, the pure torsion tests performed in this study have shown that cracks almost always prefer to grow on the $90^\circ$ maximum shear plane (Figure 5.16(a)). Although the exact reason for this preference is unknown, it may be due to some degree of anisotropy introduced by the drawing process during the production of the specimen tubes. Therefore, if there is only a small effect of normal stress on the $0^\circ$ plane, it may not be enough to overcome the preferred growth on the $90^\circ$ plane.
Additionally, the effect of compressive normal stress in a cycle is not considered in the FS parameter as long as the maximum normal stress is positive. While the STSA loading path features a constant applied tensile stress to assist in shear-mode crack growth, the compressive normal stress applied in the triangular load path may serve to counteract all or part of this effect by increasing friction between the crack faces and reducing the effective crack driving force. In turn, this would lead to a decrease in fatigue damage by restricting shear-mode crack growth during the compressive portion of the normal stress cycle. In this type of situation, the actual amount of fatigue damage experienced by the 0° and 90° planes may, in fact, be very similar. This type of behavior could also help to explain the decrease in life prediction accuracy observed for tests with $R$ ratios less than $-1$ (compressive mean stress) in the ductile cast iron data presented in Figure 5.25. The conservative nature of these predictions supports the idea that compressive normal stresses reduce fatigue damage to a greater extent than reflected in the FS parameter. More in depth analysis and discussions concerning the effects of normal stress on shear mode crack growth are presented in Section 7.2.

It should also be noted that a similar effect due to the time/load path dependence of normal-shear stress/strain interaction is suggested by the results of Wu and Yang (1987) and Shamsaei (2010), as discussed at the beginning of Section 5.4.2. In the tests performed by Shamsaei, the constant applied tensile stress in the STSA load path acts to reduce the effects of crack face friction and interlocking over the entire shear strain cycle. As a result, reduced resistance to shear-mode damage mechanisms can be used to explain the shorter fatigue life observed for this test. From Figure 5.19, on the other hand, it can be seen that the tensile stress is applied independently from the shear strain in the
pulsating tension load path. In this case, the tensile stress does not have a direct influence on crack face interaction and shear-mode damage mechanisms, which would lead to an increase in fatigue life. Therefore, the difference in experimental life observed for these two loading paths makes sense from a physical standpoint.

Similarly, in the tests performed by Wu and Yang (1987), for the entire shear strain cycle applied in path A, the normal stress/strain is at its maximum value. As a result, the tensile stress can act to reduce the effects of crack face friction and interlocking during the entire shearing process, thus leading to an increase in fatigue damage. For path B, on the other hand, the normal stress/strain is only present when the shear strain is at its maximum value, and not when the actual shearing processes are occurring. Therefore, it makes sense from a physical standpoint, again, that fatigue lives observed for the path B tests were always longer than those for the path A tests. The FS parameter, however, in either its original or modified form, does not account for these time dependent interaction effects. As a result, neither parameter is able to reflect the experimentally observed differences in fatigue life for these loading paths.

While much more data and analysis would be required to adequately investigate the effects of time dependent normal-shear stress/strain interaction on fatigue damage, these limited results suggest that this is an area where improvements are still needed with respect to damage calculation. However, until such issues are resolved, both literature data and experimental data generated in this study show that the modified FS parameter is capable of producing accurate fatigue life predictions under a wide variety of multiaxial loading conditions which are relevant to the variable amplitude fatigue life analyses discussed in the next chapter.
5.4.3 Notched Specimen Life Predictions

Considering the drawbacks and deficiencies of the equivalent stress- and strain-based approaches, and the improvement in life predictions observed for un-notched fatigue data, the notched fatigue data generated in this study were also analyzed using a critical plane-based approach. Since it was shown that the Fatemi-Socie shear-based damage parameter resulted in better correlation of un-notched fatigue data than the other critical plane parameters investigated, it is the only parameter considered in this section. Analysis was performed using the FS parameter in its original form, as well as with the modified parameter proposed in the previous section.

5.4.3.1 Comparison of Analysis Methods

Given the importance of considering the effects of stress gradients on fatigue damage, as demonstrated in Section 5.3.2, both fatigue notch factor and Theory of Critical Distances approaches were evaluated for the critical plane analyses. When implementing the TCD approach to account for stress gradient effects, both the point method and line method were considered. The impact on multiaxial fatigue life predictions was also studied with respect to the two different interpretations of critical distance path orientation discussed in Section 2.3.4. Additionally, in order to assess the general applicability of these approaches, fatigue data from the AISI 1141 notched shaft specimens discussed in Section 4.8.2 (Fang, 2001) were also analyzed.

The goal of this current section was to evaluate and compare the ability of the different stress gradient models to correlate experimental fatigue data together under a variety of loading conditions. Therefore, since the focus is not necessarily on overall life prediction accuracy, only the original FS parameter was used in the following analyses.
for the sake of brevity. All life predictions are based on uniaxial fatigue properties computed using $k$ values of 1 and 0.5 for the 2024-T3 and AISI 1141 analyses, respectively. A summary of relevant deformation and fatigue properties for the AISI 1141 material can be found in Table 5.9. After obtaining appropriate local stress-strain values, stress and strain transformations were again performed in $5^\circ$ increments to identify the plane experiencing the largest amount of fatigue damage. Once the best life prediction methods were established, life predictions were then performed for the 2024-T3 notched specimen fatigue tests using both forms of the FS parameter, based on both uniaxial and shear fatigue properties. The results of these latter analyses, however, are presented in the following section.

- **Neuber’s Rule Life Predictions**

  Similar to the equivalent stress- and strain-based analyses presented in Section 5.3.2, critical plane life predictions were first performed based on local stresses and strains derived using Neuber’s rule in conjunction with both equivalent stress concentration factor, $K_{\sigma}$, and fatigue notch factor, $K_{f\sigma}$. Equivalent stress concentration factors were defined in terms of von Mises stress and were computed from local elastic stress components superimposed for each nominally applied loading (i.e. axial and torsion). Since the local stress state at the notch root for the 2024-T3 notched tubular specimen is uniaxial regardless of the nominally applied loading conditions, Neuber’s rule was applied, in the form of Equation (2.24), without the need for additional relations to compute individual stress and strain components. For the AISI 1141 notched shaft specimens, however, principal stresses and strains were computed from equivalent stress and strain using the assumptions of Equations (2.25) and (2.26). Once appropriate stress
concentration factors were known for each nominal loading condition, local elastic-plastic notch root stresses and strains were computed for each test performed. Fatigue life predictions based on these stress and strain values are shown in Figure 5.34(a) for the 2024-T3 specimens, and in Figure 5.35(a) for the AISI 1141 specimens. Scatter bands shown are for factors of ±3 and ±10.

Next, to consider stress gradient effects using the traditional application of Neuber’s rule, fatigue notch factor, $K_{f(q)}$, was used in place of $K_{t(q)}$ in Equation (2.24). For the 2024-T3 specimens, $K_{f(q)}$ was computed based on Equation (2.28) for each respective equivalent stress concentration factor based on a notch root radius, $r$, of 1.6 mm and a material characteristic length, $\rho$, of 0.5 mm. Similarly, for the AISI 1141 specimens, $K_{f(q)}$ was computed based on a material characteristic length of 0.054 mm and notch root radii of 0.4 mm and 0.79 mm for the stepped and grooved geometries, respectively. The material characteristic length was estimated based on ultimate tensile strength using Equation (2.29).

Although values for $\rho$ generally can vary depending on the nominally applied stress state, because the local stress state at the notch is always uniaxial for the 2024-T3 tubular specimen used in this study, a constant value was assumed. Additionally, because the ultimate goal is to apply these findings to multiaxial variable amplitude fatigue life analyses, where calculation of a multiaxial $\rho$ value is difficult due to the possibility of a continuously changing local stress state under non-proportional loading conditions, the value of $\rho$ was assumed to be constant for the AISI 1141 notched shaft specimens as well.

Similarly, although the value of $K_{f(q)}$ generally increases (becomes closer to $K_{t(q)}$) as fatigue life decreases, a result of decreasing stress gradients at the notch due to localized
yielding, $K_{f_q}$ was considered constant for the analyses performed in this study. Without a function describing the variation of $K_{f_q}$ with fatigue life or nominally applied loading level, this variation cannot be accounted for in a local stress/strain-based fatigue life analysis. Additionally, such a function would change depending on the nominally applied stress state. Therefore, similar to $\rho$, calculation of a variable $K_{f_q}$ value would be difficult when varying degrees of loading non-proportionality exist. Once the constant $K_{f_q}$ was determined for each nominal loading condition, local elastic-plastic notch root stresses and strains, along with corresponding fatigue damage values, were computed. The resulting fatigue life predictions are shown in Figure 5.34(b) for the 2024-T3 specimens, and in Figure 5.35(b) for the AISI 1141 specimens.

By comparing 2024-T3 notched tubular specimen life predictions in Figure 5.34(a) to those in Figure 5.34(b), it is easy to see the impact that stress gradient consideration can have on fatigue life predictions. While life predictions are consistently conservative by a factor of 3 to 10 when based on maximum stress/strain values (22% and 83% of predictions are within factors of $\pm 3$ and $\pm 10$, respectively), nearly all predictions (89%) improve to within a factor of $\pm 3$ of experimental results when using $K_{f_q}$ to account for gradient effects. For the AISI 1141 notched shaft specimen results, shown in Figure 5.35, although the difference between life predictions based on $K_{f_q}$ and $K_{f_q}$ are not as pronounced, incorporating fatigue notch factor in local stress-strain analyses still improves life predictions from 18% and 53% of data within factors of $\pm 3$ and $\pm 10$ of experimental results (Figure 5.35(a)), respectively, to 41% and 76% of data within factors of $\pm 3$ and $\pm 10$ (Figure 5.35(b)). It should also be noted that had a variable $K_{f_q}$ been considered in these analyses, fatigue life predictions at shorter lives would have been
between results based on $K_{tq}$ and constant $K_{jq}$. Therefore, the consideration of a variable $K_{fq}$ would not have significantly changed the results of these analyses. Summaries of fatigue life predictions for all loading conditions and analysis approaches considered, along with the percentage of predictions within scatter bands of ±3 and ±10, are provided in Tables 5.10 and 5.11 for the 2024-T3 notched tubular and AISI 1141 notched shaft specimens, respectively.

It is important to note that while life predictions for the AISI 1141 specimens tend to be more accurate at shorter life, Neuber’s rule was found in Section 4.8.2 to underpredict stress-strain values for these specimens at higher load levels. This is especially true for the pure torsion loading conditions. Therefore, the derivation of more accurate local stress-strain estimations would have led to shorter life predictions at these loading levels, and produced conservative trends more consistent with those observed at longer lives.

While fatigue notch factor is easy to implement in cases where nominal and local stress states are easily defined, as mentioned in Section 2.3.4, this approach has a number of drawbacks in more complex loading situations. Namely, for components where either the nominal stress is not clearly defined, the notch root radius is considered to be zero, or for materials where the empirical characteristic length constant is not available, the application of these equations becomes challenging. Additionally, for multiaxial non-proportional cyclic loadings, where the combined stress concentration effect from each applied loading can continuously vary throughout a cycle, the definition of the fatigue notch factor can become vague.
Aside from these potential difficulties in determining an appropriate fatigue notch factor, a stress-strain analysis performed using Neuber’s rule is typically based on the location at the notch experiencing the maximum stress and strain values. However, as mentioned previously, for the 2024-T3 notch tubular specimen used in this study, this would result in the consideration of a uniaxial stress state regardless of the nominally applied loading. In reality, as the distance from the edge of the hole is increased, the local stress state becomes multiaxial. Therefore, a stress gradient approach such as fatigue notch factor cannot account for changes in stress state for different loading conditions within a small volume of material surrounding the notch root.

One specific example of this type of effect is shown in Figure 5.36. This figure compares fatigue lives for 2024-T3 notched tubular specimens subjected to the same nominal axial stress amplitude under uniaxial and 90° OP loading conditions. Since the stress concentration effect is at its maximum for the axial channel of the 90° OP loading cycle, the predicted and experimentally observed failure location for this loading condition is the same as for uniaxial loading (0° location). Additionally, since the stress concentration effect due to the nominal shear stress is zero at this location, the variation of stresses at the notch root is only affected by the applied axial stress. Therefore, for the same nominally applied axial stress amplitude, the local stress variation, and thus fatigue life prediction (regardless of damage parameter), is identical for axial and 90° OP loading conditions based on a Neuber’s rule analysis. However, Figure 5.36 shows that experimentally observed fatigue lives for 90° OP notched specimen tests are consistently shorter than those for axial loading at the same nominal stress amplitude. Average
differences in life are a factor of 2.3 for the 115 MPa nominal stress level and 7.6 for the 100 MPa nominal stress level.

In contrast to a typical Neuber’s rule analysis, an approach such as the Theory of Critical Distances is advantageous in that it considers stress gradient effects by evaluating stress-strain values a short distance away from the edge of the hole. As a result, for the example presented in Figure 5.36, there would be a non-zero effect from nominally applied shear stress on the local stress variation at the 0° location around the hole. Therefore, fatigue life predictions for axial and 90° OP loading conditions would be different, and such an approach would be expected to better reflect changes in fatigue damage due to the presence of local multiaxial stress states. Thus, a major goal of this study was to evaluate the accuracy of TCD approaches, as well as potential challenges in their implementation, when applied to multiaxial loading situations for various component geometries.

- Theory of Critical Distances: Modeling Procedures and Sensitivity Analysis

To derive local steady-state elastic-plastic stresses and strains for use with the TCD approaches, the pseudo stress approach, described in Section 4.8, was implemented for the 2024-T3 specimen. This approach was used for fatigue life calculation due to the excellent agreement demonstrated between predicted and FEA derived local stresses and strains at the loading levels relevant to the testing program. For the AISI 1141 notched shaft specimens, however, nonlinear FEA stress-strain solutions were used in fatigue life analyses due to the error in stress-strain predictions observed for the higher loading levels applied to these specimens.
The characteristic length, $L$, which is the fundamental quantity in the application of TCD methodologies, was calculated from Equation (2.30) to be approximately 0.350 mm for the 2024-T3 aluminum. This is a rounded number based on a fatigue limit stress of 100 MPa ($\Delta\sigma_o = 200$ MPa) and a mode I threshold stress intensity factor range of $\Delta K_{th} = 6.5$ MPa$\sqrt{m}$. Since the uniaxial fatigue data generated for the un-notched specimens in this study do not extend past $10^6$ cycles (excluding runout data), this fatigue limit stress was based on experimental data generated by Mayer et al. (2013) in the very high cycle fatigue regime (up to $10^{10}$ cycles). The threshold SIF, on the other hand, was derived from the crack growth data generated in this study for both notched tubular and plate specimens tested under fully-reversed uniaxial loading conditions (Section 7.3). Only the tensile portion of the loading cycle was used in computing the threshold SIF range.

Although Equation (2.30) was also used, with respect to the Kitagawa-Takahashi diagram, to compute the definition of notched specimen crack initiation for the tests performed in this study, the value of $L$ calculated here differs from the 0.2 mm initiation length. This is due to a lack of experimental data at the time when crack initiation definition was determined. Subsequent testing and literature review have resulted in updated material properties which should more accurately represent crack growth behavior. Since crack initiation definition can be rather arbitrarily defined, however, this difference does not cause any conflicts in subsequent fatigue life analysis results.

Similarly, for the AISI 1141 steel alloy, $L$ was estimated to be 0.013 mm based on a fatigue limit stress of 292 MPa ($\Delta\sigma_o = 584$ MPa) and a mode I threshold SIF range of $\Delta K_{th} = 3.79$ MPa$\sqrt{m}$. The fatigue limit stress was calculated from uniaxial fatigue properties at a life of $10^7$ cycles. The threshold SIF, on the other hand, was taken as the
value reported in (Fatemi et al., 1996) for uniaxial crack growth tests of the same material, performed on compact tension specimens in the LT orientation, under $R = 0$ loading conditions.

As mentioned in the Section 2.3.4, although it was shown that the critical distance value is generally smaller under uniaxial loading than under pure shear loading, Susmel and Taylor (2006) found that assuming a constant value of $L$, equal to that under uniaxial loading, resulted in fatigue life prediction errors mostly within a factor of $\pm 25\%$ (for both point and line methods). Therefore, they concluded that the constant $L$ assumption was reasonable. This, again, has significant implications in multiaxial variable amplitude fatigue life analyses where the possibility exists for a continuously changing local stress state. Additionally, calculating $L$ for AISI 1141 based on a limited amount of torsion fatigue and crack growth data produced an almost identical $L$ value (0.012 mm) to that calculated using uniaxial fatigue properties.

To demonstrate the sensitivity of the computed value of $L$ to the variation of material property inputs, the reader is referred to Figure 5.37. This figure shows the value of $L$ resulting from several combinations of fatigue limit stress range and threshold SIF range. From this figure, it can be seen that in the range of interest for many metals, a change in $\Delta K_{th}$ of 1.0 MPa$\sqrt{m}$ can cause a change in critical distance value of anywhere from around 0.002 to 0.2 mm, depending on the fatigue limit strength of the material. Similarly, a change in $\Delta \sigma_o$ of just 50 MPa can change the critical distance value anywhere from around $<0.001$ to 0.4 mm, depending on the threshold SIF range. Given this fact, the sensitivity of fatigue life predictions to the variation of $L$ was studied based on analyses performed using the point method. It should be noted, however, that the effect of
on fatigue life predictions is dependent on the stress gradients surrounding a notch. Therefore, such a comparison will vary depending on specimen and notch geometry, as well as the applied loading conditions and the degree of local plasticity. Nonetheless, performing such comparisons for tests performed under a variety of loading conditions gives an idea of the sensitivity of the TCD approach for the particular materials and notch geometries of interest in this study.

To evaluate the effect of $L$ on fatigue life predictions, analyses were performed for all 2024-T3 notched specimens tested in this study using critical distance values of 0.300 mm, 0.350 mm, and 0.420 mm. Using the point method, local stress and/or strain values are considered at a distance of $L/2$ from the maximum stress location. This means that stress concentration factors for the following analyses were evaluated at distances of 0.150 mm, 0.175 mm, and 0.210 mm from the notch root, respectively, for each applied loading channel (i.e. axial and torsion). Then, these stress concentration factors for each local stress component were used to scale the nominally applied loading, so that local plasticity corrections could be performed using the pseudo stress approach described in Section 4.8. With local stresses and strains known, fatigue life was then estimated using the FS damage parameter. The results of these analyses for critical distance values of $L = 0.300$ mm and $L = 0.350$ mm, in terms of experimental versus predicted fatigue life, are presented in Figures 5.38(a) and 5.38(b), respectively. Results for the $L = 0.420$ mm analyses are presented in Figure 5.39(a).

From these figures, it can be seen that while the overall correlation of data from different nominal loading conditions does not change significantly, the accuracy of life predictions shifts with a change in $L$. As expected, an increase in $L$ results in more non-
conservative life predictions, as the magnitude of local stress decreases when moving away from the notch root. By comparing Figures 5.38(a) and 5.39(a), it can be reasonably concluded that for the aluminum specimens used in this study, a change in \( L \) of 0.120 mm will produce an overall shift in fatigue life predictions of just under a factor of 3. Such a change corresponds to a variation in fatigue limit stress range of around 30 MPa or a variation in threshold SIF range of around 1 MPa√\( m \). This emphasizes the sensitivity of the TCD approaches to changes in material properties inputs.

A similar comparison for the AISI 1141 notched shaft specimens is shown in Figure 5.40 for critical distance values of \( L = 0.013 \) mm and \( L = 0.400 \) mm. Unlike the tubular specimen, where changing the critical distance value did not significantly affect the correlation of data from different nominal loading conditions, it is observed in this figure that fatigue life predictions for nominal torsion and in-phase axial-torsion loading conditions tend to diverge as the value of \( L \) is increased. While this is true for both notched shaft geometries, data from the different geometries are correlated fairly well for the same nominal loading condition. While the tubular specimen always produces a uniaxial or uniaxial-dominated stress state near the notch root, the shaft specimens result in a pure shear local stress state for nominal torsion loading, and a multiaxial stress state for in-phase nominal loading. This suggests that while the critical distance length may be selected independent of specimen geometry, the proper \( L \) value may have at least some dependence on the local stress state. Similar to fatigue notch factor calculations, however, due to the possibility of a continuously changing local stress state under non-proportional multiaxial variable amplitude loading conditions, determining appropriate \( L \) values under
such conditions is difficult, and the critical distance is often assumed to remain constant for the sake of simplicity.

It is also worth noting that while a 40% increase in $L$ resulted in a shift in fatigue life predictions of around a factor of 3 for the 2024-T3 tubular specimen, a 30 times increase in $L$ was required to achieve a similar shift for the AISI 1141 specimens. This is likely due to large local stress concentrations in the small radii notched shaft specimens. Large local stresses result in the reduction of local stress gradients due to significant localized yielding and stress redistribution, especially as the loading level is increased. Additionally, even at distances from the notch root of 0.400 mm and greater, where local stresses and strains begin to approach nominal values, fatigue life predictions are conservative for the notched shaft specimens. This so called “notch strengthening” behavior, where notched specimen fatigue lives are greater than smooth specimen lives at the same nominal stress levels, has also been reported for similar specimen geometries (featuring small notch root radii) in other studies (e.g. (Berto et al., 2011; Tanaka, 2012)) and has been attributed to differences in crack growth mechanisms between sharp and blunt notches.

- Theory of Critical Distances: Life Prediction Results

Knowing the influence of the critical distance value on resulting fatigue life predictions, more meaningful comparisons can now be made between different interpretations of the TCD methods. The first of these comparisons was made between the point method and line method. In contrast to the point method, the line method averages stress and/or strain components through integration of their variation along a linear path, of length $2L$, starting at the maximum stress location and moving away from
the notch. When comparing the two methods for the 2024-T3 specimen, life predictions were computed using an assumed critical distance value of $L = 0.420$ mm. This means that for the line method, stress concentration variation was integrated along a straight path extending 0.840 mm outward from the notch root, for each stress component, prior to performing local plasticity corrections. A comparison of experimental versus predicted fatigue lives using the line method is presented in Figure 5.39(b). Discussion of these results is provided later in this section along with comparisons to other TCD interpretations. Again, the same comparison was also performed for the AISI 1141 steel specimens. For these specimens, however, at the critical distance value of 0.013 mm, differences in life predictions between the point and line methods are negligible. Therefore, comparison plots are not shown for the sake of brevity, although the resulting fatigue life predictions are included in Table 5.11.

To this point, all notch fatigue analyses performed using TCD concepts have assumed that the path along which both the point and line methods are evaluated is a straight path extending outward from the maximum stress location at the notch root in a direction perpendicular to the notch curvature. However, as discussed in the Section 2.3.4, there are different interpretations for what the most correct orientation of this path should be. This first interpretation (perpendicular to the notch curvature) is the easiest to implement, but doesn’t necessarily reflect experimentally observed crack initiation mechanisms. For example, Figure 5.41 shows that although cracks were observed to grow macroscopically in a direction perpendicular to the notch curvature (mode I) for the 2024-T3 aluminum specimens tested in this study, for around the first 100 μm or so of growth, cracks are orientated along planes ±45° to the curvature normal. Given this fact,
the alternative TCD interpretation is to evaluate the point and line methods along a path starting at the maximum stress location and moving outward at an angle aligned with planes of maximum shear stress. Doing so may be able to better represent the stress variation for the path along which a crack is likely to nucleate, and could potentially result in improved fatigue life correlations between various multiaxial loading conditions.

In order to evaluate the influence of critical distance path on resulting fatigue life correlations, the maximum shear path interpretation was used, in conjunction with the point method, to reanalyze the 2024-T3 notched fatigue data generated in this study. The critical distance value used was again chosen as $L = 0.420$ mm. For each loading condition, the maximum shear path resulting in the largest fatigue damage value at the analysis location (if not symmetric) was chosen as the critical distance path. For the tubular specimen, because of the boundary conditions of the hole, the stress state around its perimeter is always uniaxial under plane stress conditions. This means that no matter the maximum stress location around the hole, which will vary depending on nominally applied loading conditions, the maximum shear planes are always at ±45° angles to the curvature normal direction. This is illustrated in Figure 5.42, where maximum shear path, AC, is at a 45° angle to path AB (normal to the notch curvature). For the specific analysis location (point A) shown in this figure, the stress distribution is symmetric about path AB. Therefore, if path AC were chosen at an angle of −45° to path AB, the same fatigue damage values would be predicted.

The influence of the stress analysis location on resulting stress concentration tensors is shown in Figure 5.42 at the 0° hole location for unit applied nominal stresses. The distance from point A in the figure to point B and from point A to point C is 0.210
mm. It is clear from this figure that even small changes in analysis location can have a large impact on the resulting local stress history in terms of both stress state and stress magnitude. For uniaxial nominal loading, the von Mises equivalent stress is only 70% and 76% of its maximum value (at point A) for points B and C, respectively. This reduction effectively accounts for stress gradient effects, but similar to variations in $L$, the difference in magnitude of the reduction, depending on the chosen critical distance path, can largely affect the accuracy of fatigue life predictions for the same critical distance length. For a nominal torsion load, on the other hand, it can be seen that the von Mises equivalent stress is greater at point B than point C. The fact that these differences are opposite for nominally applied axial and torsion loading can have a potentially large impact on the correlation of fatigue life data between axial and 90° OP loading conditions, as discussed earlier, depending on which critical distance path is selected for the analysis.

The fatigue life predictions resulting from the maximum shear path analyses are presented in Figure 5.39(c). Since the magnitude of the stress concentration effect at the analysis location will have an impact on the overall accuracy of fatigue life predictions, the primary focus of subsequent comparisons is how well fatigue data from different multiaxial loading conditions are correlated together. From this figure, which can be directly compared to Figures 5.39(a) and 5.39(b), it can be seen that the overall correlation between predictions for different loading conditions is similar. Therefore, it is difficult to tell visually which method produces the best results. In order to provide a quantitative means of comparison, a best fit line was generated for all experimental versus predicted fatigue life data. Then, a coefficient of determination ($R^2$ value) was
calculated to serve as an indication of the quality of the fit. As a basis for comparison, $R^2$ values were also generated for life predictions based on Neuber’s rule. For the 2024-T3 notched fatigue data, $R^2$ values of 0.873, 0.892, 0.906, and 0.910 were obtained for Neuber’s rule (with $K_{fq}$), point method (normal to curvature), line method (normal to curvature), and point method (along maximum shear plane), respectively. A summary of $R^2$ values for all analysis approaches considered is given in Table 5.10 for the 2024-T3 notched fatigue data.

For the AISI 1141 notched shaft specimens, analyses based on the maximum shear path interpretation of the point method were not performed. Unlike the notched tubular specimens, where the maximum shear plane is always 45° to the notch curvature normal at the maximum stress location, the orientation of the maximum shear plane for the notched shaft specimens changes with the nominally applied loading conditions. Therefore, similar to the dependence of $L$ on local stress state, determining an appropriate critical distance path orientation under multiaxial variable amplitude loading conditions would be a difficult task. Additionally, given the three-dimensional stress states that can exist due to local plane strain conditions in the notched shaft specimens, several additional maximum shear planes would need to be analyzed in order to find the one producing the greatest fatigue damage.

Nonetheless, quantitative comparisons for multiaxial fatigue data correlation were still made for the AISI 1141 specimens based on life predictions computed using Neuber’s rule and the TCD point and line methods. For these comparisons, $R^2$ values were generated using the same methodology used for the 2024-T3 life predictions. For the AISI 1141 fatigue data, $R^2$ values of 0.850, 0.934, and 0.931 were obtained for
Neuber’s rule (with $K_n$), point method (normal to curvature), and line method (normal to curvature). TCD analysis results used in these comparisons were based on a value of $L = 0.013$ mm. The point method analyses performed at $L = 0.400$ mm, used to evaluate the sensitivity of life predictions due to variations in $L$, resulted in an $R^2$ value of 0.819. Due to the divergence of life predictions for different nominal loading conditions at longer lives, this is considerably worse than the correlations observed for the other analyses. However, $L = 0.400$ mm is significantly greater than the value calculated based on the AISI 1141 material properties. As such, it is likely not an accurate representation of the ability of TCD concepts to correlate multiaxial fatigue data and will be disregarded in subsequent discussions. Again, a summary of all fatigue life predictions and $R^2$ values is given in Table 5.11 for the notched shaft specimens.

- Theory of Critical Distances: Discussion of Results

Based on these results, it can be concluded that all of the TCD interpretations considered resulted in improved fatigue life correlations between different loading conditions when compared to Neuber’s rule with fatigue notch factor. Although the improvement is only slight for the 2024-T3 fatigue data, a more substantial improvement was observed for the AISI 1141 notched shaft data. It should be noted, however, that some of the improvement observed for the AISI 1141 fatigue life correlations is not necessarily related to stress gradient consideration, but rather to differences in local stress-strain estimation error between Neuber’s rule and nonlinear FEA solutions. Therefore, a true comparison between fatigue notch factor and TCD approaches is difficult to ascertain from these results. It should be noted, however, that because the traditional application of fatigue notch factor requires an approach based on nominal
stresses, such as Neuber’s rule, to be used for stress-strain estimation, errors in life prediction due to local stress-strain values are indirectly a result of the fatigue notch factor approach, which generally lacks flexibility to be used with more advanced stress-strain estimation techniques such as nonlinear FEA.

The difference in fatigue life correlations between the different TCD analyses, however, can be compared directly. In this study, differences between point method, line method, and maximum shear path point method fatigue life correlations were found to be insignificant (at a fixed $L$ value) for all specimen geometries investigated. This observation has important implications in a variable amplitude multiaxial fatigue life analysis. Since the point method, evaluated along a path normal to the notch curvature, is more consistent and simpler to implement than both the line method and maximum shear path interpretation, it is valuable to know that the choice of simplicity does not necessarily come at the expense of the quality of fatigue life correlations. This is especially true when considering complex loading conditions.

Aside from the correlation of fatigue data from different loading conditions, some observations can also be made regarding the overall accuracy of fatigue life predictions based on the different TCD methods. For example, it can be seen from the 2024-T3 fatigue data in Figure 5.39 that for the same critical distance value, the line method produces the most non-conservative life predictions of the three different analysis approaches considered. Additionally, the point method evaluated normal to the notch curvature is slightly more non-conservative than when evaluated along the maximum shear plane. Similarly, as observed in Table 5.11, the line method was found to produce slightly longer fatigue life predictions than the point method for both AISI 1141 notched
shaft geometries. This is consistent with trends reported in (Susmel, 2008; Susmel and Taylor, 2011) for a variety of materials and notch geometries, and suggests some generality in these findings. Additionally, the overall life prediction trends produced using each TCD method, for the most part, tend to run parallel to the perfect prediction line when plotted in terms of experimental versus predicted life. This suggests that the assumption of a constant $L$ value (with respect to fatigue life) is, in fact, a reasonable assumption as long as local plasticity corrections are performed prior to fatigue damage calculation.

Although these observations may change depending on the notch geometry and analysis approaches under investigation, similar observations between the analysis results for the different specimen geometries, local stress states, and materials used in the current study are an encouraging sign. For the 2024-T3 tubular specimen, the dimensions of the hole are large compared to the critical distance values being considered. Therefore, stress gradients are distributed over a larger area and changes in stress state due to analysis location and/or path tend to be mitigated. For the sharp notches in the AISI 1141 shaft specimens, on the other hand, the stress state can change much more dramatically within the length scale of interest. Regardless of this fact, overall life predictions trends, along with differences between different TCD interpretations, were found to be consistent regardless of the specimen geometry considered. Therefore, these observations can be considered to be relatively general in nature.

Finally, based on the results of the constant amplitude analyses performed in this study, the impact of some key findings can be summarized with respect to multiaxial variable amplitude fatigue life analyses. Due to differences between constant and variable
amplitude multiaxial loading conditions, and their resulting local stress state variations, several simplifying assumptions were made with respect to calculating modeling parameters. These include constant $\rho$ and $K_{eq}$ values in fatigue notch factor analyses, and a constant $L$ value, with respect to both fatigue life and local stress state, in TCD analyses. Despite making these assumptions, the correlation of constant amplitude fatigue data for different multiaxial loading conditions was still very reasonable, with $R^2$ values for all analysis approaches ranging from 0.850 to 0.934. Therefore, extending these assumptions to variable amplitude loading conditions would not be expected to have a significant effect on the overall quality of fatigue life predictions.

Additionally, because they only depend on local stress distributions and a characteristic length parameter calculated from material properties, TCD approaches are able to overcome many of the challenges associated with the determination of fatigue notch factor for complex component geometries and/or loading histories. Because any means of local stress-strain calculation can be used with TCD, and different TCD interpretations produce similar fatigue life correlations, they also offer an increased level of flexibility that is desirable in performing complex fatigue analyses.

### 5.4.3.2 Notched Specimen Life Prediction Results

Based on the results presented in the previous section, TCD approaches were found to provide the best correlation of fatigue data for tests performed under a variety of multiaxial nominal loading conditions. Additionally, the point method, evaluated along a path normal to the notch curvature, was found to produce fatigue life correlation very similar to those based on the other TCD interpretations investigated. Therefore, since this approach is the most consistent and simplest to implement under the multiaxial variable
amplitude loading conditions central to this study, it was the approach chosen for the critical plane-based analyses performed in this study. As a result, this section evaluates the quality of constant amplitude notched specimen fatigue life predictions, based on both the original and modified FS parameters, when using this approach to consider stress gradient effects.

Fatigue life analyses were performed for all 2024-T3 notched specimens tested in this study using a critical distance values of \( L = 0.350 \) mm, as computed for this material in the previous section. The pseudo stress approach was then used to determine the plasticity corrected local stress and strain values from which fatigue damage was calculated. For all analyses, a \( k \) value of 1 was used with both the original and modified FS parameters, and stress/strain transformations were performed in 5° increments in order to determine the critical (maximum damage) plane.

The resulting life predictions are presented in Figures 5.43 and 5.44 based on the original FS parameter along with uniaxial and shear strain-life properties, respectively. In these figures, 100% of predictions are within a factor of ±3 of experimental results based on uniaxial properties, while 78% and 100% are within factors of ±3 and ±10, respectively, based on shear properties. Similarly, Figures 5.45 and 5.46 show notched specimen fatigue data correlations based on the modified FS parameter using uniaxial and shear properties, respectively. Using the modified parameter, 89% and 100% of predictions are within factors of ±3 and ±10, respectively, of experimental results based on uniaxial properties, while 39% and 94% are within factors of ±3 and ±10, respectively, based on shear properties.
Similar to un-notched specimen life predictions, although good correlation between data from different loading conditions is observed for both forms of the FS parameter, there is a conservative tendency for predictions based on shear strain-life properties. This may again be attributed to damage calculation on the maximum damage plane as opposed to the maximum shear strain plane. Since the difference between the maximum shear plane and maximum damage plane is slightly larger based on the modified FS parameter, this conservatism is more pronounced for life predictions based on this version of the parameter. A summary of fatigue life predictions for all 2024-T3 notched specimen fatigue tests, based on all analysis approaches considered, is provided in Table 5.8.

Overall, the critical plane-based analysis procedures implemented were able to correlate all of the notched specimen fatigue data from the different nominal loading conditions fairly well. Additionally, as was the case for un-notched specimens, both versions of the FS parameter were found to be similar in terms of overall accuracy for the fully-reversed loading conditions applied in this study. Although life predictions tend to be slightly conservative based on shear fatigue properties, those based on uniaxial fatigue properties were in excellent agreement with experimental results. Therefore, similar results should be expected for the variable amplitude fatigue life analyses presented in the following chapter.

5.4.4 Discussion on Critical Plane-Based Life Prediction Results

When reviewing the results of the critical plane-based fatigue life predictions presented in the previous sections, one of the first observations that can be made is that the shear-based FS parameter was able to provide significantly better fatigue life
correlations than the tensile-based SWT parameter. While a maximum of 28% of un-notched specimen life predictions were within a factor of ±3 of experimental results based on the different versions of the SWT parameter investigated, this number increased to as high as 88% of predictions within a factor of ±3 for analyses based on both the FS and modified FS parameters. These results are not surprising, however, as crack replicas from un-notched specimen fatigue tests indicated that the nucleation and growth of small cracks took place on or near planes of maximum shear for the 2024-T3 aluminum alloy investigated in this study.

Although the excellent life predictions observed for the 2024-T3 fatigue data indicate a general ability of the FS parameter to accurately estimate fatigue damage for the loading conditions investigated in this study, the correlation of fatigue data for some materials from literature suggests that the parameter may be less reliable under certain other loading conditions. By changing $\sigma_y$ to $G\Delta\gamma$ in the FS parameter, however, improved fatigue life predictions were obtained for loading paths containing large tensile mean stresses, and for paths containing different ratios of applied normal and shear stress/strain components. Nevertheless, additional experimental results from literature suggest that more work is still needed in quantifying fatigue damage for loading paths containing different time dependent normal-shear stress/strain interactions. Currently, neither version of the FS parameter is able to account for these effects despite the impact they could potentially have in multiaxial service loading situations.

For notched specimen fatigue life analyses, in addition to determining local stress-strain variations, there are also several different methods by which stress gradient effects can be accounted for. As a result, some of the more popular approaches were evaluated in
Section 5.4.3.1. Given the emphasis placed on multiaxial variable amplitude fatigue life analysis in this study, the ability of these approaches to be applied under more complex loading conditions was taken into consideration both when making modeling assumptions, and when interpreting the analysis results.

While analyses based on Neuber’s rule and fatigue notch factor produced reasonable fatigue life correlations for the different specimen geometries and loading conditions investigated, all TCD approaches considered were found to result in improved correlations. Little difference in the quality of fatigue life correlations, however, was observed between the different TCD approaches. Additionally, because they only depend on local stress distributions and a characteristic length calculated from material properties, TCD approaches offer an increased level of flexibility, as compared to fatigue notch factor methods. This allows them to be used in conjunction with a wider variety of life prediction methodologies when performing a complex fatigue life analysis.

Despite the advantages of TCD approaches for stress gradient consideration, however, they are not without their shortcomings. In comparing life prediction results based on different TCD interpretations, it was shown that although the correlation of data from different loading conditions remained similar, the overall accuracy of the predictions shifted depending on the analysis approach used. Similarly, it was shown for the 2024-T3 tubular specimen geometry that even for somewhat small changes in material properties, the resulting change in $L$ can have a fairly significant impact on life prediction accuracy. As a result, determining the correct $L$ value and analysis approach that will produce the most reliable fatigue life predictions could be a potentially difficult task. Although additional TCD interpretations, such as area-based and volume-based
approaches, could possibly offer some improvements in this respect, such methods were not investigated in this study.

In general, although excellent correlation of 2024-T3 fatigue data from different loading conditions was observed for both un-notched and notched specimen analyses, life predictions based on shear fatigue properties had the tendency to be more conservative than those based on uniaxial properties. This was true for both the original and modified FS parameters. It was suggested, however, that this may be attributed to damage calculation on the maximum damage plane as opposed to the maximum shear strain plane. While the life prediction curves are based on damage values on the maximum shear plane, the difference between the maximum shear plane damage and the maximum damage on any plane increases with an increase in $k$ for each parameter. Since $k$ appears on both sides of the life prediction equation for analyses based on uniaxial fatigue properties, however, these predictions are less sensitive to this difference.

Overall, for both the un-notched and notched specimen constant amplitude fatigue data generated in this study, 91% and 88% of life predictions were within a factor of ±3 of experimental results based on the original and modified FS parameters, respectively, when computed using uniaxial fatigue properties. Plots showing combined fatigue life correlations for both un-notched and notched specimen data are shown in Figure 5.47 based on the original FS parameter, and in Figure 5.48 based on the modified FS parameter. Life predictions based on shear fatigue properties, however, had the tendency to be slightly more conservative. For these analyses, only 64% and 46% of un-notched and notched specimen predictions were within a factor of ±3 of experimental results based on the original and modified FS parameters, respectively. As a result, more
accurate life predictions can generally be expected for the variable amplitude fatigue life analyses, presented in the next chapter, when based on uniaxial fatigue properties.

Additionally, in order to better characterize the overall error in constant amplitude fatigue life predictions, normal probability plots of the ratio of predicted to experimental life were generated for the critical plane analyses performed using uniaxial fatigue properties. These plots are shown in Figure 5.49(a) for predictions based on the original FS parameter, and in Figure 5.49(b) for predictions based on the modified FS parameter. Fatigue data for both un-notched and notched specimens were included for all loading conditions tested. The mean value and standard deviation of predicted to experimental life ratio are also given in each figure.

From these results, the error in fatigue life predictions is observed to follow a normal distribution (straight line) relatively well for both parameters. While the original parameter predictions show a tendency to be slightly right skewed (non-conservative bias), less deviation from the normal line is observed for the modified parameter. Together with mean and median (value at 50% cumulative probability) life ratios close to 1 for both parameters, this indicates that life prediction errors are fairly evenly distributed around the perfect prediction line. Additionally, as evidenced in previous figures, the lower standard deviation of life ratios for the modified parameter indicates an overall improvement in the correlation (tighter grouping) of the multiaxial fatigue data generated in this study.
5.5 Constant Amplitude Plate Specimen Fatigue Tests

In order to further evaluate the critical plane analysis procedures outlined in the previous section, the same techniques were used to predict fatigue crack initiation lives for the notched plate specimen tests performed in this study. Although both tubular and plate specimen notches were circular holes, the different diameters relative to the constant critical distance value, $L$, give some additional insight into the ability of TCD methodologies to account for stress gradient effects for different notch geometries. Consistent with tubular specimen predictions, the TCD point method was used with $L = 0.350$ mm, and evaluated normal to the notch curvature.

All plate specimen fatigue tests were performed in load-control under fully-reversed ($R = -1$) uniaxial constant amplitude loading conditions. Loads were applied in the form of sinusoidal waveforms at cyclic frequencies ranging from $2.0 - 10$ Hz. Like the notched tubular specimens, nominal stresses were computed based on gross cross sectional area. In all, seven tests were performed. The corresponding loading conditions and resulting fatigue lives are summarized in Table 5.12. The table lists Specimen ID, average gage section width, $W$, average gage section thickness, $t$, testing frequency, applied load amplitude, $P_a$, nominal stress amplitude, $S_{A,a}$, and experimental fatigue life to crack lengths of 0.2 mm, 1.0 mm, and 15 mm. Fatigue lives are reported in terms of cycles to failure. Crack lengths of 0.2 and 1.0 mm refer to the length of the longest crack length measured from the tip of a crack to the edge of the hole, while the 15 mm crack length refers to tip-to-tip crack length and includes the diameter of the hole.

Fatigue life predictions for the notched plate specimens were performed based on the modified FS parameter and uniaxial fatigue properties. The results of these analyses
are presented in Figure 5.50. From this figure, all predictions (excluding runout tests) were found to be within a factor of ±3 of experimental results, and it is clear that the plate specimen predictions agree well with those for tubular specimens. Additionally, although not shown, nearly identical results were obtained for analyses performed using the original version of the FS parameter. Although the notch geometries are similar for the two specimens, these results lend further credibility to the notch analysis models used in this study, in terms of both accuracy and general applicability.

5.6 Summary and Conclusions

Although there are many different steps involved in the fatigue life estimation process, relating the variation of stresses and strains to the fatigue damage that occurs within a material is the most fundamental part of any fatigue life analysis. Due to the abundance of engineering components subjected to multiaxial loading histories, being able to accurately estimate fatigue damage under multiaxial stress states is especially important. As a result, this chapter was focused on evaluating the ability of different multiaxial fatigue life prediction methodologies to accurately estimate fatigue damage under a variety of constant amplitude loading conditions. In doing so, life predictions based on both equivalent stress/strain-based and critical plane-based damage parameters were compared to experimental data from both un-notched and notched specimen fatigue tests.

Based on the experimental results and analysis presented for equivalent stress- and strain-based constant amplitude fatigue life predictions, some key findings and conclusions can be summarized as follows:
1) Equivalent stress- and strain-based analyses were unable to predict more than 50% of all un-notched specimen fatigue lives within a factor of ±3 of experimental results. Additionally, life prediction errors were not consistent between different loading conditions, resulting in poor correlation of the experimental data.

2) For non-proportional loading of notched components, the magnitude and location of maximum local stresses can change with a change in nominal stress ratio. As a result, local fatigue life analysis approaches must be considered in order to properly account for the variation of stresses and strains at the notch root.

3) All local equivalent stress- and strain-based analyses, based on local stresses and strains computed using Neuber’s rule, resulted in fairly good correlation of notched specimen fatigue data for all loading conditions considered.

4) The consideration of stress gradient effects, through the use of fatigue notch factor in Neuber’s rule, effectively reduced the conservatism and brought notched specimen life predictions together with those for un-notched specimens.

5) As a simplified analysis approach for notched specimen life prediction, which does not require the execution of local plasticity corrections, the equivalent pseudo stress-based approach produced life predictions of similar accuracy to those for the plasticity corrected equivalent stress approach.

6) The high quality of equivalent stress- and strain-based notched specimen fatigue life correlations was attributed to the notch geometry used in this study, which produces a local uniaxial stress state regardless of the nominally applied loading.
conditions. Similar results should not be expected when local stress states are multiaxial, as evidenced by the analysis results for un-notched specimens.

7) Although equivalent stress- and strain-based approaches may work well in certain situations, they lack a general robustness. More advanced fatigue life analysis techniques, on the other hand, provide an opportunity to more accurately model various aspects of the fatigue failure process under a wide variety of loading conditions.

Similarly, based on the experimental results and analysis presented for un-notched specimen critical plane-based constant amplitude fatigue life predictions, some key findings and conclusions can be summarized as follows:

8) Crack replicas from un-notched specimen fatigue tests indicated nucleation and growth of small cracks on or near planes of maximum shear for the 2024-T3 aluminum alloy investigated in this study. As such, the shear-based FS damage parameter was found to produce better fatigue life correlations than the tensile-based SWT parameter.

9) When analyzing literature data for 7075-T651 aluminum alloy and a ductile cast iron, fatigue life predictions based on the FS parameter were found to be non-conservative in cases where significant tensile mean stress was present. While increasing the $k$ value can improve uniaxial mean stress data correlation, life predictions for other multiaxial loading conditions become worse.

10) Changing $\sigma_y$ to $G\Delta\gamma$ in the FS parameter resulted in improved mean stress data correlation without adding any new material constants for damage calculation. Additionally, the parameter retains all the same physical interpretations.
11) The variation in experimental $k$ value was found to reduce at longer life for several different materials based on the modified FS parameter, resulting in improved correlation between uniaxial and pure shear fatigue data.

12) A limited number of discriminating load path tests, performed using 2024-T3 aluminum alloy, suggest that the modified FS parameter may also be better able to account for some aspects of normal and shear stress interaction on multiaxial fatigue damage.

13) Comparisons between predicted and experimental crack paths seem to suggest that the influence of normal stress increases with crack length.

14) Some experimental results from literature suggest that more work is still needed in quantifying fatigue damage for load paths containing different time dependent interactions of normal and shear stress/strain.

15) Although life prediction results based on the modified FS parameter are encouraging, more experimental data and analysis are still needed, for different materials and loading conditions, to verify its general applicability.

Finally, based on the experimental results and analysis presented for notched specimen critical plane-based constant amplitude fatigue life predictions, some key findings and conclusions can be summarized as follows:

16) Cracks in notched specimens were observed to nucleate and grow for a few hundred microns on planes of maximum shear stress/strain before transitioning to mode I growth.

17) Considering stress gradient effects in a fatigue life analysis can offer significant improvements in life prediction accuracy.
18) Theory of Critical Distances (TCD) approaches provide the ability to overcome many of the shortcomings associated with traditional stress gradient models such as fatigue notch factor, and resulted in improved fatigue life correlations for the different specimen geometries, materials, and multiaxial loading conditions investigated in this chapter.

19) While the TCD critical distance value was shown to be relatively independent of specimen geometry, some extent of dependence on local stress state was observed, especially at lower stress levels.

20) Differences in multiaxial fatigue data correlation between the different TCD approaches considered in this study were found to be insignificant, although shifts in overall accuracy were observed depending on the approach chosen.

21) The assumption of a constant critical distance value with respect to fatigue life was found to be reasonable when plasticity corrections were performed prior to fatigue damage analysis.

22) TCD approaches offer the flexibility to be used in conjunction with a variety of fatigue life prediction methodologies and provide a solid framework for the consideration of stress gradient effects in complex fatigue life analyses.

23) All 2024-T3 notched specimen fatigue data generated in this study were correlated well using the TCD point method, evaluated normal to the notch curvature with $L = 0.350$ mm, in conjunction with both the FS and modified FS damage parameters.
- Looking forward:

Of the constant amplitude fatigue life analysis approaches investigated in this chapter, those based on the FS and modified FS critical plane damage parameters were found to produce significantly better overall life predictions than those based on equivalent stress- and strain-based approaches. The von Mises stress-life approach resulted in the best overall life predictions of the equivalent stress/strain analyses with 62% and 93% of all constant amplitude life predictions (2024-T3 un-notched and notched specimens) within factors of ±3 and ±10 of experimental results, respectively. For the critical plane-based approaches, on the other hand, up to 92% of predictions were within a factor of ±3, and nearly all were within a factor of ±10 of experimental results.

For the von Mises equivalent stress/strain-based approaches, although notched specimen life predictions were fairly accurate for all nominal loading conditions considered, the correlation of un-notched specimen fatigue data was poor. Life predictions for in-phase axial-torsion loading had a slight conservative tendency, while predictions for torsion loading were, on average, even more conservative. This is in contrast to the consistently non-conservative life predictions observed for 90° OP loading conditions. Therefore, the accuracy of equivalent stress/strain-based variable amplitude fatigue life analyses for un-notched specimens could vary significantly depending on the characteristics of the applied loading history. For notched specimens, however, given the uniaxial local stress state present at the notch root for the specimens tested in this study, reasonable variable amplitude life predictions can be expected regardless of the applied loading conditions.
For the critical plane-based analyses, the FS and modified FS parameters were found to produce similar life prediction trends for the 2024-T3 fatigue data (both notched and un-notched) generated in this study. Additionally, excellent correlation of experimental data was observed for tests performed under different multiaxial loading conditions. No life predictions for any single loading path were found to be consistently more conservative or non-conservative than those for the other loading paths investigated. As a result, similar fatigue life prediction trends should also be expected between the various loading conditions investigated in the variable amplitude fatigue life analyses performed in the next chapter. However, analysis of some literature data showed that when significant tensile mean stresses are present, the modified FS parameter can be expected to produce more accurate results.

Additionally, the overall accuracy of the critical plane-based life predictions was found to vary slightly depending on whether uniaxial or shear-based fatigue properties were used in the analyses. Predictions based on uniaxial properties were found to have a relatively high level of accuracy, while those based on shear properties were more conservative (for reasons previously discussed). Therefore, similar trends should also be expected for the variable amplitude fatigue life predictions. Based on the results of all constant amplitude fatigue life analyses performed, the best overall variable amplitude life predictions should be expected when using shear-based critical plane damage parameters, uniaxial strain-life properties, and when considering notch effects through the application of TCD concepts.
Table 5.1 Summary of fully-reversed constant amplitude uniaxial fatigue testing of tubular specimens.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Specimen ID</th>
<th>Loading Conditions</th>
<th>Cycles to Failure (Nf)</th>
<th>Calculated or Controlled</th>
<th>Un-Notched Tubular Specimens – Load-Controlled</th>
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</thead>
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<tr>
<td>d_i (mm)</td>
<td>d_o (mm)</td>
<td>Freq. (Hz)</td>
<td>P_a (kN)</td>
<td>P_m (kN)</td>
<td>S_A,a (MPa)</td>
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</table>

* Crack length is half of the overall tip-to-tip crack length (0.2 mm or 0.5 mm) for un-notched specimens, and individual crack length from edge of hole (0.2 mm or 1.0 mm) for notched specimens.
** Final Nf for un-notched specimens is 3% change in position/rotation amplitude compared to stable cycle (10-15 mm tip-to-tip crack length), notched specimen definition is 15 mm tip-to-tip length.
† Denotes runout test due to specimen failure in grip section.
Table 5.2  Summary of fully-reversed constant amplitude pure torsion fatigue testing of tubular specimens.

