2015

Crack propagation analysis of a pre-stressed L-shaped spandrel parking garage beam

Seyedowjan Hashtroodi
University of Toledo

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entitled

Crack Propagation Analysis of a Pre-stressed L-shaped Spandrel Parking Garage Beam

by

Seyedowjan Hashtroodi

Submitted to the Graduate Faculty as partial fulfillment of the requirements for the

Master of Science Degree in Civil Engineering

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May 2015
Recently, a pre-stressed spandrel L beam of a parking garage at Northwest Ohio failed. Upon visiting the site and inspecting the beam, it was illustrated that the beam had experienced a sudden, brittle failure. This was concluded due to the fact that there was only one dominant crack which had propagated through the depth of the beam at the mid-span. There was no sign of any other equivalently distributed micro-cracks, which would normally form in ductile structures prior to failure.

In order to further investigate and understand the sudden failure of the pre-stressed spandrel L-shaped parking garage beam, it was decided to perform analytical and numerical analysis of the beam. Primary, hand calculations was performed by use of MathCAD to compute ultimate moment capacity and the cracking moment of the beam based on PCI and ACI 318 codes. According to ACI 318-11, if the ratio of ultimate moment capacity over cracking moment exceeds 1.2, then the structure should be ductile enough to undergo considerable deflection before failure. Moreover, in a ductile member, equivalently distributed cracks visible to naked eye would warn when the member’s
nominal strength is approached, so that immediate occupancy and safety regulations could be applied.

Secondly, Response2000 was used for cross-sectional analysis of the beam and validation of hand calculation results by MathCAD in order to conduct the rest of the study in a more efficient, less time consuming manner. Moreover, a few damage scenarios have been proposed for the beam; in which, a number of strands have been considered fully corroded by removing them from the model. For each scenario, using Response2000, the ultimate moment capacity over cracking moment ratio have been calculated to figure out the exact number of corroded strands needed for the beam to experience brittle failure. Crack patterns throughout the length of the beam and the existence of a dominant macro-crack was also studied and validated.

Finally, after conducting extensive literature review, Abaqus software was chosen for simulation purposes of the crack propagation along the depth of the beam. A three-dimensional finite element model of the pre-stressed parking garage beam was created by use of SolidWorks and Abaqus software. One initial crack was defined at mid-span of the beam by modelling a cohesive segment surface which would allow the crack to open up and propagate through the depth of the beam when the stress exceeds the corresponding limit.
Acknowledgements

First and foremost, I would like to thank my precious wife, Dr. Asma Sharafi for her support, encouragement and endless unwavering love during my research and studies which provided me with the bedrock upon which I could rise. I thank my parents, Abdolvahab and Mina, for their faith in me and providing me with their infinite love and support my entire life. It was under their guidance and supervision that I gained so much drive and determination to overcome any challenge in my life.

Personally, I would like to thank my advisor Dr. Douglas K. Nims for his patience, motivation, immense knowledge and continuous support throughout my study and research. I could not have imagined having a better advisor than him.

I would like to thank Dr. Mark A. Pickett and Dr. Liangbo Hu for honoring me and accepting to be on my thesis committee member.

To all my friends, thank you for your understanding and encouragement in my many, many moments of crisis. Your friendship makes my life a wonderful experience. I cannot list all the names here, but you are always on my mind.
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Chapter 1

Introduction

1.1. Problem Statement

Recently, a pre-stressed spandrel L beam of a parking garage in Northwest Ohio failed. After visiting the parking garage and inspecting the failed beam, it was concluded that the failure of the beam was due to corrosion of the steel strands embedded in the beam which was not visible to naked eye until the failure occurred. There are many other similar beams in parking garages of the same age and construction at Northwest Ohio that may suffer from the same phenomenon.

Upon visiting the site and inspecting the beam, it was concluded that the beam had experienced a sudden, brittle failure. This was determined due to the fact that there was only one dominant crack which had propagated through the depth of the beam at the mid-span. There was no sign of any other equivalently distributed micro-cracks, which would normally form in ductile structures.

The sudden brittle failure of the beam, with no prior warning is the problem which should be studied as it may as well be a severe safety concern. This brittle behavior of the beam could be due to various reasons. The ratio of the ultimate moment capacity over the cracking moment of the initial design of the beam needed to be calculated and compared
with the 1.2 threshold recommended by ACI 318-11. Moreover, the parking garage was constructed more than fifty years ago, during which it was exposed to harsh environmental conditions. Broken tendons and signs of corrosion was observed while visiting the failed beam on-site. Therefore, the need to study and analyze the beam by considering one, two or more corroded tendons became of interest, as the initial design of the beam could have been ductile enough, however, fifty years of harsh environmental conditions and strand corrosion may have weakened and decreased the ultimate moment capacity of the beam. Therefore, it was decided to study this matter if more depth by studying the effect of cross sectional area loss of steel strands due to corrosion on ultimate flexural strength of the beam, cracking moment, the corresponding ratio and crack patterns of the beam.

1.1. Failed Pre-stressed Beam
1.2. Potential Solution

Magnetic Inspection of pre-stressing strands of the spandrel parking garage beams was proposed to the University of Toledo to access hidden corrosion which led to failure of one of the beams. Magnetic inspection of the beams would enable us to estimate the section loss of the embedded strands due to corrosion. Therefore, evaluation of the remaining strength of the beams and if necessary, the extent of the rehabilitation would be feasible. However, due to severe weather conditions in winter and time conflict, magnetic inspection could not be arranged and it was postponed for the time being.

After magnetic inspection of the beams was put on hold and the chance of performing a field experiment was gone, it was decided to apply numerical analysis on the failed beam to better understand the reason for such brittle and sudden failure of the beam.
1.3. **Research Approach**

In order to further investigate and understand the sudden failure of the pre-stressed spandrel L-shaped parking garage beam, it was decided to perform analytical and numerical analysis of the beam. It was decided to perform a crack propagation analysis by a finite element analysis.

After literature review, ABAQUS was chosen for its specific analysis capabilities regarding crack growth. Moreover, hand calculations was performed by use of MathCAD to calculate ultimate moment capacity and the cracking moment of the beam. According to ACI 318-11, if ultimate moment capacity over cracking moment ratio exceeds 1.2, then the structure should be ductile enough to undergo considerable deflection prior to failure. Moreover, a few damage scenarios