<p>| Specimen ID |</p>
<table>
<thead>
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<th>$d_i$ (mm)</th>
<th>$d_o$ (mm)</th>
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<th>$T_w$ (N·m)</th>
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<th>$S_{T.w}$ (MPa)</th>
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Un-Notched Tubular Specimens – Load-Controlled

Un-Notched Tubular Specimens – Strain-Controlled

Machined Precrack Tubular Specimens – Load-Controlled

Notched Tubular Specimens – Load-Controlled

* Crack length is half of the overall tip-to-tip crack length (0.2 mm or 0.5 mm) for un-notched specimens, and individual crack length from edge of hole (0.2 mm or 1.0 mm) for notched specimens

** Final $N_f$ for un-notched specimens is 3% change in position/rotation amplitude compared to stable cycle (10-15 mm tip-to-tip crack length), notched specimen definition is 15 mm tip-to-tip length

347
### Table 5.3  Summary of fully-reversed constant amplitude in-phase axial-torsion fatigue testing of tubular specimens.

<table>
<thead>
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<th>Specimen ID</th>
<th>d_i (mm)</th>
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<th>T_a (N·m)</th>
<th>S_{L,a} (MPa)</th>
<th>S_{T,a} (MPa)</th>
<th>S_{vm,a} (MPa)</th>
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<th>0.2 mm*</th>
<th>0.5 mm/1.0 mm*</th>
<th>Final/15 mm**</th>
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* Crack length is half of the overall tip-tip crack length (0.2 mm or 0.5 mm) for un-notched specimens, and individual crack length from edge of hole (0.2 mm or 1.0 mm) for notched specimens

** Final N_f for un-notched specimens is 3% change in position/rotation amplitude compared to stable cycle (10-15 mm tip-tip crack length), notched specimen definition is 15 mm tip-tip length
Table 5.4  Summary of fully-reversed constant amplitude 90° out-of-phase axial-torsion fatigue testing of tubular specimens.

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* Crack length is half of the overall tip-to-tip crack length (0.2 mm or 0.5 mm) for un-notched specimens, and individual crack length from edge of hole (0.2 mm or 1.0 mm) for notched specimens

** Final $N_f$ for un-notched specimens is 3% change in position/rotation amplitude compared to stable cycle (10-15 mm tip-to-tip crack length), notched specimen definition is 15 mm tip-to-tip length
Table 5.5 Summary of additional axial-torsion fatigue tests performed on tubular specimens.

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<th>Sₚ,a (MPa)</th>
<th>Sₚm,a (MPa)</th>
<th>Sₚ,a (MPa)</th>
<th>εₚ,a</th>
<th>εₚm</th>
<th>γₚ,a</th>
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* Crack length is half of the overall tip-to-tip crack length (0.2 mm or 0.5 mm) for un-notched specimens

** Final Nₚ for un-notched specimens is 3% change in position/rotation amplitude compared to stable cycle (10-15 mm tip-to-tip crack length)
Table 5.6  Summary of experimentally determined fatigue life constants for 2024-T3 aluminum test material.

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Table 5.7: Summary of 2024-T3 un-notched tubular specimen constant amplitude fatigue life predictions based on equivalent stress/strain-based and critical plane-based (max damage plane) approaches.

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Table 5.7  (Continued) Summary of 2024-T3 un-notched tubular specimen constant amplitude fatigue life predictions based on equivalent stress/strain-based and critical plane-based (max damage plane) approaches.

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Table 5.8  Summary of 2024-T3 notched tubular specimen constant amplitude fatigue life predictions based on equivalent stress/strain-based and critical plane-based (max damage plane) approaches.

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<th>Specimen ID</th>
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<th>Nominal Loading</th>
<th>Exp. Life</th>
<th>Predicted Life</th>
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Predictions within ±3x of experimental life
- 89% 11% 89% 33% 89% 100% 78% 89% 39%

Predictions within ±10x of experimental life
- 100% 50% 100% 89% 100% 100% 100% 100% 94%

*Life predictions are based on local stress-strain values occurring at the notch root

**Life predictions are based on local stress-strain values determined according to the TCD point method with $L = 0.350$ mm
Table 5.9  Summary of material deformation and fatigue life properties reported in literature for AISI 1141, 7075-T651, and ductile cast iron.

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<th>AISI 1141</th>
<th>7075-T651*</th>
<th>Ductile Cast Iron</th>
<th>Unit</th>
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<td>1006</td>
<td>MPa</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>$E$</td>
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<td>71.7</td>
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<td>GPa</td>
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<td>Shear Modulus</td>
<td>$G$</td>
<td>80.2</td>
<td>27.5</td>
<td>63.5</td>
<td>GPa</td>
</tr>
<tr>
<td>Elastic Poisson’s Ratio</td>
<td>$\nu$</td>
<td>0.290</td>
<td>0.306</td>
<td>0.275</td>
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</tr>
<tr>
<td>Elongation at Fracture</td>
<td>%EL</td>
<td>26.1</td>
<td>9.7</td>
<td>9.1</td>
<td>%</td>
</tr>
<tr>
<td><strong>Cyclic Deformation Properties</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>0.2% Cyclic Axial Yield Strength</td>
<td>$\sigma_y'$</td>
<td>564</td>
<td>518</td>
<td>640</td>
<td>MPa</td>
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<tr>
<td>Cyclic Axial Strength Coefficient</td>
<td>$K'$</td>
<td>1205</td>
<td>845</td>
<td>1266</td>
<td>MPa</td>
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<tr>
<td>Cyclic Axial Hardening Exponent</td>
<td>$n'$</td>
<td>0.122</td>
<td>0.079</td>
<td>0.110</td>
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<td><strong>Uniaxial Fatigue Properties</strong></td>
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<tr>
<td>Fatigue Strength Coefficient</td>
<td>$\sigma_f'$</td>
<td>1297</td>
<td>1235</td>
<td>1748</td>
<td>MPa</td>
</tr>
<tr>
<td>Fatigue Strength Exponent</td>
<td>$b$</td>
<td>-0.089</td>
<td>-0.138</td>
<td>-0.120</td>
<td></td>
</tr>
<tr>
<td>Fatigue Ductility Coefficient</td>
<td>$\varepsilon_f'$</td>
<td>1.027</td>
<td>0.243</td>
<td>14.2</td>
<td></td>
</tr>
<tr>
<td>Fatigue Ductility Exponent</td>
<td>$c$</td>
<td>-0.687</td>
<td>-0.710</td>
<td>-1.06</td>
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<tr>
<td><strong>Shear Fatigue Properties</strong></td>
<td></td>
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<tr>
<td>Shear Fatigue Strength Coefficient</td>
<td>$\tau_f'$</td>
<td>489</td>
<td>797</td>
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<td>MPa</td>
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<tr>
<td>Shear Fatigue Strength Exponent</td>
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<tr>
<td>Shear Fatigue Ductility Coefficient</td>
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<td>5.42</td>
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<tr>
<td>Shear Fatigue Ductility Exponent</td>
<td>$c_o$</td>
<td>-0.456</td>
<td>-1.173</td>
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* All cyclic deformation and fatigue properties for 7075-T651 were not reported, but fit to reported data
Table 5.10  Summary of 2024-T3 notched tubular specimen constant amplitude fatigue life predictions based on the Fatemi-Socie damage parameter ($k = 1$) and uniaxial strain-life properties.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Loading Path</th>
<th>Nominal Loading</th>
<th>Exp. Life (Cycles)</th>
<th>Predicted Life (Cycles)</th>
<th>Predictions within ±3x of experimental life</th>
<th>Predictions within ±10x of experimental life</th>
<th>Coefficient of determination, $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_a$ (MPa)</td>
<td>$\tau_a$ (MPa)</td>
<td>Neuber, $K_{pq}$</td>
<td>Neuber, $K_{fq}$</td>
<td>PM $L=0.300$ mm</td>
<td>PM $L=0.350$ mm</td>
<td>PM $L=0.420$ mm</td>
</tr>
<tr>
<td>T 19</td>
<td>Axial</td>
<td>145</td>
<td>0</td>
<td>9500</td>
<td>3840</td>
<td>17300</td>
<td>12400</td>
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</tbody>
</table>
Table 5.11 Summary of AISI 1141 notched shaft specimen constant amplitude fatigue testing conditions and life predictions based on the Fatemi-Socie damage parameter \((k = 1)\) and uniaxial strain-life properties.

<table>
<thead>
<tr>
<th>Specimen Geometry</th>
<th>Loading Path</th>
<th>Nominal Loading</th>
<th>Exp. Life (Cycles)</th>
<th>Predicted Life (Cycles)</th>
<th>PM</th>
<th>LM</th>
<th>PM</th>
</tr>
</thead>
<tbody>
<tr>
<td>stepped shaft</td>
<td>Torsion</td>
<td>0 344</td>
<td>1499</td>
<td>484 499 991</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stepped shaft</td>
<td>Torsion</td>
<td>0 294</td>
<td>2796</td>
<td>4757 1757 1819 4370</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stepped shaft</td>
<td>Torsion</td>
<td>0 294</td>
<td>2796</td>
<td>4757 1757 1819 4370</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stepped shaft</td>
<td>Torsion</td>
<td>0 240</td>
<td>7136</td>
<td>13738 7785 8200 30673</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stepped shaft</td>
<td>In-Phase</td>
<td>260 274</td>
<td>7136</td>
<td>13738 7785 8200 30673</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stepped shaft</td>
<td>In-Phase</td>
<td>201 214</td>
<td>2904</td>
<td>5508 3733 4047 28931</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stepped shaft</td>
<td>In-Phase</td>
<td>201 214</td>
<td>2904</td>
<td>5508 3733 4047 28931</td>
<td></td>
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<tr>
<td>stepped shaft</td>
<td>In-Phase</td>
<td>201 214</td>
<td>2904</td>
<td>5508 3733 4047 28931</td>
<td></td>
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<tr>
<td>stepped shaft</td>
<td>In-Phase</td>
<td>140 154</td>
<td>499081</td>
<td>29127 33624 1071477</td>
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<tr>
<td>grooved shaft</td>
<td>Torsion</td>
<td>0 344</td>
<td>1834</td>
<td>2569 499 505 773</td>
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<tr>
<td>grooved shaft</td>
<td>Torsion</td>
<td>0 344</td>
<td>1834</td>
<td>2569 499 505 773</td>
<td></td>
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<tr>
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<td>Torsion</td>
<td>0 294</td>
<td>3509</td>
<td>5155 1960 1993 3448</td>
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<td>grooved shaft</td>
<td>In-Phase</td>
<td>201 214</td>
<td>3674</td>
<td>5877 4572 4745 16518</td>
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<tr>
<td>grooved shaft</td>
<td>In-Phase</td>
<td>201 214</td>
<td>3674</td>
<td>5877 4572 4745 16518</td>
<td></td>
<td></td>
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<tr>
<td>grooved shaft</td>
<td>In-Phase</td>
<td>140 154</td>
<td>789999</td>
<td>10139 16518 23662 3448</td>
<td></td>
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</table>

Predictions within ±3x of experimental life: 18% 6% 6% 53%
Predictions within ±10x of experimental life: 53% 59% 59% 88%
Coefficient of determination, \(R^2\): 0.8294 0.9314 0.8187
Table 5.12  Summary of fully-reversed constant amplitude uniaxial fatigue testing of notched plate specimens.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>$W$  (mm)</th>
<th>$t$  (mm)</th>
<th>Freq. (Hz)</th>
<th>$P_a$ (kN)</th>
<th>$S_{\Delta a}$ (MPa)</th>
<th>Cycles to Failure ($N_f$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2 mm*</td>
</tr>
<tr>
<td>P 3</td>
<td>40.01</td>
<td>3.14</td>
<td>2.0</td>
<td>18.87</td>
<td>150.0</td>
<td>5000</td>
</tr>
<tr>
<td>P 2</td>
<td>40.01</td>
<td>3.16</td>
<td>5.0</td>
<td>14.54</td>
<td>115.0</td>
<td>55000</td>
</tr>
<tr>
<td>P 7</td>
<td>40.02</td>
<td>3.14</td>
<td>3.0</td>
<td>14.46</td>
<td>115.0</td>
<td>36600</td>
</tr>
<tr>
<td>PM 10</td>
<td>40.00</td>
<td>3.14</td>
<td>8.0</td>
<td>12.56</td>
<td>100.0</td>
<td>-</td>
</tr>
<tr>
<td>PM 11</td>
<td>40.01</td>
<td>3.14</td>
<td>5.0</td>
<td>10.68</td>
<td>85.0</td>
<td>154350</td>
</tr>
<tr>
<td>PM 12</td>
<td>40.01</td>
<td>3.14</td>
<td>8.0</td>
<td>9.43</td>
<td>75.0</td>
<td>1796000</td>
</tr>
<tr>
<td>P 4</td>
<td>40.02</td>
<td>3.16</td>
<td>10.0</td>
<td>9.49</td>
<td>75.0</td>
<td>-</td>
</tr>
</tbody>
</table>

* 0.2 mm and 1.0 mm are individual crack lengths from edge of hole
** Final $N_f$ for notched specimens defined as 15 mm tip-to-tip crack length (including hole diameter)
Figure 5.1  Uniaxial stress-life fit, with corresponding experimental data, for 2024-T3 aluminum alloy.

\[ S_{A,N_f} = 1089.2 \, MPa \,(N_f)^{-0.133} \]

Figure 5.2  Shear stress-life fit, with corresponding experimental data, for 2024-T3 aluminum alloy.

\[ S_{T,N_f} = 416.2 \, MPa \,(N_f)^{-0.078} \]
Figure 5.3  Uniaxial strain-life fit, with corresponding experimental data, for 2024-T3 aluminum alloy.

\[ \varepsilon_a = \frac{1194.2 \, MPa}{73700 \, MPa} (2N_f)^{-0.133} + 0.066 (2N_f)^{-0.445} \]

Figure 5.4  Shear strain-life fit, with corresponding experimental data, for 2024-T3 aluminum alloy.

\[ \gamma_a = \frac{439.3 \, MPa}{27400 \, MPa} (2N_f)^{-0.078} + 0.834 (2N_f)^{-0.705} \]
Figure 5.5  Fatigue life correlations in terms of (a) equivalent stress amplitude vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 aluminum alloy constant amplitude un-notched multiaxial fatigue data based on maximum principal stress-life approach.
Figure 5.6  Fatigue life correlations in terms of (a) equivalent stress amplitude vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 aluminum alloy constant amplitude un-notched multiaxial fatigue data based on von Mises equivalent stress-life approach.
Figure 5.7  Fatigue life correlations in terms of (a) equivalent strain amplitude vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 aluminum alloy constant amplitude un-notched multiaxial fatigue data based on von Mises equivalent strain-life approach.
Figure 5.8  Schematic showing the construction of a pseudo S-N curve for use in 2024-T3 aluminum alloy notched specimen fatigue life prediction.

\[ S_{Nf} = A_A (N_f)^b \]

Figure 5.9  Fatigue life correlations in terms of experimental vs. predicted reversals to failure for 2024-T3 aluminum alloy constant amplitude notched multiaxial fatigue data based on the local von Mises pseudo stress-life approach.

\[ \sigma_o \]

\[ 1.0 \times 10^3 \]

\[ 1.0 \times 10^4 \]

\[ 1.0 \times 10^5 \]

\[ 1.0 \times 10^6 \]

\[ 1.0 \times 10^7 \]
Figure 5.10  Fatigue life correlations in terms of (a) equivalent local stress amplitude vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 aluminum alloy constant amplitude notched multiaxial fatigue data based on von Mises equivalent stress-life approach and Neuber’s rule with stress concentration factor, $K_{eq}$. 
Figure 5.11  Fatigue life correlations in terms of (a) equivalent local stress amplitude vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 aluminum alloy constant amplitude notched multiaxial fatigue data based on von Mises equivalent stress-life approach and Neuber’s rule with fatigue notch factor, $K_{f_q}$. 
Figure 5.12 Fatigue life correlations in terms of (a) equivalent local strain amplitude vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 aluminum alloy constant amplitude notched multiaxial fatigue data based on von Mises equivalent strain-life approach and Neuber’s rule with stress concentration factor, $K_{tq}$.
Figure 5.13  Fatigue life correlations in terms of (a) equivalent local strain amplitude vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 aluminum alloy constant amplitude notched multiaxial fatigue data based on von Mises equivalent strain-life approach and Neuber’s rule with fatigue notch factor, $K_{fq}$. 
Figure 5.14  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 aluminum alloy constant amplitude un-notched multiaxial fatigue data based on Smith-Watson-Topper critical plane approach.
Figure 5.15  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 aluminum alloy constant amplitude un-notched multiaxial fatigue data based on alternate Smith-Watson-Topper critical plane approach.
Figure 5.16 Observed cracking orientation in 2024-T3 aluminum alloy under (a) pure torsion, (b) in-phase axial-torsion, and (c) 90°OP axial-torsion loading conditions. Specimen numbers followed by an asterisk indicate secondary fatigue cracks, i.e. not the crack that caused final failure.
Figure 5.17 Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 aluminum alloy constant amplitude un-notched multiaxial fatigue data based on Fatemi-Socie critical plane approach and uniaxial strain-life properties.
Figure 5.18  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 aluminum alloy constant amplitude un-notched multiaxial fatigue data based on Fatemi-Socie critical plane approach and shear strain-life properties.
Figure 5.19  Torsion with pulsating tension load path in terms of normalized (a) load-time history and (b) shear strain vs. axial stress path.
Figure 5.20  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 7075-T651 aluminum alloy constant amplitude un-notched multiaxial fatigue data from (Zhao and Jiang, 2008) based on Fatemi-Socie critical plane approach, with $k = 1$, and uniaxial strain-life properties.
Figure 5.21 Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 7075-T651 aluminum alloy constant amplitude un-notched multiaxial fatigue data from (Zhao and Jiang, 2008) based on Fatemi-Socie critical plane approach, with $k = 4$, and uniaxial strain-life properties.
Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for ductile cast iron constant amplitude un-notched uniaxial fatigue data from (Meyer, 2014) based on Fatemi-Socie critical plane approach, with $k = 1$, and uniaxial strain-life properties.
Figure 5.23  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for ductile cast iron constant amplitude un-notched uniaxial fatigue data from (Meyer, 2014) based on Fatemi-Socie critical plane approach, with $k = 4$, and uniaxial strain-life properties.
Figure 5.24  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 7075-T651 aluminum alloy constant amplitude un-notched multiaxial fatigue data from (Zhao and Jiang, 2008) based on the modified Fatemi-Socie critical plane approach, with k = 1, and uniaxial strain-life properties.
Figure 5.25  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for ductile cast iron constant amplitude un-notched uniaxial fatigue data from (Meyer, 2014) based on the modified Fatemi-Socie critical plane approach, with $k = 1$, and uniaxial strain-life properties.
Figure 5.26  Variation of $k$ with fatigue life for several different materials based on (a) the Fatemi-Socie damage parameter, (b) the modified FS damage parameter with $G\Delta\gamma$, and (c) the modified FS damage parameter with $\Delta\tau$.  

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Figure 5.27 Triangular load path in terms of normalized (a) stress-time history and (b) shear vs. axial stress path.

Figure 5.28 Experimental and predicted fatigue lives for 2024-T3 aluminum alloy tested under triangular (Tri) and torsion with static tension (STSA) loading paths at different nominal stress ratios.
Figure 5.29  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 constant amplitude un-notched multiaxial fatigue data based on the modified Fatemi–Socie critical plane approach and uniaxial strain-life properties.
Figure 5.30  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 constant amplitude un-notched multiaxial fatigue data based on the modified Fatemi–Socie critical plane approach and shear strain-life properties.
Figure 5.31  Torsion with static tension critical plane analysis in terms of (a) damage parameter value vs. plane orientation and (b) experimentally observed crack paths in 2024-T3 aluminum alloy (0° is horizontal, 90° is specimen axis).
Figure 5.32 Triangular load path ($\lambda = 0.5$) critical plane analysis in terms of (a) damage parameter value vs. plane orientation and (b) experimentally observed crack paths in 2024-T3 aluminum alloy ($0^\circ$ is horizontal, $90^\circ$ is specimen axis).
Figure 5.33 Triangular load path ($\lambda = 2.0$) critical plane analysis in terms of (a) damage parameter value vs. plane orientation and (b) experimentally observed crack paths in 2024-T3 aluminum alloy ($0^\circ$ is horizontal, $90^\circ$ is specimen axis).
Figure 5.34 Experimental vs. predicted fatigue life for 2024-T3 notched tubular specimens with local stress-strain values based on (a) Neuber’s rule with stress concentration factor, $K_{tq}$, and (b) Neuber’s rule with fatigue notch factor, $K_{fq}$.

Figure 5.35 Experimental vs. predicted fatigue life for AISI 1141 notched shaft specimens with local stress-strain values based on (a) Neuber’s rule with stress concentration factor, $K_{tq}$, and (b) Neuber’s rule with fatigue notch factor, $K_{fq}$. 
Figure 5.36  Comparison of fatigue crack initiation lives for notched specimens subjected to the same nominal axial stress level under uniaxial and 90° OP loading conditions.

Figure 5.37  Variation of critical distance length, $L$, for different combinations of fully-reversed fatigue limit stress range and threshold SIF range.
Figure 5.38 Experimental vs. predicted fatigue life for 2024-T3 notched tubular specimens with local stress-strain values based on TCD point method evaluated using a critical distance value of (a) $L = 0.300$ mm and (b) $L = 0.350$ mm.
Figure 5.39 Experimental vs. predicted fatigue life for 2024-T3 notched tubular specimens with local stress-strain values based on TCD (a) point method, (b) line method, and (c) maximum shear plane point method, evaluated using a critical distance value of $L = 0.420$ mm.
Figure 5.40 Experimental vs. predicted fatigue life for AISI 1141 notched shaft specimens with local stress-strain values based on TCD point method evaluated using a critical distance value of (a) $L = 0.013$ mm and (b) $L = 0.400$ mm.

Figure 5.41 Typical crack path and nucleation orientations for notched specimens subjected to axial (left) and pure torsion (right) nominal loading conditions.
Figure 5.42 Effect of local stress analysis location on stress concentration tensor for the 0° hole location under axial and torsion loadings. The distance from point A to point B and from point A to point C is 0.210 mm.

<table>
<thead>
<tr>
<th></th>
<th>Unit Axial Load</th>
<th></th>
<th>Unit Torsion Load</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_A$</td>
<td>[0.00 0.00 0.00]</td>
<td>2.94</td>
<td>$\sigma_{vm} = 2.94$</td>
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<tr>
<td></td>
<td>$\sigma_B$</td>
<td>[0.24 0.00 -0.03]</td>
<td>2.06</td>
<td>$\sigma_{vm} = 2.06$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_C$</td>
<td>[0.20 -0.12 -0.02]</td>
<td>2.23</td>
<td>$\sigma_{vm} = 2.23$</td>
</tr>
</tbody>
</table>

$\sigma_A = [0.00 0.00 0.00]$, $\sigma_B = [0.00 0.00 -0.03]$, $\sigma_C = [0.00 0.00 0.00]$

$\sigma_{vm} = 2.94$, $\sigma_{vm} = 2.06$, $\sigma_{vm} = 2.23$

$\sigma_A = [0.00 0.00 0.00]$, $\sigma_B = [0.00 -0.96 0.00]$, $\sigma_C = [0.00 0.12 0.00]$

$\sigma_{vm} = 0.00$, $\sigma_{vm} = 1.68$, $\sigma_{vm} = 1.43$
Figure 5.43  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 constant amplitude notched multiaxial fatigue data based on the Fatemi-Socie critical plane approach and uniaxial strain-life properties.
Figure 5.44  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 constant amplitude notched multiaxial fatigue data based on the Fatemi-Socie critical plane approach and shear strain-life properties.
Figure 5.45 Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 constant amplitude notched multiaxial fatigue data based on the modified Fatemi-Socie critical plane approach and uniaxial strain-life properties.
Figure 5.46  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 constant amplitude notched multiaxial fatigue data based on the modified Fatemi-Socie critical plane approach and shear strain-life properties.
Figure 5.47  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 constant amplitude un-notched and notched multiaxial fatigue data based on the Fatemi-Socie critical plane approach and uniaxial strain-life properties.
Figure 5.48  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 constant amplitude un-notched and notched multiaxial fatigue data based on the modified Fatemi-Socie critical plane approach and uniaxial strain-life properties.
Figure 5.49  Normal probability plots of the ratio of predicted to experimental life for all 2024-T3 constant amplitude un-notched and notched tubular specimen fatigue data based on (a) the Fatemi-Socie parameter and (b) the modified Fatemi-Socie parameter, in conjunction with uniaxial fatigue properties.
Figure 5.50  Fatigue life correlations in terms of (a) damage value vs. reversals to failure and (b) experimental vs. predicted reversals to failure for 2024-T3 constant amplitude notched tubular and plate data based on the modified Fatemi-Socie critical plane approach and uniaxial strain-life properties.
Chapter 6

Variable Amplitude Fatigue Behavior and Life Predictions

Understanding the fatigue failure process under complex multiaxial variable amplitude loading conditions was one of the core objectives of this study. As a result, a variety of variable amplitude fatigue tests, the results from which are presented in Section 6.2, were performed using a simulated aircraft service loading history, which is described in Section 6.1. These tests, along with their corresponding fatigue life analyses, combine the synergistic effects of all the fatigue damage mechanisms and modeling techniques addressed up to this point. In addition, aspects specific to variable amplitude loading conditions, such as cycle identification and damage summation, must also be considered. As such, these tests provided key experimental data against which the effectiveness of various fatigue life analysis procedures could be evaluated.

Even in the absence of a notch, fatigue life analyses under multiaxial variable amplitude loading conditions can be quite complex. Therefore, in order to remove the extra complication that notch effects add to such an analysis, tests were performed first using un-notched tubular specimens. Applied loading conditions included pure axial, pure torsion, and combined axial-torsion loading. These tests allow for a more direct comparison of life prediction results obtained from both constant and variable amplitude
fatigue life analyses and, as such, provide a better means to evaluate analysis procedures specific to variable amplitude loading conditions. Results for un-notched specimen life predictions are presented in Section 6.3.1 based on equivalent stress/strain approaches, and in Section 6.4.1 based on critical plane analysis approaches.

Finally, fatigue tests were performed on notched tubular specimens under axial only, torsion only, and combined axial-torsion variable amplitude loading conditions. From the previous constant amplitude tests performed, the effects of multiaxial stress states on fatigue damage were evaluated using un-notched specimens, and notch effects were studied using notched specimens. Then, variable amplitude testing of un-notched specimens was used to study the effects of variable amplitude loading conditions on both uniaxial and multiaxial fatigue damage accumulation. These final notched specimen tests, which include the combined effects of all of these aspects, reflect a majority of fatigue loading situations encountered in industry. As such, they allow for the evaluation of state-of-the-art fatigue life analysis techniques in their ability to provide accurate life predictions under such conditions. Results for the notched specimen variable amplitude fatigue life analyses are presented in Section 6.3.2 based on equivalent stress/strain approaches, and in Section 6.4.2 based on critical plane approaches.

Similar to other chapters, the variable amplitude fatigue life analysis results presented in this chapter were also published in a number of journal papers. Results from the equivalent stress/strain-based analyses, for both un-notched and notched specimens, were published in (Gates and Fatemi, 2016e, 2015), while results from critical plane-based analyses were published in (Gates and Fatemi, 2016f, 2016g).
6.1 Variable Amplitude Loading History

As stated in Section 3.5.3, there was only one original service loading history from which all other variable amplitude loading histories used in this study were derived. The original history contains both normal and shear nominal stress histories derived from recorded flight test data on the lower wing skin area of a military patrol aircraft (tension-dominated). A variety of take-off, landing, and in-flight maneuvers are represented in the history. In its entirety, the loading history contains around 915,000 data points, with each point approximately corresponding to one loading reversal on the axial stress channel.

The maximum and minimum axial stresses in the unscaled history are 144.8 MPa and −51.3 MPa, respectively, while the maximum and minimum shear stresses are 67.0 MPa and −15.9 MPa, respectively. Plots showing the time history of a 1000 reversal segment taken from the loading history, along with the axial-shear stress path for this same segment, are shown in Figure 6.1. The loading segment shown in this figure is representative of the loading patterns repeated throughout the remainder of the entire history. From the stress path, it can be seen that the loading history contains significant non-proportional loading events.

To help illustrate the overall variation of stresses, Figure 6.2 shows the time history of the axial and shear stress channels over the course of the entire loading history. From this figure, it is clear that frequency and magnitude of peak stresses in each channel is fairly consistent throughout the loading history. Therefore, fatigue damage should also be fairly evenly distributed throughout the history. Different loading conditions were obtained in testing by applying the axial loading channel only, the torsion channel only, or the full combined axial-torsion loading. Prior to testing, original stress values for each
loading history were uniformly scaled to obtain fatigue lives which would result in reasonable test durations.

A limited number of notched specimen tests were also performed using an edited version of this same loading history. Not only do the edited history tests result in reduced testing and analysis time, they also provide a means for evaluating the effect of smaller loading cycles on overall fatigue damage. Two separate edited histories were produced with the goal of retaining approximately 90% of the fatigue damage contained in the full history. Damage was calculated in the editing procedure based on stress-based analysis approaches and summed using the commonly used linear damage rule.

The first edited history was used for both uniaxial and combined loading fatigue tests. Editing for this history was performed based on the combined loading history, uniformly scaled by a factor of 3.0, using a von Mises equivalent stress-life approach in conjunction with the SWT mean stress correction model. All cycles producing an equivalent fully-reversed stress amplitude of less than 125 MPa were removed from the loading history, while preserving the sequence of the larger cycles. A 500 reversal segment of the edited combined loading history is shown in Figure 6.3 along with the corresponding axial-shear stress path. The length of the original history was reduced by 94%, to approximately 58,000 reversals, in this case.

The second edited history was used for pure torsion tests only, and was obtained based on damage calculated using a shear stress-life approach and no mean stress correction. The shear channel of the full loading history was uniformly scaled by a factor of 4 prior to the editing procedure. Then, any cycle with a shear stress amplitude of less than 50 MPa was removed from the loading history. A 500 reversal segment of the edited
shear history is shown in Figure 6.4. For this history, approximately 95% of the cycles were eliminated from the original history, resulting in an edited history length of approximately 41,500 reversals.

6.2 Variable Amplitude Fatigue Test Summary

Un-notched specimen variable amplitude fatigue tests were performed by repeatedly applying the entire load history in nominal load-control until a tip-to-tip crack length of 15 mm was reached. While nominal axial stresses were computed by standard methods, the nonlinear distribution of shear stress across the thickness of the specimen under elastic-plastic loading conditions again makes nominal shear stresses computation difficult. For constant amplitude fatigue tests, a procedure was used to calculate outer surface nominal shear stress based on material deformation properties and the linear distribution of shear strain across the thickness of the specimen. For variable amplitude loading conditions, however, the level of plastic deformation induced by each applied cycle can be different. Therefore, the procedure used to calculate nominal constant amplitude shear stresses cannot be used here. Instead, maximum nominal shear stress was computed by simply assuming elastic stress conditions on the outer surface of the specimen. Although this results in an overestimation of the actual applied shear stress for cycles exceeding the yield strength of the material, this was assumed to have a negligible effect on fatigue life predictions, in this case, due to the predominately elastic nature of the loading histories applied in this study (discussed later).

Similar to constant amplitude tests, despite the fact that crack growth was monitored for un-notched specimen variable amplitude tests, crack initiation on the inner
surface of some specimens made judging crack initiation life difficult. However, since the failure criterion could not be set to a specific change in rotation or displacement amplitude, as was the case for constant amplitude loadings, tight limits were kept on maximum and minimum displacement/rotation values throughout testing to ensure that final crack lengths for variable amplitude tests were consistent with each other, as well as with those for constant amplitude tests (i.e. 10-15 mm tip-to-tip length).

All un-notched tests were performed at a data sampling rate of 10 Hz (resulting in an approximate cyclic frequency of 5 Hz). For each test performed, the entire loading history was scaled by an appropriate factor to obtain stress levels that would produce fatigue lives ranging from less than one block to around 10 blocks. A total of 10 variable amplitude fatigue tests were performed using un-notched specimens. These tests represent 3 different load levels of axial only loading (1 level repeated), 1 load level for torsion only loading (repeated), and 2 different loading levels for combined axial-torsion loading (both levels repeated). All un-notched specimen tests were performed using full (un-edited) loading histories. A summary of the applied loading conditions for each test, along with corresponding experimental fatigue lives, is presented in Table 6.1.

Notched specimen variable amplitude fatigue tests were performed in a similar nominal load-controlled manner as those for un-notched specimens. While nominal axial stresses were again computed as the ratio of applied force to gross cross sectional area, the calculation of nominal shear stresses was not as straightforward. For un-notched specimen variable amplitude tests, maximum nominal shear stress was computed by simply assuming elastic stress conditions on the outer surface of the specimen. While this assumption greatly simplified applied stress calculations for these specimens, the
presence of a notch already necessitates additional calculations in order to determine local elastic-plastic stresses and strains for use in fatigue life analyses. Therefore, nominal shear stresses for notched specimen variable amplitude fatigue tests were defined as the average shear stress at the mid-thickness of the specimen, as calculated from Equation (4.1). Then, for subsequent local stress-strain estimations, stress concentration factors were redefined based on a unit applied shear stress at the specimen mid-thickness location. This ensures that nominal stress values, as well as local elastic stress values, are consistent regardless of whether or not nominal yielding occurs.

Similar to constant amplitude notched specimen fatigue tests, crack initiation life for variable amplitude tests was defined as the life when the first growing crack reached a distance of 0.2 mm from the notch root, as observed on the specimen outer surface. Tests were then stopped when tip-to-tip crack lengths of approximately 15 mm were reached. Variable amplitude notched specimen tests were performed at a data sampling rate of either 10 Hz (approximately 5 cycles per second) or 16 Hz (approximately 8 cycles per second), depending on the applied loading level. For each test performed, the entire nominal loading history was again scaled by an appropriate factor to obtain stress levels that would produce fatigue lives ranging from less than one block to around 10 blocks.

A total of 17 variable amplitude fatigue tests were performed using notched specimens. These tests represent 3 different load levels of axial only loading (1 level repeated for the full history and 1 level repeated for the edited history), 2 load levels of torsion only loading (1 level repeated for the full history and 1 level repeated for the edited history), and 2 different load levels of combined axial-torsion loading (2 levels repeated for the full history and 1 level repeated for the edited history). A summary of the
applied loading conditions for each test, along with the resulting experimental fatigue lives, can be found in Table 6.2. Similar to constant amplitude fatigue test results, analysis of this variable amplitude test data will be presented in the following sections using various multiaxial fatigue damage parameters.

6.3 Equivalent Stress/Strain-Based Life Predictions

Due to relatively limited amounts of research and experimental data for such conditions, no generally accepted procedures exist for performing a multiaxial variable amplitude fatigue life analysis. Of the methods that are available, they can often be complex and time consuming to implement. Therefore, as with constant amplitude loading, classical equivalent stress or strain approaches, such as those based on von Mises, maximum shear, and maximum principal stress yield criteria, are commonly extended to situations involving multiaxial variable amplitude loading histories in order to simplify the analysis. Stress-based approaches are, again, typically used in situations where the material behavior is primarily elastic, while strain-based approaches can better account for the presence of plastic deformation (Stephens et al., 2000).

Equivalent stress- and strain-based life predictions performed in this study are similar in concept to those discussed in Section 5.3.1 for constant amplitude fatigue testing of un-notched specimens. Although it was shown these classical approaches were unable to adequately correlate experimental fatigue life data for a variety of multiaxial loading conditions, the methodologies are extended to variable amplitude loading in this section in order to evaluate techniques representative of those commonly used in industry. Because equivalent stress/strain-based fatigue life analyses are still commonly
used in situations involving multiaxial variable amplitude loadings, being able to assess
the accuracy of these methods when applied to complex fatigue life analysis problems is
a topic of interest for many practical applications.

The first major difference between the constant and variable amplitude fatigue life
analyses performed in this study is the need to account for mean stresses in the variable
amplitude loading histories. Since all constant amplitude fatigue tests were performed
under fully-reversed loading conditions, this issue was not discussed in the previous
chapter with respect to equivalent stress/strain-based life predictions. For these
approaches, when mean stresses are present in a multiaxial loading history, an equivalent
mean stress, based on criteria such as von Mises effective mean stress or hydrostatic
stress, is typically computed so that it may be used directly with conventional uniaxial
mean stress correction models. For non-proportional variable amplitude loading histories,
however, the calculation of an equivalent mean stress is not a straightforward task. This
will be discussed in further detail in the following sections.

The second major difference between constant and variable amplitude fatigue life
analyses is the need to perform cycle counting in order to identify individual loading
cycles within a random or variable amplitude multiaxial loading history. Because loading
components remain in a constant ratio to one other under linear elastic proportional
loading conditions, traditional uniaxial cycle counting methods may be used in such cases
to identify cycles based on the variation of any stress or strain component, or an
equivalent stress/strain history. Then, the range and mean of each stress/strain component
can subsequently be determined through appropriate scaling of the counting variable. For
non-proportional loading, however, loading components may be applied out-of-phase, at
different frequencies (asynchronous), or randomly with respect to one another. In the case of asynchronous and/or random loadings, which are often encountered in service loading histories, there may be no clear definition of a cycle. Therefore, the amplitude and mean stresses of the different loading components, required as input for equivalent stress/strain-based mean stress correction and damage models, can change depending on the component used for cycle counting. This is true even for the equivalent non-proportional stress criteria of Sonsino (1995) and Lee et al. (2007) (discussed in Section 2.4), thus making the application of these approaches vague under such conditions.

One way to deal with this issue is to compute equivalent stress/strain values for the entire loading history first, and then cycle count the single equivalent stress/strain history. However, there are a number of drawbacks to this approach. The most obvious is the need to assign a positive or negative value to the equivalent stress quantity, so that stress ranges and reversals can be properly identified. Although there are multiple methods for doing this, the most common is to use the sign corresponding to the principal stress with the highest absolute value (“Multiaxial Fatigue Theory,” 2002). However, in the case of non-proportionally varying stress components, this has the potential to produce stress histories which are not representative of the actual fatigue damage experienced by a component (Khosrovaneh et al., 2004).

An easy way to illustrate this point is to consider the case of axial-torsion loading. A look at Mohr’s circle reveals that, in this situation, the sign of the equivalent stress quantity is always equal to that of the axial stress component. Based on this fact, Figure 6.5 shows a short loading history in terms of applied axial stress, applied shear stress, and the resulting signed von Mises stress. When looking at this figure, if the small axial stress
cycles are considered insignificant compared to the much larger shear cycle, one would expect the predicted fatigue damage (from von Mises equivalent stress) to be representative of that due to the single shear cycle. However, due to the presence of the axial cycles, the computed equivalent stress history predicts several large magnitude cycles which result in significant fatigue damage. The same would also be true for the common case of combined bending-torsion loading. Therefore, the application of the signed equivalent stress approach, in this case, would lead to overly conservative fatigue life predictions.

Additionally, the consideration of mean stress effects for a signed equivalent stress history loses some physical relevance. In traditional equivalent stress-based approaches, an equivalent mean stress is computed from the mean values of the normal, or principal, stress components over a given cycle. This mean stress value is then used, along with the equivalent stress amplitude, in traditional uniaxial mean stress correction models to compute a new equivalent fully-reversed stress amplitude from which fatigue damage is computed. However, for a signed equivalent stress history, the mean value of the equivalent stress quantity is used directly in the mean stress corrections. As a result, this could lead to the prediction of increased fatigue damage due to the presence of mean shear stresses. However, mean shear stresses have been shown to have little effect on fatigue damage so long as they remain below the material yield strength (Socie and Marquis, 2000).

An alternative approach to performing a variable amplitude equivalent stress/strain-based fatigue analysis is to use the multiaxial cycle counting technique first proposed by Wang and Brown (Wang and Brown, 1996a). The Wang-Brown method
identifies loading reversals based on a relative equivalent stress or strain history. By redefining the reference point and relative stress history for each reversal counted, this approach is able to avoid the sign problem encountered when computing equivalent stress ranges from non-proportional loading histories. Individual stress and strain components between the start and end points of each identified reversal can then be used to compute equivalent stress amplitudes and mean values for use in damage calculation. Implementing such an approach, however, can add a considerable amount of complexity and/or computation time to a fatigue life analysis.

Despite the drawbacks, the variable amplitude equivalent stress- and strain-based analyses performed in this study were done so by cycle counting signed von Mises stress/strain histories to determine equivalent stress and strain ranges and mean values. This approach was chosen due to its increased simplicity and popularity as compared to the alternative Wang-Brown approach. The sign of the equivalent stress value at each point in the loading history was computed based on the sign of the principal stress with the largest absolute value at that particular point in time. Cycle counting for all variable amplitude fatigue analyses was performed using the simplified rainflow counting technique outlined in ASTM Standard E1049 (ASTM Standard E1049-85, 2009) for repeating load histories.

The final major difference between constant and variable amplitude fatigue life analyses is the need to sum the fatigue damage calculated for each cycle identified in a variable amplitude loading history. This allows a final life prediction to be obtained based on the total amount of fatigue damage contained in the loading history. Although there have been many models proposed to address various trends observed in the damage
accumulation process, as discussed in Section 2.6, the Palmgren-Miner linear damage rule (LDR) remains the simplest and most commonly used. Although some nonlinear rules may work better for certain materials and/or loading conditions, they lack general applicability and often cannot provide consistently better life predictions than LDR. Therefore, for the analyses performed in this study, damage from each cycle was computed on a cycle-by-cycle basis and summed using LDR. The inverse of the cumulative fatigue damage produced by one block of loading was taken as the number of loading blocks to failure for each testing condition.

To summarize the general progression of the equivalent stress and strain-based variable amplitude fatigue life analyses performed in this study, a flowchart is presented in Figure 6.6. Included in this figure are the specific analysis steps for each life prediction procedure implemented. More details on each analysis step, along with the variable amplitude life prediction results and comparisons to experimental data, are given in the following sections.

### 6.3.1 Un-Notched Specimen Life Predictions

For the variable amplitude equivalent stress-based approach implemented in this study, mean stress corrections were performed subsequent to cycle counting based on the identified equivalent stress ranges and mean values. Two different models were considered: modified Goodman and stress-based Smith-Watson-Topper (SWT). The modified Goodman mean stress correction is given as follows:

\[ S_{N_f} = \frac{\sigma_{qa}}{\sigma_u - \sigma_{qm}} \]  

\[ (6.1) \]
where $S_{Nf}$ is the effective fully-reversed stress amplitude for use in fatigue damage computation, $\sigma_{qa}$ and $\sigma_{qm}$ are the equivalent stress amplitude and mean stress, respectively, and $\sigma_u$ is the material ultimate tensile strength.

The SWT mean stress correction, on the other hand, takes the following form:

$$S_{Nf} = \sqrt{\sigma_{qa}\sigma_{q,max}}$$

(6.2)

where $\sigma_{q,max}$ is the maximum equivalent stress value corresponding to a particular cycle.

It should be noted that when the maximum stress value is negative, i.e. compression-compression loading, Equation (6.2) will produce imaginary roots. In this situation, no fatigue damage is assumed to occur. As a result, this model can lead to non-conservative life predictions for compression-dominated loadings.

Once the value of $S_{Nf}$ was determined for each counted cycle, fatigue damage was computed using the same fully-reversed uniaxial stress-life equation used for constant amplitude fatigue life predictions (Equation (5.1)). If the resulting fatigue life prediction for any given cycle exceeded that corresponding to the material fatigue limit (assumed to be $5(10^8)$ cycles for an aluminum alloy), the cycle was considered to be non-damaging.

Comparisons between experimental and predicted lives for variable amplitude un-notched specimen fatigue tests using the von Mises equivalent stress approach are shown, using solid symbols, in Figures 6.7(a) and 6.7(b) for the modified Goodman and SWT mean stress corrections, respectively. For both mean stress correction models, 3 of 10 (30%) and 8 of 10 (80%) un-notched specimen fatigue lives were predicted within factors of ±3 and ±10 of the experimental results, respectively. These results, along with subsequently presented results for additional life prediction approaches, are discussed in more detail in Section 6.3.3.
For the equivalent strain-based fatigue life analyses, since strains were not recorded during testing, and since maximum applied stress values often exceeded the material yield strength, equivalent strain histories were computed from equivalent stress based on a Ramberg-Osgood type cyclic stress-strain relation. Using this method, equivalent stresses and strains take the place of uniaxial quantities in Equation (2.4). When applying this technique to a variable amplitude stress history, however, appropriate memory rules for identifying reference turning points must first be implemented (Chu, 1995b; Socie and Marquis, 2000). These memory rules ensure that the formation of hysteresis loops is properly modeled based on the loading and unloading processes within a given sequence of stress reversals. Equivalent strain range, \( \Delta \varepsilon_q \), can then be determined for each successive loading reversal and used to construct the complete equivalent strain history. Relevant cyclic deformation properties needed for these calculations are given in Table 4.1.

As with the equivalent stress-based analyses, mean stresses were accounted for in the strain-based analyses by using common mean stress correction models. In this case, the two mean stress corrections considered were the SWT and modified Morrow models. Fatigue damage based on the SWT mean stress correction is computed from Equation (2.39) with the product of equivalent strain amplitude, \( \varepsilon_{qa} \), and the maximum equivalent stress, \( \sigma_{q,\text{max}} \), in any given cycle taking the place of the principal stress/strain quantities used in the critical plane interpretation.

However, like its stress-based counterpart, the strain-based version of the SWT mean stress correction is unable to properly quantify fatigue damage in situations where the maximum stress in a cycle is less than zero. Additionally, as mentioned in Section
5.4.1, in cases where loading is fully-reversed, this parameter will fail to predict fatigue lives equal to those obtained without the consideration of mean stress effects. Although an alternative formulation of the SWT parameter was presented, it was not considered in these variable amplitude fatigue life analyses due to the tension-dominated nature of the loading history under consideration.

The modified Morrow mean stress correction, on the other hand, accounts for mean stress effects by modifying the coefficients in the fully-reversed uniaxial strain-life equation. While the original Morrow correction only modifies the elastic term of the strain-life equation, the modified approach considers the mean stress effect on both elastic and plastic terms as follows (Manson and Halford, 1981):

$$\varepsilon_{qa} = \frac{\sigma'_f - \sigma_{qm}}{E} (2N_f)^b + \varepsilon'_f \left(\frac{\sigma'_f - \sigma_{qm}}{\sigma'_f}\right)^{c/b} (2N_f)^c$$

(6.3)

where $\sigma_{qm}$ is the equivalent mean stress in a cycle. When mean stress is equal to zero, Equation (6.3) reduces to the traditional strain-life equation, and when maximum stresses are compressive, fatigue damage is still predicted. Therefore, the issues discussed regarding the formulation of the SWT model do not affect life predictions based on the modified Morrow approach.

All material properties required for predicting fatigue life based on the von Mises equivalent strain approach are given in Table 5.6 and are the same as those used for constant amplitude strain-life analyses. Again, cycles producing fatigue life predictions exceeding $5(10^6)$ cycles were considered to be non-damaging. Comparisons between experimental and predicted fatigue lives for variable amplitude un-notched specimen tests are shown in Figures 6.8(a) and 6.8(b) for modified Morrow and SWT mean stress corrections, respectively. For analyses based on the modified Morrow mean stress
corrections, only 1 of 10 (10%) and 4 of 10 (40%) un-notched specimen life predictions were within factors of ±3 and ±10 of the experimental results, respectively, while the SWT approach resulted in 2 of 10 (20%) predictions within a factor of ±3 and 8 of 10 (80%) predictions within a factor of ±10 of experimental results. A summary of all equivalent stress- and strain-based variable amplitude fatigue life prediction results for un-notched specimen tests is provided in Table 6.3. Again, these results are discussed in more detail in Section 6.3.3.

6.3.2 Notched Specimen Life Predictions

For the notched specimen variable amplitude fatigue tests performed in this study, fatigue lives were predicted using the same local von Mises stress, local von Mises strain, and local elastic/pseudo von Mises stress approaches used for the constant amplitude fatigue life analyses presented in Section 5.3.2. As was the case for constant amplitude loading, the location at which stress-strain variations are computed is an important consideration in a variable amplitude notch analysis as well. Since the magnitude and location of maximum local stresses can change with a change in the nominal stress ratio, the critical analysis location can potentially change for each cycle in a variable amplitude loading history. This makes fatigue life prediction for loading histories with varying degrees of non-proportionality particularly complex.

Under these conditions, if the failure location is unknown prior to damage calculation, fatigue life analyses may need to performed at multiple potential failure locations in order to determine which will produce the minimum fatigue life prediction. In the case of this study, however, crack initiation was observed to occur at the 0° and 180° locations around the hole for axial and combined loading conditions, while pure
torsion loading resulted in failures at the −45° and 135° locations. As such, all subsequent fatigue life analyses were performed at these locations. Life prediction results, along with comparisons to experimental data, are presented in the following sections for each analysis approach considered.

6.3.2.1 Equivalent Pseudo Stress-Based Approach

With the proper location for fatigue life analysis known, the next step was to derive the local elastic stress history at this location for each applied loading condition. This was done, as explained in Section 4.7, by using linear elastic stress concentration factors, along with the nominally applied loading history and principle of superposition, to compute time histories for the variation of each local elastic stress component. Finally, for each time point in the resulting stress histories, a signed elastic/pseudo von Mises equivalent stress value was computed.

After local equivalent stress variations were determined, cycle counting and mean stress corrections were then performed using the same stress-based procedures outlined in the previous section for un-notched specimens. Finally, fatigue damage for each cycle was computed based on the same local pseudo stress-life curve constructed in Section 5.3.2.1, and summed using the linear damage rule.

Comparisons between experimental and predicted variable amplitude notched specimen fatigue lives based on the local pseudo stress approach are shown using open symbols in Figures 6.9(a) and 6.9(b) for Modified Goodman and SWT mean stress corrections, respectively. The un-notched specimen predictions (solid symbols) in these figures are the same as those shown in Figures 6.7(a) and 6.7(b) for von Mises equivalent stress-based predictions. Results for edited history tests are indicated by gray shaded
symbols. For the modified Goodman mean stress correction, only 3 of 17 (18%) and 7 of 17 (41%) notched specimen life predictions fell within factors of ±3 and ±10 of the experimental results, respectively. For the SWT approach, on the other hand, all notched fatigue lives were predicted within a factor of ±3. As was the case for un-notched specimen analyses, these results, along with subsequently presented results for additional life prediction approaches, are discussed in more detail in Section 6.3.3.

6.3.2.2 Equivalent Stress-Based Approach

While life predictions in the previous section were computed based on local elastic stress variations, if the maximum applied loading is large enough, localized plasticity corrections should also be performed prior to damage calculation in order to more accurately reflect material behavior at the notch. As discussed in Section 4.8, notch stress-strain estimation models, such as Neuber’s rule, are often used for this purpose. However, it was also pointed out that notch rules which correct for plasticity based on an equivalent stress range cannot be directly applied to non-proportionally varying stress histories without the use of plasticity modeling techniques. Although they can be applied to a signed equivalent local stress/strain history, these models remain unable to account for increased fatigue damage due to non-proportional loading in such cases.

Nevertheless, for the variable amplitude fatigue life analyses performed in this study, the local von Mises pseudo stress history was used in conjunction with a Ramberg-Osgood type cyclic stress-strain relation, Equation (2.4), to determine local elastic-plastic equivalent stress histories based on the following expression of Neuber’s rule:

\[
\frac{(\Delta \sigma_q^e)^2}{E} = \left(\frac{\Delta \sigma_q}{E}\right)^2 + 2\Delta \sigma_q \left(\frac{\Delta \sigma_q}{2K'}\right)^{1/n'}
\]  

(6.4)
where $\Delta \sigma_q^e$ is the local equivalent pseudo stress range, $K'$ is the material cyclic axial strength coefficient, and $n'$ is the cyclic axial strain hardening exponent. By applying the same memory rules mentioned in Section 6.3.1 for reference turning points, local equivalent stress range, $\Delta \sigma_q$, was determined for each successive loading reversal and used to construct the complete variable amplitude elastic-plastic stress history. Once the local stress history was established for each applied loading condition, cycle counting, mean stress correction, and fatigue damage calculation (from Equation (5.1)) were then performed using the same stress-based procedures outlined in Section 6.3.1 for un-notched specimen analyses.

It should be noted that in using this approach, the effects of stress gradients are not considered in fatigue damage calculation. As discussed previously, a typical method of accounting for stress gradients, and thus reducing the conservatism in resulting fatigue life predictions, is to substitute fatigue notch factor, $K_{fq}$, for stress concentration factor, $K_{tq}$, when implementing Neuber’s rule. However, due to the continuous variation of $K_{tq}$ at any given location around the notch root (a result of changing nominal stress ratios), the definition, as well as physical interpretation, of $K_{fq}$ becomes vague under non-proportional multiaxial variable amplitude loading conditions. Additionally, because nominal stress values, and thus $K_{fq}$, cannot be meaningfully defined for more complex component geometries, fatigue notch factors are often ignored in practice when performing complex fatigue life analyses (Khosrovaneh et al., 2004). For these reasons, only maximum stress values at the notch root were used to compute fatigue damage for the variable amplitude analyses performed in this study.
The resulting local equivalent stress-based variable amplitude notched specimen life predictions are compared to experimental fatigue lives, using open symbols, in Figures 6.7(a) and 6.7(b) for modified Goodman and SWT mean stress corrections, respectively. Again, data for edited history tests are indicated by gray shaded symbols. For both mean stress corrections considered, all notched fatigue lives were predicted within a factor of ±3.

6.3.2.3 Equivalent Strain-Based Approach

In addition to the local stress-based approach, local strain-based fatigue life analyses were also performed using von Mises equivalent strain values derived from Neuber’s rule. Using the equivalent stress ranges obtained from Equation (6.4), local elastic-plastic strain values were subsequently determined for each loading reversal based on Equation (2.4). Therefore, local equivalent strain histories for each analysis were constructed concurrently with the equivalent stress histories discussed in the previous section.

Having computed the variation of local equivalent strain for each testing condition, fatigue life was then calculated using identical procedures, including mean stress corrections, to those used for the equivalent strain-life analyses presented in Section 6.3.1 for un-notched specimens. As in the case of the local stress approach, no consideration was given to stress gradient effects under variable amplitude loading conditions.

Comparisons between experimental and predicted fatigue lives for the variable amplitude notched specimen tests are shown, using open symbols (gray shaded symbols for edited history tests), in Figures 6.8(a) and 6.8(b) for the modified Morrow and SWT
mean stress corrections, respectively. For the modified Morrow mean stress correction, 16 of 17 (94%) and 17 of 17 (100%) notched specimen life predictions fell within factors of ±3 and ±10 of the experimental results, respectively, while for the SWT approach, all predictions were within a factor of ±3. A summary of all equivalent stress- and strain-based variable amplitude fatigue life prediction results for notched specimen tests is provided in Table 6.4. Additionally, these results are discussed in more detail in the following section.

6.3.3 Discussion on Equivalent Stress- and Strain-Based Life Predictions

Similar to the constant amplitude equivalent stress/strain-based analyses in Section 5.3, because they show the best agreement with experimental data, attention was first be focused on notched specimen fatigue life predictions when interpreting the analysis results presented in the previous sections. For the variable amplitude notched specimen tests, fatigue lives were predicted using the same three approaches used for the constant amplitude analyses, but with the addition of mean stress corrections. For both the local von Mises equivalent stress-based and strain-based analyses, all life predictions fell within a factor of ±3 of experimental results, except when using the modified Morrow mean stress correction in conjunction with the strain-life approach. In this case, 94% of predictions were still within ±3. This compares to 89% of constant amplitude notched specimen fatigue life predictions within a factor of ±3 for both the stress- and strain-based approaches.

For the local pseudo stress approach, however, large differences in predicted fatigue lives were obtained depending on the mean stress correction model used. For the
SWT approach, results were similar to those for the local stress- and strain-based approaches, with all lives predicted within a factor of ±3. This compares to 89% of predictions within the same error factor for constant amplitude fatigue data. For the modified Goodman mean stress correction, however, it can be seen from Figure 6.9(a) that predictions become increasingly more conservative as loading levels are increased. As a result, only 18% of the predicted fatigue lives are within a factor of ±3 of the experimental results. Due to the formulation of the modified Goodman equation, however, large pseudo stress values (resulting from the absence of plasticity corrections) can force unrealistically high equivalent stress amplitude values as they approach the ultimate strength of the material. Therefore, it can be concluded that this mean stress correction is not suitable for use with the local pseudo stress-based approach when such conditions exist.

As discussed in Section 5.3.3 for constant amplitude notched specimen analysis results, the fairly high level of accuracy of the equivalent stress- and strain-based fatigue life predictions is not entirely surprising. Not only does the specimen geometry used in this study result in a uniaxial stress state around the perimeter of the notch regardless of the nominally applied loading, but because of the nature of the variable amplitude loading history, a large percentage of the applied stress amplitudes, even at the notch, are elastic despite large values of maximum stress (corresponding to occasional overloads).

To help illustrate this point, box and whisker plots of the local elastic von Mises equivalent stress amplitude for each cycle in each notched specimen fatigue test are shown in Figure 6.10(a). In this figure, the y-axis labels give the maximum axial stress (for axial and combined loadings) or maximum shear stress (for torsion), in MPa, along
with the type of loading (A: Axial, T: Torsion, AT: Axial-Torsion). The shaded region shown over the box and whisker plots represents the range of stress amplitudes for 99.5% of the applied cycles. With a monotonic yield stress, \( \sigma_y \), of 330 MPa and a cyclic yield stress, \( \sigma'_y \), of 415 MPa, it is clear that the majority of cycles, even for the highest applied loading levels, remain completely elastic. For this special case of uniaxial and primarily elastic local stresses, it is expected that the equivalent stress/strain approaches would produce reasonable life predictions. Additionally, the sign problem for equivalent stresses and strains is not a factor for the resulting uniaxial stress history. However, it should be emphasized that these conditions are not always present in multiaxial service loading histories (with or without a notch) and the equivalent stress/strain approaches would be expected to break down in the more complex cases.

It should also be noted that, as mentioned previously, the local stress and local strain-based approaches investigated in this study do not consider stress gradient effects in the notched specimen variable amplitude fatigue life analyses. This is despite the fact that results from constant amplitude fatigue life analyses indicate the importance of doing so. Complications with applying fatigue notch factors under multiaxial non-proportional loading conditions, combined with their lack of implementation in industrial applications, were the primary reasons for not considering them. However, if the effect of stress gradients on variable amplitude life predictions is assumed to be similar to that observed for constant amplitude analyses, a shift of close to an order of magnitude to the non-conservative side would be expected. This would effectively reduce the accuracy of the notched specimen life predictions and place them together with those for un-notched specimens.
When considering un-notched specimen tests, equivalent stress- and strain-based fatigue life prediction trends are consistent between both constant and variable amplitude loading conditions. Although the variable amplitude combined loading tests contained non-proportional loading events, the loading history is dominated by tensile stress. Therefore, the fact that combined loading life predictions were similar to those for pure axial loading was expected. However, similar to the constant amplitude predictions, the variable amplitude pure shear loadings resulted in consistently more conservative and/or less non-conservative life predictions as compared to those for the other loading conditions.

For the equivalent stress-based approach, neither mean stress correction was able to predict more than 30% of the variable amplitude fatigue lives within a factor of ±3 of experimental results. This can be compared to 43% of lives predicted within a factor of ±3 for constant amplitude analyses. The equivalent strain-based approach, on the other hand, resulted in even more non-conservative life predictions. This approach, combined with the SWT mean stress correction, was only able to predict 20% of un-notched specimen fatigue lives within a factor of ±3. This can be compared to 50% of predictions within a factor of ±3 for the constant amplitude equivalent strain-life analyses. Despite the fact that over 99.5% of all equivalent stress amplitudes remain completely elastic for all un-notched specimen tests performed (as seen in Figure 6.10(b)), predictions are still considerably worse than in the case of the notched specimens. As mentioned previously, this is most likely due to the presence of multiaxial stress states in the un-notched specimen tests.
In general, one trend observed for both un-notched and notched specimen analyses is that variable amplitude fatigue life predictions tend to be more non-conservative than those for constant amplitude loadings, regardless of applied loading conditions and/or analysis approach considered. This would be especially evident if stress gradient effects had been considered in the variable amplitude notched specimen analyses. Given the difference between the constant and variable amplitude analysis procedures, this discrepancy can be narrowed down to three likely sources: cycle counting, mean stress correction, and/or the summation of cumulative fatigue damage. However, since rainflow counting is a well-established method of identifying loading cycles, and since all cycle counting was performed based on a single equivalent stress/strain history, it is not very likely that these differences stem from the cycle counting procedure. Similarly, because multiple well-established mean stress correction models produced similar fatigue life predictions for all loading conditions and all analyses approaches considered (excluding the pseudo-stress approach), it is also not likely that these differences resulted from the presence of mean stresses.

This leaves cumulative damage summation/linear damage rule as the most likely cause of the non-conservative life predictions observed for tests performed under variable amplitude loading conditions. It has been well established, through numerous studies, that linear damage rules often cannot properly account for the load sequence effects encountered in many variable amplitude loading histories. An example commonly used to demonstrate this effect is the case of two-step high-low loadings. Even for this simple loading case, LDR tends to predict non-conservative fatigue lives (Colin and Fatemi, 2010; Jiang and Kurath, 1997a), a trend often attributed to cyclic hardening and/or the
development and growth of microcracks. Conversely, fatigue lives for low-high loadings are frequently longer than those predicted by LDR. Typically, to overcome these issues, either nonlinear cumulative damage rules are employed (Fatemi and Yang, 1998), or critical damage sums (assumed damage value at failure) other than unity are used with LDR (Lee et al., 2007; Miller, 1970). However, as mentioned previously, no single nonlinear damage rule has been shown to be generally applicable under a wide range of variable amplitude loading spectra. Additionally, an appropriate value for critical damage sum is difficult to define for LDR, as it can change depending on characteristics of the applied loading history. Therefore, LDR is still the most widely used damage rule in fatigue life analysis.

As discussed in Section 2.6, however, employing more advanced fatigue life prediction methodologies has been shown to provide the ability to overcome some of the limitations typically attributed to LDR. For example, Colin and Fatemi (2010) performed life predictions for specimens made of an aluminum alloy and a stainless steel alloy under two-level step, periodic overload, and random variable amplitude uniaxial loadings. By using a critical plane-based fatigue damage parameter, which considers both stress and strain terms to account for changes in material constitutive behavior, life predictions were found to significantly improve compared to those based on more traditional stress-life and strain-life approaches. This is despite the fact that LDR was used for damage summation in both cases. Similar improvements in life predictions were observed for two-level step tests by Jiang and Kurath (1997a), using a steel and a stainless steel alloy, when considering transient material deformation behavior, along with the Fatemi-Socie
damage parameter (stress and strain terms), as compared to those obtained from a stress-based critical plane parameter.

Additionally, when studying the applicability of LDR to situations involving alternating blocks of axial, torsion, and combined axial-torsion loading, Shamsaei et al. (2010b) (titanium and titanium alloy) and Gladskyi and Fatemi (2013) (low carbon steel) found most life predictions to be within a factor of ±2 of experimental results when the Fatemi-Socie parameter was used to compute fatigue damage. Based on the results of these four studies, it is evident that by considering transient material behavior during cyclic plasticity modeling and/or fatigue damage calculation, at least some of the deficiencies attributed to LDR can be overcome through more appropriate modeling of the entire fatigue damage process.

Aside from general load sequence effects, Pompetzki et al. (1990) also showed that due to the effects of periodic overloads and/or underloads on microcrack growth, smaller cycles below the material fatigue limit have the potential to significantly contribute to fatigue damage in a variable amplitude loading history. However, in this study, fatigue life predictions for notched specimen tests conducted with edited load histories (no small cycles) are only slightly less non-conservative than those for the full history tests. This suggests that although the damage caused by small cycles was somewhat higher than predicted, the effect did not significantly impact the overall quality of fatigue life predictions. This is true for all of the analysis approaches and loading conditions considered.

In summary, although equivalent stress- and strain-based fatigue life prediction approaches are relatively simple to implement under multiaxial variable amplitude
loading conditions, they are not capable of producing consistently accurate fatigue life predictions under such conditions. While more complexity may be involved in their implementation, critical plane-based analysis approaches, combined with advanced cyclic plasticity models and fatigue damage parameters, provide the ability to more accurately reflect damage mechanisms and account for the more challenging aspects of the fatigue failure process. As a result, it is reasonable to expect improved variable amplitude fatigue life predictions when using such methods, similar to what was observed in Section 5.4 for the constant amplitude fatigue tests performed in this study.

Additional benefits of critical plane-based fatigue life analyses not only include the ability to identify a crack initiation orientation, but damage parameters containing a normal stress term are also able to directly account for the effects of mean stress on fatigue damage. This eliminates the need for the additional mean stress correction models, along with any associated shortcomings, required when using equivalent stress/strain-based approaches. Furthermore, for notched components, multiaxial notch deformation (Section 4.8) and stress gradient effects (Section 5.4.3) were shown to be better accounted for through more robust modeling approaches such as pseudo stress-based cyclic plasticity corrections and Theory of Critical Distances approaches, respectively. Performing multiaxial variable amplitude fatigue life analyses using such techniques was a major area of focus for this research and is addressed in the following section.
6.4 Critical Plane-Based Life Predictions

Similar to the results for constant amplitude loading, the fatigue life correlations presented in Section 6.3 demonstrate an inability of the equivalent stress/strain approaches to properly quantify fatigue damage under multiaxial loading conditions. Unlike the constant amplitude analyses, however, there are several additional factors that can contribute to the overall life prediction error in a variable amplitude fatigue life analysis. Among these are: load history effects, varying degrees of loading non-proportionality, mean stress consideration (in the case of this study), cycle counting, and fatigue damage accumulation. While many of these issues can only be addressed to a limited extend in an equivalent stress/strain-based fatigue life analysis, critical plane-based analysis approaches offer the ability to more accurately reflect their impact on fatigue damage evolution. More detailed discussion is provided in the following paragraphs.

Under constant amplitude loading conditions, changes in material constitutive behavior occur at the beginning of a loading sequence, generally over a relatively small fraction of the total fatigue life. Therefore, the majority of fatigue damage is accumulated under stable material conditions, and transient effects are typically considered to be negligible in a fatigue life analysis. For variable amplitude loading histories, however, intermittent large loading cycles and/or changes in the loading non-proportionality can result in transient hardening and/or softening behavior occurring throughout the entire loading history. This is especially true in situations where large plastic deformations occur, such as in the low cycle fatigue regime and at stress concentrations. As a result,
material properties and/or deformation behavior can be highly dependent on the previously applied loading cycles.

For damage parameters which only consider stress or strain terms, such as equivalent stress and equivalent strain approaches, the effect of this load history dependence on fatigue damage cannot be captured. Therefore, there is the potential for inaccurate fatigue damage to be calculated over significant portions of the total fatigue life of a component. Similarly, for fatigue damage parameters which rely on both stress and strain terms to compute fatigue damage, ignoring transients and only considering stabilized material behavior can have potentially significant implications in a variable amplitude fatigue life analysis.

In order to overcome this deficiency, incorporating the modeling of transient material behavior into the cyclic plasticity modeling process, and calculating fatigue damage using parameters which consider both stress and strain terms (e.g. the FS damage parameter) provides the ability to more accurately reflect the progression of fatigue damage throughout a given loading history. For this reason, fatigue life analyses in the following sections are performed using both the assumption of stable material behavior, and based on stress-strain estimations which consider transient material response. Comparisons are then made between the results and some discussion is provided.

In addition to changes in material constitutive behavior, critical plane damage parameters including a maximum normal stress term are also able to implicitly account for the detrimental effect of mean tensile stresses on fatigue life. By doing so, this eliminates the need to implement the additional mean stress correction models (along with any associated shortcomings) required for equivalent/stress strain approaches. By
making sure the modified FS damage parameter was able to simultaneously account for the effects of mean stress and multiaxial stress states, as demonstrated by the constant amplitude fatigue life predictions in Section 5.4.2, significant confidence is gained in this respect when evaluating the variable amplitude life predictions presented in the following sections.

When performing a fatigue analysis using a critical plane-based approach, additional considerations must be given when choosing a cycle counting procedure. These approaches often require the use of specialized multiaxial cycle counting techniques in order to properly track the variation of multiple stress and/or strain components, required for damage calculation, over any given cycle. The two most popular multiaxial cycle counting methods, the Bannantine-Socie (BS) and Wang-Brown (WB) approaches, were discussed in Section 2.5 along with the advantages and disadvantages of each respective approach.

For the critical plane-based variable amplitude fatigue life analyses performed in this study, the BS multiaxial cycle counting approach was the method of choice. This approach first projects the individual stress and strain component histories onto several candidate planes at the desired analysis location. Then, cycle counting is performed, for each plane, on whichever stress or strain component drives fatigue damage for the chosen damage parameter (e.g. shear strain for the FS parameter). For each identified cycle, any secondary quantities required for damage calculation (e.g. maximum normal stress for the FS parameter) are identified between the starting and ending points of each cycle. After fatigue damage is calculated and summed, the plane experiencing the greatest amount of fatigue damage is considered to be the critical plane.
The BS approach was chosen over the alternative WB approach for a few reasons. Perhaps the most significant is the fact that the WB approach cannot identify a unique failure plane in situations where the degree of non-proportionality changes throughout a loading history. Because of this, the physical interpretation of the critical plane concept is lost, and an initial crack orientation is not available for subsequent crack growth analyses. Additionally, since the FS parameter assumes that alternating shear strain is the primary driving force behind crack nucleation and small crack growth, it makes sense from a physical standpoint to identify cycles based on the shear strain variation as opposed to an equivalent stress/strain variation, as is the case for the WB approach.

Since cycle counting is performed on a single strain component using the BS approach, uniaxial cycle counting techniques such as rainflow counting are still applicable, so long as the variation of any secondary quantities needed for damage calculation are properly tracked. The exact type of counting technique best suited for a particular fatigue life analysis, however, can change depending on the nature of the stress-strain history being analyzed. For example, the signed von Mises stress/strain histories analyzed in the previous sections were cycle counted using the simplified rainflow cycle counting procedure. Simplified rainflow counting, which only identifies complete cycles, was chosen because of the repeating nature of the variable amplitude loading histories applied in this study.

The major difference between traditional and simplified rainflow counting is that in the simplified version, the loading is rearranged to begin with the largest absolute value counting variable in the history. By closing the history with the same value (since the history is assumed to repeat), it is ensured that the most damaging event in the
loading history will be counted as a complete cycle. If a loading history already begins with the largest magnitude peak or valley, both traditional and simplified rainflow counting will produce the same cycle count (ASTM Standard E1049-85, 2009). However, in certain situations, traditional rainflow counting may still identify pairs of half cycles as opposed to only whole cycles.

Similar to an equivalent stress/strain analysis, when considering the repetitive nature of a stabilized stress-strain history, simplified rainflow counting is a logical approach to cycle counting in a critical plane-based analysis. When transient material behavior is accounted for in an analysis, however, this is not the case. Because of the load history dependence on material deformation behavior, rearranging the stress-strain sequence before cycle counting can result in cycle counts which do not reflect the actual stress-strain variations experienced by the material. In addition, counting the largest magnitude loading event as a whole cycle may lead to overly conservative damage predictions when large non-reversed plastic deformations occur.

To help illustrate this point, stress-strain predictions considering both transient and stabilized material behavior are compared in Figure 6.11 for the loading history used in this study. In this figure, predicted stress-strain responses for loading blocks 1 and 2, assuming transient material behavior, are compared to those for an entire stable block of loading (assuming stabilized material constitutive behavior). Only results for the highest loading level combined axial-torsion test (3.0× scale factor) are shown in the figure, but stress-strain responses for other loading conditions and loading levels exhibit the same type of behavior. From this figure, it is easy to see that the predicted stress-strain
variation for the first applied loading block (Figure 6.11(a)) is significantly different than that derived under stable material assumptions (Figure 6.11(c)).

Additionally, since the largest applied loading cycles for both the axial and shear channels have significant mean stress, there is little to no reversed deformation for any of the loading histories considered in this study. Therefore, once the maximum stress in the history is reached for the first time, the remaining blocks of loading are primarily elastic (nearly all subsequent stress states remain within the translated yield surface). This also means that after the first loading block is applied in the transient analysis, the stress-strain history for all subsequent blocks is nearly identical to that derived for the stable loading block. This can be observed by comparing the stress-strain response from the second transient loading block (Figure 6.11(b)) to that derived for a stable loading block (Figure 6.11(c)). As the number of applied loading blocks increases, the transient material response becomes closer and closer to the stable block prediction.

Given the large permeant plastic deformations experienced in the first block of applied loading, as evidenced by Figure 6.11(a), and the fact that these deformations do not form closed hysteresis loops (i.e. the deformation is not reversed), it is clear that traditional rainflow counting should be used for the transient fatigue life analyses performed in this study. Since traditional rainflow counting is able to identify half cycles in addition to whole cycles, it is able to properly consider the non-reversed nature of these deformations. As mentioned earlier, implementing simplified rainflow counting could possibly lead to overly conservative fatigue damage predictions in such a case.

Even the traditional rainflow counting technique, however, is not without its disadvantages when applied to a critical plane-based fatigue life analysis. One of the
more obvious issues with this technique is the identification of half cycles and whole cycles as opposed to only whole cycles. Because critical plane parameters such as SWT and FS rely on the maximum normal stress per cycle to compute fatigue damage, this can lead to inconsistencies in damage calculations. Figure 6.12 illustrates a simple example of such a case. In this figure, for the shear strain variation shown, both rainflow and simplified rainflow counting produce an identical cycle count in terms of shear strain ranges. For simplified rainflow counting, because only whole cycles are identified, the largest maximum normal stress occurring during each half of a particular cycle is taken to be the maximum normal stress occurring over the entire cycle. For traditional rainflow counting, on the other, because the larger cycle is counted as two half cycles, significantly different maximum normal stresses are associated with each half of the cycle. As a result, the cumulative fatigue damage computed from the traditional rainflow count (based on linear damage accumulation) will be less that that computed based on the simplified rainflow count. However, which damage value is most representative of the actual fatigue damage induced by the loading history is not easy to determine.

In an attempt to quantify some of the effects that cycle counting procedures can have on a critical plane-based fatigue life analysis, life predictions based on different cycle counting techniques are compared in the following section. These comparisons are made based on stabilized stress-strain response and are aimed at evaluating the effect of closing the largest hysteresis loop in the loading history, as well as the effect of half versus whole cycles on fatigue damage.

Since it was shown in Section 5.4 that the Fatemi-Socie shear-based damage parameter was consistent with experimentally observed cracking mechanisms and
resulted in excellent correlation of constant amplitude fatigue data, it is the only parameter considered in this chapter. For comparison purposes, damage analysis was performed using the parameter in both its original form, as well as with the modified formulation.

The last step in a variable amplitude fatigue analysis is to sum the fatigue damage caused by individual loading cycles in order to produce a total fatigue life prediction. As discussed previously, although some nonlinear damage rules may more accurately reflect experimentally observed cumulative fatigue damage trends for certain materials and/or loading conditions, they lack general applicability and often cannot provide consistently better life predictions than LDR. Additionally, evidence found in literature (discussed in Section 6.3.3) suggests that some of the typical drawbacks of LDR may, in fact, be due to inadequate modeling of other areas of the fatigue life analysis process. For these reasons, and in light of the measures taken to overcome typical deficiencies in multiaxial variable amplitude life predictions, LDR was chosen for the critical plane-based fatigue analyses performed in this study. In these analyses, consistent with the equivalent stress/strain-based analyses, damage from each cycle was computed on a cycle-by-cycle basis prior to summation. Any individual cycle resulting in a life prediction exceeding the material fatigue limit life (assumed to be 5(10^8) cycles) was considered to be non-damaging. The inverse of the cumulative fatigue damage produced by one block of loading was taken as the number of loading blocks to failure for each testing condition.

6.4.1 Un-Notched Specimen Life Predictions

For the un-notched specimen variable amplitude fatigue tests performed in this study, critical plane-based fatigue life analyses were performed based on assumptions of
stable material stress-strain response, transient stress-strain response, and using different definitions for fatigue crack initiation. The results of these analyses, including comparisons between experimental and predicted cracking orientations, are presented in the following sections.

6.4.1.1 Life Predictions Based on Stable Stress-Strain Response

The first set of critical plane fatigue life analyses presented are based on the consideration of only stabilized material constitutive behavior. To clarify, this means that the effects of initial cyclic hardening and initial deformation behavior (prior to any load history dependence) were ignored in the plasticity modeling process. However, because the variable amplitude combined axial-torsion loading history used in this study contains varying degrees of loading non-proportionality, changes in material constitutive behavior were still considered when changes in non-proportional stressing occurred. This was done through incorporation of the plasticity modeling procedures outlined in Section 4.5.

Instead of using the monotonic stress-strain curve as the initial material condition in the plasticity model, however, the stabilized in-phase axial-torsion stress-strain curve was used instead. Although the cyclic axial deformation curve could have also been used, the in-phase curve was chosen due to its slightly better overall agreement with the deformation data presented in Figure 4.7 for all stress states considered. Then, plasticity modeling was performed twice for the entire loading history. The final state of the plasticity model from the first analysis, in terms of backstress components and strain state, was used as the initial conditions for the second analysis. This way, load history dependence from previous loading blocks was properly accounted for in the stabilized stress-strain response. Stress transformations were then performed on the resulting stress-
strain histories, in 5° increments, prior to cycle counting. Individual loading cycles were identified on each plane using the BS approach.

In order to evaluate the effect of cycle counting technique on fatigue damage calculations, stable block life predictions were first compared based on cycles identified using three different rainflow counting routines: simplified rainflow counting, traditional rainflow counting, and traditional rainflow counting with the loading history reordered to start, but not end, with the largest magnitude peak or valley. As discussed previously, there are two main differences between the simplified and standard rainflow techniques: simplified rainflow counting always closes the largest hysteresis loop in the loading history, and while simplified rainflow only counts complete cycles, regular rainflow counts half cycles as well. To evaluate the effect of closing the largest hysteresis loop, life prediction results can be compared between simplified rainflow counting and the reordered traditional rainflow counting. By not ending the loading history with the largest magnitude peak or valley in the latter case, the only difference between the cycle counts is that the traditional rainflow algorithm will count the largest magnitude loading event as a half cycle rather than a complete cycle. Finally, by comparing life predictions based on the two different traditional rainflow counting algorithms, the effect of half cycles on damage calculation can be evaluated.

The results of these analyses revealed that there was not a significant difference in life predictions depending on which cycle counting technique was employed. This was true for analyses performed based on both the original and modified FS parameters, using a $k$ value of 1 and uniaxial fatigue properties. As mentioned previously, however, due to the tension/positive shear-dominated nature of the loading history used in this study, not a
significant amount of reversed plastic deformation occurs for any of the applied loading conditions. Therefore, stress-strain behavior subsequent to the first loading block is predominately linear elastic in all cases. As a result, the effect of closing the largest magnitude loading event in the simplified rainflow count was negligible. The largest observed difference in any life prediction between the simplified rainflow and reordered traditional rainflow analyses was less than 0.3%.

While these results are specific to the loading history analyzed in this study, similar results would also be expected for histories which include reversed plastic deformation. This is because when yielding occurs in both the negative and positive stress directions, closed hysteresis loops are formed. Therefore, if a closed, or nearly closed loop is naturally created for the largest amplitude loading cycle in the history, there should again be a negligible difference between fatigue damage computed from reordered and simplified rainflow counts. A situation where a large difference might be expected, however, is for a short loading history containing periodic overloads, or a loading history which causes significant ratcheting behavior. Because ratcheting would lead to the one directional accumulation of large permanent strains, closing the largest cycle for such a strain history would not be an accurate representation of the actual deformation behavior.

The second comparisons made were between life predictions based on the two different versions of traditional rainflow counting. Because the reordered count starts with the highest magnitude loading event, but leaves the largest cycle open, the largest reversal is the only half cycle identified in the entire loading history. However, this approach should identify the same shear strain cycle count as traditional (not reordered)
rainflow counting. Therefore, since the non-reordered count will identify a larger number of half cycles, any differences in predicted fatigue life can be attributed to differences in fatigue damage calculated for whole cycles versus half cycles.

Although the effect of half cycles appeared to have a larger impact on fatigue life predictions than that due to closing the largest loading cycle, the effect was still negligible for the loading histories used in this study. The largest observed difference in predicted life between the two traditional rainflow counting analyses was still less than 0.9%. However, it should be noted that the number of half cycles identified in the loading history was small for each nominal loading condition considered. A total of 22 half cycles were identified for both levels of the axial only loading history, compared to 20 for the combined histories and 14 for the shear only history. While these half cycles tended to correspond to more damaging loading events, where the effect of half versus whole cycle is greatest, the fact that they make up such a small percentage of the total reversals limits their effect on resulting fatigue life predictions. For loading histories where half cycles make up a larger percentage of the total number of cycles, however, a larger difference in fatigue damage calculations would be expected.

Since the influence of cycle counting procedure on stable block life predictions was found to be negligible for the loading histories analyzed in this study, experimental versus predicted fatigue life comparisons were made based on analyses performed using the simplified rainflow counting technique to identify loading cycles. The results of these comparisons are shown in Figure 6.13(a) based on the original FS parameter, and in Figure 6.13(b) based on the modified form of the parameter. Again, life predictions were computed based on uniaxial fatigue properties and $k = 1$. Uniaxial fatigue properties were
chosen due to the fact that constant amplitude life predictions, presented in Section 5.4, showed better agreement with experimental results when based on uniaxial properties as opposed to shear strain-life properties.

Each data point in these figures represents the total experimental fatigue life of a particular specimen (up to a tip-to-tip crack length of approximately 10-15 mm). However, for specimens with available crack growth data, bars were added to the corresponding data points which reflect the total crack growth life (from 0.2 mm to failure). These will be explained in more detail in following discussions. It should also be noted that one test, performed at the highest pure axial loading level, is not included in any of the subsequent analysis results. This is because the large applied stress levels in this test resulted in very large plastic deformations, which could not be accurately accounted for using the plasticity modeling procedures implemented in this study. Additionally, because fatigue life predictions were computed based on the cumulative damage sum from an entire block of loading, fatigue life comparisons are not as meaningful for this loading level since the entire loading history was not applied during the short life of this test.

From the results in Figure 6.13, it is clear that the analysis approach implemented was not able to quantify fatigue damage under variable amplitude loading conditions to a level of accuracy comparable to that observed for constant amplitude life predictions. Predictions are observed to be consistently non-conservative for all loading conditions analyzed, although the modified FS parameter results in life predictions which are non-conservative to a lesser extent than those based on the original form of the parameter. This difference between the parameters was expected, however, due to the significant
tensile mean stress present in the applied loading histories. It was shown in constant amplitude fatigue life analyses (Section 5.4.2) that FS parameter can have a tendency to underestimate fatigue damage in such cases, and that the modified parameter is better able to account for the effects of mean stress on fatigue damage.

The mean stress effect is further evidenced by the fact that the smallest difference in predicted life between the two damage parameters is observed for the pure shear loading history, where mean shear stresses do not affect fatigue damage values. The difference in life predictions that does exist between the parameters for pure shear loading, however, is the result of different normal stress contributions on the maximum damage plane. Since the majority of cycles in the shear history are small, they correspond to the region of the fatigue life curve where the slope can be relatively flat. Therefore, even for small differences in damage values, the difference in life predictions can be notable.

Despite the improvements in analysis results obtained by using the modified FS parameter for damage calculation, still only 2 of 9 life predictions were within a factor of ±3 of experimental lives. All lives, however, were predicted within a factor of ±10. This can be compared to 0 of 9 and 3 of 9 fatigue life predictions within factors of ±3 and ±10, respectively, when based on the original form of parameter. Again, as a clarification, when the “±” sign precedes a factor of difference expression in this dissertation, this means a factor of 3 or 10 greater than, or a factor of 3 or 10 less than the experimental result.
6.4.1.2 Life Predictions Based on Transient Stress-Strain Response

Given the non-conservative nature of these predictions, combined with the potential for large plastic deformations to be ignored when only considering stabilized material response, fatigue analyses were performed again for all loading conditions after considering transient material response. The same plasticity model formulation presented in Section 4.5, along with the same modeling parameters, was used in deriving the stress-strain response for these analyses. The implications of considering transient material behavior are emphasized by comparing the stress-strain predictions presented in Figure 6.11. Since significant plastic deformations not present in the stable block predictions exist when considering transient material behavior, a corresponding increase in fatigue damage/decrease in life prediction can be expected for such an analysis.

To properly carry out a full transient fatigue life analysis would require stress-strain analysis, stress transformations, cycle counting, damage calculation, and damage summation to be performed in an “on the fly” manner, so that the analysis could be ended when failure is predicted. However, while rainflow counting algorithms have been proposed for this purpose (Downing and Socie, 1982), which do not require the entire loading history to be known prior to cycle counting, such an analysis has some drawbacks. Perhaps the largest drawback is that for each combination of plasticity model, cycle counting procedure, damage parameter, and damage rule investigated, the entire analysis would need to be repeated from beginning to end. Therefore, for the analyses performed in this study, given the similarities between the stress-strain response, and thus fatigue life predictions, for all loading blocks subsequent to the first (compare Figures 6.11(b) and 6.11(c)), predictions were calculated as a composite of transient and stable
damage calculations. This not only simplifies the analysis procedure, but it also allows for a variety of different analysis models to be implemented without repeating time consuming calculations common to all of the models.

In order to compute the composite life prediction, $N_{f,c}$, stable block life predictions, $N_{f,s}$, were combined with life predictions based on only the first block of transient stress-strain response, $N_{f,t}$. This was done using the following equation, which is based on linear damage accumulation:

$$N_{f,c} = \frac{1 - \Sigma D_t}{\Sigma D_s} + 1 = N_{f,s} - \frac{N_{f,s}}{N_{f,t}} + 1 \quad (6.5)$$

where the relation can also be expressed in terms of the stable block cumulative damage sum, $\Sigma D_s$, and first transient block damage sum, $\Sigma D_t$. For reasons previously discussed, all stable block life predictions were computed based on simplified rainflow counting, while transient response predictions were computed based on traditional rainflow counting (not reordered).

To illustrate the effect of transient material behavior on fatigue damage calculations, Figure 6.14 shows comparisons of predicted fatigue lives based on stable block stress-strain predictions, the 1st block of predicted transient stress-strain response, and the composite life prediction (considering both transient and stable material behavior) for each loading condition tested. Results based on the original FS parameter are shown in Figure 6.14(a), while those based on the modified parameter are shown in Figure 6.14(b).

From this figure, it is clear that while predictions based on stable material response can vary significantly from those based on the 1st block of transient material response, the increased fatigue damage in the transient block has a negligible effect on
the overall composite life predictions in most cases. This can likely be attributed to the very small percentage of overall cycles which result in significant plastic deformation in the transient stress-strain histories. This is especially true at lower loading levels, where little plastic deformation is present even in the first block of applied loading. For the torsion only loading history, which contains the highest equivalent stress levels during overloads, the effect of transient material response is greatest. However, there is still less than a factor of 1.5 difference in predicted life for this case. Plots comparing the composite life predictions and experimental fatigue lives for each test performed are shown in Figure 6.15(a) based on the original FS parameter, and in Figure 6.15(b) based on the modified form of the parameter. As expected from the results of Figure 6.14, the results presented in Figure 6.15 look very similar to those in Figure 6.13 based on only stable material stress-strain response. A summary of these critical plane-based variable amplitude life predictions, along with the percentage of predictions within factors of ±3 and ±10 of experimental results, can be found in Table 6.5.

6.4.1.3 Effect of Crack Initiation Definition on Life Predictions

Since the critical plane-based variable amplitude life predictions still tend to be non-conservative, even when considering transient stress-strain response, the difference in accuracy observed between constant and variable amplitude life predictions can likely be attributed to other factor(s). One possible cause for this discrepancy, which is often overlooked, is in the definition of fatigue failure. Due to the additional resources needed to monitor crack length during testing, crack initiation is often defined in terms of a change in material stiffness, which accompanies the presence of a crack, rather than crack length itself. This is indicated in a fatigue test by a percentage change in load/torque or
position/rotation amplitude. However, depending on the material, specimen geometry, applied loading, etc., this can result in a relatively wide range of crack lengths at “crack initiation.”

For constant amplitude loading conditions and/or small specimen cross sections, differences in fatigue life from one crack length to another may not be so pronounced due to the continuous manner in which cracks grow. For variable amplitude loadings, on the other hand, load sequence effects can produce varying degrees of crack growth acceleration and/or retardation. As a result, this can have different effects with respect to the fraction of total fatigue life required to grow a crack from one length to another. Therefore, given the inconsistency in final crack lengths, defining crack initiation based on changes in material stiffness, rather than a small finite crack length, can have a potentially large impact on resulting fatigue life correlations.

In this study, the definition of fatigue crack initiation used for un-notched specimens, which was based on a percentage change in position/rotation amplitude, typically resulted in final crack lengths between 10-15 mm. As a result, the potential for differences in crack growth behavior between constant and variable amplitude loading conditions, especially for different nominal loading conditions, could have led to similar inconsistencies in the experimentally observed crack initiation life. Therefore, additional analyses were performed for the variable amplitude fatigue tests in order to evaluate the effect of crack initiation definition on life predictions.

Although crack growth data could not be collected for a large number of constant amplitude un-notched specimen fatigue tests due to inner surface crack initiation, a limited amount of data were available to construct an un-notched specimen shear strain-
life curve based on a 0.2 mm crack initiation definition. The resulting fatigue life data and shear strain-life curve are shown in Figure 6.16 along with the curve corresponding to fatigue lives at final failure. The resulting strain life properties are as follows: \( \tau'_f = 523.8 \) MPa, \( b_o = -0.099 \), \( \gamma'_f = 2.810 \), and \( c_o = -0.855 \). While this curve lacks the data required for a sufficiently accurate fit in the low cycle regime, it correctly predicts a shorter crack initiation life than the final failure curve, at a given strain amplitude, as fatigue life increases. As such, although it may not provide the most accurate fatigue damage estimates, it should be able to at least give some basic insight into the effect of crack initiation definition on life predictions. Similar data were not available, however, to construct a 0.2 mm fatigue life curve based on un-notched uniaxial test results.

Unlike the previous life predictions presented in this section, these 0.2 mm crack initiation predictions are based on shear strain-life properties. Since comparing predictions based on shear strain-life properties to those based on uniaxial strain-life properties may not give the best indication of any improvements gained in life predictions due to changes in crack initiation definition, Figure 6.17 is also presented. This figure shows comparisons between life predictions based on uniaxial final failure, shear final failure, and 0.2 mm shear strain-life curves. Predictions in Figure 6.17(a) are based on the original FS parameter, while those in Figure 6.17(b) are based on the modified FS parameter. Again, these are composite predictions based on a combination of fatigue damage values computed from blocks of both transient and stable material stress-strain behavior.

From this figure, it is clear that a fairly significant reduction in fatigue life is predicted when modifying the crack initiation definition. When comparing results for the
FS parameter in Figure 6.17(a) based on the two different shear strain-life curves, the reduction in predicted fatigue lives ranges from a factor of 1.5 to 2.8 with an average factor of 2.1 difference. Comparing predictions based on the uniaxial final failure strain-life curve to those based on the 0.2 mm shear curve, a similar or slightly lesser reduction is observed in all cases, except for the pure torsion loading condition. In this case, the large number of small cycles contained in the pure torsion loading history, combined with the increasing difference between life predictions based on uniaxial and pure shear fatigue properties in the high cycle fatigue regime (as evidenced by Figure 5.17), lead to a shorter life prediction based on uniaxial fatigue properties.

Similarly Figure 6.17(b) shows that, for the modified FS parameter, differences in shear strain-life predictions for the 0.2 mm and final failure curves range from a factor of 1.5 to 2.6 with an average of 2.0. Unlike the original FS parameter, however, reductions in predicted life based on the 0.2 mm shear curve are similar or slightly larger in all cases when compared to those based on uniaxial strain-life properties. The fact that life prediction trends based on uniaxial and shear fatigue properties are more consistent for the modified parameter supports the findings of Section 5.4.2, which show improved correlation of fatigue damage between shear and uniaxial stress states for the 2024-T3 testing material. When analyzing the results of Figures 6.17, however, it is important to note that while the difference in crack initiation definition results in shorter predicted fatigue lives, the experimental lives must also be recalculated to match this initiation definition. Therefore, the shorter experimental lives will counteract some of the effect of the more conservative life predictions when comparing experimental versus predicted lives.
Variable amplitude experimental versus predicted fatigue lives based on the 0.2 mm crack initiation definition are shown in Figure 6.18(a) based on the original FS parameter, and in Figure 6.18(b) based on the modified FS parameter. Since experimental crack growth data were not available for all tests, only 0.2 mm crack initiation lives are shown using filled symbols. The crack growth lives for these tests, up to final failure, are represented by the bars extending from each data point. For any test from which crack growth data were not available, the experimental life is shown as an open symbol, and is based on the life recorded at final fatigue failure.

In these figures, although not much improvement in life prediction accuracy is observed for the original version of the FS parameter, predictions based on the modified parameter improved to mostly within a factor of ±3 of experimental lives when based on the 0.2 mm crack initiation definition. The only data that fall outside of the ±3 scatter bands are for the pure shear loading tests. Additionally, based on the available crack growth lives observed for tests performed under axial and combined loading conditions, life predictions for most specimens where crack growth data were not available would also be expected to fall within a factor of ±3 of experimental lives.

As stated previously, however, it is difficult to gage the improvement in life prediction accuracy when comparing analyses based on shear fatigue properties to analyses based on uniaxial fatigue properties. Figure 6.17(a) shows that for the FS parameter, life predictions based on final failure have the tendency to be shorter (in 3 out of 5 cases) when based on uniaxial properties than when based on shear properties. Conversely, Figure 6.17(b) shows that for the modified FS parameter, predictions based on uniaxial properties tend to be longer (in 4 out of 5 cases) than those based on shear
properties. As a result, it is not surprising that the difference in fatigue life correlations between Figures 6.15(b) and 6.18(b) are more pronounced than those observed in Figures 6.15(a) and 6.18(a) for the original parameter. To provide a more direct comparison, Figure 6.19(a) and 6.19(b) show life predictions based on the final failure shear strain-life curve for both the FS and modified FS parameters, respectively.

By comparing Figures 6.18 and 6.19 it can be concluded that although changing the crack initiation definition to 0.2 mm results in more accurate variable amplitude fatigue life predictions, the overall effect of the change is relatively small. However, it was also shown in Figure 6.16 that although the 0.2 mm shear strain-life curve correctly predicts shorter fatigue lives in the high cycle regime, due to a lack of crack growth data in the low cycle regime, this curve also predicts longer fatigue lives than the final failure shear strain-life curve as shear strain amplitude increases. This obviously should not be the case and, as a result, the lower fatigue damage values predicted for larger loading cycles applied in the variable amplitude loading histories may be counteracting some of the increased fatigue damage calculated for the smaller magnitude loading cycles. Therefore, if a more accurate fatigue life curve had been available based on the 0.2 mm crack initiation definition, life predictions may have improved even further. Again, a summary of un-notched specimen critical plane-based variable amplitude life predictions, along with the percentage of predictions within factors of ±3 and ±10 of experimental results, is included in Table 6.5.

In all, despite the challenges involved in a variable amplitude multiaxial fatigue life analysis, the modified FS parameter was able to produce life predictions within a factor of ±10 of experimental results for all loading conditions and analysis procedures.
investigated. In addition, when accounting for effects such as those due to transient material behavior and crack initiation definition, the modified FS parameter was able to predict all but two (corresponding to the pure torsion tests) experimental lives within a factor of ±3. Although there is still room for improvement in these results, the critical plane-based fatigue life analyses were able to provide improved life predictions, in terms of both accuracy and quality of correlation between different loading conditions, than the equivalent stress/strain-based approaches investigated in Section 6.3.

6.4.1.4 Cracking Orientation Comparisons

In addition to fatigue life predictions, experimental failure planes can also be compared to critical plane orientations in order to further evaluate the analysis procedures implemented in this study. For the axial only variable amplitude loading histories, the predicted failure planes (to the nearest 5°) were the ±35° planes based on the FS parameter, and the ±25° planes based on the modified FS parameter. Available experimental crack paths are shown, along with a diagram of predicted initiation planes, in Figures 6.20 and 6.21 for variable amplitude axial tests performed at scale factors of 3.1 and 2.9, respectively.

Due to the existence of additional maximum damage planes, which vary with respect to the specimen thickness direction, comparisons between predicted and experimental crack paths are more difficult to interpret for the pure axial tests. Nonetheless, Figures 6.20 and 6.21 show that for specimens tested at both load levels, when cracks were small (on the order to 100-200 μm), their orientations agree fairly well with predictions based on both the FS and modified FS parameters. Since this is the length scale relevant to crack initiation mechanisms (a crack length on the order of a few
grains is needed to satisfy continuum assumptions), cracking comparisons at this scale are the most meaningful. An average grain size value for 2024-T3 aluminum alloy is approximately 75 μm (Merati, 2005). Then, after a few hundred microns of growth, cracks were observed to turn and grow in the 0° orientation, perpendicular to the specimen axis. However, since cracks could have turned to grow on a shear plane inclined to the specimen thickness direction as well, it is unclear whether or not this transition corresponds to mode I crack growth. More information on the mechanisms which could produce this type of crack branching behavior is presented in the following chapter (Section 7.2).

For pure torsion variable amplitude tests, predicted failure planes were oriented at 105° (−75°) and 165° (−15°) based on the FS parameter, and 110° (−70°) and 160° (−20°) based on the modified FS parameter. Since the larger applied shear stresses in these tests were always positive, only two failure planes produce the largest cumulative damage values. Available experimental crack paths are shown, along with the predicted initiation planes, in Figure 6.22 for torsion tests performed at a scale factor of 4.0. For each specimen, crack images are shown for both the failure crack (fc) and a secondary fatigue crack (s1) growing in the specimen gage section.

When cracks were on the order to a few hundred microns in length, all crack paths in Figure 6.22 are observed to be either vertical (90° orientation), or nearly vertical. While maximum damage planes were different by 15° to 20° from vertical, only one crack path (TM107-TRS (fc)) tends to agree better with the maximum damage planes as opposed to the maximum shear plane. Despite the limitation to maximum damage plane-based life predictions in multiaxial variable amplitude fatigue life analyses, the fact that
most cracks agree better with maximum shear planes for these tests is consistent with constant amplitude crack growth observations in Section 5.4.1. Additionally, as cracks grew longer, crack branching/changes in growth direction were found to result in complex crack paths under the loading conditions investigated.

Finally, for the combined axial-torsion variable amplitude tests, failure plane orientations of 155° (−25°) and 160° (−20°) were predicted based on the FS and modified FS parameters, respectively. Again, because the larger applied shear stresses in these tests were always positive, only a single failure plane produced the largest cumulative damage value. Available experimental crack paths are shown, along with the predicted initiation planes, in Figures 6.23 and 6.24 for combined axial-torsion tests performed at scale factors of 3.0 and 2.5, respectively. For the specimens shown in Figure 6.23, crack images are included for both the failure crack (fc) and a secondary fatigue crack (s1).

For the combined loading tests, the crack orientations observed for all specimens in Figures 6.23 and 6.24 are generally found to be consistent with the critical planes predicted by the FS and modified FS parameters for all crack lengths represented. Combined with the good agreement between predicted and experimental crack initiation planes for the axial and torsion loading tests, it is reasonable to conclude that the shear-based FS damage parameter is able to produce fairly accurate crack orientation predictions even for these complex loading histories. This not only supports the idea that the physical basis of the damage parameter is able to capture the damage mechanisms induced by such loading histories relatively well, but also provides some verification for the suitability of the cycle counting and damage accumulation procedures implemented.
6.4.2 Notched Specimen Life Predictions

For the notched specimen variable amplitude fatigue tests performed in this study, fatigue lives were predicted using the same critical plane-based procedures used for the constant amplitude analyses presented in Section 5.4.3. Additionally, variable amplitude considerations such as cycle counting and damage accumulation were addressed using the same procedures outlined in the previous section for the un-notched specimen analyses.

In deriving local stress-strain histories for variable amplitude notched specimen fatigue analyses, the first step was to determine the critical analysis location(s). As mentioned in Section 6.3.2, since the magnitude and location of maximum local stresses can change with a change in the nominal stress ratio, the critical analysis location has the potential to change for each cycle in a non-proportional variable amplitude loading history. Under these conditions, if the failure location is unknown prior to damage calculation, fatigue life analysis may need to be performed at multiple potential failure locations in order to determine which will produce the minimum fatigue life. Additionally, when using a critical plane-based fatigue life analysis approach, for each analysis location considered, several planes may also need to be analyzed to determine the orientation of the critical plane. As a result, a significant amount of analysis time may be required in situations where the experimental failure location is not known. In this study, however, crack initiation was observed/expected to occur at the 0° and 180° locations around the hole for axial and combined loading conditions, while pure torsion loading resulted in failures at the −45° and 135° locations. As such, all subsequent fatigue life analyses were performed at these locations.
In order to account for the effects of stress gradients, the TCD point method, evaluated normal to the curvature of the hole, was again used in all analyses. The $L$ value was taken to be 0.350 mm, consistent with the constant amplitude analyses in Section 5.4.3. After determining the critical analysis location, the pseudo stress approach was used to calculate local elastic-plastic stress-strain histories from the local elastic stress variation for each loading condition. Then, stress and strain transformations were performed in $5^\circ$ increments to identify the plane experiencing the largest amount of fatigue damage based on both the original and modified versions of the FS parameter. A $k$ value of 1 was used in both parameters and damage summation was performed using the linear damage rule for all analyses performed.

6.4.2.1 Life Predictions Based on Stable Stress-Strain Response

Similar to the un-notched specimen variable amplitude fatigue life analyses, the first life predictions performed for the notched specimens were based on stabilized local stress-strain response. Since the influence of cycle counting procedure on stable block life predictions was found to be negligible in Section 6.4.1, the simplified rainflow counting technique was used to identify loading cycles. The results of these comparisons are shown in Figure 6.25(a) based on the original FS parameter, and in Figure 6.25(b) based on the modified form of the parameter. Similar to the equivalent stress- and strain-based analysis results presented in Section 6.3.2, data corresponding to tests performed using the edited service loading histories are denoted using gray shaded symbols. Additionally, as with the un-notched specimen analyses, the results for one notched specimen test, corresponding to the highest pure axial loading level, are not included in
any of the subsequent analysis results due to excessive plastic deformation at the notch root, which could not be properly accounted for.

Life predictions were computed based on uniaxial fatigue properties, as opposed to shear properties, due to the fact that the constant amplitude notched specimen life predictions, presented in Section 5.4.3, showed better agreement with experimental results when based on uniaxial properties. Also consistent with constant amplitude fatigue life comparisons, each data point represents the fatigue life of a particular specimen corresponding to life when the first surface crack growing from the hole reached a length of 0.2 mm. However, for reference, bars have also been added to each data point which reflect the total crack growth life of the specimen (from 0.2 mm to failure).

The results shown in Figure 6.25 for notched specimens fatigue life predictions are similar to those shown in Figure 6.13 for the un-notched life predictions based on the same analysis procedures. Although life predictions based on the modified FS parameter were found to be more accurate than those based on the original parameter, in nearly all cases, life predictions were non-conservative regardless of the loading condition and/or damage parameter. While the degree of non-conservatism is less for the notched specimen analyses based on both damage parameters investigated (compared to the un-notched analyses), the variable amplitude life predictions are still not comparable, in terms of overall accuracy, to those obtained for constant amplitude loading conditions. Based on the original FS parameter, 1 of 16 and 14 of 16 fatigue life predictions were within factors of ±3 and ±10 of experimental results, respectively, while 13 of 16 and 16
of 16 were within factors of ±3 and ±10 of experimental data, respectively, based on the modified parameter.

6.4.2.2 Life Predictions Based on Transient Stress-Strain Response

As mentioned in the previous section for the un-notched specimen analyses, the potential for large initial plastic deformations to be ignored when only considering stabilized material response could lead to some of the errors observed in the variable amplitude life predictions. Therefore, all notched specimen analyses were performed again, for all loading conditions, with the consideration of transient material response. Since significant plastic deformation, not present is the stable block predictions, exists when considering transient behavior, a corresponding change in fatigue damage/life prediction can be expected for such an analysis. However, some key differences in deformation behavior exist between notched and un-notched specimens which result in different transient hardening characteristics in each case.

Although all variable amplitude fatigue tests were performed in load-control in this study, the local loading/deformation behavior at a notch tends to more closely reflect strain-controlled conditions (Stephens et al., 2000). This is a result of the constraint that surrounding linear elastic deformation imposes on the material at the notch root. While the differences in load versus strain-controlled conditions do not have a significant impact on fatigue damage calculations for a material exhibiting stable stress-strain response, these conditions produce different transient hardening characteristics which have opposite effects on fatigue damage calculation.

For a cyclic hardening material plastically deforming under load-controlled loading conditions, initial strain values will be large, but reduce as the material hardens.
For the same material under strain-controlled conditions, initial stress values will be low, but tend to increase as the material hardens. Therefore, for loading conditions which result in the same stabilized stress-strain response, material in a strain-controlled test will experience less initial fatigue damage (due to lower initial stress values), while material in a load-controlled test will experience more initial fatigue damage (due to higher initial strain values). After the material stabilizes, however, both loading conditions will result in identical fatigue damage predictions.

For the un-notched specimen analyses presented in the previous section, this meant that the consideration of transient material behavior produced larger fatigue damage predictions for the initial loading blocks, which eventually converged to the stabilized damage values as the number of applied loading blocks increased. For the notched specimens, however, the effect is generally opposite. Due to smaller initial stress values under strain-controlled notch deformation, fatigue damage values should be lower for the initial loading blocks before eventually increasing to levels predicted based on stabilized material response. Therefore, considering transient material behavior in the notched specimen analyses should actually produce more non-conservative life predictions than those based on stable material response.

In reality, however, local loading conditions at a notch are not purely strain-controlled. For example, it is common for material at a notch root to develop a mean plastic strain, even under constant amplitude loading conditions, due to initial plastic deformation and/or loading conditions which are not fully-reversed. This mean deformation is responsible for the presence of local residual stresses at the notch, and can be relaxed away or altered following subsequent plastic deformation of the notch root.
material. If local deformation conditions in notched components were purely strain-controlled, this would not be possible, as the local strain variation should be identical for each applied loading cycle/block. Therefore, there is still the potential for increased initial fatigue damage to occur in notched specimen fatigue tests due to the possibly for large initial mean plastic deformations to exist. Correctly modeling this behavior, however, is a challenging task.

Unlike the un-notched specimen stress-strain analyses, the plasticity modeling procedure is applied twice in the notched specimen analyses. The plasticity model is first used to derive local elastic-plastic strains from elastic stresses, and then a second time to derive plasticity corrected stress components from the corresponding elastic-plastic strain history. Therefore, it is not a straightforward task to determine when and where to incorporate transient material behavior into the plasticity modeling process. However, because local deformation at the notch is assumed to more closely resemble strain-controlled conditions than load-controlled, the local strain history should not change significantly as the material hardens at the notch root. As a result, when computing material response using the pseudo stress approach, stable material response was assumed when first converting elastic stresses to elastic-plastic strains. Then, once the local strain history was determined, transient material behavior was incorporated into the plasticity model when computing the corresponding elastic-plastic stress history.

While this modeling procedure reflects the strain-controlled characteristics of cyclic hardening behavior, it can also be implemented in a way that allows for the consideration of initial mean plastic deformations. In this study, this was done by modeling the response for the first transient block of stress-strain response separately
from all subsequent blocks. For the first block of loading, plasticity modeling was performed only once to calculate local strains from local pseudo stresses, and once to calculate local stresses from the resulting local strains. Again, stable stress-strain behavior was assumed in the derivation of the structural stress-strain curve, used to relate pseudo stress to local strain, while transient material response was assumed in calculating local stresses from strains. Similar to predictions for un-notched specimens, because there is initially no influence from prior loading on the deformation behavior experienced in the first block of applied loading, large initial plastic deformations are predicted in these stress-strain analyses.

Then, once initial stress-strain predictions were established for the first loading block, the stabilized local strain variation was calculated by performing pseudo stress to local strain plasticity calculations twice. The final state of the plasticity model from the first analysis, in terms of backstress components and strain state, was used as initial conditions for the second analysis. Again, this ensured that load history dependence from first loading block was accounted for in the stress-strain predictions for subsequent blocks. Then, using this stabilized local strain history, the transient local stress response was calculated by repeatedly performing plasticity calculations for loading blocks applied subsequent to the first. The final state of the plasticity model from the local strain to local stress calculations for the first block of loading was used as initial conditions for modeling the transient deformation behavior of the second block of loading. Similarly, all subsequent loading blocks were modeled using the final state of the plasticity model from the previous block as initial conditions. While the accuracy of the assumptions used in this approach are unclear without experimental results for comparison, it generally
provides a close representation of the strain-controlled characteristics expected for the actual local deformation behavior of a notched component.

It is also important to note that because cyclic hardening behavior is dependent on plastic strain accumulation, a material will generally reach a stabilized state sooner under load-controlled conditions than under strain-controlled conditions. This is because of the larger initial plastic deformations that can exist in load-control. Due to the nature of the loading history used in this study, where relatively little plastic deformation occurs in blocks applied subsequent to the first, this hardening behavior was found to result in situations where little change was predicted in transient stress-strain response from block to block, even though values had not yet converged to those obtained under the assumption of fully stabilized stress-strain behavior.

For this reason, computing transient life predictions based on a combination of fatigue damage computed from the first transient block and stable block stress-strain histories, as was done for un-notched specimen analyses, does not represent the actual material behavior predicted for the notched specimen analyses. As a result, an alternative approach was considered for computing transient life predictions for the notched specimen tests. Instead of using stable block fatigue damage in Equation (6.5) to compute composite life predictions, it was decided that using fatigue damage sums from the third block of transient stress-strain response, in combination with those from the first block, would produce more realistic life predictions. Transient stress-strain predictions for all loading blocks subsequent to the first were very similar for all loading conditions (similar to Figure 6.11), and produced negligible differences in fatigue damage. Therefore, the
stress-strain response from the third block of loading was considered a reasonable approximation of the behavior for all blocks excluding the first.

A comparison of fatigue life predictions based on stable material response, the first block of transient stress-strain response, the third block of transient response, a composite of the first and third transient block response, and a composite of the first transient and stable block stress-strain response is shown in Figure 6.26. Loading conditions are again designated using an alphanumerical format, where letters A, T, and AT represent the nominally applied axial, torsion, and combined axial-torsion loading channels, respectively. When these letters are followed by “e” they represent edited versions of each respective loading history. Finally, the numbers are used to denote the nominally applied load scale factor. Similar to un-notched specimen analyses in the previous section, all stable block life predictions were computed based on simplified rainflow counting, while transient response predictions were computed based on traditional rainflow counting. Uniaxial fatigue properties were used in all analyses.

From this figure, it is clear that for individual blocks of stress-strain response, stable material assumptions result in the shortest life predictions, followed by the first block of transient loading, and finally, the third block of transient loading. For shorter life tests, however, the difference between life predictions for each stress-strain history is small. This is likely due to the fact that as the loading level increases, the increased plastic deformation at the notch root results in more cyclic hardening, which reduces the difference in fatigue damage between the transient and stable blocks. As loading levels decrease, however, the amount of cyclic hardening decreases as well, resulting in the increasing difference observed between the different life predictions. Regardless of the
loading level, composite life predictions based on first block transient and stable damage values are consistently very similar to those for the stable block only predictions. Similarly, predictions based on a combination of fatigue damage from the first and third transient loading blocks are very similar to those for the third loading block only. Despite the differences in predicted stress-strain response, however, the largest difference in life predictions for any given loading condition is less than a factor of 2.

While transient life predictions based on a combination of fatigue damage from the first and third transient loading blocks are more non-conservative than both the stable block life predictions and alternative composite life predictions, these predictions should theoretically provide the most accurate representation of fatigue damage accumulation in the notched specimen fatigue tests. Therefore, these life predictions were plotted against experimentally measured fatigue lives in Figure 6.27. Results based on the original FS parameter are shown in Figure 6.27(a), while those based on the modified parameter are shown in Figure 6.27(b).

Although the consideration of transient stress-strain behavior did not improve the accuracy of fatigue life predictions for the notched specimen tests performed in this study, the fact that these predictions are slightly more non-conservative than those based on stabilized material response produces better agreement with the life predictions presented in the previous section for un-notched specimens. Additionally, the predictions based on transient material response generally tend to run more parallel with the perfect prediction line when plotted in terms of experimental versus predicted life. This suggests that the transient stress-strain analyses are able to more consistently reflect the variation of fatigue damage with changes in applied loading conditions. Overall, 0 of 16 and 13 of
16 fatigue life predictions based on the original FS parameter were within factors of ±3
and ±10 of experimental results, respectively, while 6 of 16 and 16 of 16 were within
factors of ±3 and ±10 of experimental data, respectively, based on the modified
parameter. A summary of these, as well as other critical plane-based notched specimen
variable amplitude life predictions is included in Table 6.6.

6.4.2.3 Effect of Crack Initiation Definition on Life Predictions

As discussed in Section 6.4.1.3, the potential for differences in crack growth
behavior between constant and variable amplitude fatigue tests, especially for different
nominal loading conditions, can lead to inconsistencies in the experimentally observed
crack initiation life. Therefore, in order to evaluate the effect of crack initiation definition
on notched specimen life predictions, damage analyses were again performed using the
shear strain-life curve based on the 0.2 mm crack initiation definition. It should be noted,
however, that unlike the un-notched specimen analyses, where experimental fatigue lives
also had to be redefined in terms of the 0.2 mm initiation definition (instead of final
failure), all experimental fatigue lives for the notched specimens were already defined
based on the 0.2 mm crack initiation definition. Therefore, while notched specimens life
predictions based on the 0.2 mm shear strain-life curve decrease, the experimental fatigue
lives they are compared to do not change.

Results for the notched specimen variable amplitude fatigue life analyses based
on the 0.2 mm crack initiation definition are shown in terms of experimental versus
predicted life in Figure 6.28(a) based on the original FS parameter, and in Figure 6.28(b)
based on the modified FS parameter. As stated previously, however, it is difficult to gage
the improvement in life prediction accuracy when comparing analyses based on shear
fatigue properties to analyses based on uniaxial fatigue properties. As a result, Figures 6.29(a) and 6.29(b) are also included, and show life predictions based on the final failure shear strain-life curve for both the FS and modified FS parameters, respectively. This provides a more direct comparison from which the effect of crack initiation definition on life predictions can be evaluated.

From these figures, it is clear that the effect from changing life prediction curves is larger for the uniaxial versus shear final failure curves (Figures 6.27 and 6.29) than for the 0.2 mm versus final failure shear curves (Figures 6.28 and 6.29). Using shear fatigue properties based on final failure reduces notched specimen life predictions by an average of around a factor of 2.5, for both parameters, as compared to those based on uniaxial properties. This results in the prediction of all notched fatigue lives within a factor of ±3 based on the modified FS parameter. The average difference between predicted fatigue lives based on the 0.2 mm and final failure shear curves, however, is only around a factor of 1.5 for both parameters. While the final failure shear curve produces very accurate notched specimen life predictions based on the modified FS parameter, those based on the 0.2 mm shear curve tend to be on the slightly conservative side. All predictions based on the original FS parameter, on the other hand, are still non-conservative despite the increase in fatigue damage predicted by both shear strain-based fatigue life curves. Again, a summary of critical plane-based notched specimen variable amplitude life predictions, along with the percentage of predictions within factors of ±3 and ±10 of experimental results, can be found in Table 6.6.

Some useful additional comparisons can also be made between the life prediction results for full and edited versions of the variable amplitude loading history. For each of
the lowest loading levels of the axial, torsion, and combined axial-torsion notched specimen fatigue tests, duplicate tests were performed for both full and edited versions of the loading history. This allows for the evaluation of the effect of smaller loading cycles on both experimental and predicted fatigue damage accumulation. Despite the fact that the edited histories were 94-95% shorter than the full histories, the average increase in fatigue life was only a factor of 2.3, 2.7, and 1.3 for axial, torsion, and combined axial-torsion edited history tests, respectively. This gives insight into the effect of smaller cycles on variable amplitude crack growth behavior as well as their implications with respect to load history editing for the purpose of accelerated fatigue testing.

In order to evaluate the ability of the critical plane-based analysis procedures to reflect these changes in fatigue damage, the experimental increase in edited history fatigue life was then compared to the predicted increase in life. This was done by comparing how parallel full and edited history data run to the perfect prediction line when plotted in terms of experimental versus predicted fatigue life for each respective analysis approach. On an average basis, the predicted variation in fatigue life between full and edited history tests is slightly less than the experimental variation in all cases. This suggests that the life prediction procedures may have the tendency to slightly underestimate the fatigue damage caused by the smaller cycles. The difference, however, is small enough to remain within the range of scatter observed in experimental fatigue lives. As such, it is reasonable to conclude that the critical plane-based life prediction procedures implemented in this study were able to reflect experimental variations in fatigue damage relatively well when considering the effect of small cycles and/or changes in loading sequence.
Overall, when considering the effects of transient material behavior and crack initiation definition on fatigue damage calculation, life predictions based on the modified FS parameter and 0.2 mm shear strain-life curve (Figure 6.28(b)) were slightly conservative to a similar extent as the constant amplitude notched specimen life predictions presented in Figure 5.46. The fact that notched specimen fatigue data from such a variety of constant and variable amplitude loading conditions are correlated within a relatively narrow band is encouraging, regardless of the relative accuracy of the predictions. This suggests that differences in fatigue damage accumulation between the different loading histories and/or stress states are being properly reflected in the life prediction procedure. Possible reasons for the shift in overall accuracy could then stem from factors unrelated to the modeling of fatigue damage itself. More information on some of these potential factors is included in the following discussion section.

6.4.3 Discussion on Critical Plane-Based Life Predictions

To better summarize the results presented in the previous sections, plots comparing critical plane-based variable amplitude fatigue life predictions for both un-notched and notched specimen tests are presented in Figure 6.30 based on transient stress-strain response and uniaxial fatigue properties, and in Figure 6.31 based on transient stress-strain response and 0.2 mm shear strain-life properties. Predictions based on both the original and modified FS parameters are included.

From these figure, it is clear that the modified FS parameter resulted in improved life predictions, in terms of both accuracy and overall scatter, when compared to those based on the original form of the parameter. This shift in accuracy was expected, however, due to the better mean stress data correlation observed for the modified
parameter in Section 5.4.2. Additionally, the improved correlation of constant amplitude fatigue data generated under different multiaxial stress states is likely related to the reduced scatter observed between the different variable amplitude loading conditions for life predictions based on the modified parameter.

The agreement between un-notched and notched specimen life predictions for the variety of variable amplitude loading conditions investigated in this study also suggests that differences in fatigue damage accumulation were reflected relatively well in the modeling procedures implemented. This is especially true for predictions based on the modified FS parameter. For the original parameter, however, it can be noted that the un-notched specimen life predictions tend to be slightly, but consistently, more non-conservative than those for notched specimens. Since this difference is diminished in life predictions based on the modified parameter, however, it is likely not an issue related to the modeling of notch deformation and/or stress gradient effects.

Instead, when considering the differences between nominal and local deformation behavior for the variable amplitude fatigue tests performed in this study, the most likely explanation for this shift in accuracy is related to mean stress effects. Because local notch deformation tends to more closely reflect strain-controlled conditions, some of the large mean stresses experienced in the load/stress-controlled un-notched specimen tests were likely reduced in the notched specimens due to local mean stress relaxation. This is supported by the fact that lower mean stress levels were observed, to some extent, in the local stress-strain analyses performed for these tests. Therefore, the fact that notched specimen life predictions based on the original FS parameter are more accurate than those for the un-notched specimens is not surprising. This behavior is consistent with the life
prediction trends reported in Section 5.4.2.1 for constant amplitude fatigue data (from literature) generated under various mean stress levels.

Although un-notched and notched specimen life predictions are in better agreement for analyses performed using the modified FS parameter, all life predictions based on both parameters still have the tendency to be more non-conservative than those computed for the constant amplitude loading conditions investigated in this study. Therefore, there are likely still additional factors, related specifically to the variable amplitude loading conditions applied in this study, which are not being properly accounted for in these analyses. The following sections aim to highlight some of these factors, along with their potential impact on life prediction accuracy, with respect to different areas of the fatigue life analysis procedures implemented herein.

6.4.3.1 Influence of Life Prediction Curve Accuracy

As mentioned in the previous sections, one possible reason for the decrease in accuracy for the variable amplitude life predictions is due to crack initiation definition. Due to differences between constant and variable amplitude loading histories, not having a sufficiently short crack length at the defined crack initiation life can cause the load history dependence of crack growth to affect the correlation of life predictions between the two types of loading. Although this effect was studied by performing life predictions based on a 0.2 mm crack initiation definition (instead of final failure), due to difficulties in recording experimental crack growth in un-notched specimen fatigue tests, not enough crack growth data were available to construct a sufficiently accurate 0.2 mm shear (or uniaxial) strain-life curve. As a result, the 0.2 mm shear strain-life curve used in these
analyses predicted more fatigue damage than the final failure curve at longer life, but incorrectly predicted less fatigue damage at shorter lives.

While slight improvements in life prediction accuracy were observed for both un-notched and notched specimen fatigue tests based on the 0.2 mm crack initiation definition, lower damage values predicted for the more damaging cycles in the variable amplitude loading histories may have cancelled out some of the improvements gained from the increased damage predicted for lower amplitude cycles. This is also a likely cause for the increased scatter observed in the life predictions presented in Figure 6.31 based on the 0.2 mm shear strain-life properties, as compared to those presented in Figure 6.30 based on uniaxial fatigue properties. Therefore, had a more accurate 0.2 mm life prediction curve been available, further improvements in life prediction accuracy may have been observed.

On a related note, due to the length of the loading history utilized in this study and the high percentage of smaller cycles contained within it, even for tests with a life of just a few blocks, millions of cycles of loading were applied. This means that the fatigue damage caused by the majority of cycles was well into the high cycle fatigue regime. However, due to a lack of experimental data in this regime for the constant amplitude un-notched specimen fatigue tests used to derive the life prediction curves, the accuracy of these curves is questionable at lives above approximately $10^6$ cycles. Because the slopes of the life prediction curves are very shallow in the high cycle fatigue regime, even small changes in their fit can have a large effect on resulting life predictions.

Since smaller cycles were eliminated from the notched specimen tests performed using the edited variable amplitude loading histories, the fact that life predictions tend to
be slightly more accurate for these tests as compared to those for full history tests may provide some evidence of this type of effect. Additionally, to verify that the assumed fatigue limit of $5 \times 10^8$ cycles was not excluding a notable amount of fatigue damage from the full history variable amplitude life predictions, a limited number of analyses were also performed with an assumed fatigue limit of $10^{10}$ cycles. However, these analyses produced nearly identical fatigue lives to those based on the $5 \times 10^8$ cycle assumption. Therefore, the fatigue limit definition likely did not contribute to the non-conservative nature of the variable amplitude life predictions computed in this study.

### 6.4.3.2 Factors Related to Fatigue Damage Calculation

Aside from improved life prediction curves, there are some additional factors related to the modeling of fatigue damage processes which could also potentially help to improve both the accuracy, and the consistency, of the variable amplitude fatigue life analyses. Most of these factors have been mentioned to some extent previously, but will be summarized again here in light of the variable amplitude life prediction results presented in the previous sections.

One of these factors is the selection of the $k$ value used in both the original and modified FS parameters. Although it was shown in Figure 5.26 that the experimental value of $k$, calculated from uniaxial and pure shear fatigue properties, varies with fatigue life, a constant value was assumed in all analyses performed in this study. Although the $k$ value chosen was a reasonable approximation of the experimental value in the intermediate to long life regime, the value of $k$ tends to increase with life for the 2024-T3 aluminum alloy investigated. Therefore, the high percentage of smaller cycles contained in the loading histories, combined with the possibility for the $k$ value corresponding to
these cycles to be larger than the value used for damage calculation, could have resulted in the underestimation of fatigue damage in the variable amplitude fatigue life analyses.

Since the effect of $k$ value on fatigue damage under pure torsion loading (on the maximum damage plane) is small, however, this gives some insight into what effects the constant $k$ assumption may have had on fatigue life predictions. By analyzing the variable amplitude analysis results, in most cases it can be seen that the relative accuracy of unnotched specimen life predictions under pure torsion loading is similar to that observed for axial and combined loading conditions. Since the effect of $k$ value on damage calculation is considerably larger for axial and combined loading, this suggests that the variation in $k$ was likely not a large contributor to the overall error in life predictions.

Additionally, in order to further study the effect of $k$ value assumptions, an attempt was made at performing variable amplitude fatigue life analyses using a variable $k$ value. As a result, a function describing the variation of $k$ with shear strain range was fitted, for both the original and modified FS parameters, to be used in damage calculation. In these analyses, however, large $k$ values were computed for the large number of small cycles applied in the very high cycle fatigue regime, i.e. $k$ values as large as 11 and 2.6 at $5 \times 10^8$ cycles for the original and modified FS parameters, respectively. This resulted in maximum damage values and plane orientations that were significantly different from those corresponding to the maximum shear plane(s). When the critical planes are no longer close to maximum shear planes, this results in a loss of the physical interpretation of the shear-based damage parameter. Additionally, because life prediction curves are based on lower damage values computed on the maximum shear plane(s), overly conservative life predictions were produced as well. While these
challenges could be overcome by performing damage analysis on the maximum shear plane, this is difficult to do for multiaxial variable amplitude loading conditions due to previously discussed issues. For these reasons, further investigations into analyses considering the variable $k$ assumption were not performed.

Another factor that can influence the calculation of fatigue damage is the consideration of shear strain magnitude. This was discussed in Section 2.5 with respect to the two shear components that may simultaneously be acting on any given plane. Since the magnitude of these two components cannot be used for cycle counting because of the sign problem this would create, their combined effect is usually ignored. Instead, individual strain components are cycle counted with the assumption that one component is dominant to the other. For proportional loading histories, this generally does not present a problem since the magnitude of both shear components will be equal to the magnitude of a single component at a particular plane/coordinate system orientation. For non-proportional variable amplitude loading histories, however, not considering the combined effect of both shear components could lead to a potential underestimation of fatigue damage and result in non-conservative life predictions.

While non-conservative life predictions were observed for the variable amplitude fatigue tests performed in this study, the degree of non-conservatism was similar for loading histories featuring fixed principal directions (i.e. axial and torsion) as well as non-proportional histories where the principal directions varied (i.e. combined axial-torsion). Therefore, this type of shear strain effect is not expected to have had a significant influence on the analysis results presented herein. The combined axial-torsion loading history used, however, is tension-dominated and, as such, the assumption of a
single dominant shear strain component was likely reasonable. In non-proportional loading histories which result in similar contributions from individual shear strain components, on the other hand, failing to properly account for their combined effect could have more significant implications.

Another issue related to the calculation of fatigue damage in a critical plane-based variable amplitude fatigue life analysis is the effect of half cycles versus whole cycles identified during cycle counting. As discussed at the beginning of Section 6.4, counting the same loading cycle as two half cycles instead of one whole cycle could lead to potential differences in the amount of fatigue damage calculated for that particular cycle. Similarly, for loading histories with a large one directional accumulation of plastic strain, there is the possibility that one half cycle could be identified over the course of the entire loading history. As a result, the maximum normal stress to be used in damage calculation for this cycle would be the maximum normal stress in the entire loading history, regardless of when or for how long it was applied. In situations such as these, the physical interpretation of the maximum normal stress per cycle becomes unclear with respect to damage calculation. Therefore, as mentioned previously, the consideration of time dependent normal-shear stress/strain interactions is also an area where improvements could be made in the quantification of multiaxial fatigue damage. More information on possible methods for improvement can be found in Section 9.4.

6.4.3.3 Effects due to Material Deformation Modeling

The variable amplitude fatigue life predictions presented in this chapter could also potentially benefit from more accurate modeling of material deformation behavior. As noted in Section 4.9, the cyclic plasticity modeling techniques used in this study for un-
notched specimens were shown, on average, to result in a slightly non-conservative stress-strain predictions. The same was also true for pseudo stress-based local plasticity corrections at a distance from the notch root similar to that considered using the TCD point method. Since the variable amplitude life predictions were also found to be non-conservative, some of the prediction error could be a result of material deformation predictions.

Additionally, as previously discussed, it is difficult to apply transient plasticity modeling techniques using the pseudo stress-based notch stress-strain estimation procedures implemented in this study. This is due to the fact that actual notch deformation behavior likely falls somewhere between that which would be calculated based on local load/stress-controlled conditions, and that which would be calculated based on local strain-controlled conditions. As such, transient hardening behavior, as well as residual stress development for unbalanced loading histories (for both transient and stable material assumptions), may not be properly accounted for in the resulting stress-strain output. Without experimental notch deformation data to verify predictions, however, it is difficult to improve upon these modeling techniques.

Potential improvements in life predictions considering transient material behavior could also be attained by performing fatigue life analyses in an “on the fly” manner. This way, changes in material deformation behavior from block to block could be more accurately represented in such an analysis. Another advantage of an “on the fly” type analysis is that fatigue damage is computed for each cycle as it is applied. Because of this, the analysis can be immediately terminated once the cumulative damage sum reaches the value defined for failure. This is especially beneficial for long loading
histories, such as the ones used in this study, where only a small number of loading blocks are applied before failure occurs. Calculating fatigue life predictions based on damage summed for entire blocks of loading can result in the consideration of damage corresponding to a large number of cycles that were not actually applied. Although the significance of this effect would vary depending on loading history profile, it should generally reduce as the number of applied loading blocks increases.

6.4.3.4 Potential Improvements from Damage Summation Rule

Last but not least, it should again be noted that only one damage summation rule, the Palmgren-Miner linear damage rule, was used in all variable amplitude fatigue life analyses presented in this chapter. This rule was chosen due to its simplicity, its popularity, and the lack of a robust alternative approach for nonlinear damage summation. Although it has been shown that some deficiencies typically attributed to the linear damage rule can be overcome through more accurate modeling of other aspects of the fatigue failure process, as was the goal in this study, damage accumulation still may not be properly captured by a simple linear summation. As a result, other damage rules, or a damage sum at failure other than unity, may be required in order to produce variable amplitude life predictions in the same range of accuracy as those computed for the constant amplitude fatigue tests performed in this study.

For example, it has been shown in numerous studies, e.g. (Buxbaum, 1992; Haibach, 2003b; Sonsino, 2005; Sonsino and Franz, 2016), that the linear damage rule, when assuming fatigue failure at a damage sum equal to unity, consistently results in non-conservative life predictions, particularly for aluminum alloys. As a result, a critical damage sum of 0.3 is recommended instead in various fatigue design guidelines, e.g.
(Haibach, 2003a; Rice, 1997). While the damage sum of 0.3 is empirically derived, its applicability has been supported by a large amount of experimental data. As mentioned previously, however, the critical damage sum was assumed to be unity in the current study because some of the factors that may contribute to a lower experimental damage sum at failure, e.g. transient material deformation behavior and load history effects on crack growth (through crack initiation definition), were explicitly considered in the analyses. If all factors were properly accounted for, fatigue failure should theoretically occur when the damage sum is equal to unity, as this is the case for constant amplitude loading. However, it is clear that the variable amplitude fatigue life predictions presented in this chapter were still not as accurate as those for the constant amplitude loading conditions investigated.

Although any combination of the previously mentioned factors may have also contributed to the non-conservative nature of the variable amplitude life predictions, again, at least some of this error could be a result of the damage summation procedure used. As a result, fatigue life predictions were recomputed for all of the variable amplitude loading conditions using a critical damage sum of 0.3, along with the linear damage rule. Additionally, since the consideration of transient material deformation behavior and crack initiation definition can have different effects on the damage sum at failure, analyses were performed for three different cases. These correspond to life predictions based on stable stress-strain response and uniaxial (final failure) fatigue properties, shown in Figure 6.32, predictions based on transient stress-strain response and uniaxial (final failure) fatigue properties, shown in Figure 6.33, and predictions based on transient stress-strain response and 0.2 mm initiation definition shear fatigue properties,
shown in Figure 6.34. All figures show results based on damage calculated using both the original and modified versions of the FS parameter.

While the difference between previous life predictions is simply a factor of 0.3 for analyses based on stable stress-strain response, the difference is larger for the composite life predictions incorporating transient deformation behavior. This is because Equation (6.5) does not vary linearly with respect to changes in both initial and stable block life predictions. This becomes especially notable as the difference between initial and stable block life predictions increases, as was the case for the un-notched torsion loading condition in analyses based on uniaxial (final failure) fatigue properties (Figure 6.33).

Based on the results presented in Figures 6.32 to 6.34, variable amplitude life predictions were found to improve considerably when using a critical damage sum of 0.3. While predictions based on the original FS parameter still tend to favor the non-conservative side, this is likely due to other factors, such as the consideration of large tensile mean stress, which were discussed previously. For analyses based on the modified FS parameter, on the other hand, the variable amplitude life predictions are of a similar accuracy to those obtained for constant amplitude loading conditions in all cases. However, it should also be noted that the overall correlation of data from different loading conditions becomes slightly worse for these analyses (e.g. compare Figure 6.30 to Figure 6.33). Therefore, although changing the critical damage sum was able to improve life prediction accuracy, it is likely that additional factors related to life prediction curves, damage calculation, and/or deformation behavior predictions had at least some impact on the variable amplitude fatigue life analysis results as well.
6.5 Summary and Conclusions

One of the key objectives of the research performed in this study was to be able to better understand and predict fatigue crack initiation in components subjected to complex service loading histories. Therefore, in this chapter, the constant amplitude fatigue life analysis techniques evaluated in Chapter 5 were applied to situations involving variable amplitude loading of both un-notched and notched specimens. These included signed equivalent stress- and strain-based life prediction techniques, as well as critical plane-based approaches. In addition to the effects of multiaxial stress states and notches, discussed in Chapter 5, variable amplitude loading conditions introduce additional factors such as cycle identification, damage summation, and load history effects. As a result, issues related to cycle counting techniques, transient material deformation behavior, and crack initiation definition were evaluated in this chapter as well.

Based on the experimental results and analysis presented for the equivalent stress- and strain-based variable amplitude fatigue life predictions, some key findings and conclusions can be summarized as follows:

1) Signed equivalent stress- and strain-based analyses were unable to predict more than 3 of 10 un-notched specimen fatigue lives within a factor of ±3 of experimental results. Additionally, life predictions for axial and combined axial-torsion loading conditions were consistently more non-conservative than those for torsion loading, resulting in poor correlation of experimental fatigue data.

2) As expected, given the local uniaxial stress state for the notched tubular specimens investigated in this study, all equivalent stress- and strain-based
analyses resulted in good correlation of variable amplitude notched fatigue data for all loading conditions considered.

3) While a high level of accuracy was observed for the notched specimen life predictions, stress gradient effects were not considered in equivalent stress/strain-based fatigue life analyses. Due to the continuous variation of $K_t$ at any given location around the notch root under non-proportional variable amplitude loadings, the definition, as well as physical interpretation, of fatigue notch factor becomes vague in such cases.

4) Had stress gradient effects been accounted for, notched specimen life predictions would have been considerably more non-conservative, similar to the predictions for un-notched specimens.

5) For equivalent stress-based approaches, the modified Goodman and SWT mean stress corrections resulted in similar life predictions. The same is true for the equivalent strain-based approaches, with the modified Morrow corrections producing only slightly more non-conservative life predictions than SWT.

6) As a simplified analysis approach for notched specimen life prediction, which does not require the execution of local plasticity corrections, the equivalent pseudo stress-based approach, coupled with SWT mean stress corrections, produced life predictions of similar accuracy to those for the plasticity corrected equivalent stress approach.

7) Equivalent stress/strain approaches were unable to produce variable amplitude fatigue life predictions within the same range of accuracy as those observed under constant amplitude loading conditions.
8) For both un-notched and notched specimens (had gradient effects been accounted for), consistently non-conservative life predictions were observed under variable amplitude loading conditions. This is likely a result of load history effects not accounted for in the analyses procedures.

9) Similar to observations for constant amplitude loading conditions, equivalent stress- and strain-based analysis approaches may work well in certain situations, but lack a general robustness and offer little room for improvement. More advanced fatigue life analysis techniques, however, provide an opportunity to more accurately account for various aspects of the fatigue failure process under variable amplitude loading conditions.

Next, from the experimental results and analysis presented for un-notched specimen critical plane-based variable amplitude fatigue life predictions, some key findings and conclusions can be summarized as follows:

10) While rainflow and simplified rainflow counting techniques can result in different life predictions due to the identification of half cycles versus whole cycles and/or the closing of the largest cycle in a loading history, these factors were found to have a negligible impact on variable amplitude life predictions for the loading history considered in this study.

11) Life predictions based on the assumption of stable material stress-strain response were found to be non-conservative in all cases based on both the original and modified FS damage parameters.

12) Due to only a small percentage of cycles experiencing plastic deformation, a result of significant mean stresses causing non-reversed yielding, overall
improvements in life prediction accuracy gained through the consideration of transient material stress-strain response were negligible for the loading history investigated.

13) While changing crack initiation definition from final failure to 0.2 mm resulted in shorter variable amplitude fatigue life predictions by an average factor of 2, corresponding decreases in experimental lives made the overall improvement in life prediction accuracy relatively small for all loading conditions considered.

14) Although little improvement in variable amplitude life predictions was gained by changing the crack initiation definition in this study, more significant improvements may have been obtained had sufficient data been available to generate a more accurate 0.2 mm strain-life curve.

15) Overall, the modified FS parameter was found to produce more accurate fatigue life predictions than the original version of parameter for all analysis procedures investigated. This was expected based on constant amplitude analysis results, however, due to the presence of significant tensile mean stresses in the variable loading history investigated.

16) Constant amplitude fatigue life predictions based on shear fatigue properties had a tendency to be more conservative than those based on uniaxial properties. While this was also generally true for the variable amplitude analyses based on the modified FS parameter, shear-based predictions for the original parameter were either similar to or more non-conservative than those based on uniaxial fatigue properties.
17) Experimental cracking orientations were generally found to be consistent with the critical planes predicted by both versions of the FS parameter for all loading conditions considered. This suggests that the physical basis of these parameters, along with the cycle counting and damage accumulation procedures implemented in this study, were able to capture the damage mechanisms induced by these complex loading histories relatively well.

Similarly, based on the experimental results and analysis presented for notched specimen critical plane-based variable amplitude fatigue life predictions, some key findings and conclusions can be summarized as follows:

18) Similar to results from un-notched specimen analyses, notched specimen life predictions based on the assumption of stable material stress-strain response were found to be non-conservative, in all cases, based on both the original and modified FS parameters. Notched specimen predictions, however, were non-conservative to a slightly lesser extent.

19) Given the tendency of local deformation behavior at a notch to more closely resemble strain-controlled conditions, incomplete material hardening behavior when considering transient stress-strain response resulted in variable amplitude life predictions which were more non-conservative than those based on the assumption of stable stress-strain response.

20) Although considering the effects of transient deformation behavior on fatigue damage had a relatively small overall effect on analysis results for the loading histories investigated, the slight shift in prediction accuracy produced variable
amplitude notched specimen fatigue life predictions which were more consistent with those obtained from the un-notched specimen analyses.

21) Changing the crack initiation definition from final failure to 0.2 mm had a larger impact on notched specimen life predictions than that observed for un-notched specimen analyses. This is because the same 0.2 mm experimental crack initiation definition was used for the notched specimens in both sets of analyses.

Finally, based on the comparisons and discussions presented for all critical plane-based variable amplitude fatigue life predictions, some key findings and conclusions can be summarized as follows:

22) As expected from constant amplitude analysis results, the modified FS parameter resulted in improved variable amplitude fatigue life predictions for the loading histories investigated in this study, in terms of both overall accuracy and data correlation, when compared to those based on the original form of the parameter.

23) When accounting for the effects of transient stress-strain response and crack initiation definition on fatigue damage, nearly all (22 of 25) variable amplitude life predictions were within a factor of ±3 of experimental results based on the modified FS parameter.

24) When using similar analysis procedures, even the best variable amplitude fatigue life predictions still had the tendency to be more non-conservative than those computed for the constant amplitude loading conditions investigated in this study.

25) Some of the error observed in variable amplitude life predictions may be due to inaccuracies in life prediction curves, resulting from a lack of experimental
fatigue data. This is true for both the 0.2 mm crack initiation definition, and for final failure curves in the high cycle fatigue regime.

26) Another possible source for life prediction errors is in the modeling of material deformation behavior. Deformation analyses in Chapter 4 showed slightly non-conservative trends in stress-strain predictions for both the un-notched and notched specimens (near the TCD analysis location) tested in this study.

27) Other areas for potential improvements in the variable amplitude fatigue life analyses are related to the consideration of normal-shear stress/strain interaction effects, the combined effect of multiple shear stress/strain components, and linear versus nonlinear damage accumulation.

28) Although changing the critical damage sum used with the linear damage rule from 1 to 0.3, as suggested in literature, considerably improved the accuracy of variable amplitude fatigue life predictions, the correlation of data from different loading conditions became slightly worse.

29) Improvements are most likely needed in several of the aforementioned areas in order to further increase the accuracy and robustness of the fatigue life prediction process under general multiaxial variable amplitude loading conditions.
### Table 6.1 Summary of variable amplitude un-notched specimen fatigue tests.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>$d_i$ (mm)</th>
<th>$d_o$ (mm)</th>
<th>Samp. Rate (Hz)</th>
<th>$P_{max}$ (kN)</th>
<th>$T_{max}$ (N·m)</th>
<th>$S_{L,\text{max}}$ (MPa)</th>
<th>$S_{T,\text{max}}$ (MPa)</th>
<th>Channels</th>
<th>Scale</th>
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<th>0.5 mm*</th>
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<td>-</td>
<td>506.6</td>
<td>-</td>
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<td>-</td>
<td>-</td>
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<td>-</td>
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<td>-</td>
<td>-</td>
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<td>-</td>
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<td>419.8</td>
<td>-</td>
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<td>3.84</td>
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<td>-</td>
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<td>9.24</td>
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<td>201.0</td>
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<td>3.53</td>
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<td>361.9</td>
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<td>Combined</td>
<td>2.5x</td>
<td>-</td>
<td>-</td>
<td>2.03</td>
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</tbody>
</table>

* Crack length is half of the overall tip-to-tip crack length for un-notched specimens

** Final $N_f$ for un-notched specimens resulted in approximately 10-15 mm tip-to-tip crack lengths

---

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Table 6.2  Summary of variable amplitude notched specimen fatigue tests.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>$d_i$ (mm)</th>
<th>$d_o$ (mm)</th>
<th>Samp. Rate (Hz)</th>
<th>$P_{max}$ (kN)</th>
<th>$T_{max}$ (N·m)</th>
<th>$S_{A,max}$ (MPa)</th>
<th>$S_{T,max}$ (MPa)</th>
<th>Channels</th>
<th>Scale</th>
<th>Blocks to Failure</th>
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</thead>
<tbody>
<tr>
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<td>25.47</td>
<td>10</td>
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<td>-</td>
<td>376.4</td>
<td>-</td>
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<td>2.6x</td>
<td>0.18</td>
</tr>
<tr>
<td>T 30</td>
<td>29.16</td>
<td>25.44</td>
<td>10</td>
<td>50.87</td>
<td>-</td>
<td>318.5</td>
<td>-</td>
<td>Axial</td>
<td>2.2x</td>
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</tr>
<tr>
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<td>-</td>
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<td>-</td>
<td>Axial</td>
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<td>4.64</td>
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<td>16</td>
<td>38.38</td>
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<td>253.3</td>
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<td>2.00</td>
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<td>25.46</td>
<td>10</td>
<td>37.93</td>
<td>-</td>
<td>253.3</td>
<td>-</td>
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<td>7.92</td>
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<td>10</td>
<td>37.53</td>
<td>-</td>
<td>253.3</td>
<td>-</td>
<td>Edited Axial</td>
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<td>7.67</td>
</tr>
<tr>
<td>T 31</td>
<td>29.05</td>
<td>25.43</td>
<td>10</td>
<td>-</td>
<td>495.1</td>
<td>-</td>
<td>234.5</td>
<td>Shear</td>
<td>3.5x</td>
<td>0.39</td>
</tr>
<tr>
<td>T 34</td>
<td>28.98</td>
<td>25.47</td>
<td>10</td>
<td>-</td>
<td>341.9</td>
<td>-</td>
<td>167.5</td>
<td>Shear</td>
<td>2.5x</td>
<td>3.09</td>
</tr>
<tr>
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<td>25.46</td>
<td>10</td>
<td>-</td>
<td>343.3</td>
<td>-</td>
<td>167.5</td>
<td>Shear</td>
<td>2.5x</td>
<td>2.52</td>
</tr>
<tr>
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<td>25.46</td>
<td>10</td>
<td>-</td>
<td>345.5</td>
<td>-</td>
<td>167.5</td>
<td>Edited Shear</td>
<td>2.5x</td>
<td>2.74</td>
</tr>
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<td>10</td>
<td>-</td>
<td>345.8</td>
<td>-</td>
<td>167.5</td>
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<td>134.0</td>
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<td>0.56</td>
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<td>43.54</td>
<td>274.2</td>
<td>289.5</td>
<td>134.0</td>
<td>Combined</td>
<td>2.0x</td>
<td>0.71</td>
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<td>16</td>
<td>32.72</td>
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<td>217.1</td>
<td>100.5</td>
<td>Combined</td>
<td>1.5x</td>
<td>6.07</td>
</tr>
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<td>32.88</td>
<td>207.1</td>
<td>217.1</td>
<td>100.5</td>
<td>Combined</td>
<td>1.5x</td>
<td>6.34</td>
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<td>32.77</td>
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<td>217.1</td>
<td>100.5</td>
<td>Edited Combined</td>
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<td>32.97</td>
<td>207.7</td>
<td>217.1</td>
<td>100.5</td>
<td>Edited Combined</td>
<td>1.5x</td>
<td>23.90</td>
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</table>

* Crack length is individual crack length from edge of hole (0.2 mm or 1.0 mm) for notched specimens
** Final $N_f$ for notched specimens defined as 15 mm tip-to-tip length
Table 6.3 Summary of von Mises equivalent stress/strain-based variable amplitude un-notched specimen fatigue life predictions.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Loading History</th>
<th>Exp. Life (Blocks)</th>
<th>Predicted Life (Blocks)</th>
<th>von Mises Stress</th>
<th>von Mises Strain</th>
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<tbody>
<tr>
<td></td>
<td>Channels</td>
<td>Scale</td>
<td>Final*</td>
<td>Mod. Goodman</td>
<td>SWT</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>Alt. Morrow</td>
<td>SWT</td>
</tr>
<tr>
<td>TM 75</td>
<td>Axial</td>
<td>3.5×</td>
<td>0.56</td>
<td>5.72</td>
<td>8.43</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>10.29</td>
<td>11.31</td>
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<td>Axial</td>
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<td>4.33</td>
<td>18.48</td>
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<td>28.93</td>
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<tr>
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<td>Axial</td>
<td>3.1×</td>
<td>3.28</td>
<td>18.48</td>
<td>21.03</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>73.85</td>
<td>28.93</td>
</tr>
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<td>5.19</td>
<td>34.56</td>
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<td>Shear</td>
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<td>12.27</td>
<td>4.18</td>
<td>7.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td>34.51</td>
<td>11.12</td>
</tr>
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<td>2.53</td>
<td>5.48</td>
<td>5.68</td>
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<td></td>
<td>13.28</td>
<td>8.21</td>
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<td>0.89</td>
<td>5.48</td>
<td>5.68</td>
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<td></td>
<td></td>
<td>13.28</td>
<td>8.21</td>
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<td>5.28</td>
<td>23.43</td>
<td>22.51</td>
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<td></td>
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<td>45.43</td>
<td>29.41</td>
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<td>Combined</td>
<td>2.5×</td>
<td>2.03</td>
<td>23.43</td>
<td>22.51</td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td>45.43</td>
<td>29.41</td>
</tr>
</tbody>
</table>

Predictions within ±3x of experimental life: 30% 30% 10% 20%
Predictions within ±10x of experimental life: 80% 80% 40% 80%

* Final \( N_f \) for un-notched specimens resulted in approximately 10-15 mm tip-to-tip crack lengths
Table 6.4  Summary of von Mises equivalent stress/strain-based variable amplitude notched specimen fatigue life predictions.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Loading History</th>
<th>Exp. Life (Blocks)</th>
<th>Predicted Life (Blocks)</th>
<th>Predictions within ±3x of experimental life</th>
<th>Predictions within ±10x of experimental life</th>
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<tbody>
<tr>
<td></td>
<td>Channels</td>
<td>Scale</td>
<td>0.2 mm*</td>
<td>von Mises Pseudo Stress</td>
<td>von Mises Stress</td>
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<tr>
<td>T 35</td>
<td>Axial</td>
<td>2.6×</td>
<td>0.18</td>
<td>&lt; 0.01</td>
<td>0.18</td>
</tr>
<tr>
<td>T 30</td>
<td>Axial</td>
<td>2.2×</td>
<td>0.51</td>
<td>&lt; 0.01</td>
<td>0.64</td>
</tr>
<tr>
<td>T 36</td>
<td>Axial</td>
<td>1.75×</td>
<td>4.64</td>
<td>0.63</td>
<td>3.59</td>
</tr>
<tr>
<td>T 47</td>
<td>Axial</td>
<td>1.75×</td>
<td>2.00</td>
<td>0.63</td>
<td>3.59</td>
</tr>
<tr>
<td>T 69</td>
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<td>1.75×</td>
<td>7.92</td>
<td>0.56</td>
<td>5.06</td>
</tr>
<tr>
<td>T 90</td>
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<td>1.75×</td>
<td>7.67</td>
<td>0.56</td>
<td>5.06</td>
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<tr>
<td>T 31</td>
<td>Shear</td>
<td>3.5×</td>
<td>0.39</td>
<td>&lt; 0.01</td>
<td>0.27</td>
</tr>
<tr>
<td>T 34</td>
<td>Shear</td>
<td>2.5×</td>
<td>3.09</td>
<td>&lt; 0.01</td>
<td>3.36</td>
</tr>
<tr>
<td>T 46</td>
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<td>2.5×</td>
<td>2.52</td>
<td>&lt; 0.01</td>
<td>3.36</td>
</tr>
<tr>
<td>T 91</td>
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<td>2.74</td>
<td>0.23</td>
<td>3.38</td>
</tr>
<tr>
<td>T 93</td>
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<td>2.5×</td>
<td>4.48</td>
<td>0.23</td>
<td>3.38</td>
</tr>
<tr>
<td>T 39</td>
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<td>2.0×</td>
<td>0.56</td>
<td>0.06</td>
<td>1.31</td>
</tr>
<tr>
<td>T 38</td>
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<td>2.0×</td>
<td>0.71</td>
<td>0.06</td>
<td>1.31</td>
</tr>
<tr>
<td>T 41</td>
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<td>1.5×</td>
<td>6.07</td>
<td>4.01</td>
<td>11.46</td>
</tr>
<tr>
<td>T 48</td>
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<td>6.34</td>
<td>4.01</td>
<td>11.46</td>
</tr>
<tr>
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<td>23.90</td>
<td>4.13</td>
<td>16.15</td>
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</table>

* Crack length is individual crack length from edge of hole for notched specimens
Table 6.5  Summary of Fatemi-Socie critical plane-based variable amplitude un-notched specimen fatigue life predictions considering the effects of transient stress-strain response.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Loading History</th>
<th>Exp. Life (Blocks)</th>
<th>Predicted Life (Blocks)</th>
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<td></td>
<td>Channels</td>
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<td>Uniaxial ε-N Properties</td>
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<td></td>
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<td>-</td>
</tr>
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<td>TM 83</td>
<td>Axial</td>
<td>3.1×</td>
<td>-</td>
</tr>
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<td>TM 108</td>
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<td>Axial</td>
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<td>3.84</td>
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<td>-</td>
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<td>-</td>
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<td>3.53</td>
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<tr>
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<td>Combined</td>
<td>2.5×</td>
<td>-</td>
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</table>

Predictions within ±3x of experimental life: 11%**, 78%**, 0%, 67%, 0%, 33%.
Predictions within ±10x of experimental life: 33%**, 100%**, 11%, 100%, 33%, 100%.

* Crack length is half of the overall tip-to-tip crack length for un-notched specimens.
** Final \( N_f \) for un-notched specimens resulted in approximately 10-15 mm tip-to-tip crack lengths.
Table 6.6  Summary of Fatemi-Socie critical plane-based variable amplitude notched specimen fatigue life predictions considering the effects of transient stress-strain response.

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Loading History</th>
<th>Exp. Life (Blocks)</th>
<th>Predicted Life (Blocks)</th>
<th>Predictions within ±3x of experimental life</th>
<th>Predictions within ±10x of experimental life</th>
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<td>Shear γ-N Properties</td>
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<td></td>
<td></td>
<td>FS</td>
<td>Mod. FS</td>
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<td>0.18</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>T 30</td>
<td>Axial</td>
<td>2.2×</td>
<td>0.51</td>
<td>3.12</td>
<td>0.52</td>
</tr>
<tr>
<td>T 36</td>
<td>Axial</td>
<td>1.75×</td>
<td>4.64</td>
<td>17.69</td>
<td>2.75</td>
</tr>
<tr>
<td>T 47</td>
<td>Axial</td>
<td>1.75×</td>
<td>2.00</td>
<td>17.69</td>
<td>2.75</td>
</tr>
<tr>
<td>T 69</td>
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<td>7.92</td>
<td>27.13</td>
<td>3.96</td>
</tr>
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<td>7.67</td>
<td>27.13</td>
<td>3.96</td>
</tr>
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<td>3.5×</td>
<td>0.39</td>
<td>2.54</td>
<td>0.37</td>
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<td>3.09</td>
<td>33.12</td>
<td>3.60</td>
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<td>2.52</td>
<td>33.12</td>
<td>3.60</td>
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<td>2.74</td>
<td>33.20</td>
<td>3.67</td>
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<td>4.48</td>
<td>33.20</td>
<td>3.67</td>
</tr>
<tr>
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<td>2.0×</td>
<td>0.56</td>
<td>5.54</td>
<td>0.91</td>
</tr>
<tr>
<td>T 38</td>
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<td>2.0×</td>
<td>0.71</td>
<td>5.54</td>
<td>0.91</td>
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<td>50.29</td>
<td>8.24</td>
</tr>
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<td>6.34</td>
<td>50.29</td>
<td>8.24</td>
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<td>71.09</td>
<td>11.00</td>
</tr>
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<td>1.5×</td>
<td>23.90</td>
<td>71.09</td>
<td>11.00</td>
</tr>
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* Crack length is individual crack length from edge of hole for notched specimens
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In addition to crack initiation, fatigue crack growth often represents a significant portion of a component’s total fatigue life as well. Therefore, a fundamental understanding of the mechanisms that govern crack propagation and the development of accurate crack growth modeling techniques are essential to a complete fatigue life analysis. This is especially true in situations where a damage tolerant design philosophy has been employed. As such, crack growth mechanisms and their influence on crack growth rate have been researched extensively over the years for uniaxial loadings and mode I crack extension. Most of these studies, however, utilize specimens which promote ideal crack growth conditions. In practical applications, where components are often subjected to complex multiaxial loading histories, these ideal conditions usually don’t exist. Instead, naturally initiated fatigue cracks, such as the ones investigated in this study, can grow in a complex and mixed-mode manner which is not easy to predict or quantify.

For example, as mentioned in Section 2.8.5, there sometimes exists a discrepancy between the preferred crack growth mode for un-notched and notched specimens. Cracks growing from notches, even if initiated on maximum shear planes, typically always turn
to grow in mode I on planes of maximum tensile stress, regardless of the nominally applied stress state. Cracks in un-notched specimens, on the other hand, have been shown to propagate on maximum shear planes, maximum tensile planes, or a combination of both.

This same discrepancy between notched and un-notched specimen crack growth behavior was also observed in the current study. As such, characterizing and modeling the effects of friction and roughness induced crack closure on shear-mode crack growth and branching mechanisms were topics of much interest in the un-notched specimen crack growth analyses presented in Section 7.2. In addition, Section 7.3 also aims to evaluate the effects of notches and multiaxial nominal stress states on the mode I crack growth behavior observed in notched specimens. These results were also published in (Gates and Fatemi, 2016h) for the un-notched specimen crack growth analyses, and in (Fatemi et al., 2014; Gates and Fatemi, 2016i) for the notched specimen analyses. Although the results and analyses presented in this chapter are with respect to constant amplitude loading conditions, where loading sequence effects are not a factor, the knowledge gained provides valuable insight that can be applied to more complex variable amplitude crack growth problems as well.

Since SIF solutions were not readily available for several crack geometries and loading modes analyzed, Section 7.1 begins this chapter by outlining and verifying the accuracy of the computational methods used to generate such solutions in this study. Additionally, since both one- and two-dimensional crack geometries were of interest, the crack length nomenclature used in the following sections will be clarified here. Although \( a \) is most commonly used as the crack length variable for one-dimensional through crack
geometries, when referring to two-dimensional geometries, \(a\) is often used to denote crack depth. Therefore, to be consistent in the following discussions, crack depth is always denoted by \(a\), while surface crack length (or half length) is always denoted by \(c\). As a result, \(c\) is the variable used when referring to the crack length of one-dimensional crack geometries.

### 7.1 Stress Intensity Factor Determination

Calculating stress intensity factor range for a given set of loading conditions, crack geometry, and component/specimen geometry is a fundamental step in any LEFM crack growth analysis. In order to accurately obtain these values, proper geometry and crack length dependent correction factors, \(Y_I\), \(Y_{II}\), and/or \(Y_{III}\), must first be known for the geometry of interest. These factors account for the redistribution of remote stresses due to the presence of the crack. While handbooks are available containing geometry factor functions for a variety of the most common specimen and component geometries, most solutions are only available for cracks growing in mode I. As a result, an alternative method of SIF calculation was needed in this study in order to determine shear-mode geometry factors for the specimen and crack geometries of interest.

Linear elastic FEA was used in this study to generate any necessary geometry factors for which commonly available solutions were not found. In particular, the extended finite element method (XFEM) technique was used for modeling cracks within Abaqus/CAE 6.14-5. Eight node linear hexahedral brick elements (C3D8R) were used in all analyses and SIF outputs for each loading mode were calculated automatically within the software using a contour integral method. Five different paths were used for the
evaluation of the contour integral. Excellent convergence/path independence of the results was generally observed starting with the third contour. Consequently, the SIF values used from each analysis were an average of those generated from contours 4 and 5. Averaging SIF values resulted in slightly more consistent results as compared to using values from a single contour. Appropriate geometry factor functions for each loading mode were then obtained through fitting of the FEA results for several crack lengths in the range of interest.

In order to verify the accuracy of the FEA-derived geometry factor functions generated in this study, FEA solutions were first compared to SIF values computed for simple crack/specimen geometries for which well-established geometry factor functions are readily available. These included the following two cases: a single edge through crack in a flat plate under mode I loading (SENT) and a through crack in the center/middle of a flat plate under mode I loading (MT). Two different FEA models were used to compute SIFs for both the SENT and MT specimens. Both were flat plates with a height of 80 mm, but one had a thickness of 20 mm while the second had a thickness of 2 mm. This was done so that the effect of plane stress versus plane strain conditions on the resulting SIF solution could be evaluated. The widths, $W$, of the SENT models were 30 mm, while the widths of the MT models were 40 mm. For the 20 mm thickness models, 20 elements were generated across the thickness of the specimen, while for the thin models, 10 elements were used. Along the width and height of the specimen, the meshes for all models were refined down to a minimum node spacing of 0.20 mm, at the origin of the crack, and allowed to gradually increase. The mesh for the 20 mm thick SENT model is
shown in Figure 7.1(a). The model is sectioned about the crack plane, which is shown as the top surface in this figure, for clarity.

For the SENT specimen, crack lengths, \( c \), of 1.5 mm, 3 mm, 6 mm, 10 mm, and 20 mm were analyzed using the thick plate model, while crack lengths of 3 mm and 10 mm were analyzed using the thin model. A typical von Mises stress output is shown in Figure 7.1(b) for the 20 mm thick SENT specimen containing a 3 mm crack. Geometry factors, computed from resulting SIF outputs, were then compared to two readily available formulations: Equation (7.1) (Anderson, 2005) and Equation (7.2) (Brown, Jr. and Srawley, 1966).

\[
Y_I = \sqrt{\frac{2\tan\left(\frac{\pi c}{2W}\right)}{\cos\left(\frac{\pi c}{2W}\right)}} \sqrt{\frac{W}{\pi c}} \left[ 0.752 + 2.02\left(\frac{c}{W}\right) + 0.37\left(1 - \sin\left(\frac{\pi c}{2W}\right)\right)^3 \right] 
\]  

\[
Y_I = \frac{1}{\sqrt{\pi}} \left[ 1.99 - 0.41\left(\frac{c}{W}\right) + 18.7\left(\frac{c}{W}\right)^2 - 38.48\left(\frac{c}{W}\right)^3 + 53.85\left(\frac{c}{W}\right)^4 \right] 
\]  

where \( c \) is the crack length and \( W \) is the specimen width (in the direction of crack growth). When computing SIF, all geometry factors in this section are rearranged to fit the form of Equation (2.48).

Comparisons of geometry factor values between both the 20 mm and 2 mm XFEM models and Equation (7.1) are shown in Figures 7.2(a) and 7.2(b) for the 3 mm and 10 mm crack lengths, respectively. These plots show the variation of geometry factor across the thickness of each specimen. An \( x \)-axis value of zero denotes the outer surface of the specimen, while an increasing \( x \) value moves towards the center of the specimen. Results are only plotted for half of the specimen thickness, as the distribution is symmetric about the specimen mid plane. It can be seen from this figure that the SIF
solution for the SENT geometry is highest at the middle of the specimen and decreases slightly at the specimen outer surface. At a distance from the surface of approximately 15% of the specimen thickness, the geometry factor solution stabilizes. The mid-thickness value for \( Y_f \) is slightly higher for the 2 mm thick specimen when compared to the 20 mm thick specimen. This is likely due to a plane stress-dominated condition, resulting in a larger crack tip plastic zone, in the thin specimen. The difference, however, is only a 4.6% increase for the 3 mm crack length and 2.4% for the 10 mm crack. It can also be seen that the mid-thickness geometry factors derived using the XFEM technique tend to be slightly higher than those calculated from Equation (7.1).

Assuming plane strain conditions are considered for the derivation of most closed form geometry factor solutions, the mid-thickness \( Y_f \) values derived from XFEM analysis of the 20 mm thick SENT specimen were plotted versus crack length, along with Equations (7.1) and (7.2), in Figure 7.3. This figure shows good agreement between all geometry factor solutions considered. Comparing the XFEM results to Equation (7.1), which produces values almost identical to Equation (7.2), the error in geometry factor solution for all crack lengths considered ranged from \(-1.0\%\) to 9.0% with an average error of 4.5%.

Similarly for the MT specimen, half crack lengths, \( c \), of 1.5 mm, 3 mm, 5 mm, and 10 mm were analyzed using the thick plate model, while half crack lengths of 1.5 mm and 5 mm were analyzed using the thin model. Again, FEA geometry factor solutions were compared to two commonly available formulations: Equation (7.3), from ASTM Standard E647 (2009), and Equation (7.4) (Anderson, 2005).

\[
Y_f = \sqrt{sec\left(\frac{\pi c}{W}\right)} \tag{7.3}
\]
\[ Y_\text{I} = \sqrt{\sec \left( \frac{\pi c}{W} \right)} \left[ 1 - 0.025 \left( \frac{2c}{W} \right)^2 + 0.06 \left( \frac{2c}{W} \right)^4 \right] \]  

(7.4)

where \( c \) is the half crack length and \( W \) is the entire specimen width (in the direction of crack growth).

Comparisons of geometry factor solutions between both the 20 mm and 2 mm XFEM models and Equation (7.3) are shown in Figures 7.4(a) and 7.4(b) for the 1.5 mm and 5 mm half crack lengths, respectively. Unlike the SENT specimen, this figure shows that the SIF solution for the MT geometry is highest at the outer surface of the specimen, and decreases slightly at the specimen mid-thickness. This is likely due to differences in stress distributions along the crack front for each specimen geometry. While the centered crack in the MT specimen always results in symmetric stressing conditions, the SENT specimen produces an increasing bending stress as the crack grows. Additionally, geometry factor solutions for the MT specimen take longer to stabilize across the thickness of the specimen than those for the SENT geometry, requiring around 30% of the total thickness. The mid-thickness value for \( Y_\text{I} \) is, again, slightly higher for the 2 mm thick specimen when compared to the 20 mm thick specimen, likely due to the plane stress-dominated conditions in the thin specimen. The difference for the MT specimen is a 9.7% increase in SIF for the 1.5 mm half crack length and a 6.0% increase for the 5 mm half crack.

Similar to the results for the SENT specimen, mid-thickness \( Y_\text{I} \) values derived from XFEM analysis of the 20 mm thick MT specimen were plotted versus crack length, along with Equations (7.3) and (7.4), in Figure 7.5. This figure again shows good agreement between all geometry factor solutions considered. Comparing the XFEM results to Equation (7.3), which produces values almost identical to Equation (7.4), the
error in geometry factor solution for all crack lengths considered ranged from \(-1.7\%\) to 5.3\% with an average error of 2.1\%.

Based on these analyses, it can be concluded that the Abaqus XFEM technique is a reliable method of generating geometry factor functions for use in subsequent SIF calculations. For the two common specimen geometries studied, the maximum error between XFEM and handbook solutions for any of the analysis conditions considered was 9.0\%, with an average error of around 3.4\%. Therefore, similar results should be expected for more complex geometries and loading conditions as well. It is also worth noting the effect that specimen thickness and plane stress versus plane strain assumptions can have on the resulting SIF values. This can be an important consideration when studying small cracks (e.g. semi-elliptic surface cracks) growing into the depth of a component. As such, determining geometry factor functions using the XFEM technique has the added advantage of being able to implicitly account for this effect.

### 7.2 Un-Notched Specimen Crack Growth

For the constant amplitude fatigue tests performed in this study using un-notched specimens, crack initiation and growth was observed primarily on planes of maximum shear stress/strain. However, under certain conditions, cracks were also observed to branch and grow in mode I for part of their growth life. Since being able to accurately predict the orientation of a growing crack is an essential step in being able to compute accurate crack driving forces, the mechanisms governing this crack branching phenomenon were a topic of much interest in this study.
To study shear-mode (i.e. mode II, mode III, or mixed-mode) crack growth and/or crack branching in un-notched specimens, the work performed in this study is similar in some regards to studies mentioned previously in Section 2.8.5. However, it also addresses some aspects typically not considered in shear-mode crack growth studies. For example, nearly all of the studies available in literature deal exclusively with mode II cracks growing from precracks or circumferential notches. This study, on the other hand, places a strong emphasis on the behavior of naturally initiated fatigue cracks in the uniformly stressed gage section of un-notched specimens. One problem with studying growth from precracks is that residual stresses in the crack tip region, resulting from the precracking procedure, have the potential to influence the subsequent mode II crack growth. Even if specimens are annealed following precracking to eliminate these effects, the crack face topography still varies from that of a naturally occurring crack. In addition, growing a mode II crack from a precrack can significantly alter the length scale required for branching mechanisms to occur and eliminates any influence on crack path from microcrack coalescence.

The present study evaluates crack growth mechanisms, in part, by comparing crack path evolution between both natural cracks and cracks growing from artificial precracks. This is done in order to evaluate the effect of microcrack coalescence on overall crack path. The role of crack face friction and roughness induced crack closure on shear-mode crack growth is also evaluated through torsion tests performed with static mean normal stresses. A model in then proposed in an attempt to quantify the experimental observations and to account for reductions in effective mode II SIF due to crack face interaction effects. An emphasis is placed on crack growth in the low to
intermediate fatigue life regime where shear-mode cracks may or may not branch to grow in mode I.

7.2.1 Crack Path Comparisons

Although surface replicas have shown the development of small microcrack networks in a number of the un-notched specimen fatigue tests performed in this study, crack coalescence was only observed in a limited number of these tests and usually occurred relatively early in the crack growth life. Therefore, questions were raised on whether or not crack coalescence played a large role in determining overall crack path for these tests. Thus the effect of microcrack networks and coalescence was investigated through a limited number of additional constant amplitude fatigue tests performed on un-notched specimens containing a small “precrack.” For these precracked specimens, the term precrack is used to refer to a small notch machined in the specimen surface meant to resemble, as closely as possible, the profile of a naturally occurring fatigue crack. No actual precracking procedure was performed on this notch prior to applying the testing loads. It is referred to as a precrack simply as a means of differentiating it from the other notched specimens, containing a circular hole, which are referenced throughout this dissertation.

All precrack notches were machined by means of a 0.127 mm diameter ball mill and were semi-elliptic in shape. The precracks were of a length and depth equal to approximately 1 mm and 0.2 mm, respectively, and were aligned with a plane of maximum shear stress for each applied loading condition. A sectioned view of a precrack notch is shown in Figure 7.6. Precracked specimens were polished similar to the un-
notched specimens, but external polishing was only performed in the crack growth region surrounding the precrack.

Two load levels each were used to perform precracked specimen constant amplitude fatigue tests under fully-reversed axial, torsion, in-phase axial-torsion, and 90° out-of-phase axial-torsion loading. Loading conditions for each tests, along with corresponding experimental lives, are given in Tables 5.1 to 5.4 for each respective loading path. Each applied loading level corresponds to an identical level used for a fatigue test of an un-notched specimen. However, results from the axial and 90° OP loading conditions are not presented in this section because the difference between mode I and mode II growth cannot be observed from the outer surface of the specimen. A future three-dimensional fracture surface analysis would be required to interpret those results. Therefore, only the experimentally observed crack paths for the two torsion loading levels and two in-phase axial-torsion load levels are shown in Figure 7.7.

By comparing crack paths between the un-notched and precracked specimens in Figure 7.7, it is easy to see that they are very similar. This is true even for complex crack paths where cracks initiate in shear-mode, branch into mode I cracks, and eventually transition back to shear-mode after growing for some distance (Figure 7.7(b-d)). This suggests that microcrack networks and their coalescence did not play a significant role in determining the crack paths for these tests. If dominant cracks, growing without the influence of crack coalescence, were expected to always branch into mode I growth regardless of the applied loading (as predicted by traditional crack growth direction criteria), then the crack path for the precracked specimen subjected to the higher level pure torsion load (Figure 7.7(a)) would certainly not have remained vertical for its entire
growth life. This, combined with the transition back to shear-mode growth after a period of mode I growth observed for the other precracked specimen tests, supports findings from many un-notched specimen surface replicas where shear-dominated crack growth occurred even in the absence of any observable crack coalescence.

Although the existence of the type R (i.e. coalescence driven) crack growth mechanism is certainly not being rejected, it is clear that in the case of this study, coalescence was most likely not responsible for the shear-dominated crack paths observed in the un-notched specimen tests. Therefore, there must be some other mechanism by which crack growth behavior transitions from being shear-mode to mode I-dominated. By studying the crack paths in Figure 7.7, a correlation between the loading level and/or SIF and the crack growth direction is observed. To help illustrate this point, Table 7.1 was constructed and contains all available crack orientation data from pure torsion, torsion with static axial stress, and in-phase axial-torsion fatigue tests of un-notched specimens. Nominally applied loading conditions are given along with shear stress amplitude and maximum normal stress on the maximum shear plane. Specimen suffixes indicate the loading conditions as follows: ST are pure torsion tests, STSA are torsion tests with static axial stress, and SC are in-phase axial-torsion tests. The table is sorted based on decreasing shear stress amplitude on the maximum shear plane. Within each shear stress level, tests are sorted by decreasing level of maximum normal stress on the maximum shear plane. Approximate crack plane orientation is then listed for different ranges of overall tip-to-tip crack length, $2c$ (in mm), as well as the total number of cycles to final failure, $N_f$. 
The dependence of crack growth direction on loading level is easy to see from Table 7.1. Above a shear stress amplitude of 188 MPa, all significant ranges of crack growth for all loading conditions took place on planes of maximum shear. Below this level, cracks began to transition to mode I growth shortly after initiation and continued growing in mode I until near failure. The only exception to this trend comes when a tensile normal stress existed on the maximum shear plane. In these cases, cracks spent more time in shear-mode growth than specimens tested at the same shear stress amplitude without a tensile normal stress. In all but one case, both crack initiation and final failure were observed to be shear-mode processes.

These observations support the idea that friction and roughness induced crack closure, caused by crack face interaction, play a key role in determining crack path. At the lower loading levels, the combined effect of friction and roughness seem to reduce the effective shear-mode driving force at the crack tip until it drops below a critical level. This critical level could either represent a mode II threshold condition, or a value at which the potential for mode I growth exceeds that of the existing shear-mode crack. At this point, the crack then turns to maximum tensile planes where there is less resistance to mode I crack propagation. This idea echoes the conclusions drawn in several of the studies reviewed in Section 2.8.5.

At the higher loading levels, however, these effects do not appear to play as large of a role in determining crack path. This is likely due to a combination of factors. For example, at higher stress levels, the effect of friction may not be large enough to drop the effective shear-mode driving force below its critical value. Additionally, increased plasticity at the crack tip can result in the deformation and destruction of the crack face
asperities which would otherwise restrict the effective shear-mode driving force. Figure 7.8 shows that as the shear stress amplitude was increased in the pure torsion tests, a longer mode II crack length was observed before branching occurred. This trend was also reported in (Doquet and Bertolino, 2008a; Makabe and Socie, 2001) and suggests that the mode II driving force increases at a faster rate than the attenuation effect due to crack face interaction, which eventually leads to a non-branching condition.

Such a dependence on load level is consistent with the observed differences between crack paths in un-notched and notched specimens. Because of the stress concentration effect in notched specimens, they are often tested at much lower nominal stress amplitudes than un-notched specimens. Therefore, once an initial shear-mode crack propagates out of the notch affected zone and into the lower stressed gage section, frictional effects would significantly restrict its growth and lead to a tendency for the crack to transition into mode I, where there is less resistance to crack extension. The same is true for crack growth tests where crack growth is evaluated from a precrack. Because data for a large range of growth rates and stress intensity factors are typically desired in these types of test, they are performed at the lower nominal loadings conducive to mode I growth.

### 7.2.2 Mean Stress Effects on Crack Growth

The normal stress effect on crack path that was observed in these tests further emphasizes the role of crack face interaction in the crack growth process. For two different values of shear stress amplitude, specimens were tested with a static compressive stress, no normal stress, and a static tensile stress. It should be noted that static normal stresses were used so as to not introduce mixed-mode growth effects
through a nonzero mode I SIF range. The fatigue lives for these tests, to various tip-to-tip crack lengths, are shown in Figure 7.9. For both shear stress levels, the addition of the tensile normal stress reduced the overall fatigue life by around an order of magnitude. Much of this difference can be attributed to a decrease in crack growth life which would logically stem from a reduction in crack face friction and roughness induced closure effects. As a result, crack paths for these tests were always observed to be in the specimen circumferential direction, perpendicular to the applied tensile stress (Figure 7.10(a)).

For the tests in which a static compressive stress was applied, the crack growth plane switched from the specimen circumferential direction to the longitudinal direction (Figure 7.10(b)). Although both are planes of maximum shear stress, the compressive normal stress acts to increase crack face interaction and inhibit crack growth on the circumferential plane. As a result, longitudinal cracks developed and grew under the same nominal loading and in a similar manner to those for torsion only tests. For specimens tested at the lower shear stress amplitude, crack branching and mode I growth was observed over the same range of crack lengths for both the compressive normal stress and pure torsion cases (although difficult to see at the scale presented in Figure 7.10, this will be better illustrated in Section 7.2.4.1).

The decrease in fatigue life observed in Figure 7.9(b) for the static compression tests, as compared to pure torsion tests, may be due to an increase in plastic zone size and crack driving force as a result of the additional compressive tangential stress (T-stress) at the crack tip. Although also present for tests performed at the lower shear stress amplitude, the effect of T-stress is expected to have a larger effect as stress levels
increase. Also, the addition of the static stresses at the higher shear stress amplitude results in a general yielding condition throughout the specimen gage section, which greatly increases the probability of crack initiation compared to the pure torsion test. For the lower shear stress amplitude, all loading conditions result in nominally elastic stresses throughout the gage section.

As mentioned in Section 2.4, similar observations were made by Kaufman and Topper (2003) when studying the effect of static mean stress on shear-mode crack growth in 1045 steel. They found that by increasing the tensile normal stress on the maximum shear plane, fatigue life continually decreased until a critical level of mean stress was applied. Above this level, no significant difference in experimental life was observed. Conversely, the effect of compressive mean stress, normal to both maximum shear planes, was found to increase fatigue life. After studying crack front and fracture surface asperity profiles, these trends were attributed to changes in crack face friction and roughness induced closure effects resulting from varying degrees of crack closure. The critical value of tensile normal stress, above which no significant difference in fatigue life was observed, was found to correspond to a condition where cracks were fully open and no interference was present between opposing crack faces.

7.2.3 Modeling of Friction Effects

As evidenced by the experimental results, the role of friction and roughness induced closure effects on shear-mode crack growth is a complex, yet significant, issue. Because of the many factors involved, and their inherent variability, quantification of these effects is challenging. This section will aim to reproduce some of the crack growth
trends observed in experiments by presenting a simplified model to predict and quantify crack growth attenuation due to crack face friction and roughness.

The proposed model takes as its starting point the idea that friction and roughness induced crack face interaction allow a portion of the nominally applied loading to be transferred through a crack. Consider two extremes for pure torsion loading: an uncracked volume of material can transmit all of the nominal loading and creates no stress concentration effect, while a geometrically ideal mode II crack cannot transfer any load between crack faces and produces the theoretical mode II SIF value at its crack tip. Therefore, it would seem logical that an effective mode II SIF could be determined by taking into account the amount of loading transferred between crack faces in an actual cracked component. To be effective, the model should be able to account for experimentally observed trends such as normal stress effect and loading level dependence, and should also depend on quantities relevant to crack face friction and roughness such as coefficient of friction and crack face asperity angle. For ease of implementation, it should also ideally only depend on readily available material properties and not require the use of geometry dependent functions, other than those used in SIF calculations.

Because the proposed model is primarily concerned with frictional attenuation in the shear-mode crack growth of small cracks, it is taken as an initial assumption that a crack has nucleated and is initially growing on a plane of maximum shear. Additionally, since the model relies on the concept of reduced remote stresses for SIF calculation, it is assumed in the analysis that no large scale plasticity is present, and that LEFM principles are applicable. Finally, despite the complex manner in which actual cracks grow on the
microscopic level, it is assumed in the model that overall crack growth is planar in nature and that micro level crack path deviations (i.e. zig-zagging) are consistent in terms of both angle and height.

In order to compute the proposed effective SIF values, the first step is to determine the nominal stress state. Nominal, in this case, refers to the stress state that would exist in the volume of material surrounding a crack if the crack were not present. This is assumed to be a two-dimensional stress state aligned with the direction of overall crack growth. Loads transferred through the crack face are considered on an averaged basis along the entire length of the crack. If significant stress gradients exist along the length and/or depth of the crack, average stress values should be considered. The nominal coordinate system ($x$-$y$) is shown schematically in Figure 7.11 along with other expressions relevant to the following discussions. Once the appropriate stress state is known, it is transformed into a coordinate system (subsequently named $x'$-$y'$) aligned with the average effective crack face asperity angle, $\alpha_{eff}$, the calculation of which will be explained later. Of the stresses in the transformed coordinate system, the crack can only directly transfer a compressive stress, $\sigma_{x'}$, normal to the asperity face. However, a resulting friction induced stress component allows for additional loads to be transferred as well. It should be noted that the model will not correctly predict frictional attenuation for asperity angles exceeding 45°. Above this angle, attenuation starts to become more of a function of mechanical interlocking than friction. Additionally, shielding effects from adjacent asperities will begin to have an effect on stresses at the crack interface above this angle.
The friction stress acting at the crack interface is calculated by multiplying the compressive normal stress, \( \sigma_{x'} \), by a coefficient of friction, \( \mu \), which represents the friction between opposing crack faces under ideal contact conditions. Coefficients of friction for various materials can be readily found in machinery handbooks such as (Oberg et al., 2000) and typical reported values for static coefficients include 1.1-1.3 for aluminum on aluminum contact, 0.7-0.8 for steel on steel, and 1.0 for copper on copper. Although sliding coefficients of friction are harder to find, typical values range from 50-80% of the value of the static coefficient for the same material. Since relative motion occurs between crack faces in shear-mode crack growth, a sliding coefficient of friction should be assumed.

This friction stress then serves to react all or part of the transformed shear stress, \( \tau_{xy'} \). If the value of the friction stress is greater than the transformed shear stress, then the entire value of \( \tau_{xy'} \) is considered to be acting at the crack interface as the local shear stress component, \( \tau_{nt} \). If the available friction stress is less than the transformed shear stress, on the other hand, then \( \tau_{nt} \) assumes the value of the friction stress. However, if local normal stress component, \( \sigma_{x'} \), is tensile, then no contact occurs between opposing crack faces, no load is transferred through the crack, and both \( \sigma_n \) and \( \tau_{nt} \) are zero. Additionally, because crack faces perpendicular to local stress component \( \sigma_{y'} \) are not in contact, this stress component is not transferred through the crack (i.e. \( \sigma_t = 0 \)). This can be expressed mathematically by Equations (7.5) and (7.6):

\[
\sigma_n = \begin{cases} 
\sigma_{x'} & \text{if } \sigma_{x'} < 0 \\
0 & \text{if } \sigma_{x'} \geq 0
\end{cases}
\]  

(7.5)
\[ \tau_{nt} = \begin{cases} 
0 & \text{if } \sigma_{x_t} \geq 0 \\
-\mu \sigma_{x_t} \frac{\tau_{xy'}}{|\tau_{xy'}|} & \text{if } \sigma_{x_t} < 0 \text{ and } -\mu \sigma_{x_t} \leq |\tau_{xy'}| \\
\tau_{xy'} & \text{if } \sigma_{x_t} < 0 \text{ and } -\mu \sigma_{x_t} > |\tau_{xy'}| 
\end{cases} \] (7.6)

With local crack face stress components \( \sigma_n, \sigma_t, \) and \( \tau_{nt}, \) known, these values can then be subtracted from their corresponding transformed stress components, \( \sigma_{x'}, \sigma_{y'}, \) and \( \tau_{xy'}, \) in order to determine the portion of the nominally applied loading not transmitted through the crack. These effective stress values, i.e. \( \sigma_{x'}:\text{eff} = \sigma_{x'} - \sigma_n, \sigma_{y'}:\text{eff} = \sigma_{y'}, \) and \( \tau_{xy'}:\text{eff} = \tau_{xy'} - \tau_{nt}, \) can then be transformed back into the nominal coordinate system (aligned with the overall crack growth plane) in order to compute the effective SIFs responsible for driving crack growth.

To this point, the proposed model only relies on two parameters in addition to the nominal loading: the average effective crack face asperity angle and the coefficient of friction. The coefficient of friction is assumed to be a constant, while the average effective asperity angle is allowed to evolve in order to reflect changes in crack face contact conditions. An equation describing the variation in effective asperity angle is proposed as follows:

\[ \alpha_{\text{eff}} = \alpha \left(1 - e^{-\left(\frac{c}{l}\right)^4}\right) \left(1 - \beta \frac{K_{q,\text{nom}}}{K_{IC}}\right) \] (7.7)

where \( \alpha \) is the natural/undeformed asperity angle, \( c \) is the crack length, \( l \) is a material characteristic length, \( K_{q,\text{nom}} \) is the nominally applied equivalent SIF (not accounting for friction effects), \( K_{IC} \) is the material plane-strain fracture toughness, and the parameter, \( \beta, \) describes the influence of loading level on crack face interaction. The Macaulay brackets in Equation (7.7) represent the following function: \( \langle x \rangle = (x + |x|)/2. \) Although Equation (7.7) is phenomenological in nature, it is designed to reflect the complex changes that
occur at the crack interface as a crack grows and provides a simple means of quantifying these changes.

The undeformed asperity angle, $\alpha$, is assumed to be independent of the applied loading conditions and can either be determined through experimental measurements, or estimated based on crystallographic structure (Pokluda et al., 2014). Assuming a constant value of average asperity angle, however, would result in a nearly constant frictional effect on effective shear stress. In reality, variations in asperity angle will cause the value of friction stress to change, which will affects the load transfer through the crack interface. This phenomenon is reflected in the current model through a variation in asperity angle with crack length and/or loading level.

The idea behind the first bracketed term in Equation (7.7) is that initially, when cracks are on the order or a grain size or two, there is not much deviation in their ideal path. This is because there has not been a sufficient amount of growth to encounter slip system misalignment from one grain to the next and/or other microstructural obstacles which lead to crack meandering and the development of crack face asperities. Therefore, this term, which takes the form of a Weibull cumulative distribution function, reduces the effective asperity angle at zero crack length to zero, and allows it to gradually increase with crack length until it approaches its saturated value at a length equal to that of a few grains. This produces a behavior which agrees with the decreasing crack growth rates observed for short cracks in the constant SIF-controlled tests reported in (Tschegg, 1983).

The material characteristic length, $l$, can be considered equal to the average grain size in the direction of crack growth. The Weibull shape parameter and scale parameter were chosen to be 4 and 2, respectively, so that the full value of asperity angle is
progressively reached between crack lengths of 1 and 3 grains. This reflects the idea that over the length of a single grain, the crack is planar in nature and no crack face asperities exist. Then, as the crack grows across the second and third grains, deviations in its path can result in the formation of a peak and a valley, respectively, in the crack face topography. This, in turn, would result in a complete set of alternating crack face asperities to impede crack growth. The evolution of the first term of Equation (7.7) with normalized (by grain size) crack length is shown, as the solid black line, in Figure 7.12(a). The function is plotted in this figure with other values of Weibull shape and scale parameter as well to illustrate the effect of these parameters on the resulting output. A schematic illustrating the previously described development of crack face asperities with crack length is also included at the bottom of this figure. Evidence of this type of crack behavior, observed in short crack replicas from pure torsion tests, is shown in Figure 7.13(a)-(b).

Similar to short cracks, long cracks may also experience a reduction in frictional stresses due to changes in crack face asperities. Unlike short cracks, however, these changes are brought about as a result of asperity destruction due to plasticity and fretting along the crack interface. Additionally, the actual coefficient of friction is more prone to a reduction in long cracks due to the formation of oxide and/or debris layers between crack faces. Experimental evidence of fretting, and the resulting debris formation, is shown in Figure 7.13(c) for a test conducted under pure torsion loading. It can also be noted in this figure that the fretting debris is only visible in portions of the crack growing vertically, along planes of maximum shear. Since the effects of these processes generally increase with an increase in local stresses and/or crack length, the second bracketed term
in Equation (7.7) reflects changes in frictional attenuation by decreasing the effective asperity angle as the ratio of nominal SIF to fracture toughness increases. A linear relationship was chosen based on trends reported in (Doquet and Bertolino, 2008b). The variation of this term with normalized (by fracture toughness) equivalent SIF is shown in Figure 7.12(b). The function is plotted for several values of the coefficient $\beta$.

Finally, to illustrate how the entirety of Equation (7.7) varies with crack length and stress intensity, Figure 7.14 is presented. This plot shows the ratio of effective to undeformed asperity angle on the height axis, which results from various combinations of normalized crack length and normalized SIF, shown on the remaining axes. The coefficient, $\beta$, was assumed to be 3 for the purpose of generating this plot. In an actual crack growth situation, the surface of the plot would be traversed starting from its back left corner and moving across to the front right corner.

In order to investigate how the variation of each model parameter influences the effective SIF output, a sensitivity analysis was performed for the case of mode II crack growth under pure torsion loading. A set of baseline parameters was first chosen from which individual parameters could be varied one at a time. These baseline values were $\alpha = 36^\circ$, $\beta = 3$, $l = 0.075$, and $\mu = 0.8$. A nominally applied shear stress value of 180 MPa was used for the sensitivity analysis. The variation of predicted frictional attenuation, $K_{II,eff}/K_{II,nom}$, with increasing crack length is shown in Figure 7.15(a) for several different values of average undeformed asperity angle, $\alpha$, in Figure 7.15(b) for different values of friction coefficient, $\mu$, in Figure 7.15(c) for different values of the load level influence coefficient, $\beta$, and in Figure 7.15(d) for the variation of average grain size, $l$. The results presented in Figure 7.15 indicate a fairly strong dependence of the model output on the $\beta$.
value, especially at longer crack lengths, a more moderate dependence on coefficient of friction and asperity angle, which gradually decreases with increasing crack length, and a smaller effect of the average grain size only at shorter crack lengths. These sensitivities should be kept in mind, along with the uncertainty involved in choosing the value of each parameter, when evaluating model predictions.

Although the application of this model is most easily discussed with respect to cases of fully-reversed pure torsion loading, it can also be applied to cases where nonzero mean and/or mixed-mode loading conditions exist. Because it is based on the stress state at the crack location, it is applicable to any type of loading and the predicted frictional attenuation is sensitive to both the applied shear stress and normal stress components. A simple Mohr’s circle analysis reveals that the model will correctly predict a decrease or increase in crack face friction/interaction due to the presence of an applied tensile or compressive stress, respectively. Effective SIFs can then be computed for both mode I and mode II/mode III loading components based on the friction reduced effective stress components. For situations where crack face friction stresses are not symmetric over the course of a loading cycle, the model can be applied at both the minimum and maximum loadings in the cycle to compute the effective SIF range. However, care should be taken when determining the sign of the stress transformation angle to ensure that the transformed stress, $\sigma_x'$, is perpendicular to the crack faces in contact (related to the shear stress direction on crack growth plane) at each loading state considered.
7.2.4 Model Predictions and Comparisons with Experimental Results: 2024-T3 Aluminum Alloy

Regardless of how well a model qualitatively agrees with experimental observations, its real value comes from the ability to predict crack growth behavior in a quantitative manner. The following section will evaluate the proposed model’s ability to do so by analyzing the experimental data presented earlier. Predictions and comparisons will be made concerning crack branching, crack growth life, and crack growth rate under constant amplitude loading conditions.

The first step in analyzing the experimental data was to gather the relevant material parameters for the proposed model. The average undeformed crack face asperity angle was measured from the surface crack replicas of several specimens when crack lengths, 2c, were less than 1 mm. Fifteen measured values ranged from 16° to 60° with an average of 36° and standard deviation of 14°. A value for sliding coefficient of friction under ideal conditions could not be found and was instead estimated as being 0.67 times the average reported static values for aluminum on aluminum contact, i.e. 0.67(1.2) = 0.8.

The parameter, β, in Equation (7.7), was assumed to be 4 loosely based on data and trends reported in (Doquet and Bertolino, 2008b). This predicts that the frictional attenuation due to crack face interaction disappears after the nominal SIF exceeds one quarter of the value of the material’s plane strain fracture toughness. The value for latter was considered to be 34 MPa√(m) (Military Handbook: Metallic Materials and Elements for Aerospace Vehicle Structures, 1998), and an average grain size of 0.075 mm was used based on data reported in (Merati, 2005) for 2024-T3 aluminum alloy.

To compute nominal and effective SIF values, the ideal shear-mode crack geometry was considered to be a semi-elliptic surface crack growing with a constant
aspect ratio \((a/c)\) of 0.5. This aspect ratio corresponds to a condition where the mode II SIF at the specimen surface is approximately equal to the mode III SIF at the crack’s maximum depth, \(a\), (Murakami et al., 2008). Therefore, an ideal crack growing in shear-mode would be expected to maintain this aspect ratio throughout its growth life. This agrees well with the experimental crack shape data presented in (Beer, 1984) for pure torsion and combined axial-torsion loading of Inconel 718. With this aspect ratio, cracks do not become through-thickness for any of the LEFM applicable crack lengths considered in this analysis. Appropriate geometry factor functions were obtained through the fitting of linear FEA results, derived using the XFEM crack technique presented in Section 7.1, for several crack lengths. The resulting fits, for both mode I and mode II SIF at the specimen surface, are given in Table 7.2 along with other relevant parameters used in the subsequent crack growth analyses.

With all of the model inputs known, calculations for effective SIF were carried out using the previously described procedure. SIF range was determined by applying the model incrementally throughout a loading cycle and computing the difference between the maximum and minimum effective SIF values over any given cycle.

7.2.4.1 Crack Branching Predictions

In order to predict crack branching based on the effective SIF output of the proposed model, an appropriate crack branching criterion must first be chosen. Based on the results of available literature studies, reviewed in Section 2.8.5, a maximum growth rate criterion was selected for the current analysis. Crack branching potential was evaluated at the outer surface location of the specimen where the growth was either pure mode II or mixed-modes I and II. Since reliable data on shear-mode crack growth
kinetics (in the absence of closure effects) were not readily available for the tested material, an equivalent SIF formulation was used as the basis to compare crack growth potential for each mode. Equivalent SIF (not to be confused with the friction reduced effective SIF) was computed using Equation (2.55), which is based on the summation of energy release rates due to each crack extension mode. According to this equation, mode I and mode II SIFs have the same contribution to the equivalent SIF value. Therefore, for these analyses, whichever loading mode produced the highest SIF was considered the preferred crack growth mode. For loading conditions in which the nominal loading on the maximum shear plane was mixed-mode, e.g. axial and in-phase axial-torsion, equivalent SIFs were computed from effective mode I and mode II SIF components before comparisons were made to the potential mode I SIF of a branch crack.

Of the more popular equivalent SIF parameters (i.e. Equations (2.54) and (2.55)), Equation (2.55) was chosen for these analyses because it showed slightly better correlation of growth rate data from different loading conditions, when computed using nominally applied stresses on the maximum shear plane, than did Equation (2.54). The equivalent SIF ranges calculated using Equation (2.55) also demonstrated better agreement with the uniaxial long crack growth rate trends reported in (Newman Jr. and Edwards, 1988). To illustrate these observations, a comparison of mixed-mode crack growth rate correlations using both of these equivalent SIF parameters is shown in Figure 7.16.

It should be noted that for crack growth in 90° out-of-phase tests, the calculation of equivalent SIF range is not straight forward. Because of the ratio of nominal axial to shear stress used in this study, the full range of nominal mode I and mode II SIFs occur
on the same plane. However, the maximum and minimum mode I and mode II SIF do not occur simultaneously. Since equivalent SIF, from Equation (2.55), is a function of SIF component ranges, it is computed subsequent to the determination of the individual mode I and mode II SIF ranges over a given cycle. For 90° OP loading, however, this would disregard the phase difference between the two loading modes and overpredict the equivalent SIF range. Computing the variation of equivalent SIF over the entire cycle in order to determine equivalent SIF range is not feasible either. This is because the even exponents, in both Equations (2.54) and (2.55), will always produce a positive value of SIF, thus preventing the determination of a meaningful equivalent SIF range. Due to these difficulties, in the following analyses, equivalent SIF for 90° OP loading conditions is considered to be the maximum of either the mode I or mode II SIF range acting on the crack growth plane.

Mode I SIF values for a potential branch crack were based on the normal stress component, as calculated from the nominally applied stresses, acting on the maximum principal stress plane at the location of the crack tip. Nominal stress values were used, as opposed to effective values, because the effects of frictional attenuation were assumed to be small on the mode I opening of the branch crack tip. To account for differences in effective crack length due to plane orientation, the overall crack length was projected onto the maximum principal plane prior to calculating the mode I SIF. Only the tensile portion of the normal stress cycle was considered for the calculation of SIF range, as the crack is assumed to be closed under compression. The mode I geometry factor function was obtained, via the XFEM technique, assuming the same crack geometry and aspect ratio as the shear-mode crack. Although there is likely some degree of error in these
calculations due to the difference in profile between the projected shear-mode crack and an ideal mode I crack, this effect has not yet been evaluated.

Model predictions for crack branching were compared to experimental results for each un-notched constant amplitude fatigue test for which appropriate data were available. Crack branching lengths were determined by plotting the variation of effective mode I, effective mode II, effective equivalent (mixed-mode), and mode I branch SIF with crack length. The intersections of the mode I branch and effective equivalent SIF traces indicate crack lengths where crack growth is predicted to transition from the maximum shear plane to the maximum principal/tensile plane, or vice versa. Example plots showing the variation of each SIF component with crack length are given in Figure 7.17(a) and 7.17(b) for pure torsion and torsion with static tension loadings, respectively.

A summary of experimental and predicted crack branching lengths for all loading conditions is given in Table 7.3. From these results, it can be noted that the predicted crack length at the beginning of branching is within a factor of 1.75 of the experimentally measured value for nearly all loadings cases considered. It is also worth noting that a further increase in applied loading would have resulted in the correct prediction of a non-branching crack condition. Additionally, a transition back to shear-mode growth after a period of mode I crack growth, which was observed in experiments, was also predicted in these analyses. In the latter case, however, quantitative predictions for branch ending length are not as accurate as those for branch starting lengths, although nearly all are still predicted within a factor of 2.5. This could be due to the potential for a mode I branch to cause significant deviation from the idealized shear-mode crack path assumed by the
model. Therefore, the longer a crack grows in mode I, the less accurate the model predictions are expected to be.

To provide another means for evaluating these branching predictions, experimental crack paths, from surface replicas, were superimposed on top of predicted crack paths. These comparisons are shown in Figures 7.18 to 7.21 for pure torsion, torsion with static axial stress, in-phase axial-torsion, and axial and 90° OP loading conditions, respectively. From these figures, it can be seen that the model is able to reproduce the experimentally observed crack path relatively well for all loading conditions considered. This is true both when crack branching was present, as well as in cases where either mode I or mode II/mixed-mode growth dominated the entire crack growth life.

Finally, plots showing the variation of experimental and predicted crack branch starting and ending lengths versus applied shear stress amplitude (on the maximum shear plane) are presented in Figure 7.22(a) for pure torsion and torsion with static compression loading conditions, and in Figure 7.22(b) for in-phase axial-torsion loading. By studying this figure, it can be seen that this type of analysis is able to reflect the trend observed in Figure 7.8 of an increase in initial mode II crack length before branching with increasing loading level. Similarly, a trend of decreasing crack branch ending length with increasing loading level is also correctly predicted.

7.2.4.2 Crack Growth Life Predictions

In addition to crack branching predictions, experimentally measured crack growth lives were also compared to predicted growth lives based on two different ranges of crack length. These include the life from a half crack length of 0.1 mm to failure, and from 0.2
mm to failure. The goal of computing crack growth life from two different initial crack lengths was to evaluate the ability of the proposed model to accurately reflect crack growth behavior for small cracks, where the effects of frictional attenuation are greatest. Along with predictions based on the output of the proposed effective SIF model, life predictions were also performed based on equivalent SIF values computed from nominally applied stresses. For all nominal stress-based life predictions, mixed-mode crack growth was assumed to occur on planes of maximum shear for all crack lengths and loading conditions considered. Axial and 90° OP life predictions were not performed, however, because the proposed model predicts mode I crack growth for the entire growth life under these loading conditions. Since the model has not been adapted to accommodate mode I-dominated growth, it would not be expected to provide any improvements in life prediction for these cases.

In order to predict crack growth life, an appropriate $dC/dN$ versus $\Delta K$ curve must first be obtained. Since the proposed model was formulated to account for the effects of friction and crack closure on small crack growth, the selected curve should be representative of crack growth rates in the absence of closure effects. Also, since an equivalent SIF parameter was used to compute mixed-mode crack driving force in these analyses, a crack growth curve generated under uniaxial loading conditions was acceptable.

The crack growth curve used for life prediction was input in the form of a single Paris equation (Equation (2.49)), without considering a threshold region for crack growth (nearly all SIF values considered were above the closure free threshold region). To determine “closure free” crack growth properties, 2024-T3 crack growth data were first
collected from the Federal Aviation Administration Fatigue Crack Growth Database (FCGD) (Williams, 2004). This database is a collection of crack growth data compiled from various sources for different materials. Along with the data, fits are also provided for both the Walker equation, which accounts for mean stress effects on crack growth by modifying the Paris equation coefficient, and for the NASGRO sigmoidal growth curve.

Since the effects of crack closure generally diminish as the $R$ ratio is increased in uniaxial crack growth tests, a “closure free” Paris equation was generated by evaluating the Walker equation provided in the FCGD at an $R$ ratio of 0.8. However, since mean stress can also affect crack growth rates independent of crack closure, the resulting Paris equation was also compared to closure corrected crack growth data reported by Newman et al. (1999). These closure corrected data, shown in Figure 7.23(a), were generated under $R$ ratios ranging from −2 to 0.7 and are effectively collapsed into a single crack growth curve when crack opening stress levels are accounted for in SIF calculation. A Paris equation fit representing this curve is shown in Figure 7.23(b), along with a comparison to the Paris equation derived from the FCGD Walker equation at $R = 0.8$.

From this figure, it can be concluded that the FCGD Walker equation-based growth curve is a reasonable approximation of closure free crack growth conditions for this material. As such, the resulting Walker fit Paris equation ($C = 2.33(10^{-10})$, $m = 3.273$) was used when computing fatigue life predictions from the proposed effective SIF model output. To give an idea of the effect of closure on crack growth rates, a Paris equation fit for crack growth rate data generated under $R = 0$ conditions (Newman Jr. and Edwards, 1988) is also included in Figure 7.23(b). This $R = 0$ fit ($C = 4.39(10^{-11})$, $m = 3.282$) was used for all fatigue life predictions based on nominally applied SIF ranges.
Units for all Paris equation fits are m/cycle and MPa(m). To compute crack growth life, discrete integration of the Paris equation was carried out based on 0.050 mm crack growth increments. Again, a summary of the parameters used in these analyses can be found in Table 7.2.

Predicted and experimental crack growth lives for each crack length range considered are listed in Table 7.3. Additionally, results are shown graphically in Figures 7.24(a) and 7.25(a), for lives from 0.1 mm to failure, and in Figures 7.24(b) and 7.25(b) for lives from 0.2 mm to failure. Figure 7.24 compares the ratio of predicted to experimental crack growth life, based on both nominally applied SIF ranges and friction reduced effective SIF ranges, for each loading condition considered. Figure 7.25, on the other hand, is a more general comparison between nominal and model-based crack growth analyses which shows the accuracy of predictions with respect to crack growth life. It is clear from these figures that significant improvements in life predictions were obtained when accounting for the effects of crack face interaction on crack growth. This is especially true for the case of torsion with static tension stress, where nominal life predictions were consistently non-conservative by up to a factor of 10.

For the 0.1 mm to failure analyses, only 30% of crack growth lives were predicted within a factor of ±2 of experimental results based on nominally applied SIF ranges. This only improved to 36% of nominal predictions within a factor of ±2 for the 0.2 mm to failure analyses. For effective SIF ranges computed from the proposed model, however, 60% and 73% of predicted crack growth lives were within a factor of ±2 of experimental results for the 0.1 mm to failure and 0.2 mm to failure comparisons, respectively. Furthermore, all model predictions were within a factor of ±3 of experimental results for
the case of crack growth from 0.2 mm to failure. The fact that life prediction errors for the 0.1 mm to failure and 0.2 mm to failure crack growth ranges are similar suggests that the fraction of fatigue life spent in growing cracks from 0.1–0.2 mm was predicted well by the model. This is an important observation because not only do crack face interaction effects have the largest impact when cracks are small, but the majority of crack growth life is spent in small crack growth as well.

Although a trend of increasingly non-conservative life predictions is observed in Figures 7.24 and 7.25 as loading levels increase (loading levels in Figure 7.24 increase from left to right for each loading path), this trend exists for predictions based on both nominal and effective SIF values. This suggests that this behavior is likely not due to deficiencies in the formulation of the proposed model. Instead, due to the high loading levels applied in these un-notched specimen tests, this may be the result of large scale plasticity and/or general yielding conditions brought about by nominal stresses near or exceeding the material yield strength. Consequently, the assumptions for LEFM crack growth would be violated in these situations, and a corresponding underestimation of crack driving forces would be expected. For reference, the ratio of nominal von Mises equivalent stress to yield stress for each loading condition considered is listed in Table 7.3, along with the respective crack growth life predictions.

7.2.4.3 Crack Growth Rate Correlations

Finally, from experimental data for crack length versus number of cycles (c vs. N), crack growth rates were computed for all un-notched specimen constant amplitude fatigue tests for which sufficient crack growth data were available. These crack growth rates were then plotted against both nominally computed and friction reduced equivalent
SIF ranges to observe differences in data correlation trends between different loading conditions. Crack growth rates were computed using a three point polynomial reduction technique (outlined in ASTM standard E 647 (ASTM Standard E647-05, 2009)) so as to provide some level of smoothing of the experimental \( c \) vs. \( N \) results while eliminating as few data points as possible.

Crack growth rate correlations for pure torsion and torsion with static axial stress tests are shown in Figure 7.26(a) based on nominally applied equivalent SIF ranges, and in Figure 7.26(b) based on effective equivalent SIF ranges computed using the proposed model. As mentioned in the previous section, however, for some of the higher applied loading levels used in this study, nominal equivalent stress values were very near to, or above, the material yield strength. Since these conditions violate the assumptions of LEFM, some degree of error in growth rate predictions should be expected. Therefore, only the data in Figure 7.26 from tests performed at nominal equivalent stress levels below the material yield strength are re-plotted in Figure 7.27. This plot presents a better representation of crack growth rate correlations in the LEFM applicable growth regime. Similar to Figures 7.26 and 7.27, Figures 7.28 and 7.29 compare crack growth rate correlations based on both nominal and effective SIF ranges computed for all loading conditions considered. Figure 7.28 shows results for all tests, while Figure 7.29 excludes data from tests performed at nominal equivalent stress levels at or above the material yield strength. Any data violating cyclic plastic zone size versus crack length conditions for LEFM (crack length should be at least four times the plastic zone size) are also excluded from all figures.
Although a fair amount of scatter exists in these crack growth rate correlations, this is somewhat expected in un-notched specimen tests due to microstructural influences and/or crack interaction effects (e.g. shielding) on small crack growth. These effects can be random in nature and result in periods of accelerated or retarded growth as a crack extends through a region affected by such factors. The scatter for any given loading condition (e.g. all torsion tests or all in-phase axial-torsion tests) can be considered indicative of that resulting from these effects and should be kept in mind, in relation to the overall scatter between different loading conditions, when evaluating crack growth rate correlations.

Additionally, in order to evaluate the accuracy of the various correlations, the Paris equation fits discussed in the previous section for mode I long crack growth data were also included in these figures. Again, since the proposed effective SIF model was formulated to account for the effects of friction and closure on small crack growth, the curves shown in Figures 7.26(b) to 7.29(b) are those based on closure corrected crack growth data reported in (Newman Jr. et al., 1999), and as estimated from the Walker equation fit reported in (Williams, 2004) at $R = 0.8$. Similarly, for crack growth rate correlations based on nominally applied equivalent SIF ranges, shown in Figures 7.26(a) to 7.29(a), the Paris equation fit for crack growth rate data generated under $R = 0$ loading conditions in (Newman Jr. and Edwards, 1988) is included with the results.

It can be seen from these figures that the model predicts a significant effect from frictional attenuation on the effective SIF values. This is especially true for pure torsion (ST) and torsion with static compression (STSA (-)) loading conditions, shown in Figures 7.26 and 7.27, where there is no normal stress on the maximum shear plane. For torsion
loading, the effective SIF decreases by around a factor of 3 at the lower crack growth rates and begin to merge with the nominal values at around $10^{-7}$ m/cycle. Reductions in effective SIF of a similar order were measured experimentally in (Tong et al., 1995b; Vaziri and Nayeb-Hashemi, 2005). A lesser, but still significant, frictional effect is predicted for torsion with static tension (STSA (+)) and in-phase axial-torsion (SC (IP)) loading conditions.

Overall, it is clear from these figures that a substantial improvement in growth rate correlations between the various loading conditions is obtained when using the proposed model to calculate effective SIF ranges, rather than using nominally computed SIFs. This is especially evident at lower crack growth rates, where the largest effects from frictional attenuation are experienced. Even for axial and 90° OP loadings, where mode I growth (under nominally applied stresses) is predicted for the entire crack growth life, SIF ranges agree with those for computed for shear-mode cracks. Furthermore, all growth rates correlations are found agree fairly well with the “closure free” Paris equation derived for the testing material.

At higher SIF ranges, however, although still below the material fracture toughness, experimental growth rates tend to be higher than predicted. This is true for both nominal and model based SIF ranges (model predictions converge to nominal values at higher SIF ranges) and the reasons for such behavior are currently not clear. One potential cause could be a lack of recorded crack growth data, due to rapid crack growth, towards the end of these specimens’ fatigue lives, which would result in the calculation of higher than actual growth rates. Additionally, increasing net section stresses/plasticity in the specimen gage section could have led to a violation of LEFM assumptions as the
cracks extended, which would result in an underestimation of crack driving force. Although LEFM validity checks were performed during these crack growth analyses, computing net section stress for mixed-mode loadings and cracks inclined to the applied loading direction is not a straightforward task. As a result, these checks are only an approximation of the actual stress state within the specimen.

Finally, although the semi-elliptic crack aspect ratio assumed in these analyses does not result in through-thickness cracks for any of the LEFM applicable crack growth data presented, there is the possibility for this assumption to be incorrect. A larger aspect ratio would result in through-thickness cracks developing at smaller than predicted surface crack lengths. Therefore, if larger aspect ratio cracks were present in these experiments, higher SIF ranges, corresponding to through-thickness cracks, could be expected at higher growth rates. If accounted for in the crack growth analyses, this would shift the higher growth rate SIF predictions to be more in line with their expected values.

### 7.2.5 Model Predictions and Comparisons with Experimental Results: AISI 1050 Medium Carbon Steel

In order further test the general applicability of the proposed effective SIF model, literature crack growth data from a medium carbon steel (Shamsaei, 2010) were also analyzed in this study. The steel, SAE/AISI 1050, was tested in the normalized condition with a yield strength (0.2% offset) of 421 MPa, an ultimate tensile strength of 709 MPa, and a Brinell hardness of 198. Experimental crack paths were reported for thin-walled tubular specimens tested under three different in-phase axial-torsion loading levels.

For the required inputs to the model, a grain size of 0.025 mm was estimated based on microstructure images, and a coefficient of friction of 0.55 was selected based
on values reported in (Oberg et al., 2000). Additionally, an average undeformed asperity angle of 38° was determined from 10 measurements taken from cellulose acetate replicas of small shear cracks growing in the material. Finally, a plane strain fracture toughness of 60 MPa√(m) was estimated based on values reported for similar materials, and $\beta = 4$ was assumed to be consistent with the 2024-T3 aluminum alloy.

Crack branching predictions for the 1050 steel were performed using the same procedures and SIF geometry factor solutions discussed in the previous sections for the 2024-T3 specimens tested in this study. Experimental crack paths, from surface replications, are superimposed on top of predicted crack paths in Figure 7.30. From this figure, it can be seen that the model is again able to reproduce the experimentally observed crack branching trends for this carbon steel. Similar to the 2024-T3 alloy, an increasing amount of shear-mode crack growth before branching is correctly predicted by the model with an increase in loading level. Furthermore, quantitative predictions for crack branch starting length agree well with the experimental observations. For branched cracks transitioning back to shear-mode growth, the same trend of decreasing branch ending length with increasing loading level was also observed, and is predicted correctly by the model. Branch ending length, however, is typically overpredicted (similar to the in-phase prediction results for 2024-T3). Again, this could potentially be attributed to a violation of model assumptions as a crack experiences increasing periods of mode I growth.

At the highest applied loading level, where the maximum von Mises equivalent stress exceeds the material yield strength by 19%, no clear branching was observed for the failure crack. This is in contrast to the model predictions, which indicate a transition
from shear-mode to mode I, followed by a transition back to shear-mode. For this test, however, the development of dense microcrack networks is clearly visible in crack replicas, as shown in Figure 7.31. Due to the presence of these crack networks, the experimental crack path was likely influenced by phenomena such as microcrack interaction and/or coalescence. Therefore, a deviation from the model prediction, in this case, is not necessarily evidence of deficiencies in the model’s theoretical framework.

The results presented in this section are promising because they demonstrate the ability of the proposed friction and roughness induced crack closure model to predict experimentally observed trends in shear-mode crack growth for materials other than the 2024-T3 aluminum alloy tested in the current study. However, because this model is new, further investigation is still needed, for various materials and loading conditions, to verify its robustness and general applicability.

7.3 Notched Specimen Crack Growth

As mentioned previously, crack growth in the notched specimens tested in this study, for all relevant crack lengths beyond initiation, was observed to be in mode I on planes of maximum principal stress. This is true regardless of the nominally applied loading conditions. Figures 7.32, 7.33, and 7.34 show the final crack paths observed for all notched specimens tested under axial, torsion, and combined axial-torsion (both in-phase and 90° OP) loading conditions, respectively. Similarly, crack paths for all plate specimens, tested under axial loading conditions, are shown in Figure 7.35. For reference, maximum principal stress planes are shown as dotted lines in all figures.
When collecting crack growth data from notched specimen fatigue tests, image capture intervals were chosen so that approximately 40 to 50 crack length measurements were recorded during each test. For tubular specimen tests, after crack lengths were measured from each image, they were input into a curvature correction program to account for the 2D image’s inability to capture the 3D nature of the actual crack path. A description of the curvature correction program is included in Appendix A. The resulting crack growth curves, in terms of curvature corrected surface half crack length (half of the tip-to-tip crack length) versus cycles from crack initiation, $N-N_i$, are shown in Figures 7.36(a), 7.36(b), and 7.36(c) for axial, pure torsion, and combined axial-torsion (in-phase and 90° OP) loading of the notched tubular specimens, respectively.

Because crack initiation lives based on the 0.2 mm crack initiation definition were not available for all specimens, initiation in these crack growth plots refers to the first cycle at which a crack length was recorded, regardless of the length of the crack. In the majority of cases, however, initial half crack lengths ranged between 1.62 mm and 1.75 mm (including the 1.6 mm hole radius), and the 0.2 mm crack initiation life was reached shortly thereafter. For crack growth data under pure torsion loading, where multiple cracks were observed to initiate and grow from the hole, individual cracks are denoted in each plot by either Q1,3 or Q2,4. The Q1,3 designation refers to cracks growing in quadrants 1 and 3 (from the lower left and upper right regions of the hole), while the Q2,4 designation refers to cracks growing in quadrants 2 and 4 (from the upper left and lower right regions of the hole). For combined loading plots, in-phase and 90° OP loading conditions are designated by (IP) and (OP), respectively.
The same crack growth data, plotted in terms of half crack length versus normalized cycles to failure, are shown in Figures 7.37(a), 7.37(b), and 7.37(c) for axial, pure torsion, and combined axial-torsion loading, respectively. Similarly, Figures 7.38(a) and 7.38(b) show crack growth data for the notched plate specimens in terms of half crack length versus cycles from crack initiation and half crack length versus normalized cycles to failure, respectively. While the data points in these figures correspond to measured crack lengths, the lines for each set of data correspond to crack length fits smoothed according to the ASTM seven point polynomial reduction technique (ASTM Standard E647-05, 2009). Overall, it is clear from these figures that both crack initiation and crack growth make up significant portions of the total fatigue life of each specimen. This is true regardless of the nominally applied loading condition and/or stress level, and emphasizes the importance of considering crack growth behavior as part of a complete fatigue life analysis.

Unlike the shear-dominated crack growth observed for un-notched specimens, mode I crack growth is not largely affected by frictional attenuation and/or the mechanical interlocking of crack face asperities. This is due to the lack of sliding displacement between opposing crack faces. Therefore, roughness induced closure effects are not expected to play a significant role in the crack growth analyses performed for the notched specimens tested in this study. The effects of notches and multiaxial nominal stress states on mode I growth, on the other hand, can have a large impact on crack growth predictions. As a result, these topics will be investigated in the following sections.
7.3.1 Crack Growth Rate Correlations

The first crack growth rate correlations performed were for the notched plate specimens tested in this study. In calculating the nominal mode I SIF range for each test, the ASTM solution for a center through crack in a finite width flat plate (MT) geometry (Equation (7.3)) was used. The half crack length used in the computation was considered to be half of the tip-to-tip crack length for any given crack measurement, which includes the diameter of the hole. Crack growth rates were calculated from crack length data smoothed using the seven point polynomial technique. Only the tensile portion of the loading cycle was considered in calculating SIF range, as cracks were assumed to be closed under compression. Calculations were also performed during each analysis, per (Stephens et al., 2000), to ensure that all crack growth data included in subsequent figures do not violate LEFM assumptions regarding plastic zone size versus crack length (\( r_y < c/4 \)), plastic zone size versus uncracked ligament length (\( r_y < (W-c)/8 \)), and net section yielding (\( S_{net} < 0.8\sigma_y \)). The resulting crack growth rate correlations are shown in Figure 7.39 for the notched plate specimen tests.

Also included in this figure are Paris equation fits from uniaxial crack growth data reported in literature for the same aluminum alloy. These fits are included for a number of reasons. Not only do they help to evaluate the accuracy of SIF solutions based on different specimen geometry assumptions (relevant to subsequent analyses), but they also serve as useful reference lines when comparing crack growth rate correlations between the different specimen geometries and loading conditions investigated in this study.

Three different Paris equation fits are included in Figure 7.39. The first, based on data reported by Hudson in NASA TN D-5390 (1969), can be described by Paris
equation constants $C = 3.18(10^{-12})$ and $m = 4.139$ and corresponds to crack growth data obtained from 305 mm wide by 2.3 mm thick MT specimens tested in the LT orientation under $R = 0$ loading conditions. The next fit ($C = 4.39(10^{-11}), m = 3.282$), based on data reported by Newman Jr. and Edwards in AGARD R-732 (1988), corresponds to crack growth data obtained from 2.3 mm thick MT specimens, of varying widths, tested in the LT orientation under $R = 0$ loading conditions. The final curve ($C = 5.72(10^{-13}), m = 4.566$), based on data reported by Illg and McEvily Jr. in NASA TN D-52 (1959), was fit to crack growth data obtained from 305 mm wide by 2.1 mm thick MT specimens tested in the LT orientation under $R$-ratios ranging from 0.02–0.05. Differences between these crack growth curves also help to give some indication of the amount of variability that might be expected in crack growth properties for the 2024-T3 testing material. A more comprehensive analysis of crack growth rate variability for this material can be found in (Oldersma and Wanhill, 1996).

In performing crack growth analyses for the notched tubular specimens tested in this study, a number of assumptions can be made concerning the idealized crack geometry. The simplest geometry that can be assumed is that of the MT specimen, with a width equal to the mean circumference of the tubular specimen. The MT assumption has a few drawbacks, however, including a failure to account for the increased bending moment induced in a tubular specimen as a mode I crack grows around its circumference. Although this can have a significant impact on resulting SIF values, the effect is initially small before gradually increasing with crack length. Additionally, for the same crack length, the effect of the induced bending moment will be smaller for a larger diameter specimen. Therefore, for relatively short cracks growing in a larger diameter specimen,
the assumption of the MT geometry may be a reasonable approximation of the tubular specimen SIF solution.

Although SIF solutions exist for various crack geometries in tubular specimens, they are typically meant for mode I crack growth under uniaxial loading conditions. Therefore, cracks are assumed to be either growing in the specimen circumferential direction, due to axial loading, or in the specimen longitudinal direction, due to hoop stresses from internal pressurization. For torsion and combined axial-torsion loadings, however, where mode I cracks may be growing in any orientation with respect to the curvature of the tube, these solutions are no longer accurate. For these situations, it is nearly impossible to find a readily available SIF solution for each specific crack orientation. Since the induced bending moment for a mode I crack growing in a tube decreases as the crack orientation becomes closer to the specimen longitudinal direction, however, it may be reasonable to assume the much simpler MT crack geometry in such cases. If the resulting error is within reason, the MT SIF solution can be applied to mode I growth regardless of crack orientation.

In order to test the validity of the MT crack geometry assumption, resulting uniaxial mode I SIF values were compared to those computed from a tubular specimen SIF solution. The tubular specimen solution, as reported in (Miedlar et al., 2000), was formulated by Forman et al. (1985) for a through crack growing around the circumference of a thin-walled tube (TT). A detailed description of this SIF solution is included in Appendix B (Figure B.1). The MT plate solution, on the other hand, was again calculated from Equation (7.3), with the specimen width set equal to the mean circumference of the tubular specimen.
A comparison between the two SIF solutions for the tubular specimen geometry investigated in this study is shown in Figure 7.40. The percentage error versus crack length is also included in the figure. From this comparison, it can be seen that the error in SIF resulting from the MT geometry assumption increases from around 1.4% to approximately 13% as the crack grows from a half crack length of 1.6 mm (hole radius) to a length 7.5 mm (considered failure for the notched specimens tested in this study). At a half crack length of 3.1 mm, the difference between the two solutions is 5%, and at 4.6 mm, the difference in 10%. Again, this error should tend to reduce for cracks growing inclined to the specimen circumferential direction. Therefore, for the majority of the crack growth life of the notched tubular specimens tested in this study, the maximum expected error in SIF range should be less than 10%.

The effect of this error on crack growth rate correlations can be observed by plotting experimental crack growth rates against mode I SIF range calculated using each SIF solution. The results for such comparisons are shown in Figure 7.41 for the MT geometry assumption, and in Figure 7.42 based on the TT geometry. In each figure, plots (a)-(c) show results for pure axial, pure torsion, and combined axial-torsion (in-phase and 90° OP) loading conditions, respectively. Similar to the plate specimen analyses, tubular specimen crack growth rates were calculated from crack length data smoothed using the seven point polynomial reduction technique, LEFM validity checks were performed, and only the tensile portion of the maximum principal stress variation was considered in calculating SIF range.

When comparing Figures 7.41 and 7.42, the results are as expected. SIF ranges computed using the tubular specimen solution are slightly larger than those based on the
MT assumption. Although the overall scatter of data from different specimens appears to be slightly reduced for any given loading condition based on the MT assumption, the tubular specimen solution produces SIF ranges which agree more closely with those reported in literature for a given crack growth rate. This is especially true for the axial loading condition, where the tubular specimen SIF solution directly reflects the experimentally observed cracking behavior. For the other loading conditions, however, where cracks inclined to the specimen circumferential direction were observed, actual SIF ranges are likely somewhere between those predicted based on the MT and TT geometry assumptions. Since this difference is relatively small, especially at lower crack growth rates, either assumption can be considered reasonable in these cases.

It should also be noted that crack growth rates for torsion loading tend to be higher than those for the other loading conditions, at any SIF range, regardless of crack geometry assumption. Similarly, while growth rates for combined loading conditions agree with those for axial loading at lower SIF ranges, they tend to merge with those for torsion loading as SIF increases. One possible reason for the higher growth rates observed under torsion and combined loading conditions, as discussed in Section 2.8.3.3, is the presence of a negative tangential stress (T-stress) component, acting parallel to the crack growth plane, which is not present under uniaxial nominal loadings. This T-stress has been shown to result in increased crack driving force and faster mode I crack growth for the same nominal SIF range. Additionally, the non-proportional nature of the 90° OP loading conditions introduces additional factors that can affect mode I crack growth rates as well. More discussion on these effects is included in Section 7.3.3.
One final comparison that can be made based on these analyses is between the crack growth rate correlations for the notched plate and notched tubular specimens tested in this study. By comparing Figures 7.39 and 7.42(a), crack growth rate trends for plate and tubular specimens tested under uniaxial loading conditions are found to be similar. This is especially true at low to intermediate growth rates. At higher growth rates, however, plate specimen SIF ranges tend to be slightly lower than those calculated for tubular specimens, although the difference is not large. The fact that tubular and plate specimen growth rate data agree fairly well both with each other, and with literature trends, is an important assessment. Not only does this help to verify the accuracy of the experimental crack length measurement techniques implemented, but it also lends credibility to the SIF solutions and calculation procedures used to determine mode I crack driving force in this study. Additionally, the fact that the literature data were generated under $R = 0$ loading conditions supports the idea that cracks are closed under remote compressive stress, and that only the tensile portion of the loading cycle need be considered in determining SIF range.

Overall, the amount of scatter present in any set of crack growth rate correlations is consistent with the variation in growth rate data reported in literature. While scatter is greater at low to intermediate crack growth rates, data from different specimens and loading levels tend to converge to a single narrow band as growth rates increase. Increased scatter at lower growth rates, however, could be due to factors such as initial crack geometry, small crack growth behavior, and/or notch effects, which can lack consistency in naturally initiated fatigue cracks such as the ones investigated in this study. More details on these factors are presented in the following section.
7.3.2 Notch Effects on Crack Growth

As discussed in Section 2.8.7, one of the primary effects of a notch in an engineering component is that the local stress distribution surrounding the notch is altered. Not only do notches produce a stress concentration effect, which can lead to localized regions of plastic deformation surrounding the notch, but they can also change the local stress state within a component. As a result, when a crack is growing through a notch affected zone, it will behave differently than if it were growing under the influence of the nominally applied stresses. Additionally, because notch geometries such as the hole used in this study result in multiple free surfaces at the fatigue critical location, there are multiple crack geometries that a newly initiated fatigue crack can assume.

While reasonable uniaxial crack growth rate correlations were obtained in most cases using the SIF solutions discussed in the previous section, the assumption of a through crack with the diameter of the hole included in the total crack length is typically only valid for longer cracks growing from a hole. When a crack is still small compared to the dimensions of the notch, however, this assumption does not accurately reflect the actual crack growth conditions. This is evident in Figures 7.39, 7.41, and 7.42 by the anomalous crack growth rate behavior (growth rate trends show a false threshold-like condition) observed for some specimens at lower SIF ranges/crack lengths.

In reality, cracks generally nucleate from a hole in one of two ways: a corner crack will develop at the intersection of the hole bore and the specimen surface, or one or more semi-elliptic surface cracks will develop along the bore of the hole before eventually growing into a through-thickness crack. However, in many situations, there is a chance that either type of crack may develop. For example, in a study on fatigue crack
initiation and growth in plate specimens of 2024-T3 aluminum alloy containing a central hole, Sova et al. (1976) used a four camera video system to monitor crack growth along both the surface and depth of both sides of the hole. Out of 42 tests, performed at $R$ ratios ranging from $-1$ to 0.5, cracks were found to initiate as corner cracks in 31 specimens, and as surface cracks along the bore of the hole for the remaining 11 specimens. For 10 of the 31 specimens, however, after the initiation of a corner crack, an additional surface crack initiated and grew from the hole as well. Therefore, choosing the correct crack geometry for a notched specimen crack growth analysis is not always a trivial task. For reference, Department of Defense JSSG-2006 (Joint Service Specification Guide: Aircraft Structures, 1998) recommends assuming an initial corner crack geometry ($a/c = 1$) for cracks growing from holes in metallic structures when the material thickness is greater than the assumed initial defect size (typically 1.27 mm). When material thickness is less than the initial defect size, however, a through-thickness crack assumption is recommended.

Aside from the geometry of short cracks growing from notches, accounting for notch effects such as stress concentration can also be an important consideration in a crack growth analysis. One of the simplest ways to account for elevated stresses in the vicinity of a notch is to employ an effective SIF model such as one discussed in Section 2.8.7.2. These models account for notch effects by modifying the remotely applied stress term by an appropriate stress concentration factor prior to SIF calculation. While Equations (2.68)-(2.70) consider a constant (maximum) $K_i$ value up until a crack length where notch effects are considered negligible, Equations (2.71)-(2.73) are able to account
for stress gradient effects moving away from a notch by considering an elastic notch-tip stress field distribution.

For this study, given the small amount of crack growth data recorded in the notch affected zone, it would be difficult to compare crack growth rate correlations for different crack growth models. Therefore, only a simple approximation was used to evaluate notch effects on crack growth. The approach considered is that proposed by Dowling (1979) and represented by Equations (2.68)-(2.70). To implement this model, the only additional information needed was the elastic stress concentration factor at the notch root for each nominal loading condition. Since crack growth occurred primarily in mode I for these notched specimen tests, the stress concentration factor was expressed in terms of maximum principal stress at the notch root divided by nominal maximum principal stress (used to calculate long crack SIFs). Given the slight variation in stress concentration along the depth of the hole for the specimens tested in this study (as discussed in Section 4.7), the $K_t$ values used in subsequent short crack growth analyses were taken to be the averaged values along the depth of the hole. In the case of this study, notch depth, $D$, is equal to the radius of the hole, $r$.

Since crack growth was only measured on the outer surface of the specimens tested in this study, if a crack initiated as either an inside corner crack or a semi-elliptic surface crack along the bore of the hole, its length would have only been recorded after it grew through the thickness of the specimen. Therefore, for the crack growth analyses performed in this study, it is reasonable to assume that while cracks were short, their geometry was either that of a through-thickness crack, or a corner crack growing from the outer edge of the hole. Additionally, since cracks were observed to grow from both sides
of the hole in all cases, the assumption of diametrically opposite symmetric through or corner cracks was considered to be reasonable.

While Equation (2.68) is intended for use with through-thickness cracks growing from a notch, the corner crack geometry was also considered in this study by using an appropriate geometry factor solution, \( Y_i \), in place of the free surface correction factor, 1.12, in the equation. This geometry factor solution, taken from (Anderson, 2005), was formulated for a quarter-elliptic corner crack in a finite width plate. While the geometry factor was recalculated for each crack length considered, the crack aspect ratio, \( a/c \), was assumed to remain constant at 1 for the sake of simplicity. Although this solution was derived for plate specimens, for the small crack lengths relevant to the corner crack geometry, the curvature of the tubular specimen was assumed to have a negligible effect on the resulting SIF values.

For both the through crack and corner crack assumptions, SIF ranges were calculated using both the notch affected short crack and long crack equations for each experimentally measured crack length. Then, when plotting the resulting SIF ranges against crack growth rate for a given test, the short crack solution was used up until a critical crack length was reached. This crack length corresponds to the conditions where the short and long crack solutions intersect, and was reached when cracks in tubular specimens had grown approximately 0.15 mm and 0.40 mm from the edge of the hole for the through crack and corner crack assumptions, respectively. Similarly, for plate specimens, critical crack lengths were around 0.25 mm and 0.67 mm from the edge of the hole for the through crack and corner crack assumptions, respectively. Once cracks
exceeded this length, SIF ranges were taken to be those corresponding to the long crack solution.

Results of the short-long crack growth rate correlations for notched tubular specimen tests are shown in Figures 7.43(a), 7.43(b), and 7.43(c) for axial, pure torsion, and combined axial-torsion (in-phase and 90° OP) loading conditions, respectively, based on the assumption of a short notch affected through crack transitioning to the TT long crack geometry. Similarly, results of crack growth rate correlations based on the assumption of a short corner crack transitioning to the TT long crack geometry are shown in Figures 7.44(a), 7.44(b), and 7.44(c) for axial, pure torsion, and combined axial-torsion loading, respectively. Finally, for the notched plate specimen tests, short-long crack growth rate correlations are shown in Figures 7.45(a) and 7.45(b) based on the assumptions of a notch affected through or corner crack, respectively, transitioning to the MT long crack geometry.

Based on these results, accounting for notch effects was found to have a notable impact on crack growth rate correlations. One of the most obvious improvements is that the false threshold-like behavior observed for some specimens (e.g. T24-NA, P3, P7, PM11) in Figures 7.39, and 7.42, was reduced or eliminated when accounting for notch effects. Depending on the initial crack geometry assumed, however, resulting SIF ranges were found to vary by as much as a factor of 1.5 at shorter crack lengths. This is an important observation because it highlights the significant role that initial crack geometry assumptions can have on the accuracy of crack growth predictions for naturally initiated fatigue cracks.
While the short corner crack assumption was found to produce better agreement with literature crack growth trends in some cases, e.g. T24-NA, the small through crack assumption produced the overall best growth rate correlations for both the tubular and plate specimen data generated in this study. This suggests that the cracks in these specimens likely either initiated as surface cracks along the bore of the hole, or as corner cracks on the inner surface of the specimen, before they grew through the specimen thickness and were detected by the camera system.

### 7.3.3 Multiaxial Stress State Effects on Mode I Crack Growth

In addition to notch effects, multiaxial nominal stress states can also cause mode I cracks to behave differently than similar cracks growing under uniaxial loading conditions. When a mode I crack is growing under uniaxial loading, the entire applied stress is acting perpendicular to the crack growth plane. For mode I cracks growing under multiaxial nominal stress states, however, although there may be no shear stress on the maximum principal stress plane, there is an additional stress component acting parallel to the crack growth plane. As discussed in Section 2.8.3.3, this stress, often referred to as tangential stress (T-stress), alters the stress state at the crack tip and can affect the resulting crack driving force.

As shown in the notched tubular specimen crack growth rate correlations presented thus far in this chapter, mode I crack growth rates for nominal torsion loading conditions tend to be higher than those for uniaxial nominal loading at the same applied SIF range. The same is also true for combined axial-torsion nominal loadings, but the increased growth rates were observed primarily at higher SIF ranges in these cases. In order to evaluate the effect of T-stress on constant amplitude crack growth rates, the
Brown and Miller (1985) model, represented by Equation (2.62), was used to adjust the crack growth rates calculated for tests performed under torsion and in-phase axial-torsion loading conditions. For torsion loading, the biaxial stress ratio, $A = \sigma_x/\sigma_y$, is equal to −1 while for in-phase loading, at the nominal stress ratio used in this study, the biaxial stress ratio is equal to −0.21. For the loading levels used in this study, this resulted in growth rate reduction factors ranging from 1.54 to 2.10 for torsion loading, and from 1.43 to 1.96 for in-phase axial-torsion loading. The resulting T-stress corrected crack growth rate correlations are shown in Figure 7.46. SIF ranges are the same as those presented in Figure 7.43 for the notch affected through crack to TT long crack geometry assumption.

By comparing Figure 7.46 to Figure 7.43, it is clear that correcting for the effects of T-stress on mode I crack growth under multiaxial nominal stress states resulted in improved crack growth rate correlations. For in-phase axial-torsion loading, applying Equation (2.62) effectively moved all crack growth rate data on top of the literature trend lines, where they agree very well with growth rates measured under uniaxial loading conditions. For nominal torsion loading on the other hand, while data were moved closer to the literature trend lines, growth rates still have the tendency to be higher than those corresponding to uniaxial loading at the same SIF range. The T-stress corrections did, however, result in improved crack growth rate correlations between data generated under different nominal torsion loading levels at the same applied SIF range.

In a study on crack growth behavior in similar notched tubular specimens, however, Gladskyi and Fatemi (2014) used FEA to show that in addition to T-stress, the existence of a second pair of cracks growing from the notch under nominal torsion loading will increase the crack tip stresses for the first crack pair. Therefore, since similar
cracking behavior was observed for the pure torsion tests performed in this study, it is likely that this type of crack interaction effect contributed, at least to some extent, in the higher mode I crack growth rates observed under nominal torsion loading conditions.

Although crack growth rates for the 90° OP tests were also observed to deviate from those corresponding to uniaxial loading at higher nominal SIF ranges, no T-stress corrections were performed for these data. This is because for the 90° OP tests, given the mode I-dominated growth observed for the other loading conditions, the SIF ranges used in growth rate correlations were taken to be the maximum range of mode I SIF experienced on any plane. This predicts crack growth on the 0° plane (perpendicular to the specimen axis), and agrees with experimental crack path observations when cracks were relatively short. On this plane, when the axial loading channel is at its peak (resulting in the maximum mode I SIF), the torsion loading channel is at zero, and there is no T-stress component present at the crack tip.

After cracks in the 90° OP tests experienced around a millimeter or so of growth from either side of the hole, however, Figure 7.34 shows that crack growth was observed to deviate from the 0° plane. This suggests that changes in crack growth mechanisms may have occurred as the cracks increased in length. As discussed in Section 7.2.4, because of the non-proportional nature of the 90° OP loading, both mode I and mode II SIF ranges exist on any given plane. Therefore, the change in crack path and the higher crack growth rates observed at longer crack lengths/higher SIF ranges for these tests may have been the result of mixed-mode crack growth conditions, which exist regardless of the crack orientation. Although mixed-mode crack growth was not accounted for in the 90° OP crack growth analyses performed in this study, the calculation of appropriate crack
driving forces under these conditions is not straightforward. Due to the phase difference between the loading modes, the maximum and minimum mode I and mode II SIFs do not occur simultaneously, which makes the calculation of an appropriate equivalent SIF range unclear.

Nonetheless, for the tests performed in this study, 90° OP crack growth appears to have been dominated by the mode I loading from the axial component of the applied stress when cracks were small. The fact that growth rate correlations for 90° OP tests agree well with those for axial loading at lower SIF ranges lends credibility to this statement. As the cracks grew, however, the shear stress component seems to have had an increasing influence on crack growth/path as well. Whether this was due to notch effects (e.g. differences in local and nominal stress state), mixed-mode crack growth, and/or other factors is currently unclear. It is clear, however, that at longer crack lengths and higher growth rates, mode I SIF range alone was unable to successfully relate 90° OP crack growth behavior to that observed under uniaxial nominal loading conditions.

While Figure 7.46 shows T-stress corrected crack growth rate correlations separately for axial, torsion, and combined axial-torsion loading conditions, the data from these plots were also combined in Figure 7.47. This allows for a more direct comparison of crack growth rate correlations between the different nominal loading conditions. Although the results presented in this section suggest that the effects of T-stress on mode I crack growth under multiaxial nominal stress states are significant, Figure 7.47 shows that the simple T-stress correction model used in this study was unable to collapse all of the tubular specimen crack growth data into a single narrow band. However, there is also the possibility for other factors, such as multiple crack interaction, mixed-mode crack
growth, and varying degrees of crack closure, to influence crack growth rates under different multiaxial loading conditions as well. Therefore, more advanced models are likely needed in order to fully account for the effects of multiaxial stress states on mode I crack growth. However, due to time constraints, such investigations were not performed in this study.

7.4 Summary and Conclusions

Since both crack initiation and crack growth were important aspects of this study, crack growth data recorded from the constant amplitude un-notched and notched specimen fatigue tests discussed in Chapter 5 were also analyzed in this chapter. Similar to observations reported in literature, crack growth in the notched specimens took place primarily in mode I, while cracks in un-notched specimens were observed to propagate on maximum shear planes, maximum tensile planes, or a combination of both.

However, despite the practical significance of understanding shear-mode crack growth and crack branching behavior, relatively little research is available regarding these topics. Since being able to accurately predict the orientation of a growing crack is an essential step in computing effective crack driving forces, the mechanisms that govern these phenomena were a topic of much interest in the un-notched specimen crack growth analyses performed this study. Additionally, particular attention was also paid to investigating the effects of notches and multiaxial nominal stress states on mode I crack growth in notched specimens.
Based on the experimental results and analysis presented for constant amplitude crack growth in un-notched specimens, some key findings and conclusions can be summarized as follows:

1) Crack initiation and growth in un-notched specimens was observed primarily on planes of maximum shear stress/strain. Under certain conditions, however, cracks were also observed to branch and grow in mode I for part of their growth life.

2) Microcrack networks and crack coalescence did not appear to have a significant effect on the experimentally observed crack paths for the un-notched specimen fatigue tests performed in this study, regardless of the applied loading level.

3) The preferred crack growth mode was shown to have a dependence on the applied shear stress magnitude as well as the stress normal to the crack plane. This indicates that friction and roughness induced crack closure effects play a significant role in the shear-mode crack growth process.

4) An increase in the initial mode II crack length before branching was observed with an increase in alternating shear stress. This indicates that the mode II driving force increased at a higher rate than the attenuation effect due to crack face interaction, which eventually led to a non-branching condition.

5) A simple model was proposed in an attempt to quantify the complex phenomena involved in crack growth attenuation due to friction and roughness induced closure effects. The model is based on the idea that crack face interaction reduces the effective mode II driving force by allowing a portion of the nominally applied loading to be transferred through a crack interface.
6) Crack path/branching predictions based on the proposed model were shown to agree relatively well, both qualitatively and quantitatively, with experimentally observed trends for all loading conditions considered in this study.

7) Predicted crack growth lives and the correlation of crack growth rate data for all multiaxial loading conditions investigated were significantly improved based on the friction reduced effective SIF ranges calculated using the proposed model, as compared to those based on nominally applied SIF ranges.

8) Crack branching predictions were also found to agree with experimental crack paths reported in literature for a carbon steel alloy subjected to in-phase axial-torsion loading.

9) While the proposed model shows promise in its ability to predict experimentally observed trends in shear-mode crack growth under different loading conditions, further investigation is still needed, for various materials and loading conditions, to verify its robustness and general applicability.

Similarly, based on the experimental results and analysis presented for constant amplitude crack growth in notched specimens, some key findings and conclusions can be summarized as follows:

10) Crack growth in the notched specimens, for all relevant crack lengths beyond initiation, was observed to take place in mode I on planes of maximum principal stress.

11) For mode I cracks growing inclined to the tubular specimen circumferential direction, the MT crack geometry was a reasonable assumption for computing SIF ranges, especially at smaller crack lengths.
12) Mode I crack growth rates for specimens subjected to multiaxial nominal stress states were observed to be higher than those corresponding to uniaxial loading conditions at the same SIF range.

13) Fully-reversed uniaxial crack growth rate trends for plate and tubular specimens were found to be similar both with each other, and with literature data. Since literature data were generated under R = 0 loading conditions, this supports the idea that cracks are closed under remote compressive stress, and that only the tensile portion of the loading cycle need be considered in determining SIF range.

14) Increased scatter in experimental data at lower crack growth rates is likely due to factors such as initial crack geometry, small crack growth behavior, and/or notch effects, which can lack consistency in naturally initiated fatigue cracks.

15) Using an effective SIF model to account for stress concentration and initial crack geometry effects on crack growth from notches resulted in improved crack growth rate correlations. Depending on the initial crack geometry assumption used, however, effective SIF ranges were found to vary by as much as a factor of 1.5 at shorter crack lengths.

16) For specimens subjected to in-phase axial-torsion nominal loading conditions, T-stress corrections effectively moved all crack growth rate data to where they agreed very well with growth rates measured under uniaxial loading conditions.

17) For crack growth data generated under nominal torsion loading conditions, while T-stress corrections improved correlations, growth rates still had the tendency to be higher than those corresponding to uniaxial loading at the same SIF range.
Some of this error, however, may be a result of interaction effects from multiple pairs of cracks growing from the hole.

18) Under 90° OP loading conditions, small crack growth seems to have been dominated by the mode I loading from the axial component of the applied stress. At longer crack lengths, however, mode I SIF range alone was unable to relate higher 90° OP growth rates to those observed under uniaxial nominal loading. This was likely a result of mixed-mode crack growth effects, which due to the non-proportional nature of the loading path, can occur even on the maximum principal stress plane.

- **Looking forward:**

For the un-notched specimen fatigue tests performed in this study, complex cracking behavior was observed with cracks growing in shear-mode, mode I, or a combination of both. These differences in crack growth behavior, as well as the crack branching phenomena observed in many specimens, were attributed to the effects of friction and roughness induced crack closure between opposing crack faces. As such, the preferred crack growth mode was shown to depend on crack length, applied stress amplitudes, and stress normal to the crack growth plane. However, since stress levels and nominally applied stress states can continuously change in a variable amplitude loading history, accurately predicting crack paths and crack driving forces under such conditions would be particularly challenging. This is especially true when considering the additional load history dependence of crack growth brought about under variable amplitude loading conditions.
Although crack growth in notched specimens was observed to be mode I-dominanted, factors such as initial crack geometry, notch effects, multiaxial nominal stress states, and loading non-proportionality were all found to have adverse effects on the correlation of the constant amplitude crack growth rate data generated in this study. Since any number of these factors can be present simultaneously in a variable amplitude loading history, in addition to general load sequence effects, modeling crack growth behavior under such conditions is a challenging task. In general, higher crack growth rates and shorter crack growth lives can be expected for variable amplitude fatigue tests performed under pure torsion and combined axial-torsion loading conditions, as compared to those for uniaxial loading. Additionally, since a larger portion of crack growth life can be spent in small crack growth under variable amplitude loading conditions, the effect of initial crack geometry assumptions could have a more significant impact on crack growth predictions in such cases.
Table 7.1  Loading conditions, crack orientation vs. crack length, and fatigue life (to crack lengths of approximately 15-20 mm) for un-notched specimen fatigue tests. All stresses are given in MPa and all crack lengths are given in mm.

<table>
<thead>
<tr>
<th>Specimen ID†</th>
<th>Nominal Loading (MPa)</th>
<th>On τ(_{\text{max}}) plane (MPa)</th>
<th>Crack Orientation vs. Length (mm)</th>
<th>(N_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\tau_0) (\sigma_0) (\sigma_m)</td>
<td>(\tau_0) (\sigma_{\text{max}})</td>
<td>Initiation</td>
<td>0.2 &lt; (c) &lt; 1</td>
</tr>
<tr>
<td>TM56-ST</td>
<td>248.3 0 0</td>
<td>248.3 0</td>
<td>(\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}})</td>
<td>736</td>
</tr>
<tr>
<td>TM63-SC</td>
<td>178.0 300.0 0</td>
<td>232.8 150.0</td>
<td>(\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}})</td>
<td>2009</td>
</tr>
<tr>
<td>TM100-STSA</td>
<td>187.9 0 92.0</td>
<td>187.8 92.0</td>
<td>(\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}})</td>
<td>6742</td>
</tr>
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<td>187.8 92.0</td>
<td>(\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}})</td>
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</tr>
<tr>
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<td>187.8 0 0</td>
<td>187.8 0</td>
<td>(\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}})</td>
<td>72397</td>
</tr>
<tr>
<td>TM58-ST</td>
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<td>187.8 0</td>
<td>(\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}})</td>
<td>68105</td>
</tr>
<tr>
<td>TM103-STSA</td>
<td>187.8 0 -92.0</td>
<td>187.8 0</td>
<td>(\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}})</td>
<td>15283</td>
</tr>
<tr>
<td>TM106-STSA</td>
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<td>187.8 0</td>
<td>(\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}})</td>
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</tr>
<tr>
<td>TM53-SC*</td>
<td>130.3 225.0 0</td>
<td>172.1 112.5</td>
<td>(\tau_{\text{max}}) (\sigma_1) (\tau_{\text{max}}) (\tau_{\text{max}})</td>
<td>25555</td>
</tr>
<tr>
<td>TM4-ST</td>
<td>168.2 0 0</td>
<td>168.2 0</td>
<td>(\tau_{\text{max}}) (\sigma_1) (\sigma_1) (\tau_{\text{max}})</td>
<td>180288</td>
</tr>
<tr>
<td>TM78-ST</td>
<td>150.1 0 0</td>
<td>150.1 0</td>
<td>(\tau_{\text{max}}) (\sigma_1) (\tau_{\text{max}}) (\tau_{\text{max}})</td>
<td>181299</td>
</tr>
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<td>TM96-STSA</td>
<td>140.3 0 150.0</td>
<td>140.3 150.0</td>
<td>(\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}}) (\tau_{\text{max}})</td>
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</tr>
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<td>TM99-STSA</td>
<td>140.3 0 150.0</td>
<td>140.3 150.0</td>
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</tr>
<tr>
<td>TM67-ST</td>
<td>140.3 0 0</td>
<td>140.3 0</td>
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<td>531716</td>
</tr>
<tr>
<td>TM97-STSA</td>
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<tr>
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<td>137.8 87.5</td>
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<tr>
<td>TM54-SC</td>
<td>90.3 156.0 0</td>
<td>119.3 78.0</td>
<td>(\sigma_1) (\sigma_1) (\sigma_1) (\sigma_1)</td>
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</tr>
<tr>
<td>TM62-SC</td>
<td>90.3 156.0 0</td>
<td>119.3 78.0</td>
<td>(\tau_{\text{max}}) (\sigma_1) (\sigma_1) (\tau_{\text{max}})</td>
<td>867295</td>
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</tbody>
</table>

†Suffix indicates loading condition as follows: ST (pure torsion), STSA (torsion with static axial stress), and SC (in-phase axial-torsion)

*Crack orientations refer to a secondary crack, not the failure crack
Table 7.2 Relevant parameters for un-notched crack growth analysis.

<table>
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<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2024-T3 Friction Model Parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Undeformed Asperity Angle</td>
<td>$a$</td>
<td>36</td>
<td>degrees</td>
</tr>
<tr>
<td>Average Grain Size</td>
<td>$l$</td>
<td>0.075</td>
<td>mm</td>
</tr>
<tr>
<td>Coefficient of Friction</td>
<td>$\mu$</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Load Level Influence Coefficient</td>
<td>$\beta$</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Plane Strain Fracture Toughness</td>
<td>$K_{IC}$</td>
<td>34</td>
<td>MPa$\sqrt{m}$</td>
</tr>
<tr>
<td>Semi-Elliptic Surface Crack ($a/c = 0.5$) Mode II Geometry Factor (at specimen surface)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_{II} = a_2 c^2 + a_1 c + a_0$</td>
<td>$a_0$</td>
<td>0.614</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a_1$</td>
<td>-0.029</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a_2$</td>
<td>0.016</td>
<td></td>
</tr>
<tr>
<td>Semi-Elliptic Surface Crack ($a/c = 0.5$) Mode I Geometry Factor (at specimen surface)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_I = b_2 c^2 + b_1 c + b_0$</td>
<td>$b_0$</td>
<td>0.538</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b_1$</td>
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</tr>
<tr>
<td></td>
<td>$b_2$</td>
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<td></td>
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<tr>
<td>Paris Equation Constants</td>
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<tr>
<td>FCGD Walker $R = 0.8$ Coefficient</td>
<td>$C$</td>
<td>$2.33(10^{-10})$</td>
<td>MPa$\sqrt{m}$ &amp; m/cycle</td>
</tr>
<tr>
<td>FCGD Walker $R = 0.8$ Exponent</td>
<td>$m$</td>
<td>3.273</td>
<td></td>
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<tr>
<td>Newman Jr. $R = 0$ Coefficient</td>
<td>$C$</td>
<td>$4.39(10^{-11})$</td>
<td>MPa$\sqrt{m}$ &amp; m/cycle</td>
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<tr>
<td>Newman Jr. $R = 0$ Exponent</td>
<td>$m$</td>
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Table 7.3  Crack branching and growth life predictions for un-notched constant amplitude fatigue tests.

<table>
<thead>
<tr>
<th>Specimen ID†</th>
<th>Nominal Loading (MPa)</th>
<th>Branch Starting Length, (mm)</th>
<th>Branch Ending Length, (mm)</th>
<th>Cycles from ( c = 0.1 ) mm to Failure</th>
<th>Cycles from ( c = 0.2 ) mm to Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \tau_0 )</td>
<td>( \sigma_a )</td>
<td>( \sigma_m )</td>
<td>( \sigma_{\text{min}}/\sigma_f )</td>
<td>Experiment</td>
</tr>
<tr>
<td>TM58-ST</td>
<td>187.9</td>
<td>0</td>
<td>0</td>
<td>0.99</td>
<td>n/a</td>
</tr>
<tr>
<td>TM1-ST*</td>
<td>187.8</td>
<td>0</td>
<td>0</td>
<td>0.99</td>
<td>0.235</td>
</tr>
<tr>
<td>TM4-ST</td>
<td>168.1</td>
<td>0</td>
<td>0</td>
<td>0.88</td>
<td>0.215</td>
</tr>
<tr>
<td>TM78-ST</td>
<td>150.1</td>
<td>0</td>
<td>0</td>
<td>0.79</td>
<td>0.080</td>
</tr>
<tr>
<td>TM67-ST</td>
<td>140.3</td>
<td>0</td>
<td>0</td>
<td>0.74</td>
<td>0.130</td>
</tr>
<tr>
<td>TM103-STSA</td>
<td>187.8</td>
<td>0</td>
<td>-92.0</td>
<td>1.03</td>
<td>n/a</td>
</tr>
<tr>
<td>TM106-STSA</td>
<td>187.9</td>
<td>0</td>
<td>-92.0</td>
<td>1.03</td>
<td>n/a</td>
</tr>
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<td>TM97-STSA</td>
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<td>-150.0</td>
<td>0.86</td>
<td>0.195</td>
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<td>TM100-STSA</td>
<td>187.8</td>
<td>0</td>
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<td>1.03</td>
<td>n/a</td>
</tr>
<tr>
<td>TM96-STSA</td>
<td>140.3</td>
<td>0</td>
<td>150.0</td>
<td>0.86</td>
<td>n/a</td>
</tr>
<tr>
<td>TM99-STSA</td>
<td>140.3</td>
<td>0</td>
<td>150.0</td>
<td>0.86</td>
<td>n/a</td>
</tr>
<tr>
<td>TM53-SC*(1)</td>
<td>130.3</td>
<td>225.0</td>
<td>0</td>
<td>0.95</td>
<td>0.210</td>
</tr>
<tr>
<td>TM53-SC*(2)</td>
<td>130.3</td>
<td>225.0</td>
<td>0</td>
<td>0.95</td>
<td>0.145</td>
</tr>
<tr>
<td>TM37-SC</td>
<td>106.4</td>
<td>175.0</td>
<td>0</td>
<td>0.77</td>
<td>0.155</td>
</tr>
<tr>
<td>TM54-SC</td>
<td>90.3</td>
<td>156.0</td>
<td>0</td>
<td>0.67</td>
<td>0.190</td>
</tr>
<tr>
<td>TM62-SC</td>
<td>90.3</td>
<td>156.0</td>
<td>0</td>
<td>0.67</td>
<td>0.190</td>
</tr>
</tbody>
</table>

†Suffix indicates loading condition as follows: ST (pure torsion), STSA (torsion with static axial stress), and SC (in-phase axial-torsion)

*Crack orientations refer to a secondary crack, not the failure crack
Figure 7.1  XFEM model for SIF calculation verification of SENT specimen in terms of (a) hex mesh, top surface shown is the crack plane, and (b) sample von Mises stress output for the 3 mm crack length.
Figure 7.2  Comparison of SIF geometry factor results across the specimen thickness for 2 mm thick and 20 mm thick SENT XFEM models and Equation (7.1) for (a) 3 mm crack length and (b) 10 mm crack length.
Figure 7.3 Comparison of SIF geometry factor versus crack length for the 20 mm thick SENT XFEM model, Equation (7.1), and Equation (7.2).
Figure 7.4  Comparison of SIF geometry factor results across the specimen thickness for 2 mm thick and 20 mm thick MT XFEM models and Equation (7.3) for (a) 1.5 mm half crack length and (b) 5 mm half crack length.
Figure 7.5  Comparison of SIF geometry factor versus crack length for the 20 mm thick MT XFEM model, Equation (7.3), and Equation (7.4).

Figure 7.6  Sectioned view of machined precracked notch. All dimensions are in mm.
Figure 7.7  Superimposed crack paths for precracked specimens (black) and smooth specimens (green) tested under identical loading conditions for (a) higher load level pure torsion, (b) lower load level pure torsion, (c) higher load level in-phase axial torsion, and (d) lower load level in-phase axial-torsion tests. Relative load level is indicated by the ratio of von Mises equivalent stress to yield strength.
Figure 7.8  Initial mode II crack length before mode I branching vs. shear stress amplitude for fully-reversed pure torsion tests. Vertical arrows indicate a non-branching condition.
Figure 7.9  Fatigue lives to various crack lengths for smooth specimens tested at shear stress amplitudes of (a) 140 MPa and (b) 188 MPa with and without static axial stresses. Specimens for which no crack growth data were available are shown as solid gray columns.
Figure 7.10  Final crack paths for selected smooth specimen tests under (a) torsion with static tensile stress, (b) torsion with static compressive stress, and (c) pure torsion loadings.
Figure 7.11  Schematic showing coordinate systems and nomenclature relevant to the proposed frictional attenuation model.
Figure 7.12  Function variation for (a) the first term and (b) the second term of the effective asperity angle formulation used in the crack friction model.
Figure 7.13 Shear mode crack face morphology in terms of (a) mostly planar crack growth up to a length of approximately a grain size, (b) the development of crack face asperities after growth over the length of a few grains, and (c) destruction of crack face asperities through plasticity and fretting (see fretting debris).

Figure 7.14 Surface plot showing the variation of effective asperity angle with normalized crack length and normalized SIF.
Figure 7.15  Sensitivity of friction model output to variations in (a) average undeformed asperity angle, (b) coefficient of friction, (c) \( \beta \) coefficient, and (d) average grain size. Computed for pure shear loading and baseline parameter values of: \( \alpha = 36^\circ \), \( \beta = 3 \), \( l = 0.075 \), and \( \mu = 0.8 \).
Figure 7.16  Crack growth rate versus equivalent SIF range computed using (a) Equation (2.54) and (b) Equation (2.55).
Figure 7.17 Proposed friction model SIF component outputs versus half crack length for (a) 150 MPa pure torsion loading and (b) 140 MPa torsion loading with 150 MPa superimposed static tension stress. Results are qualitatively similar for other loading levels.
Figure 7.18  Superimposed experimental and predicted crack paths for constant amplitude un-notched torsion tests.
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Figure 7.20  Superimposed experimental and predicted crack paths for constant amplitude un-notched in-phase axial-torsion tests.
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T22-NT (76 MPa)   T20-NT (91 MPa)   T21-NT (91 MPa)

T15-NT (108 MPa)
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Chapter 8

Variable Amplitude Crack Growth and Predictions

Since crack growth was monitored for all fatigue tests performed in this study, a significant amount of variable amplitude crack growth data were generated for the axial, torsion, and combined axial-torsion tests described in Chapter 6. While most of these data are for notched specimens, a few crack length histories were also recorded for un-notched specimens. Given the core objectives of this study, being able to predict how cracks will grow under these complex loading conditions was a topic of particular interest. Therefore, this chapter is focused on the analysis of these test results, including comparisons to predicted crack growth trends.

As discussed in Section 2.8.8, however, one of the key requirements for the application of fracture mechanics concepts in a fatigue crack growth analysis is that conditions of similitude should be retained. Similitude implies that for a particular value of a driving force parameter, the state of stress surrounding the tip of a crack is uniquely described by the value of that parameter. However, for variable amplitude loading conditions, load history dependence may alter the local stress state at a crack tip allowing the possibility for multiple crack growth rates to occur at the same nominal driving force value. This dependence has been attributed to a number of mechanisms including: crack
blunting, an increase in compressive residual stress in front of the crack tip, increased plasticity induced crack closure in the crack wake, crack deflection (i.e. increased roughness induced closure), and strain hardening effects. Although all of these mechanisms may contribute, in some degree, to the observed load history effects, residual stresses and changes in plasticity induced closure levels tend to be the more favored explanations.

Given the complexity of such an analysis, two state-of-the-art crack growth programs were used in order to compare the experimentally generated variable amplitude crack growth data to predictions based on fundamentally different crack growth models. These programs were UniGrow version 2014-02-09 (*UniGrow*, 2014) and FASTRAN version 5.42 (*FASTRAN*, 2013). As mentioned in Section 2.8.8, UniGrow is based on the idea that residual stress distributions, resulting from varying degrees of plastic deformation surrounding the crack tip, are responsible for causing load sequence effects in variable amplitude crack growth. FASTRAN, on the other hand, attributes these effects to plasticity induced closure and uses the distribution of residual deformations in the crack wake to compute the contact stresses and far-field opening stress level for a given crack configuration.

Section 8.1 begins this chapter by describing the material properties used as input to each crack growth model. Then, after discussing the crack geometry assumptions used in the models (Section 8.2.1), Section 8.2.3 presents the results of the notched specimen variable amplitude crack growth analyses. A limited number of constant amplitude crack growth analyses based on these models were also performed, and are presented in Section 8.2.2, in order to provide baseline results for evaluating the variable amplitude
predictions. The work presented in these sections can also be found published in (Gates and Fatemi, 2016j). Although similar crack growth analyses were not performed for the variable amplitude un-notched specimen tests, due to a violation of model assumptions, the experimental crack growth data collected from these tests are presented in Section 8.3.

8.1 Material Property Inputs

In order to predict crack growth using the aforementioned analysis software, appropriate material properties must first be selected as input. Due to differences in prediction methodologies, however, each software requires its own unique set of properties.

8.1.1 UniGrow Material Properties

As a starting point, UniGrow requires the input of several basic material properties such as Young’s modulus, Poisson’s ratio, yield strength, cyclic strength coefficient, and cyclic strain hardening exponent. However, additional model specific material properties are required as well. While universal crack growth curve (Equation (2.84)) constants $C$, $p$, and $m$ can be calculated from Equations (2.85)-(2.87), respectively, material block size, $\rho^*$, is fit to experimental constant amplitude crack growth data. While this can be done using data from a single nominally applied stress ratio, $R$, more accurate values can be obtained based on the correlation of data generated under a variety of $R$ ratios.

Because constant amplitude crack growth rate data for the specimens tested in this study were only generated under fully-reversed loading conditions and include a fair
amount of scatter due to notch effects and initial crack geometry, these data are not ideal for computing crack growth properties for the UniGrow software. As a result, the analyses performed in this study utilized material properties for a 2024-T351 aluminum alloy, which were already included in the UniGrow material library. The mechanical properties for the 2024-T351 are very similar to those measured experimentally for the 2024-T3 alloy used in this study. A comparison of these properties is included in Table 8.1, along with other relevant material properties required for the UniGrow model.

Crack growth properties for the 2024-T351 alloy were compiled from three different sources (Liu, 1998; Pang and Song, 1994; Wanhill, 1994) and were generated under $R$ ratios ranging from $-2$ to $0.7$. Using a $\rho^*$ value of $1.2535(10^{-5})$ m, these growth rate data are effectively collapsed into a single narrow band when plotted against the residual SIF-corrected two parameter crack driving force represented in Equation (2.84). However, the slope of this crack growth curve changes rather significantly with different ranges of crack growth rate. Therefore, instead of using constants $C$ and $m$ to describe a single crack growth rate curve, a four segment piecewise linear (in log-log scale) representation of the growth curve was used to more accurately represent the data. Each of these segments has its own values of $C$ and $m$ which were fit to the experimental data, rather than calculated using Equations (2.85) and (2.87), and were input into the program in a tabular format. A summary of crack growth properties is included in Table 8.1.

In order to verify that the crack growth properties for the 2024-T351 material included in the UniGrow library are similar to those for the 2024-T3 alloy tested in the current study, crack growth rate data generated under $R = 0$ and $R = -1$ loading conditions for the library material were compared, in terms of nominally applied SIF
range, to constant amplitude crack growth data generated for the notched tubular specimens tested in this study. This comparison is shown in Figure 8.1(a). For $R = -1$ loading conditions, only the tensile portion of the loading cycle was considered in calculating SIF range. From this figure, it is clear that while crack growth rates are fairly similar between the two materials, especially at higher SIF ranges, the material included in the UniGrow library tends to exhibit slightly higher growth rates at low to intermediate SIF ranges. However, a true comparison cannot be made between crack growth rates at the lower applied SIF ranges due to the possibility for notch effects and incorrect crack geometry assumptions to produce errors in SIF calculation for the tubular specimen tests performed in this study.

8.1.2 FASTRAN Material Properties

Similar to UniGrow, FASTRAN requires the input of several basic material properties such as Young’s modulus, yield strength, and ultimate tensile strength. However, several other software specific properties, including various constraint factors and fracture properties, are also required. For inputting crack growth rate data, either universal crack growth rate equations (i.e. FASTRAN equation or NASGRO equation), or a table of closure corrected effective SIF range, $\Delta K_{\text{eff}}$, versus crack growth rate can be used for this purpose.

In order to generate an effective crack growth rate curve for use in FASTRAN, constant amplitude crack growth data generated under several different $R$ ratios (preferably a wide range of ratios) are first analyzed using the closure model, along with initial assumptions for constraint factors. Once crack opening stresses and effective SIF ranges are computed, the correlation of data is checked to see if an adequate relationship
between $\Delta K_{\text{eff}}$ and crack growth rate is produced. Values for constraint factors are then varied, using a trial-and-error method, until the best correlation of data from different $R$ ratios is achieved. A subroutine is included with the FASTRAN software which helps to automate this process.

Since experimental crack growth data from tests performed at multiple $R$ ratios are again required to compute appropriate closure corrected crack growth rate curves, data for the 2024-T3 alloy tested in the current study could not be used in the FASTRAN analyses. Instead, all material properties required for the software were taken from literature. In a study by Newman Jr. et al. (1999), mechanical and effective crack growth properties for 2024-T3 were presented based on data taken from literature. Similar to the material used in UniGrow analyses, the mechanical properties presented by Newman Jr. et al. agree very well with those generated for the 2024-T3 aluminum alloy used in the current study. Again, a comparison of these properties is included in Table 8.1.

The data used by Newman Jr. et al. (1999) to generate the 2024-T3 effective crack growth rate curve were taken from three different sources. Included in these references is NASA TN D-5390 (Hudson, 1969), the data from which were shown in Section 7.3 to agree well with the constant amplitude growth rates generated in this study for notched specimens under uniaxial loading conditions. The remaining data were generated by Dubensky (1971) and Phillips (1988). In all, the effective growth rate curve was able to collapse crack growth rate data from several different $R$ ratios, ranging from $-2$ to 0.7, into a single narrow band. The resulting crack growth curve was then presented in terms of an effective (closure corrected) SIF range versus crack growth rate table. All mechanical properties, crack growth data, variable constraint factors (related to the
transition from flat to slant crack growth), and fracture properties, required as input to the FASTRAN model, are summarized in Table 8.1 as reported by Newman Jr. et al.

Once again, in order to verify that the crack growth properties used in the FASTRAN analyses are similar to those for the 2024-T3 alloy tested in the current study, crack growth data generated under $R = 0$ and $R = -1$ loading conditions for the literature material were compared, in terms of nominally applied SIF range, to the tubular specimen data generated in this study. This comparison is shown in Figure 8.1(b). Again, for $R = -1$ loading conditions, only the tensile portion of the loading cycle was considered in calculating SIF range. From this figure, all crack growth rate data are found to agree very well for all SIF ranges represented. Consequently, using the material data from literature is not expected to have a significant effect on the accuracy of the crack growth predictions presented in subsequent sections for either analysis program.

### 8.2 Notched Specimen Crack Growth

For all variable amplitude fatigue tests performed in this study, the details of which were described in Sections 6.1 and 6.2, crack growth was monitored and recorded whenever possible. For the notched specimen tests, this was done using the same digital microscope camera setup used for the constant amplitude notched specimen fatigue tests. The resulting crack growth data, in terms of surface crack length versus number of applied loading blocks, are included in the following sections along with comparisons to predicted crack growth curves based on both the UniGrow and FASTRAN crack growth models. Other experimental cracking observations, including crack path and fracture
surface pictures, are also included with these results. For a summary of the applied loading conditions, the reader is referred to Table 6.2.

### 8.2.1 Crack Geometry Assumptions

The SIF solutions used for the calculation of crack driving force in a crack growth analysis can have a significant impact on the resulting growth rate predictions. For example, it was shown in Section 7.3 for constant amplitude loading conditions that assumptions on specimen geometry, i.e. tubular versus plate, can result in up to a 15% difference in the computed SIF range for the tubular specimens tested in this study. Similarly, the assumption of initial crack geometry, i.e. corner versus through, was shown to result in differences in SIF range of up to a factor of 2. For a power law crack growth curve with an exponent 4, a 15% and factor of 2 difference in SIF range will result in predicted crack growth rates which vary by factors of 1.75 and 16, respectively.

Given the variety of crack profiles and growth planes that can exist in the tubular specimens tested in this study under various nominal loading conditions, a total of three different notched specimen crack geometries were assumed for the crack growth analyses performed in this chapter. These include the middle crack tension (MT) geometry, the circumferential through crack growing in a thin walled tubular specimen (TT), and two symmetric semi-elliptic corner cracks growing from a central hole in a finite plate subjected to remote tensile stress (CCH).

While SIF solutions for an MT specimen geometry are included in both the UniGrow and FASTRAN programs, solutions for a circumferential through crack in a tubular specimen and are not included in either. Additionally, while FASTRAN includes a solution for symmetric corner cracks growing from a hole, the solution for this
geometry included in UniGrow is not applicable for the specimen dimensions used in this study. However, both programs give the option of inputting custom user defined geometry factor solutions to be used in crack growth analyses. Both programs require the same two column input format for the geometry factors. The first column contains a list of crack lengths normalized by the specimen width ($c/W$) and the second column contains the geometry factor, in the form of $Y_i = K_i / (\sigma \sqrt{\pi c})$, corresponding to each normalized crack length. Therefore, solutions for the TT and CCH geometries were calculated for the specimen dimensions and crack length range relevant to this study, rearranged into the proper format, and used with the custom geometry option in both crack growth analysis programs. To be consistent with the MT geometry assumptions, the width of the specimen for all SIF solutions was considered to be the gross mean circumference of the tubular specimen gage section, which is approximately equal to 85 mm.

While the SIF solution for the TT crack geometry was discussed in Section 7.3 with respect to constant amplitude crack growth, the CCH solution will be explained here. In order to account for notch effects and differences in initial crack geometry, variable amplitude crack growth analyses were performed using a SIF solution for two symmetric diametrically opposite semi-elliptic corner cracks growing from a central hole in a finite plate subjected to remote tensile stress. This solution, taken from the United States Air Force Damage Tolerant Design Handbook (Gallagher et al., 1984), was formulated by Newman Jr. and Raju (1981) and can be used to calculate the SIF at any location along the crack front. Additionally, the specimen dimensions used in this study fall within the range of applicability of the solution. Although this solution was derived for plate specimens, for the small crack lengths relevant to the corner crack geometry, the
curvature of the tubular specimen is assumed to have a negligible effect on the resulting SIF values. A detailed description of this SIF solution is included in Appendix B (Figure B.2)

In computing the geometry factor solutions for the corner crack geometry, a crack aspect ratio ($a/c$) first needs to be assumed. The initial aspect ratio value chosen was 1, resulting in a surface crack length and depth both equal to 0.2 mm at the crack initiation conditions defined in this study. However, since the finite dimensions of the specimen result in a changing ratio of SIF values at the crack surface and depth locations as the crack grows, it would be incorrect to assume a constant aspect ratio as the crack front progresses across the thickness of the specimen. Therefore, a program was created to predict the progression of the crack front profile for the specific specimen geometry used in this study.

In order to compute the changing crack profile, SIF values resulting from a unit applied stress were first evaluated at the crack surface, $c$, and at its maximum depth, $a$, for the initial crack length conditions. Based on these values, crack growth rates were computed for both $da/dN$ and $dc/dN$. By assuming a small predefined crack growth increment, $da$, the number of cycles required to increase the crack length by this distance, $dN$, was calculated from $da/dN$. Then, using the same $dN$, crack growth increment $dc$ was determined from $dc/dN$. Crack growth increments were then added to the previous crack lengths to determine the new crack length in each direction. By repeating this procedure until the crack depth was equal to the thickness of the specimen, and recalculating the SIF solutions for each new set of crack lengths, the evolution of the crack profile was effectively estimated. Then, once the corner crack grew across the thickness of the
specimen, the crack geometry was assumed to immediately transition to that of a one-dimensional through-thickness crack. A schematic representing this idealized evolution of the crack front profile is shown in Figure 8.2.

It should be emphasized, however, that this is only an approximation of actual crack growth behavior. In reality, the evolution of a corner crack profile can vary for the same component geometry, especially under variable amplitude loading conditions, due to a number of factors. These include the approximate nature of SIF solutions, independent changes in crack growth behavior in each growth direction due to different degrees of retardation/acceleration effects, and the possibility of different crack growth properties in each direction due to anisotropy in the cracked material (Miedlar et al., 2000). However, since the effects of these variable are difficult to quantify and/or predict, the same crack profile evolution/SIF solution was considered for each analysis performed in this study.

Since the custom geometry factor inputs for the crack growth programs are one-dimensional, geometry factor solutions for the CCH geometry were calculated in terms of surface crack length $c$. Additionally, once the corner crack grows through the thickness of the specimen, the TT geometry was assumed for the remainder of the crack growth life. Since the surface half crack length used in the TT solution is assumed, in this study, to include the radius of the hole, normalized crack lengths for the CCH geometry were based on the hole radius plus the surface crack length in order to maintain consistency. Since the Newman Jr. and Raju SIF solution is based on crack lengths excluding the radius of the hole, SIF values calculated on the specimen surface were divided by $\sigma \sqrt{(\pi(r+c))}$ in order to compute the appropriate geometry factor values for use in the
variable amplitude crack growth analyses. From this point forward, surface half crack length, $c$, refers to the crack length, including the radius of the hole, for the variable amplitude notched specimen crack growth analyses performed in this study.

In order to help verify the calculated geometry factor solution based on the progression of the corner crack profile, a constant amplitude crack growth analysis was performed in FASTRAN using the included SIF solution for the same geometry. Knowing the applied stress level, the FASTRAN geometry factor solution was back calculated based on the SIF range and surface crack length output from the analysis. A comparison of the MT, TT, CCH, and back calculated FASTRAN CCH geometry factor solutions are shown in Figure 8.3. From this figure, it is clear that the FASTRAN CCH solution is very similar to the solution derived manually using the previously described procedure. Additionally, it can be noted that the FASTRAN solution produces significantly lower geometry factor values once the crack becomes through-thickness. This is due to the fact that FASTRAN assumes a plate specimen geometry rather than that of a tubular specimen. Therefore, using the custom SIF input option has the added benefit of allowing for a more correct representation of the crack geometry as the crack grows long.

Because stress intensity factors for short cracks are lower for the corner crack geometry, the predicted crack growth lives will be longer than those based on the TT crack geometry assumption. Since there is a possibility for actual cracks to assume either of these geometries, or a combination of both, producing crack growth predictions based on both TT and CCH SIF solutions should provide an upper and lower bounds for
experimentally observed crack growth curves, so long as the crack growth models are accurate.

While other crack geometries exist which could theoretically place crack growth curves outside of the bounds established based on the TT and CCH geometries, these are difficult to establish without first examining experimental crack growth data. For example, a single corner crack could initiate and grow on only one side of the hole for a while before another corner crack eventually initiates and grows on the opposite side of the hole. This type of behavior was observed for some of the tests performed in this study, but generally cannot be known prior to analyzing measured crack length data. Therefore, since this study is concerned with the ability to predict variable amplitude crack growth before a failure occurs, these types of geometries were not considered.

8.2.2 Constant Amplitude Baseline Crack Growth Comparisons

In order to establish some baseline crack growth prediction results, both the UniGrow and FASTRAN programs were used to analyze crack growth under simple constant amplitude loading conditions. These results are useful when interpreting the results of subsequent variable amplitude analyses. The first comparisons made, shown in Figure 8.4, were aimed at evaluating differences in predicted crack growth curves due to both the assumed crack geometry and the analysis model. The results presented in this figure are based on the tubular specimen geometry used in this study and a fully-reversed 130 MPa nominally applied axial stress amplitude. As expected, the TT geometry produces the shortest predicted crack growth life, followed by the MT and CCH geometries, respectively. While the predicted lives for the MT and TT geometries differ by around a factor of 1.35, based on life predictions for either analysis program, the CCH
geometry results in life predictions that are approximately 2.7 times longer than those for the TT geometry.

Aside from comparing life predictions for different crack geometry assumptions, life predictions between the two different crack growth analysis models can also be compared. From Figure 8.4, it is clear that UniGrow consistently produces more conservative crack growth predictions than FASTRAN. The difference is small, however, with the average life predicted for any given SIF solution being a factor of 1.1 times longer for the FASTRAN analyses. This difference is likely due to slight differences in crack growth properties for the two different materials used as input to either program. The higher growth rates for intermediate SIF ranges observed for the UniGrow material (as evidenced in Figure 8.1), as well as differences in the derivation of the universal crack growth curve for each program, could result in the slightly shorter crack growth lives predicted in the UniGrow analyses.

In order to evaluate the accuracy of the constant amplitude crack growth analyses, UniGrow and FASTRAN results are plotted along with experimental crack growth curves from tubular specimen fatigue tests in Figure 8.5. Predictions based on both TT and CCH geometries are included to provide the theoretical upper and lower bounds for experimental crack growth. Additionally, a similar comparison is provided in Figure 8.6 for constant amplitude plate specimen crack growth data based on the MT crack geometry. The tubular specimen data were generated under a fully-reversed nominal axial stress of 130 MPa, while the plate specimen data were generated for a 115 MPa fully-reversed nominal stress. Loading levels for these analyses were chosen based on stress levels at which experimental growth data were available from two duplicate tests. This
helps to demonstrate the variability inherent in experimentally measured crack growth behavior due to the influence of random variables. For other loading levels, plots showing experimental versus predicted crack growth life for all tubular specimen fatigue tests performed under constant amplitude axial loading conditions are included in Figure 8.7(a) for FASTRAN analyses, and in Figure 8.7(b) for UniGrow analyses. Results for both TT and CCH crack geometry assumptions are included.

From these figures, it is clear that both UniGrow and FASTRAN are capable of producing fairly accurate crack growth predictions under constant amplitude loading conditions. Although the growth predictions based on the TT crack geometry should theoretically provide a lower bounds for the experimental crack growth curves, for both the tubular specimen data in Figure 8.5 and plate specimen data in Figure 8.6, one experimental data set falls below the TT prediction. While this may suggest that the crack growth predictions have a slightly non-conservative tendency, Figure 8.7 shows that the majority of life predictions based on the TT geometry are conservative within a factor of 3 for both analysis programs. Crack growth life predictions based on the CCH crack geometry, on the other hand, are observed in Figure 8.7 to be mostly non-conservative within a factor of 3 for both FASTRAN and UniGrow analyses. The fact that most experimental crack growth lives fall between the TT and CCH predictions supports the idea that they can be used to represent the theoretical lower and upper bounds for crack growth life, respectively. Additionally, the fact that the majority of crack growth lives are predicted within a factor of ±3 regardless of the crack geometry assumption suggests that the material properties input from literature into each crack growth analysis program are
sufficiently accurate to model variable amplitude crack growth behavior for the specimens tested in this study.

8.2.3 Variable Amplitude Analysis Results and Discussion

Having established a baseline for evaluating crack growth predictions based on comparisons to constant amplitude crack growth data, both the UniGrow and FASTRAN programs were used to predict crack growth behavior for the variable amplitude fatigue tests performed in this study. While both crack growth models are meant for application to mode I crack growth under uniaxial loading conditions, they are subsequently applied to all nominal loading conditions investigated in this study. Although crack growth was observed to be mode I in all cases, multiaxial effects such as T-stress have the ability to affect the experimentally observed growth rates when the nominal loading is not uniaxial. As a result, crack growth predictions in this section are presented separately for axial, torsion, and combined axial-torsion loading conditions.

For the axial and torsion nominal loading histories, stresses were projected onto the maximum principal stress plane (consistent with observed crack growth planes) and input into the crack growth programs for analysis. This corresponds to the 0° plane (perpendicular to the specimen axis) for axial loading histories, and the 45° plane for pure torsion loading. Although the maximum principal stress plane changes throughout the combined axial-torsion loading history due to changing nominal stress ratios, the loading history is axial-dominated and crack growth, especially at shorter crack lengths, was observed to occur predominately on the 0° plane. Therefore, the normal stress history on this plane was used as input for these analyses. On these planes, the maximum principal and/or normal stress history is simply equal to the applied axial stress history for pure
axial and combined axial-torsion loading conditions, and to the applied shear stress history for pure torsion loading conditions. Experimental crack paths are shown in subsequent results figures for comparison with these assumptions.

For each analysis, crack growth was simulated from an initial half crack length of 1.8 mm (0.2 mm excluding the radius of the hole) up until either an arbitrarily defined crack length of 30 mm was reached, or until specimen failure was predicted. However, since most experiments were manually ended at a half crack length of around 7.5 mm, crack growth curves were only compared up to this length. When compared to predicted crack growth curves, experimental results have been adjusted so that a crack growth life of 0 blocks corresponds to the life where the first detected surface crack had grown a distance of 0.2 mm from the edge of the hole. This was done to maintain consistency with the crack initiation definition used in fatigue life predictions. This would be especially important if total life predictions were to be calculated as the sum of both crack initiation and crack growth lives.

More details on the variable amplitude crack growth analyses, including specific modeling parameters and/or analysis options used for each program, can be found in Appendix C. Here, sample input and output files from a FASTRAN crack growth analysis are presented along with a sample output file from the corresponding UniGrow analysis. The examples presented are for crack growth under axial only loading conditions, using the edited variable amplitude loading history, and are based on the TT crack geometry assumption.
8.2.3.1 Results for Axial Only Loading

The first variable amplitude loading conditions analyzed were those corresponding to the pure axial nominal loading histories. In all, a total of four different tests were performed for the full axial history, at three different loading levels, while two tests were performed for the edited history at the same loading level. Data for the full history tests, along with their corresponding crack growth predictions, are shown in Figures 8.8, 8.9, and 8.10 for nominal scale factors of 2.6, 2.2, and 1.75, respectively. Similarly, results for the edited history tests, performed at a scale factor of 1.75, are shown in Figure 8.11. Only predictions for the TT and CCH crack geometry assumptions are included in these figures, as they are directly applicable to the circumferential crack growth observed in the pure axial tests. A summary of all variable amplitude growth life predictions is also included in Table 8.2, along with experimental results.

In each figure, experimental crack length data are presented first in terms of individual left and right surface crack lengths (excluding the radius of the hole), along with tip-to-tip half crack length (including the radius of the hole), versus the number of applied loading blocks. Then, experimental versus predicted crack growth curves (based on tip-to-tip half crack length) are presented in a separate plot. Finally, the observed crack path and fracture surface pictures are included for all specimens from which they were available. No smoothing of experimental crack growth data (i.e. polynomial reduction) was performed for any of the experimental crack growth curves presented in these figures.

From these results, both UniGrow and FASTRAN are generally found to produce conservative crack growth predictions for the variable amplitude service loading history.
utilized in this study, regardless of the assumed crack geometry. Additionally, the degree of conservatism in crack growth predictions tends to increase with decreasing stress level. For the TT crack geometry assumption, the average ratio of experimental to predicted crack growth life (from 1.8 mm to 7.5 mm) for the full axial loading history is 1.7, 4.1, and 7.2 for load scale factors of 2.6, 2.2, and 1.75, respectively, based on FASTRAN analyses. For UniGrow analyses assuming the TT geometry, average ratios of experimental to predicted crack growth life are 3.1, 9.3, and 12.0 for load scale factors of 2.6, 2.2, and 1.75, respectively. This can be compared to average life ratios of 0.7, 1.8, and 3.9, for each respective load scale factor, based on FASTRAN analyses with the CCH crack geometry assumption, and to ratios of 1.0, 2.9, and 7.6 for UniGrow analyses and CCH geometry assumptions. Plots of experimental versus predicted crack growth life for each analysis performed are shown in Figure 8.12 for the TT crack geometry, and in Figure 8.13 for the CCH geometry.

By comparing these results to the constant amplitude baseline analyses, the effects of the variable amplitude loading history are found to be greater for life predictions based on the UniGrow crack growth model than for those based on FASTRAN. While FASTRAN growth life predictions for constant amplitude notched specimen tests were, on average, a factor of 1.14 times longer than UniGrow predictions based on the TT geometry assumptions, this difference increased to an average of 1.85 times longer for the axial only full service loading history tests. Similarly, FASTRAN predictions based on the CCH crack geometry increased from an average factor of 1.05 times longer than UniGrow predictions for constant amplitude loading conditions to a factor of 1.75 longer for the variable amplitude analyses. Given the differences between crack growth under
constant and variable amplitude loading conditions, this is likely due to either the consideration of load sequence effects in each model, or due to errors in the representation of near threshold crack growth behavior for the chosen material property inputs.

In addition to full history crack growth predictions, the UniGrow and FASTRAN models were also used to predict crack growth for fatigue tests performed using the edited version of the service loading history. Despite the fact that 94% of the original loading history was removed for the axial only edited history tests, Table 8.2 shows that experimental crack growth lives for these tests are only an average of 2.6 times longer than those for full history tests performed at the same nominal scale factor. This gives insight into the effect of smaller cycles on variable amplitude crack growth behavior as well as their implications with respect to load history editing for the purpose of accelerated fatigue testing. By comparing crack growth predictions for full and edited loading histories, each model’s ability to properly account for load history effects with and without the inclusion of small cycles can be evaluated. Additionally, since applied SIF ranges remain above the threshold region for the edited loading history, errors in predictions due to near threshold crack growth properties should be mitigated.

When comparing experimental and predicted crack growth curves in Figures 8.10 and 8.11 for the full and edited axial loading histories, respectively, life prediction trends are found to be relatively similar in both cases. From Figure 8.12, assuming the TT crack geometry, the average ratio of experimental to predicted crack growth life for the edited axial loading history tests is 6.1 for FASTRAN analyses (7.6 for full history), and 24.7 for UniGrow analyses (12.6 for full history). Similarly, from Figure 8.13, the average
The ratio of experimental to predicted crack growth life is 2.8 based on FASTRAN analyses (4.2 for full history) and 12.8 based on UniGrow analyses (8.0 for full history), when assuming the CCH crack geometry.

Given the fact that life prediction errors are similar between full and edited histories for the FASTRAN analyses, and edited history predictions are more conservative for the UniGrow analyses, it is unlikely that the overall conservative nature of the variable amplitude crack growth predictions is due to near threshold crack growth properties. To help verify these findings, some analyses were also performed with modified crack growth properties which increased the slope of the growth curve in the threshold region in an attempt to produce less conservative crack growth predictions. The results of these analyses, however, showed a negligible difference in growth life predictions. Additionally, the fact that full history growth life predictions for both analysis programs are closer to the experimentally measured lives at the higher loading levels suggests that increased plasticity is also not to blame for the error. Instead, the most likely cause for the difference between constant and variable amplitude crack growth prediction trends is in the consideration of load sequence effects on crack growth.

Concerning loading history effects, it should be noted that by starting an analysis from the crack length defined as initiation, load history effects on initial crack growth, a result of residual stress distributions produced by a crack growing to this initial length, are neglected. Because the initial crack length and SIF ranges in this study were small, however, these effects were assumed to be relatively small as well. This assumption was verified, for both analysis programs and geometry assumptions, by comparing crack growth lives (from a half crack length of 1.8 mm to 7.5 mm) based on analyses...
preformed with both 1.8 mm and 1.65 mm initial crack lengths. Similarly, growth lives were also computed from a half crack length of 2.6 mm (1.0 mm excluding the radius of the hole) to 7.5 mm, using the same analysis results (1.8 mm initial crack length). These results, while not presented herein, help to eliminate the effects of both load history dependence and initial crack geometry assumption on growth life comparisons. However, the resulting life predictions were almost identical, in terms of relative accuracy, to those based on crack growth from 1.8 mm to 7.5 mm. Therefore, initial load history effects were likely not a large contributor to the error observed in the variable amplitude crack growth predictions.

It should also be noted that, due to the crack initiation definition used in this study and the assumption of symmetric cracks growing from each side of the hole, the experimental crack initiation conditions are generally different than those assumed in the crack growth analyses. For example, crack length measurements for specimen T69-AEN in Figure 8.11(a) show that a crack was observed at the specimen’s outer surface on the left side of the hole approximately 2.5 blocks before a crack was observed on the right side of the hole. While this not only affects the accuracy of SIF solutions, it also creates a discrepancy in the half crack length at initiation. Because crack initiation was defined as the life when the first crack growing from the hole reached a length of 0.2 mm (in order to be consistent with fatigue life predictions), the experimental half crack length at initiation is only 1.7 mm when a single crack exists instead of the 1.8 mm defined in the crack growth analyses. Therefore, the right side crack in specimen T69-AEN would have to reach a length of 0.4 mm before the 1.8 mm half crack initiation condition is met.
From Figure 8.11(a), this corresponds to a difference in initiation life (reduction in crack growth life) of approximately 1.3 blocks.

While this type of behavior was observed, to some extent, for all of the variable amplitude tests performed in this study, it is not expected to have a significant effect on the overall crack growth prediction trends. Even if the crack growth life for specimen T69-AEN were to be reduced by 1.3 blocks, which is one of the more extreme cases in terms of percentage difference in initiation life, all predicted crack growth lives would still be conservative to a similar degree. Additionally, the same type of crack growth behavior was observed for the constant amplitude notched specimen tests, and despite the discrepancy in both crack initiation geometry and life, all growth life predictions were found to be satisfactory.

To summarize, crack growth predictions for variable amplitude fatigue tests performed under axial only loading conditions were generally found to be conservative based on both FASTRAN and UniGrow crack growth models, regardless of the assumed crack geometry. This is despite the fact that, for both programs, experimental crack growth lives under constant amplitude loading conditions were generally found to fall between predictions based on the TT and CCH crack geometry assumptions. Given the differences in applied loading level, analysis model, and full versus edited loading histories, growth life predictions for the CCH crack geometry ranged between a factor of 1.6 and 3.2 times longer than those based on the TT geometry, with an average factor of 2.3 increase in life prediction. Additionally, it can be noted from studying the fracture surface pictures presented along with analysis results, that the actual crack initiation
geometry for a particular specimen can be difficult to determine, even after the test has been completed.

For the FASTRAN analyses and TT crack geometry assumptions, only 1 of 6 growth life predictions were within a factor of ±3 of experimental lives, while all predictions were within a factor of ±10. For the UniGrow analyses, no predictions were within a factor of ±3 and only 3 of 6 predictions within a factor of ±10. Similarly, for the CCH geometry assumption, 4 of 6 and 6 of 6 predictions were within factors of ±3 and ±10 of experimental lives, respectively, based on FASTRAN analyses, while 2 of 6 and 4 of 6 predictions were within factors of ±3 and ±10 of experimental lives based on UniGrow analyses. In addition to being more accurate, analyses based on the FASTRAN crack growth model were also more consistent between full and edited versions of the axial service loading history. This suggests that FASTRAN may be better able to reflect to effects of load history dependence on variable amplitude crack growth for the particular loading history applied in this study.

8.2.3.2 Results for Torsion Only Loading

In addition to tests performed under axial only loading conditions, crack growth for the pure torsion variable amplitude loading histories was also analyzed. In all, a total of three different tests were performed for the full history, at two different loading levels, while two tests were performed for the edited history at the same loading level. The results for the full history tests are shown in Figures 8.14 and 8.15 for nominal scale factors of 3.5 and 2.5, respectively, while results for the edited history tests, performed at a scale factor of 2.5, are shown in Figure 8.16. In addition to crack growth predictions for the TT and CCH crack geometry, prediction results are also included for the MT
geometry assumption, as the actual SIF solution for a crack inclined 45° to the specimen axis is expected to fall somewhere between the TT and MT solutions. Similar to the axial results figures, individual crack length components and experimental versus predicted crack growth curves are presented along with the observed crack path and fracture surface pictures (when applicable) for each loading condition.

From these figures, crack growth predictions for the nominal torsion loading histories are found to be significantly different than those for pure axial loading. Instead of being consistently conservative, the full torsion history growth predictions tend to be non-conservative based on FASTRAN analyses, and fairly accurate based on UniGrow analyses. Additionally, the shift in conservatism is notably larger for the FASTRAN analyses than for the UniGrow analyses. While FASTRAN life predictions for the full axial loading history and TT crack geometry were an average of 1.9 times longer than UniGrow predictions, this increases to a factor of 10.4 for the full torsion loading history analyses. Similarly, for the CCH geometry, FASTRAN life predictions increased from an average of 1.8 times longer than UniGrow predictions for the axial history, to a factor of 6.5 longer for the torsion history. Based on the differences between crack growth conditions under the pure axial and pure torsion loading histories applied in this study, the cause for these shifts in prediction accuracy can be narrowed down to two likely sources: multiaxial stress state effects, such as the presence of T-stress, and/or differences in loading history profile.

Concerning loading history profile, the axial channel of the variable amplitude service loading history, as shown in Figure 6.1, is composed of smaller cycles with significant tensile mean stress mixed with occasional larger cycles at a much smaller $R$
ratio. The shear channel, on the other hand, is composed of small amplitude cycles, ranging between approximately zero to minimum and zero to maximum conditions, mixed with occasional larger amplitude cycles of an approximate zero to maximum range. Given these differences, significantly more crack closure would be expected in the case of mode I crack growth under the pure torsion loading history, when compared to the axial loading history, due to the smaller tensile mean stress values. This agrees with the more conservative FASTRAN life predictions for the full axial loading history, as compared to the full shear history.

Residual stresses, on the other hand, are most significantly affected by the maximum stress values in a loading history. Since the maximum tensile stresses in both the axial and shear loading histories are significantly larger than the maximum compressive stresses, nearly all plastic deformation at the notch root takes place in one direction for these tests. Therefore, the effect of the loading history profile is not expected to have as large of an effect on residual stress distributions so long as the frequency and magnitude of maximum stress cycles are similar, which is the case in this study. As such, the fact that growth life predictions (Figures 8.12 and 8.13) are more consistent between full axial and full torsion loading histories for UniGrow analyses, as compared to FASTRAN analyses, is not surprising.

In addition to loading history profile, it was shown in Section 7.3 (Figure 7.42) that even for constant amplitude loading conditions, mode I crack growth rates for nominal torsion loading of the notched tubular specimens were considerably higher than those for axial nominal loading at the same applied SIF range. This was attributed to the presence of compressive tangential stress (T-stress) at the crack tip, not accounted for in
mode I SIF calculation, resulting in an increased plastic zone size and crack driving force under multiaxial nominal stress states. While T-stress corrections for constant amplitude crack growth data weren’t able to completely account for the higher growth rates observed under nominal torsion loading conditions, they did provide a significant improvement in crack growth rate correlations between data generated under different nominal stress states.

Because UniGrow and FASTRAN are both meant to model crack growth under uniaxial nominal loading conditions, neither program accounts for multiaxial stress state effects, such as T-stress, on mode I crack growth rates. Therefore, for a given SIF range, both models will predict lower crack growth rates than what would be expected in experiments for torsion and combined axial-torsion nominal loadings. As a result, the overall reduction in conservativism for the pure torsion variable amplitude growth life predictions in Figures 8.12 and 8.13 is to be expected.

Similar to the pure axial crack growth predictions, additional insight can also be gained from comparing prediction results for both full and edited versions of the pure torsion loading history. Again, despite the fact that 95% of the original loading history was removed for the torsion only edited history tests, Table 8.2 shows that experimental crack growth lives for these tests are only an average of 1.6 times longer than those for full history tests performed at the same maximum stress level. Therefore, smaller cycles had an even lesser effect on crack growth behavior for the torsion loading history than for the axial history.

By comparing experimental and predicted crack growth lives in Figures 8.12 and 8.13 for pure torsion loading tests, the average ratio of experimental to predicted life for
the edited history tests is found to be 0.43 for FASTRAN analyses (0.31 for full history), and 0.29 for UniGrow analyses (3.89 for full history), when assuming the TT crack geometry. Similarly, the average ratio of experimental to predicted crack growth life is 0.24 based on FASTRAN analyses (0.17 for full history) and 0.13 based on UniGrow analyses (0.92 for full history), when assuming the CCH crack geometry. From these comparisons, life prediction errors are found to be very similar between full and edited histories for the FASTRAN analyses, while edited history predictions are significantly more non-conservative for the UniGrow analyses. This is opposite to observations for the axial only crack growth analyses, where UniGrow edited history predictions were more conservative than those for the corresponding full history tests, and suggests that UniGrow may not be able to adequately account for the differences between the full and edited torsion loading histories.

In addition to the effect of small cycles on crack growth, the difference in UniGrow predictions may also be the result of differences in loading patterns between the full loading history and each edited history. For the edited axial history, shown in Figure 6.3, the same pattern of smaller cycles with significant tensile mean stress mixed with occasional larger cycles at a much smaller R ratio is observed for both the edited and full history, although the number of smaller cycles is significantly reduced in the edited history. For the edited shear history, however, Figure 6.4 shows that essentially all smaller cycles have been removed from the full history. As a result, the majority of cycles in the edited shear history are of a similar amplitude and R ratio, resulting in a significantly different loading profile when compared to the full history. This difference
in loading profile likely has a substantial impact on the resulting crack growth acceleration and retardation predictions.

For variable amplitude crack growth analyses performed using the pure torsion loading histories, predicted crack growth curves were shown in all results figures based on the MT crack geometry in addition to those based on the TT and CCH geometry assumptions. This is because for mode I cracks growing inclined to the specimen circumferential direction, the actual SIF solution is expected to fall in between the TT and MT solutions. Therefore, growth predictions for both of these crack geometries should provide a bounds on the through crack life predictions. Similar to the other crack geometry analyses, experimental versus predicted crack growth lives (from 1.8 mm to 7.5 mm) are shown in Figure 8.17 for the MT geometry analyses from both FASTRAN and UniGrow.

The results shown in this figure are as expected, life predictions based on the MT geometry assumption fall in between those based on the other geometries. As such, no significant differences in life prediction accuracy are observed. Additionally, the shape of the crack growth prediction curves for TT crack geometry tends to resemble the experimentally observed crack growth more closely than the predictions based on the MT geometry. This is especially true when considering crack growth rates (slope of the \( c \) vs. \( N \) curve) at longer crack lengths/SIF ranges. Therefore, it is likely that the TT SIF solution is a reasonable representation of through crack growth in the notched tubular specimens, even for inclined cracks growing under pure torsion loading.

Overall, for the FASTRAN analyses and TT crack geometry assumptions, only 1 of 5 pure torsion growth life predictions were within a factor of ±3 of experimental lives,
while all predictions were within a factor of ±10. For the UniGrow analyses, 2 of 5 predictions were within a factor of ±3 and all predictions were within a factor of ±10. Similarly, for the CCH geometry assumption, 1 of 5 and 5 of 5 predictions were within factors of ±3 and ±10 of experimental lives, respectively, based on FASTRAN analyses, while 3 of 5 and 4 of 5 were within factors of ±3 and ±10 of experimental lives based on UniGrow analyses. The accuracy/lack thereof of these predictions, however, is not a true indication of each model’s predictive capability due to multiaxial stress state effects on experimental crack growth that are not accounted for in the crack growth analyses.

### 8.2.3.3 Results for Combined Axial-Torsion Loading

Finally, crack growth analyses were performed for the variable amplitude combined axial-torsion loading histories. A total of four different tests were performed for the full combined loading history, at two different loading levels, while two edited history tests were performed at the same loading level. The results for the full history tests are shown in Figures 8.18 and 8.19 for nominal scale factors of 2.0 and 1.5, respectively, while results for the edited history tests, performed at a scale factor of 1.5, are shown in Figure 8.20.

Given the tension-dominated nature of the variable amplitude loading history applied in this study, crack growth behavior for the combined axial-torsion loading history is expected to be similar to that observed for the axial only fatigue tests. By studying the results figures, this is found to be generally true. Figure 8.12 and 8.13 show, in terms of experimental versus predicted crack growth life, that like the crack growth predictions for axial only fatigue tests, predictions for the combined loading tests tend to
be mostly conservative based on both FASTRAN and UniGrow analyses. Additionally, the degree of conservatism is found to increase with decreasing loading level.

Experimental and predicted crack growth trends for combined edited history tests are also found to be similar to those observed for axial only edited history tests. While the same edited loading history was used for the combined and axial only fatigue tests (with and without the inclusion of the corresponding shear channel), however, edited history crack growth lives were found to be an average of 4.9 times longer than full history lives for the combined loading tests. This suggests that although there is no contribution from nominal shear stress to mode I crack driving force on the 0° plane, the applied shear stress still contributed to increased crack growth caused by even the smaller loading cycles in the full history tests.

For the combined loading, when assuming the TT crack geometry, the average ratio of experimental to predicted crack growth life for the edited history tests is 4.7 for FASTRAN analyses (3.3 for full history), and 29.1 for UniGrow analyses (10.5 for full history). Similarly, the average ratio of experimental to predicted crack growth life is 2.7 based on FASTRAN analyses (2.5 for full history) and 12.6 based on UniGrow analyses (5.8 for full history), when assuming the CCH crack geometry. From these comparisons, life prediction errors are found to be similar between full and edited histories for the FASTRAN analyses, while edited history predictions are more conservative for the UniGrow analyses. This is consistent with observations for the axial only crack growth analyses and, again, these differences can likely be attributed to each model’s consideration of load sequence effects for the particular loading history investigated.
While crack growth prediction trends are qualitatively very similar for the axial only and combined loading conditions, growth life predictions are generally found to be less conservative for combined loading tests than for the axial only tests. Similar to the growth life predictions for the pure torsion tests, some of this difference is likely due to the crack growth models’ inability to account for increased growth rates due to the effect of T-stress on mode I crack growth under multiaxial loading conditions. However, there are also additional factors that can contribute to this discrepancy which are only brought about under combined loading situations.

Figure 6.1(b) shows that the variable amplitude service loading history investigated in this study contains a number of significant non-proportional loading events. When non-proportionally varying stresses are present in a crack growth analysis, it becomes especially difficult to calculate crack driving forces. Because the principal stress directions are not constant under non-proportional loading conditions, a growing crack is continuously subjected to varying degrees of mixed-mode loading, regardless of its orientation. Additionally, the tendency of a mode I crack to grow under the influence of maximum principal stress can cause increased crack meandering and crack face roughness as cracks try to align with the changing principal stress direction. Despite the fact that the combined loading history applied in this study is tension-dominated, the effect of the applied shear stress on crack path and fracture surface roughness is evident when comparing Figures 8.8 to 8.11 ((c) and (d)) to Figures 8.18 to 8.20 ((c) and (d)) for axial and combined loading conditions, respectively.

The fact that cracks under variable amplitude combined loading are consistently observed to deviate from the 0° plane (perpendicular to the specimen axis) has significant
implications in a crack growth analysis. While the first 1 mm or so of crack growth in the full combined loading history tests was observed to be approximately on the 0° plane, cracks began to turn at longer crack lengths to eventually orient themselves at an angle of approximately -30°. Since the cracks growing at -30° are not perpendicular to the applied tensile stress, they are not subjected to the full stress range input into the crack growth analysis models. Therefore, higher SIF range and crack growth rate predictions would be expected under such conditions. Since the majority of crack growth life is consumed by small crack growth, however, the effect of the changing crack path on growth life predictions reduces as the crack grows longer. For this reason, combined loading crack growth analyses performed in this study considered all crack growth to occur on the 0° plane. Similar changes in crack path were observed for edited combined history tests as well, although the final crack growth plane is not as clear in these cases.

As discussed in Section 7.3.3 for 90° OP constant amplitude loading conditions, not only does a changing crack path affect the driving forces present at the crack tip, it also suggests a difference in crack growth conditions as the crack increases in length. For the combined loading tests performed in this study, when the crack is small, its growth is dominated by the mode I loading from the axial component of the applied stress history. However, as it grows, a change in growth direction suggests that the shear stress component has an increasing influence on the crack path as well. Whether this is due to notch effects, mixed-mode crack growth, increased crack face roughness, or other factors is not currently clear.

While there are many factors, in addition to plasticity induced closure and residual stress effects, which have the potential to influence crack growth behavior under
variable amplitude combined loading conditions, some of these effects act to increase
 crack growth rates (e.g. T-stress and mixed-mode loading), while others tend to hinder
 crack growth (e.g. crack path meandering and increased crack surface roughness). For the
 loading conditions and specimen geometry of interest in the current study, the combined
effect of all of these mechanisms appears to result in less conservative crack growth
predictions for combined loading histories than for axial only histories. This agrees with
the higher constant amplitude crack growth rates observed in Section 7.3, for both in-
phase and 90° OP combined loading conditions, as compared to those for axial loading.
Nonetheless, for the FASTRAN analyses and TT crack geometry assumptions, 3 of 6 and
6 of 6 variable amplitude crack growth life predictions were within a factors of ±3 and
±10 of experimental lives, respectively, while 0 of 6 and 3 of 6 UniGrow predictions
were within factors of ±3 and ±10 of experimental results, respectively. Similarly, for
the CCH geometry assumption, 4 of 6 and 6 of 6 predictions were within factors of ±3
and ±10 of experimental lives, respectively, based on FASTRAN analyses, while 0 of 6
and 5 of 6 were within factors of ±3 and ±10 of experimental lives based on UniGrow
analyses.

8.2.4 Notched Specimen Variable Amplitude Crack Growth Summary

Due to the complexity of predicting crack growth under variable amplitude
loading conditions, two state-of-the art analysis codes were used in this study in order to
compare experimentally generated crack growth data to predictions based on different
crack growth models. The UniGrow model is based on the idea that residual stress
distributions, resulting from varying degrees of plastic deformation surrounding the crack
tip, are responsible for causing load sequence effects in variable amplitude crack growth,
while FASTRAN attributes these effects to plasticity induced closure resulting from residual displacements of plastically deformed material in the crack wake. Although both analysis codes are meant for application to crack growth under uniaxial loading conditions, variable amplitude crack growth under multiaxial nominal loadings was also analyzed. Since cracks in this study were initiated naturally at a notch, the effect of initial crack geometry assumptions on crack growth was evaluated by using two different SIF solutions, meant to provide upper and lower bounds for the experimental crack growth behavior.

With regards to the effect of crack geometry assumptions, the CCH geometry was found to produce life predictions an average of 2.7 times longer than those based on the TT crack geometry under constant amplitude loading conditions. For variable amplitude loading conditions, when accounting for all combinations of nominal stress state, stress level, loading history, and analysis model, crack growth life predictions for the CCH crack geometry ranged between a factor of 1.1 and 4.2 times longer than those based on the TT geometry. The average difference in life predictions was a factor of 2.2. This is a fairly substantial difference and emphasizes the importance of crack geometry considerations in obtaining accurate crack growth predictions. Additionally, it was found that even by studying the fracture surface after the completion of a fatigue test, the actual crack initiation geometry for a particular specimen was difficult to determine. This was true regardless of the nominally applied loading conditions.

In general, variable amplitude crack growth predictions based on both FASTRAN and UniGrow analyses were found to be conservative, regardless of the initial crack geometry assumption, for both axial and combined axial-torsion loading conditions. The
accuracy of crack growth predictions for pure torsion loading conditions, however, was found to vary depending on the crack growth model and loading history profile (i.e. full history versus edited history), although all predictions were less conservative than in the case of axial and combined loadings. This is despite the fact that, for both programs, experimental crack growth lives under constant amplitude axial loading conditions were generally found to fall between predictions based on the TT and CCH crack geometry assumptions. Additionally, comparisons with constant amplitude crack growth data show that the shift in conservatism between the different nominal loading conditions can likely be attributed to multiaxial stress state effects on mode I crack growth, such as the presence of T-stress and the potential for mixed-mode crack growth conditions. These effects are not accounted for in either crack growth model investigated.

Despite the fact that at least 94% of the original loading history was removed for all edited history fatigue tests, average experimental crack growth lives for these tests were, at most, a factor of 4.9 times longer than those for full history tests performed at the same nominal scale factor. This gives insight into the effect of smaller cycles on variable amplitude crack growth behavior as well as their implications with respect to load history editing for the purpose of accelerated fatigue testing. By comparing differences in experimental crack growth life between full and edited history tests to the differences predicted by each crack growth model, it was found that FASTRAN is generally better able to account for changes in crack growth behavior due to the effects of small cycles and/or changes in loading history profile. This is evidenced in Figures 8.12(a) and 8.13(a) by the tendency of experimental versus predicted crack growth life data to run approximately parallel to the perfect prediction line for each loading condition. Edited
history predictions based on UniGrow, however, are more conservative than full history predictions for axial and combined loading conditions, and more non-conservative in the case of torsion only loading.

In all, the FASTRAN analyses and TT crack geometry assumption resulted in 5 of 17 (29%) growth life predictions within a factor of ±3 of experimental results, while all predictions were within a factor of ±10. For the UniGrow analyses, only 2 of 17 (12%) predictions were within a factor of ±3 of experimental lives and 11 of 17 (65%) predictions fell within a factor of ±10. Similarly, for the CCH geometry assumption, 9 of 17 (53%) and 17 of 17 (100%) predictions were within factors of ±3 and ±10 of experimental lives, respectively, based on FASTRAN analyses, while 5 of 17 (29%) and 13 of 17 (77%) were within factors of ±3 and ±10 of experimental lives based on UniGrow analyses. Again, a summary of all variable amplitude growth life predictions is included in Table 8.2. However, as previously noted, the overall accuracy of these crack growth life prediction results isn’t a true indication of each model’s predictive capability. This is due to the presence of multiaxial stress state effects in the torsion and combined loading tests, which are not accounted for in the crack growth analyses.

8.3 Un-Notched Specimen Crack Growth

Details of the un-notched specimen variable amplitude fatigue tests performed in this study were presented in Sections 6.1 and 6.2. For a summary of the applied loading conditions, the reader is referred to Table 6.1. Since crack growth in these tests was monitored and recorded whenever possible using a cellulose acetate surface replication technique, a reasonable amount of variable amplitude crack growth data were generated
for most specimens tested. Crack growth data were captured for failure cracks, as well as for secondary cracks found growing in the specimen gage section. These experimental crack growth data, in terms of surface crack length versus number of applied loading blocks, are subsequently presented in this section.

Given the complexities involved in variable amplitude crack growth analyses, state-of-the-art analysis programs were used in the last section to analyze the variable amplitude crack growth data generated for the notched specimens tested in this study. Because of the much larger maximum nominal stress levels applied in the un-notched specimen tests, however, the small scale yielding assumptions on which these programs are based were often violated for these tests. Additionally, unlike the mode I-dominated crack growth observed in the notched specimen tests, crack growth in un-notched specimens was often observed to be shear/mixed-mode. Since these are conditions which neither program is equipped to handle, no variable amplitude crack growth analyses were performed for the un-notched specimen tests.

For the axial only variable amplitude fatigue tests, no crack growth data were available for the highest load level test (TM75-ARS) performed. However, crack growth curves from specimens tested at the two lower load levels are shown in Figure 8.21 (3.1× scale factor) and Figure 8.22 (2.9× scale factor). No smoothing of experimental data (i.e. polynomial reduction) was performed for any of the crack growth curves presented in these figures. Crack replica pictures from near failure are also included in order to show the overall path for each crack. For additional crack replica pictures from these tests, the reader is referred to Figures 6.20 and 6.21, respectively. It should also be noted that, due to previously discussed difficulties in obtaining crack growth data for some un-notched
specimen tests (due to inner surface crack initiation), not all specimens are represented in these figures.

For the pure torsion variable amplitude fatigue tests, crack growth data were recorded for the failure crack, as well as two secondary cracks, in both of the tests performed (4.0× scale factor). The resulting crack growth curves and crack paths are shown in Figure 8.23(a) for specimen TM85-TRS, and in Figure 8.23(b) for specimen TM107-TRS. Again, some additional crack replica pictures corresponding to these tests were previously presented in Figures 6.22(a) and 6.22(b), respectively.

Finally, for the combined axial-torsion variable amplitude fatigue tests performed in this study, experimental crack growth data were only available for one un-notched specimen. The resulting crack growth curve and crack path for this specimen, tested at a load scale factor of 2.5, are shown in Figure 8.24. Additional crack path pictures corresponding to this test can also be found in Figure 6.24(a). While crack growth data were not available for the other un-notched specimens tested under combined axial-torsion loading, some crack path pictures for failure cracks and secondary cracks growing in these specimens, taken from replicas at or near the end of each test, are shown in Figures 6.23 and 6.24.

From these results, it can be seen that the majority of the fatigue life for the un-notched specimens tested under variable amplitude loading conditions was spent in the nucleation and growth of cracks up to a length of around 1 to 2 mm (2 to 4 mm tip-to-tip crack length). Once cracks reached this length, subsequent growth and final failure occurred very rapidly, typically due to the application of a large overload cycle. While no crack growth analyses were performed in this study for these tests, these experimental
results offer valuable data if future investigations into mixed-mode variable amplitude crack growth are to be performed.

8.4 Summary and Conclusions

As stated previously, the central objective of the research performed in this study was to be able to better understand and predict fatigue crack initiation and growth in components subjected to complex service loading histories. As a result, crack growth data recorded from the variable amplitude fatigue tests discussed in Chapter 6 were analyzed in this chapter. Given the complexities involved in such an analysis, however, two state-of-the-art crack growth models, UniGrow and FASTRAN, were used to compare crack growth predictions to experimental data for only the notched specimens tested in this study. Variable amplitude crack growth trends for multiaxial nominal loadings were then compared with those for constant amplitude loading conditions in order to help interpret the analysis results. Additionally, since fatigue cracks were initiated naturally at the notch, the effect of initial crack geometry assumptions on crack growth predictions was also investigated.

Based on the experimental results and analysis presented for variable amplitude crack growth in notched specimens, some key findings and conclusions can be summarized as follows:

1) Differences in initial crack geometry assumptions (i.e. corner crack versus through crack) were found to produce fairly significant differences in predicted crack growth life, ranging mostly between a factor of 2 and 3 for the notched specimen geometry tested in this study.
2) Both UniGrow and FASTRAN crack growth models were able to predict nearly all uniaxial constant amplitude crack growth lives within a factor of ±3 of experimental results, regardless of crack geometry assumptions. Additionally, for both models, most experimental data fell between predictions based on crack geometries producing theoretical upper and lower bounds for crack growth life.

3) For variable amplitude crack growth analyses of tests performed under pure axial nominal loading conditions, both UniGrow and FASTRAN were generally found to produce conservative growth life predictions, although FASTRAN resulted in slightly more accurate results.

4) For variable amplitude torsion and combined axial-torsion crack growth analyses, effects from multiaxial nominal stress states, such as T-stress and mixed-mode crack growth, likely increased experimental crack growth rates and reduced the conservatism in growth life predictions. These effects were also seen in constant amplitude crack growth rate correlations, but are not accounted for in either UniGrow or FASTRAN.

5) While at least 94% of the original loading history was removed, the largest average difference in experimental crack growth life between full and edited loading history tests was less than a factor of 5. This gives insight into the effect of smaller cycles on variable amplitude crack growth behavior, as well as their implications with respect to load history editing for the purpose of accelerated fatigue testing.

6) By comparing differences in experimental crack growth life between full and edited history tests to the differences predicted by each crack growth model, it
was found that FASTRAN is generally better able to account for changes in crack growth behavior due to the effects of small cycles and/or changes in loading history profile.

Crack growth analyses were not performed for the un-notched specimen variable amplitude fatigue tests performed in this study. This was due to the fact that many of the experimentally observed crack growth conditions violated fundamental assumptions of the crack growth models employed herein. However, based on the experimental results presented for these specimens, some key findings can be summarized as follows:

7) The majority of the fatigue life for the un-notched specimens tested under variable amplitude loading conditions was spent in the nucleation and growth of cracks up to a length of around 1 to 2 mm (2 to 4 mm tip-to-tip crack length).

8) Once cracks reached a few mm in length, crack growth and final failure occurred very rapidly, typically due to the application of a large overload cycle.

9) These experimental results offer valuable crack growth data if future investigations into mixed-mode variable amplitude crack growth are to be performed.
Table 8.1  Summary and comparison of material properties used in UniGrow and FASTRAN variable amplitude crack growth simulations.

<table>
<thead>
<tr>
<th>Property</th>
<th>Experimental</th>
<th>UniGrow</th>
<th>FASTRAN</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mechanical Properties</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>2024-T3</td>
<td>2024-T351</td>
<td>2024-T3</td>
<td></td>
</tr>
<tr>
<td>Modulus of Elasticity, $E$</td>
<td>73.7</td>
<td>73.1</td>
<td>72.0</td>
<td>GPa</td>
</tr>
<tr>
<td>Monotonic Yield Strength, $\sigma_y$</td>
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<td>$\sim$</td>
<td>360</td>
<td>MPa</td>
</tr>
<tr>
<td>Cyclic Yield Strength, $\sigma'_y$</td>
<td>415</td>
<td>428</td>
<td>$\sim$</td>
<td></td>
</tr>
<tr>
<td>Ultimate Tensile Strength, $\sigma_u$</td>
<td>495</td>
<td>$\sim$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poisson’s Ratio, $\nu$</td>
<td>0.343</td>
<td>0.330</td>
<td>$\sim$</td>
<td></td>
</tr>
<tr>
<td>Cyclic Strength Coefficient, $K'$</td>
<td>677.0</td>
<td>662.0</td>
<td>$\sim$</td>
<td>MPa</td>
</tr>
<tr>
<td>Cyclic Hardening Exponent, $n'$</td>
<td>0.070</td>
<td>0.070</td>
<td>$\sim$</td>
<td></td>
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<tr>
<td><strong>UniGrow Specific Material Properties</strong></td>
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<td></td>
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<tr>
<td>Driving Force Constant, $p$</td>
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<td></td>
<td>0.06542</td>
<td></td>
</tr>
<tr>
<td>Material Block Size, $\rho^*$</td>
<td></td>
<td></td>
<td>1.2535(10^{-5})</td>
<td>m</td>
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<td><strong>FASTRAN Specific Material Properties</strong></td>
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<td>Elastic–Plastic Fracture Toughness, $K_F$</td>
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<td>MPa√(m)</td>
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<td>Fracture Toughness Parameter, $m$</td>
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<td></td>
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<tr>
<td>Tensile Constraint Factor, $\alpha_1$</td>
<td></td>
<td></td>
<td>2.0 @ 1.0(10^{-7}) m/cycle</td>
<td></td>
</tr>
<tr>
<td>Tensile Constraint Factor, $\alpha_2$</td>
<td></td>
<td></td>
<td>1.0 @ 2.5(10^{-9}) m/cycle</td>
<td></td>
</tr>
<tr>
<td>Compressive Constraint Factor, $\beta$</td>
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<td><strong>Crack Growth Rate Properties</strong></td>
<td>UniGrow</td>
<td>FASTRAN</td>
<td></td>
<td></td>
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<tr>
<td>Segment, $i$</td>
<td>$C^{(i)}$</td>
<td>$m^{(i)}$</td>
<td>$\Delta K_{eff}^{(i)}$</td>
<td>dc/dN^{(i)}</td>
</tr>
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<td>1</td>
<td>9.7231(10^{-15})</td>
<td>17.262</td>
<td>0.80</td>
<td>1.0(10^{-11})</td>
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<td>2.228</td>
<td>1.05</td>
<td>1.0(10^{-10})</td>
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<td>3</td>
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<td>7.621</td>
<td>2.05</td>
<td>2.0(10^{-9})</td>
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<td>3.694</td>
<td>4.00</td>
<td>8.0(10^{-9})</td>
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<td>7.70</td>
<td>1.0(10^{-7})</td>
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</tr>
<tr>
<td>6</td>
<td>13.5</td>
<td>1.0(10^{-9})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>23.0</td>
<td>1.0(10^{-8})</td>
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<tr>
<td>8</td>
<td>36.0</td>
<td>1.0(10^{-8})</td>
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</table>

† Crack growth rate properties derived based on SI units: MPa√(m) and m/cycle
Table 8.2  Summary of variable amplitude notched specimen crack growth life predictions.

<table>
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<tr>
<th>Specimen ID</th>
<th>Maximum Nominal Stress</th>
<th>Loading History</th>
<th>Experimental Blocks to Failure*</th>
<th>Predicted Blocks to Failure (1.8 mm to 7.5 mm half crack length)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$S_{A,\text{max}}$ (MPa)</td>
<td>$S_{T,\text{max}}$ (MPa)</td>
<td>Channels</td>
<td>Scale</td>
</tr>
<tr>
<td>T35-ARN</td>
<td>376.4 -</td>
<td>-</td>
<td>Axial</td>
<td>2.6x</td>
</tr>
<tr>
<td>T30-ARN</td>
<td>318.5 -</td>
<td>-</td>
<td>Axial</td>
<td>2.2x</td>
</tr>
<tr>
<td>T36-ARN</td>
<td>253.3 -</td>
<td>-</td>
<td>Axial</td>
<td>1.75x</td>
</tr>
<tr>
<td>T47-ARN</td>
<td>253.3 -</td>
<td>-</td>
<td>Axial</td>
<td>1.75x</td>
</tr>
<tr>
<td>T69-AEN</td>
<td>253.3 -</td>
<td>-</td>
<td>Edited Axial</td>
<td>1.75x</td>
</tr>
<tr>
<td>T90-AEN</td>
<td>253.3 -</td>
<td>-</td>
<td>Edited Axial</td>
<td>1.75x</td>
</tr>
<tr>
<td>T31-TRN</td>
<td>- 234.5</td>
<td>-</td>
<td>Shear</td>
<td>3.5x</td>
</tr>
<tr>
<td>T34-TRN</td>
<td>- 167.5</td>
<td>-</td>
<td>Shear</td>
<td>2.5x</td>
</tr>
<tr>
<td>T46-TRN</td>
<td>- 167.5</td>
<td>-</td>
<td>Shear</td>
<td>2.5x</td>
</tr>
<tr>
<td>T91-TEN</td>
<td>- 167.5</td>
<td>-</td>
<td>Edited Shear</td>
<td>2.5x</td>
</tr>
<tr>
<td>T93-TEN</td>
<td>- 167.5</td>
<td>-</td>
<td>Edited Shear</td>
<td>2.5x</td>
</tr>
<tr>
<td>T39-CRN</td>
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<td>Combined</td>
<td>2.0x</td>
<td>0.56</td>
</tr>
<tr>
<td>T38-CRN</td>
<td>289.5 134.0</td>
<td>Combined</td>
<td>2.0x</td>
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<tr>
<td>T41-CRN</td>
<td>217.1 100.5</td>
<td>Combined</td>
<td>1.5x</td>
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<td>T48-CRN</td>
<td>217.1 100.5</td>
<td>Combined</td>
<td>1.5x</td>
<td>6.34</td>
</tr>
<tr>
<td>T94-CEN</td>
<td>217.1 100.5</td>
<td>Edited Combined</td>
<td>1.5x</td>
<td>9.62</td>
</tr>
<tr>
<td>T92-CEN</td>
<td>217.1 100.5</td>
<td>Edited Combined</td>
<td>1.5x</td>
<td>23.90</td>
</tr>
</tbody>
</table>

*0.2 mm length refers to first crack to reach 0.2 mm, as measured from the edge of the hole. 7.5 mm refers to half tip-to-tip crack length.

†TT predictions assume an initial through crack geometry, CCH predictions assume an initial corner crack growing from a hole.
Figure 8.1  Comparison of fatigue crack growth rates for constant amplitude tubular specimen data, generated in the current study, and for materials used as input to (a) UniGrow crack growth analyses and (b) FASTRAN crack growth analyses.
Figure 8.2  Schematic illustration of idealized crack profile evolution for the corner crack from hole (CCH) geometry, including the transition to a through-thickness (TT) crack.

Figure 8.3  Comparison of SIF geometry factor solutions for the tubular specimen geometry and different crack profile assumptions considered in this study. Surface half crack length, c, includes the 1.6 mm radius of the hole.
Figure 8.4  Comparison of fully-reversed constant amplitude fatigue life predictions for the different analysis programs and crack profile assumptions considered in this study. Surface half crack length, $c$, includes the radius of the hole.

Figure 8.5  Comparison of experimental and predicted crack growth curves for notched tubular specimens tested under fully-reversed constant amplitude 130 MPa nominal axial stress.
Figure 8.6  Comparison of experimental and predicted crack growth curves for notched plate specimens tested under fully-reversed constant amplitude 115 MPa nominal axial stress.

Figure 8.7  Experimental vs. predicted crack growth life from 1.8 mm to 7.5 mm half crack length for fully-reversed uniaxial constant amplitude tubular specimen tests based on TT and CCH crack geometries and (a) FASTRAN and (b) UniGrow crack growth analyses.
Figure 8.8 Variable amplitude crack growth data from full axial loading history tests (2.6× scale) in terms of (a) crack length components, (b) experimental vs. predicted crack growth, and (c) crack path.
Figure 8.9  Variable amplitude crack growth data from full axial loading history tests (2.2× scale) in terms of (a) crack length components, (b) experimental vs. predicted crack growth, (c) crack path, and (d) fracture surface appearance.
Figure 8.10 Variable amplitude crack growth data from full axial loading history tests (1.75× scale) in terms of (a) crack length components, (b) experimental vs. predicted crack growth, (c) crack path, and (d) fracture surface appearance.
Figure 8.11  Variable amplitude crack growth data from edited axial loading history tests (1.75× scale) in terms of (a) crack length components, (b) experimental vs. predicted crack growth, (c) crack path, and (d) fracture surface appearance.
Figure 8.12  Experimental vs. predicted crack growth life from 1.8 mm to 7.5 mm half crack length for variable amplitude tubular specimen tests based on TT crack geometry and (a) FASTRAN and (b) UniGrow crack growth analyses.

Figure 8.13  Experimental vs. predicted crack growth life from 1.8 mm to 7.5 mm half crack length for variable amplitude tubular specimen tests based on CCH crack geometry and (a) FASTRAN and (b) UniGrow crack growth analyses.
Figure 8.14 Variable amplitude crack growth data from full shear loading history tests (3.5× scale) in terms of (a) crack length components, (b) experimental vs. predicted crack growth, and (c) crack path.
Figure 8.15  Variable amplitude crack growth data from full shear loading history tests (2.5× scale) in terms of (a) crack length components, (b) experimental vs. predicted crack growth, (c) crack path, and (d) fracture surface appearance.
Figure 8.16  Variable amplitude crack growth data from edited shear loading history tests (2.5× scale) in terms of (a) crack length components, (b) experimental vs. predicted crack growth, (c) crack path, and (d) fracture surface appearance.
Figure 8.17 Experimental vs. predicted crack growth life from 1.8 mm to 7.5 mm half crack length for variable amplitude tubular specimen tests based on MT crack geometry and (a) FASTRAN and (b) UniGrow crack growth analyses.
Figure 8.18 Variable amplitude crack growth data from full combined loading history tests (2.0× scale) in terms of (a) crack length components, (b) experimental vs. predicted crack growth, (c) crack path, and (d) fracture surface.
Figure 8.19  Variable amplitude crack growth data from full combined loading history tests (1.5× scale) in terms of (a) crack length components, (b) experimental vs. predicted crack growth, (c) crack path, and (d) fracture surface.
Figure 8.20  Variable amplitude crack growth data from edited combined loading history tests (1.5× scale) in terms of (a) crack length components, (b) experimental vs. predicted crack growth, (c) crack path, and (d) fracture surface.
Figure 8.21  Experimental crack growth curves and crack paths for pure axial variable amplitude un-notched specimen fatigue tests performed at a scale factor of 3.1.

Figure 8.22  Experimental crack growth curves and crack paths for pure axial variable amplitude un-notched specimen fatigue tests performed at a scale factor of 2.9.
Figure 8.23  Experimental crack growth curves and crack paths for un-notched specimen (a) TM85-TRS and (b) TM107-TRS, tested under pure torsion variable amplitude loading conditions at a scale factor of 4.0.
Figure 8.24  Experimental crack growth curve and crack path for un-notched specimen TM89-CRS, tested under combined axial-torsion variable amplitude loading conditions at a scale factor of 2.5.
Chapter 9

Overall Summary and Additional Research Recommendations

The central objective of the research performed in this study was to be able to better understand and predict fatigue crack initiation and growth from stress concentrations subjected to complex service loading histories. Emphasis was placed on combining various modeling techniques for each aspect of the fatigue failure process into a single fatigue life analysis procedure which is generally applicable under any set of loading conditions. As such, major areas of focus were related to understanding and modeling material deformation behavior, fatigue damage quantification, notch effects, cycle counting, damage accumulation, and crack growth behavior under multiaxial nominal loading conditions. Since large amounts of experimental data and material properties are not always available when performing fatigue life analyses in an industry setting, an emphasis was placed on implementing and/or developing simpler modeling techniques, whenever possible, which avoid the use of empirical modeling parameters fitted to experimental data.

To accomplish this objective, experimental data and observations generated from a wide variety of material deformation and fatigue tests, along with advanced computational simulation techniques, were used to gain insight into the damage
mechanisms at work within a material for a given set of loading conditions. By doing this, it enabled various life prediction techniques to be evaluated and/or proposed based on the actual physics of fatigue failure. Although all experiments performed in this study utilized tubular and plate specimens made from 2024-T3 aluminum alloy, with and without the inclusion of a circular through thickens hole, the analyses performed were meant to be general in nature, and applicable to a wide variety of materials and component geometries. As a result, experimental data from literature were also used, when appropriate, to supplement the findings of various analyses.

The following sections outline some of the key findings of this study with respect to material deformation (Section 9.1), crack initiation (Section 9.2), and crack growth (Section 9.3) aspects of the fatigue failure process. Additionally, a final section, Section 9.4, is included to discuss areas where more research may be needed in the future to improve upon the accuracy and robustness of the fatigue life analysis procedures implemented.

9.1 Summary of Cyclic Material Deformation Aspects

Before fatigue damage can be calculated for a given set of loading conditions, it is often necessary to estimate the variation of stress and/or strain components acting at the critical analysis location(s) within a component. As a result, a cyclic plasticity model based on an Armstrong-Frederick-Chaboche style nonlinear kinematic hardening rule was implemented in order to estimate material deformation behavior, including transient hardening, under complex multiaxial loading conditions. Tanaka’s parameter was used to model the degree of non-proportionality within the material. Given the emphasis placed
on simplicity, Masing material behavior was assumed, and a new transient hardening formulation was proposed, so that all required material constants could be calculated from a relatively limited amount of experimental data.

Overall, the model was found to be fairly accurate under several complex multiaxial loading conditions, with maximum errors in any predicted stress amplitude of around 16% when considering only steady-state material response, and around 25% when including transient hardening effects. While some stress-strain prediction errors were found to be conservative (overpredictions) and some were non-conservative (underpredictions), average prediction errors were less than ±5%, and had the tendency to be slightly non-conservative, regardless of the loading history and/or the degree of hardening in the material. Although larger errors were typically found in predicted mean stresses, all axial mean stresses, which have the largest impact on fatigue damage calculation, were predicted within ±15% error, with an average error of 0.3%.

In order to estimate local notch stress-strain histories from nominally applied loading conditions, both Neuber’s rule and a pseudo stress-based plasticity modeling approach were implemented. To evaluate the accuracy of these approaches, local stress-strain predictions were compared to solutions obtained from nonlinear FEA under a variety of nominal loading conditions. Although Neuber’s rule resulted in fairly accurate local stress-strain predictions for the notched tubular specimen geometry evaluated in this study, the boundary conditions of the notch always result in a uniaxial local stress state around the perimeter of the hole. Additionally, notch estimation rules which correct for plasticity based on equivalent stress range, such as Neuber’s rule, generally cannot be applied to non-proportionally varying stress histories without the use of plasticity
modeling techniques. In addition to overcoming some of the disadvantages of Neuber’s rule, the pseudo stress-based plasticity corrections were also found to result in relatively accurate local stress-strain predictions.

While all pseudo stress-based stress-strain predictions performed were found to fall within ±20% error for the 2024-T3 notched tubular specimen geometry, all predictions were found to be within ±7% error at loading levels relevant to typical fatigue loading situations. Similarly, the majority of local stress-strain predictions for two different AISI 1141 notched shaft specimen geometries were found to be within ±15% error for typical fatigue loading levels, regardless of the local stress state. As loading levels increased, however, significant local plasticity was found to result in the overprediction of notch root stresses and strains, likely due to the redistribution of elastic-plastic stresses and strains not accounted for in linear pseudo stress distributions. Conversely, local stress-strain predictions away from the notch root had the tendency to be slightly non-conservative. For the 2024-T3 notched tubular specimen, the average error for all stress-strain predictions at a distance of 0.2 mm from the notch root was only −1.9%. The accuracy of these predictions is important because when accounting for stress gradient effects in subsequent fatigue life analyses based on the TCD point method, local stresses and strains were evaluated at a distance of 0.175 mm from the notch root.

9.2 Summary of Crack Initiation Aspects

After studying material deformation behavior, fatigue crack initiation behavior was then investigated for fatigue tests performed under constant amplitude loading conditions. Un-notched specimen tests were performed first under pure axial and pure
torsion loadings in order to generate the baseline fatigue life curves, both stress-life and strain-life, from which all subsequent fatigue life predictions were based. Tests were also performed using in-phase, 90° out-of-phase (OP), and a few other axial-torsion discriminating load paths in order to study the effects of various stress states and loading paths on fatigue damage. For these tests, classic equivalent stress- and strain-based fatigue life analysis techniques were unable to predict more than 50% of experimental lives within a factor of ±3 for all multiaxial loading conditions considered. As a result, fatigue life predictions were also performed using more robust critical plane-based analysis approaches.

Un-notched specimen life predictions based on two different versions of the tensile failure-based SWT parameter were also unable to correlate experimental data within a reasonable margin of error. A maximum of 28% of experimental lives were predicted within a factor of ±3 using either version of the parameter considered. This was somewhat expected, however, because crack initiation in the 2024-T3 aluminum alloy tested in this study was shown in surface replicas to be a shear driven process. As such, the shear-based FS parameter was found to correlate all fully-reversed multiaxial fatigue data relatively well, with up to 88% of fatigue lives predicted within a factor of ±3 of experimental results.

Experimental data from literature, however, revealed a degradation in fatigue life correlations based on the FS parameter when significant tensile mean stress and/or different ratios of applied normal and shear stress/strain components were present. As a result, a modified version of the parameter was proposed by replacing yield strength with the shear stress range (in the form of $G\Delta\gamma$) on the maximum shear plane. While fatigue
life predictions were found to improve in the presence of large tensile mean stresses and different nominal stress ratios, neither the original nor the modified FS parameter were able to completely account for differences in fatigue damage resulting from time dependent normal-shear stress/strain interaction effects.

In addition to un-notched specimen constant amplitude fatigue tests, tests were also performed under many of the same nominal loading conditions for notched tubular specimens. The primary goal of these tests was to evaluate notch effects on fatigue damage by comparing the experimental and analytical results to those generated for un-notched specimens. Notched specimen tests also allowed for the evaluation of notch stress-strain estimation models, stress gradient models, and other local life prediction techniques under a variety of nominal loading conditions.

For notched specimen fatigue life analyses, equivalent stress- and strain-based approaches were found to produce accurate life predictions so long as stress gradient effects were accounted for using fatigue notch factor in Neuber’s rule. However, this was attributed to the notched tubular specimen geometry, which results in a local uniaxial stress state regardless of the nominally applied loading conditions. Additionally, the definition of fatigue notch factor can become vague in situations involving complex multiaxial loading histories. As a result, when evaluating critical plane-based analysis approaches, multiple local stress-strain prediction and stress gradient models, including Neuber’s rule with fatigue notch factor and TCD approaches with pseudo stress-based plasticity modeling techniques, were considered. While TCD approaches resulted in the best correlation of data between different multiaxial nominal loading conditions, there
was little difference in the correlation of data based on different interpretations of the TCD approaches.

Based on the constant amplitude analysis results presented in this study, notched fatigue life predictions for the 2024-T3 specimens, based on the TCD point method \((L = 0.350)\) and both the FS and modified FS parameters, were found to agree well both with experimental results, and with predictions for un-notched specimens. Additionally, fatigue life predictions for the limited number of notched plate specimen tests were also shown to agree well with those obtained from tubular specimen analyses. In addition to producing excellent fatigue life predictions, TCD approaches also have the advantage of offering increased flexibility to be used with a variety of life prediction approaches. This is an especially desirable characteristic when performing more complex multiaxial variable amplitude fatigue life analyses.

Finally, fatigue behavior was studied for tests performed under variable amplitude axial, torsion, and combined axial-torsion loading conditions. In addition to the effects of multiaxial stress states and notches, factors such as cycle identification, damage accumulation, and load history effects must also be considered in a variable amplitude fatigue life analysis. The loading history used for these tests was derived from recorded flight test data on the lower wing skin area of a military patrol aircraft (tension-dominated), and includes a variety of take-off, landing, and in-flight maneuvers. Since the loading history was long in its entirety, a limited number of tests were also performed using edited versions of the history in order to reduce testing and analysis time, and to provide a means for evaluating the effect of smaller loading cycles on overall fatigue damage.
Similar to the constant amplitude fatigue life analyses, signed equivalent stress- and strain-based approaches were unable to predict more than 3 of 10 un-notched specimen variable amplitude fatigue lives within a factor of ±3 of experimental results. Additionally, life predictions for axial and combined axial-torsion loading conditions were consistently more non-conservative than those for torsion loading, resulting in poor correlation of the experimental fatigue data. Given the local uniaxial stress state in the notched tubular specimens investigated, however, all equivalent stress- and strain-based analyses resulted in good correlation of the variable amplitude notched fatigue data for all loading conditions considered. However, stress gradient effects were not considered in these analyses. Had stress gradient effects been accounted for, notched specimen life predictions would have been considerably more non-conservative, similar to those for un-notched specimen analyses. Overall, equivalent stress/strain approaches were unable to produce variable amplitude life predictions within the same range of accuracy as those observed under constant amplitude loading conditions.

From the critical plane-based variable amplitude fatigue life analyses, it was found that even though differences between rainflow and simplified rainflow counting techniques can result in different life predictions, this effect was negligible for the loading history considered in this study. This was attributed to the fact that only a very small percentage of cycles in the loading history were identified as half cycles based on traditional rainflow counting. Additionally, life predictions based on the assumption of stable material stress-strain response were found to be non-conservative in all cases based on both the original and modified FS parameters. Due to only a small percentage of cycles experiencing plastic deformation, however, overall improvements in life
predictions gained through the consideration of transient stress-strain behavior were negligible for the loading history investigated. While changing the crack initiation definition in un-notched specimens from final failure to a crack length of 0.2 mm resulted in a slightly larger improvement in variable amplitude fatigue life predictions, more significant improvements may have been obtained had sufficient data been available to generate a more accurate 0.2 mm life prediction curve.

Overall, the modified FS parameter was found to produce slightly more accurate fatigue life predictions than the original version of parameter for all analysis procedures investigated. This was expected based on constant amplitude analysis results, however, due to the presence of significant tensile mean stresses in the variable loading history. Additionally, experimental cracking orientations were generally found to be consistent with the critical planes predicted by both versions of the FS parameter for all loading conditions considered. This suggests that the physical basis of these parameters, along with the cycle counting and damage accumulation procedures implemented in this study, were able to capture the damage mechanisms induced by these complex loading histories relatively well.

Similar to results from the un-notched specimen analyses, notched specimen variable amplitude fatigue life predictions based on the assumption of stable material stress-strain response were found to be non-conservative, in all cases, based on both the original and modified FS parameters. Notched specimen predictions, however, were non-conservative to a slightly lesser extent. Additionally, when considering the effects of transient material deformation behavior on fatigue damage, it was again shown to have a relatively small influence on life prediction accuracy. Changing the crack initiation
definition from final failure to a crack length of 0.2 mm, however, did have a more significant impact on notched specimen life predictions. This was due to the fact that unlike the un-notched specimens, where experimental lives were recalculated to match the different crack initiation definitions, all notched specimen analyses were based on the same experimental fatigue lives determined using the 0.2 mm crack initiation definition.

Overall, when accounting for the effects of transient stress-strain response and crack initiation definition on fatigue damage, nearly all (22 of 25) un-notched and notched specimen variable amplitude fatigue lives were predicted within a factor of ±3 of experimental results based on the modified FS parameter. Although this is a relatively high level of accuracy, when using similar analysis procedures, even the best variable amplitude life predictions still had the tendency to be more non-conservative than those computed for the constant amplitude loading conditions investigated in this study.

While there are numerous factors that could have contributed to the non-conservative nature of these predictions, it was determined that some of the error in the variable amplitude fatigue life analyses may have been the result of inaccuracies in life prediction curves. This is true both for the 0.2 mm crack initiation definition, and for final failure curves in the high cycle fatigue regime. Additionally, the modeling of material deformation behavior was identified as another potential source for life prediction errors. This was due to the fact that results from deformation analyses showed slightly non-conservative trends in stress-strain predictions for both the un-notched and notched specimens tested in this study. Finally, some other areas for potential improvements were identified as being related to the consideration of normal-shear stress/strain interaction effects on fatigue damage, transient deformation modeling at notches, and linear versus
nonlinear damage accumulation. Although changing the critical damage sum used with the linear damage rule from 1 to 0.3 considerably improved the accuracy of variable amplitude fatigue life predictions, the correlation of data from different loading conditions became slightly worse. Therefore, improvements are most likely needed in several of the aforementioned areas in order to further increase the accuracy and robustness of the fatigue life prediction process under general multiaxial variable amplitude loading conditions.

9.3 Summary of Crack Growth Aspects

Fatigue crack growth often represents a significant portion of a component’s total fatigue life. Therefore, a fundamental understanding of the mechanisms that govern crack propagation and the development of accurate crack growth modeling techniques are essential to a complete fatigue life analysis. As a result, crack growth was monitored and recorded for all fatigue tests performed in this study. Based on the experimental data generated from these tests, crack growth behavior was then analyzed for both un-notched and notched specimens subjected to both constant and variable amplitude loading conditions. Similar to observations reported in literature, constant amplitude crack growth in notched specimens was observed to be a primarily mode I process, while cracks in un-notched specimens were observed to propagate on maximum shear planes, maximum tensile planes, or a combination of both. Since being able to accurately predict the orientation of a growing crack is an essential step in computing crack driving forces, this behavior was a topic of much interest in this study.
Specialized tests performed using precracked tubular specimens indicated that the preferred crack growth mode in un-notched specimens was dependent on the applied shear stress magnitude, as well as the stress normal to the crack plane. This suggested a significant role of friction and roughness induced closure effects in the shear-mode crack growth process. As a result, a simple model was proposed to account for these effects based on the idea that crack face interaction reduces the effective SIF by allowing a portion of the nominally applied loading to be transferred through a crack interface.

Crack path/branching predictions based on the proposed model were shown to agree relatively well, both qualitatively and quantitatively, with experimentally observed trends for all loading conditions considered in this study. Additionally, crack growth life predictions and the correlation of crack growth rate data for all loading conditions investigated were significantly improved based on effective SIF ranges calculated using the proposed model, as compared to those based on nominally applied SIF ranges. Crack branching predictions were also found to agree with experimental crack paths reported in literature for a carbon steel alloy subjected to in-phase axial-torsion loading.

For notched specimen constant amplitude fatigue tests, although crack growth for all loading conditions was observed to be a mode I-dominated process, crack growth rates under multiaxial nominal stress states were observed to be higher than those for uniaxial loading at the same SIF range. For specimens subjected to in-phase axial-torsion loading, T-stress corrections were able to effectively move all crack growth rate data to where they agreed very well with those measured under uniaxial loading conditions. For crack growth data generated under nominal torsion loading, however, while T-stress corrections improved data correlations, growth rates still had the tendency to be higher than those for
uniaxial loading at the same SIF range. Some of this error, however, may have been the result of interaction effects from multiple pairs of cracks growing from the hole.

For 90° OP loading conditions, crack growth behavior was more complex. For these tests, the growth of smaller cracks appeared to have been dominated by the mode I loading from the axial component of the applied stress. As such, these data were correlated well with uniaxial crack growth rates based on mode I SIF range. However, at longer crack lengths and higher growth rates, cracks turned, and mode I SIF range alone was unable to successfully correlate the crack growth rate data. This suggested an increasing mixed-mode growth effect from the applied shear stress component as the cracks grew.

Additionally, it was determined that increased scatter observed in notched specimen data at lower crack growth rates was likely due to factors such as initial crack geometry, small crack growth behavior, and/or notch effects, which can all lack consistency in naturally initiated fatigue cracks. As a result, using short crack models to account for stress concentration and initial crack geometry effects on crack growth from notches was found to improve growth rate correlations.

For variable amplitude crack growth, given the complexity of such an analysis, two state-of-the-art crack growth models, UniGrow and FASTRAN, were used to predict crack growth behavior for the notched specimens tested in this study. UniGrow is based on the idea that residual stress distributions surrounding the crack tip are responsible for causing load sequence effects in variable amplitude crack growth, while FASTRAN attributes these effects to varying degrees of plasticity induced closure in the crack wake. Variable amplitude crack growth trends for multiaxial nominal loadings were then
compared to those for constant amplitude loading conditions in order to help interpret the analysis results. Additionally, since fatigue cracks were initiated naturally at the notch, the effect of initial crack geometry assumptions on crack growth predictions was also investigated.

Differences in initial crack geometry assumptions (i.e. corner crack versus through crack) were found to produce fairly significant differences in predicted crack growth life, ranging mostly between a factor of 2 and 3 for the notched specimen geometry tested in this study. Additionally, while both the UniGrow and FASTRAN crack growth models were able to predict nearly all uniaxial constant amplitude crack growth lives within a factor of ±3, regardless of crack geometry assumptions, both programs were found to produce conservative crack growth predictions under uniaxial variable amplitude loading conditions. FASTRAN, however, resulted in slightly more accurate results. For variable amplitude torsion and combined axial-torsion crack growth analyses, the degree of conservatism in crack growth predictions was found to reduce. This was attributed to an increase in experimental crack growth rates due to multiaxial nominal stress state effects such as the presence of T-stress and mixed-mode crack growth. These effects were also seen in constant amplitude crack growth rate correlations, but are not accounted for in either UniGrow or FASTRAN.

Additionally, while at least 94% of cycles were removed, the largest average difference in experimental crack growth life between tests performed using full and edited versions of the same loading history was less than a factor of 5. This gives insight into the effect of smaller cycles on variable amplitude crack growth behavior, as well as their implications with respect to load history editing for the purpose of accelerated
fatigue testing. By comparing differences in experimental crack growth life between full
and edited history tests to the differences predicted by each crack growth model, it was
found that FASTRAN is generally better able to account for changes in crack growth
behavior due to the effects of small cycles and/or changes in loading history profile.

Variable amplitude crack growth analyses were not performed for the un-notched
specimen tests performed in this study due to the fact that crack growth conditions often
violated fundamental assumptions of the crack growth models employed in this study.
However, based on the experimental results for these tests, it was found that the majority
of the fatigue life of these specimens was spent in the nucleation and growth of cracks up
to a length of around 1 to 2 mm (2 to 4 mm tip-to-tip crack length). Once cracks reached
a few mm in length, crack growth and final failure occurred very rapidly, typically due to
the application of a large overload cycle. Additionally, the experimental results generated
from these tests can offer valuable crack growth data if future investigations into mixed-
mode variable amplitude crack growth are to be performed.

9.4 Additional Research Recommendations

Based on the analysis and discussions presented in this study, there are a few
areas where it is clear that improvements could be made with respect to the fatigue life
prediction procedures implemented herein. Included in these is the quantification of
fatigue damage for loading paths containing different time dependent interactions of
normal and shear stress/strain components. In most multiaxial fatigue damage
parameters, time dependent effects are typically ignored by only considering the
amplitude and/or maximum values of various stress and/or strain components over the
duration of a complete loading cycle. As discussed previously, however, this can be especially problematic under variable amplitude loading conditions, where single cycles can be identified over large portions of a loading history.

One possible approach to account for these interactions would be to develop a time dependent damage parameter. Such a parameter would allow a fatigue damage/driving force value to be computed independently, from individual stress and strain components on a desired plane(s), for each time point in a loading history. This way, traditional uniaxial cycle counting techniques could be applied directly to the variation of this single quantity, without the need to track auxiliary stress/strain values. As a result, this would also eliminate the problem where the identification of half cycles versus whole cycles could potentially cause differences in the cumulative fatigue damage calculated in a variable amplitude fatigue life analysis. Such a parameter would be similar in concept to the signed von Mises stress/strain quantities used for damage calculation in Section 6.3, but should also incorporate critical plane concepts to overcome the deficiencies of an equivalent stress/strain-based analysis.

One advantage of using stress-strain amplitudes and/or maximum values “per cycle” in many current damage parameters is that by ignoring any phase difference that may exist between different loading components, these terms help to account for an increase in fatigue damage caused by non-proportional loading paths. While this same type of effect may not be possible with a time dependent parameter, it is worth noting that multiple smaller “damage cycles” could potentially be identified within a single nominally applied non-proportional loading cycle. Therefore, the sum of damage from
these smaller cycles could potentially account for the increase in fatigue damage caused by the single nominal cycle.

Care must also be taken to ensure that such a damage parameter maintains good correlation of fatigue data under simple proportional loading paths, e.g. uniaxial and pure torsion, as well. For example, a time dependent evaluation of the FS parameter would yield the same fatigue damage value as the traditional parameter for pure torsion loading, but not for uniaxial loading. Under fully-reversed uniaxial loading conditions, the equal but opposite effect of the normal stress term for tension and compression portions of the cycle would result in a damage “amplitude” that is simply equal to the maximum shear strain amplitude. This is not desirable, however, since the FS parameter has been shown to produce good correlation of multiaxial fatigue data based on its traditional formulation. As a result, in depth analysis and experimental verification would be required to address such issues in future research.

Another worthwhile issue that could be addressed is the combined effect of multiple shear stress/strain components acting on a single plane. Since the magnitude of these two components cannot be used for cycle counting because of the sign problem this would create, their combined effect is usually ignored in critical plane-based fatigue life analyses. Instead, individual strain components are cycle counted with the assumption that one component is dominant to the other. For non-proportional variable amplitude loading histories, however, not considering the combined effect of both shear components could lead to a potential underestimation of fatigue damage and result in non-conservative life predictions. The impact of this effect could be studied in the future through experimental testing involving short non-proportional block loading histories.
which result in significant variations of both shear stress/strain components on the critical planes within the specimen. Experimental results could then be compared, in terms of both fatigue life and failure orientation, to predictions based on both the assumption of a single dominant shear component, and based on the combined effect of both shear components, to see which analysis method yields more accurate predictions.

Aside from fatigue damage calculation, additional work could also be performed in developing more accurate methods for estimating transient local deformation behavior in notched components. As previously discussed, it is difficult to apply transient plasticity modeling techniques using the pseudo stress-based notch stress-strain estimation procedures implemented in this study. This is due to fact that actual notch deformation behavior likely falls somewhere between that which would be calculated based on local load/stress-controlled conditions, and that which would be calculated based on local strain-controlled conditions. As such, transient hardening behavior, as well as residual stress development for unbalanced loading histories, may not be properly accounted for in the resulting stress-strain output. Obtaining experimental notch deformation data to verify different modeling techniques, however, would help to improve such predictions. With the development of more advanced digital image correlation systems, using such techniques to obtain accurate local stress-strain distributions from experiments is becoming a more feasible option. Additionally, there is also the possibility to study transient local deformation behavior through nonlinear FEA. Long computation times and limitations in transient material modeling capabilities, however, may prove prohibitive for such analyses.
Also related to transient deformation behavior, the variable amplitude loading history used in this study contained significant mean stresses in both loading channels. Therefore, most of the plastic deformation experienced in both the un-notched and notched specimen tests was non-reversed, and only occurred in a very small number of cycles. Because of this, transient material hardening was not a significant factor in these tests, and incorporating such behavior into the fatigue life analysis process did not have a significant impact on life predictions. As a result, it may be beneficial to perform additional variable amplitude fatigue tests in the future which utilize a loading history containing significant cyclic plastic deformation. This way, more meaningful comparisons could be made to study the effect of transient deformation behavior on fatigue life predictions.

With respect to fatigue damage accumulation, it was shown that changing the critical damage sum at failure, as suggested in literature, resulted in a substantial improvement in variable amplitude fatigue life predictions based on the linear damage rule. However, since the basis for this change is empirical in nature, it is difficult to determine whether or not these improvements were due to the actual damage accumulation behavior experienced in these tests, or just a result of canceling out the effects of other factors which were improperly accounted for in the life predictions. Therefore, more in depth studies could be performed in the future to try and quantify the effects of load history dependence on damage accumulation. Ideally, fatigue tests for such a study would be performed under fully-reversed linear elastic proportional loading conditions, and with a consistent and sufficiently small crack initiation definition. This way, transient deformation behavior, mean stress, time dependent normal-shear
stress/strain interaction, and crack growth effects would not have an influence on the experimental results. Then, different variable amplitude loading history profiles could be investigated, with respect to their impact on the critical damage sum predicted at failure, in order to evaluate the effectiveness of various damage accumulation procedures.

Finally, from a crack growth perspective, the most challenging issues encountered in this study involved situations where different multiaxial stress states had different influences on the resulting crack growth behavior. This was true for both mixed-mode crack growth in un-notched specimens, and for mode I crack growth in notched specimens. Given the frequent observance of mode I crack growth, especially in notched components, it is clear that more work is needed in developing or evaluating more robust models to account for the effects of multiaxial stress states in such analyses. This is true with respect to the presence of T-stress, varying degrees of mixed-mode crack growth due to non-proportional loading conditions, and any changes that may occur in crack closure mechanisms under such conditions. Developing an equivalent crack driving force model/parameter which can account for these factors, or verifying the applicability of more advanced existing parameters (e.g. J-integral), would be of great benefit for a variety of practical applications.
References


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Shamsaei, N., 2010. Multiaxial Fatigue and Deformation Including Non-proportional Hardening and Variable Amplitude Loading Effects (Ph.D. Dissertation). The University of Toledo, Toledo, OH.


Sonsino, C.M., Franz, R., 2016. Multiaxial fatigue of cast aluminium EN AC-42000 T6 (G-ALSi7Mg0.3 T6) for automotive safety components under constant and variable amplitude loading, in: Proceedings of the 11th International Conference on Multiaxial Fatigue and Fracture. Seville, Spain.


Appendix A

Description of Curvature Correction Routine for Image-Based Crack Length Measurements

Figure A.1  Schematic of curvature correction routine.

Step 1. From the crack image, measure the distance \( x \), in the specimen circumferential direction, from the center of the specimen to each crack tip.

Step 2. For each crack tip, calculate angle \( \alpha \) from \( x \) and specimen outer radius, \( R \):
\[ \alpha = \sin^{-1}(x/R) \]

Step 3. Calculate actual crack length in specimen circumferential direction, \( l \), as the arc length from point A to point B: \( l = aR \)

Step 4. Compute tip-to-tip crack length using curvature corrected \( l \) value for each crack half, along with length components in the specimen axial direction.
Appendix B

Stress Intensity Factor Solutions used in Notched Specimen Crack Growth Analyses

Figure B.1 SIF solution for a circumferential through crack growing in a tubular section (TT). This solution, shown as reported in (Miedlar et al., 2000), was formulated by Forman et al. (1985).
Figure B.2  SIF solution for diametrically opposed corner cracks growing from a hole (CCH). This solution, shown as reported in (Gallagher et al., 1984), was formulated by Newman Jr. and Raju (1981).
Appendix C

Sample Input and Output Files from Variable Amplitude Crack Growth Analyses

-FASTRAN version 5.42 Sample Input File-

AxialEdit1.75x_0.2_TT
AxialEdit_1.0x(psi)_nfopt8.txt
2024-T3
52.0  72.0  10442.8  0.0  2.0  1.0  1.0  1.0  0
1  0  0.0
1.8E-8  3.75  0.68  0.0  9999  2.0  2.0  243.0  1.0  0
8  0
0.728  3.94E-10
0.956  3.94E-9
1.866  7.87E-8
3.640  3.15E-7
7.008  3.94E-6
12.29  3.94E-5
20.93  3.94E-4
32.76  0.00394
3.94E-6  2.0  1.0  1.0  9.84E-5  1.0  1.0  1.0
0  0  1  0  5.0E-4
99  0  0  2  8  0  0  1
3.3465  0.07087  0.07087  0.07087  0.07087  0.07087  0.000  0.000  0
50
0  1
0.00235  1.00029
0.00705  1.00262
0.00941  1.00466
0.01411  1.01045
0.01647  1.01420
0.02117  1.02339
0.02352  1.02883
0.02823  1.04136
0.03058  1.04846
0.03529  1.06427
0.03764  1.07298
0.04000  1.08223
0.04235  1.09200
0.04470  1.10230
0.04705  1.11311
0.04941  1.12444
0.05176  1.13627
0.05411  1.14860
0.05647  1.16143
0.05882  1.17475
0.06117  1.18706
0.06352  1.19141
0.06588  1.19619
0.06823  1.20138
0.07058  1.20697
0.07294  1.21292
0.07529  1.21924
0.07764  1.22590
0.08000  1.23292
HALT

-FASTRAN version 5.42 Sample Output File-

*******************************************************************************
********  ***       ****  ********  ***      ***
********  *****    ***    ********  *****    *****   **    **
**      **   **  **         **    **   **  **   **  ***   **
**      **   **  **         **    **   **  **   **  ****  **
*****   *******   ***
**    ******   *******  ***** **
****    *******     ***     **    *****    *******  ** ****
**      **   **       **    **    ** **    **   **  **  ****
**      **   **       **    **    **  **   **   **  **   ***
*      **   **     ***     **    **   **  **   **  **    **
**      **   **  ****       **    **    ** **   **  **     *  5.42
*******************************************************************************

AxialEdit1.75x_0.2_TT

SPECTRUM FILE = AxialEdit_1.0x(psi)_nfopt8.txt

MATERIAL PROPERTIES:2024-T3
YIELD STRESS = 0.5200E+02     ULTIMATE STRENGTH = 0.7200E+02
ELASTIC MODULUS = 0.1044E+05

PLANE-STRAIN-TO-PLANE-STRESS SOLUTION:
(ALP = ALP1 TO ALP2; BETA = BETAT1 TO BETAT2; BETA = BETAW1 TO BETAW2)

DKEFF IS ELASTIC:
CRACK GROWTH RATES FROM TABLE LOOKUP (NDKTH = 0):

DKEFF           RATE
0.7280E+00      0.3940E-09
0.9560E+00      0.3940E-08
0.1866E+01      0.7870E-07
0.3640E+01      0.3150E-06
0.7008E+01      0.3940E-05

0.08235     1.24027
0.08470     1.24796
0.08705     1.25598
0.08941     1.26433
0.09176     1.27303
0.09411     1.28205
0.09647     1.29142
0.09882     1.30113
0.10117     1.31119
0.10352     1.32159
0.10588     1.33236
0.10823     1.34348
0.11058     1.35497
0.11294     1.36684
0.11529     1.37909
0.11764     1.39172
0.17647     1.85900
0.29411     4.42064
0.41176     17.1669

1     1000
25
2
0
0
12    12    0    12    0    0
36.741
0     0     0     0     0

HALT
0.1229E+02    0.3940E-04
0.2093E+02    0.3940E-03
0.3276E+02    0.3940E-02

THRESHOLD EQUATION:  \( D_{K0} = C_3(1+C_4R) \) for \( +R \) &  \( D_{K0} = C_3 \) for \( -R \)

THRESHOLD CONSTANTS:  \( C_3 = 0.68 \)  \( C_4 = 0.00 \)

FRACTURE TOUGHNESS (TPFC) PROPERTIES:  \( K_f = 0.243E+03 \)  \( m = 1.00 \)
(Note: \( C_6 \) and \( C_7 \) are not used)

RATE AT START OF TRANSITION = 0.394E-05
WITH  \( \alpha_1 = 2.00 \)  \( \beta_1 = 1.00 \)  \( \omega_1 = 1.00 \)

RATE AT END OF TRANSITION = 0.984E-04
WITH  \( \alpha_2 = 1.00 \)  \( \beta_2 = 1.00 \)  \( \omega_2 = 1.00 \)

USER SPECIFIED CRACK CONFIGURATION:  \( NTYP = 99 \)
STRESS-INTENSITY FACTOR FROM USER INPUT TABLE:

\[
\begin{array}{cc}
\text{c/w} & \text{F(c/w)} \\
0.00000 & 0.1000E-01 \\
0.00235 & 0.1000E-01 \\
0.00706 & 0.1003E-01 \\
0.00941 & 0.1005E-01 \\
0.01412 & 0.1010E-01 \\
0.01647 & 0.1014E-01 \\
0.02118 & 0.1023E-01 \\
0.02353 & 0.1029E-01 \\
0.02824 & 0.1041E-01 \\
0.03059 & 0.1048E-01 \\
0.03529 & 0.1064E-01 \\
0.03765 & 0.1073E-01 \\
0.04000 & 0.1082E-01 \\
0.04235 & 0.1092E-01 \\
0.04471 & 0.1102E-01 \\
0.04706 & 0.1113E-01 \\
0.04941 & 0.1124E-01 \\
0.05176 & 0.1136E-01 \\
0.05412 & 0.1149E-01 \\
0.05647 & 0.1161E-01 \\
0.05882 & 0.1175E-01 \\
0.06118 & 0.1187E-01 \\
0.06353 & 0.1191E-01 \\
0.06588 & 0.1196E-01 \\
0.06824 & 0.1201E-01 \\
0.07059 & 0.1207E-01 \\
0.07294 & 0.1213E-01 \\
0.07529 & 0.1219E-01 \\
0.07765 & 0.1226E-01 \\
0.08000 & 0.1233E-01 \\
0.08235 & 0.1240E-01 \\
0.08471 & 0.1248E-01 \\
0.08706 & 0.1256E-01 \\
0.08941 & 0.1264E-01 \\
0.09176 & 0.1273E-01 \\
0.09412 & 0.1282E-01 \\
0.09647 & 0.1291E-01 \\
0.09882 & 0.1301E-01 \\
0.10118 & 0.1311E-01 \\
0.10353 & 0.1322E-01 \\
0.10588 & 0.1332E-01 \\
0.10824 & 0.1343E-01 \\
0.11059 & 0.1355E-01 \\
0.11294 & 0.1367E-01 \\
\end{array}
\]
SPECIMEN WIDTH = 0.335E+01     THICKNESS = 0.709E-01
INITIAL CRACK LENGTH (CI) = 0.70870E-01  INITIAL CRACK DEPTH = 0.70870E-01
NOTCH LENGTH (CN) = 0.70870E-01   NOTCH DEPTH = 0.70870E-01
NOTCH HEIGHT = 0.000E+00  ELEMENTS ON HOLE AND/OR STARTER NOTCH (NS) = 1
FINAL CRACK LENGTH (CF) REQUESTED = 0.10000E+01

PROGRAM OPTIONS:  LFAST = 0   KOPEN = 1     KCONST = 0    INVERT = 0    LSTEP = 1
ERR = 0.62E+00  PDC = 0.05  NMAX = 1  NIPT = 0  NPRT = 0  NDKE = 0
DCPR = 0.500E-03
BLOCK (OR FLIGHT) LOADING:  NFOPT = 8     LPRINT = 0     MAXLPR = 12
TOTAL NUMBER OF BLOCKS (OR) FLIGHTS TO BE REPEATED = 12
NUMBER OF DIFFERENT BLOCKS (OR FLIGHTS) = 12
SPECTRUM LOAD SEQUENCE (NFOPT = 8):   MAX STRESS = 0.3674E+02

NOTE: KO/KMAX AND "ELASTIC" SIF BASED ON HIGHEST AND LOWEST APPLIED STRESS IN SEQUENCE

BLOCK   C_crack     A_crack     CYCLES  ALP  KO/KMAX  DKC   DC/DN      DKA   DA/DN
1  0.71370E+01 0.70870E-01       131 2.00  0.026  22.37 0.773E-05   0.00 0.000E+00
2  0.71870E+01 0.70870E-01       256 2.00  0.177  22.45 0.684E-05   0.00 0.000E+00
3  0.72370E+01 0.70870E-01       377 1.94  0.060  22.54 0.144E-04   0.00 0.000E+00
4  0.72870E+01 0.70870E-01       492 2.00  0.072  22.62 0.144E-05   0.00 0.000E+00
5  0.73370E+01 0.70870E-01       677 1.78  0.123  22.71 0.136E-04   0.00 0.000E+00
... [Some results removed for brevity]
16  0.61206E+00 0.70870E-01      36792 1.00  0.180 108.08 0.490E-01   0.00 0.000E+00 *

SPECIMEN FAILED:
CRACK LENGTH = 0.6121E+00  CRACK DEPTH = 0.7087E-01 TOTAL CYCLES = 36792
NFCODE = 6:  CRACK LENGTH PLUS PLASTIC ZONE EXCEEDS WIDTH
CPU (OR CLOCK) TIME = 9.7 SECONDS

*******************************************************************************
-UniGrow version 2014-02-09 Sample Output File-

Name: AxialEdit1.75x_0.2_TT
Type: Fatigue Crack Growth
Path: 
Date: 2016.01.07 08:05:54
Material:
Category: Aluminum
Name: Al 2024-T351

E: 73100 [MPa]
K: 662 [MPa]
n: 0.07
r: 1.25354e-05 [m]
v: 0.33
Sys: 428 [MPa]
p: 0.065421

Number of FCG Pairs: 4
FCG1: C1: 9.72308e-15, m1: 17.262 [SI]
FCG2: C2: 4.36048e-10, m2: 2.228 [SI]
FCG3: C3: 6.40123e-13, m3: 7.6208 [SI]
FCG4: C4: 2.51458e-10, m4: 3.694 [SI]

Load:
Name: Ae_N0deg_1.75x
Path:
Scale: 1
Type: File-Based
Number of points: 57227
Min: -64.7978 [MPa]
Max: 253.3 [MPa]

Geometry:
ID: geom_custom_y
Name: Custom SIF - Custom Y
Dimensions: 1
Width: 0.085 [m]
Initial Crack Length (first dimension): 0.0018 [m]
Final Crack Length (first dimension): 0.03 [m]
Maximum number of cycles: 0

Y Distribution:
Name: Circumferential Through Crack NG tube
Path:
Number of points: 47
Is Normalized: yes

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<th>X</th>
<th>Y</th>
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<td>1.00066</td>
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<tr>
<td>0.00823</td>
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0.0741176    1.21604
0.0764706    1.22253
0.0788235    1.22937
0.0811765    1.23655
0.0835294    1.24407
0.0858824    1.25193
0.0882353    1.26012
0.0905882    1.26864
0.0929412    1.2775
0.0952941    1.2867
0.0976471    1.29624
0.1          1.30612
0.102353     1.31635
0.104706     1.32694
0.107059     1.33788
0.109412     1.34919
0.111765     1.36086
0.114118     1.37292
0.116471     1.38536
0.14117      1.54111
0.23529      2.74774
0.29411      4.42064
0.35294      7.9048
0.41176      17.1669
1            1000

Result:
Dimensions: 1
Number of values:968

VALUES:

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... [Some results removed for brevity]...

9630       0.0147745   -20.1525  48.2656  48.7485  91.0926
9640       0.0150918   -21.9101  49.5767  50.1422  100.559
9650       0.01541     -20.6762  54.5544  39.6939  124.072
9660       0.0157481   -18.6989  61.2568  52.0484  136.298
9666       0.03         0       373.48   223.628  148.71
End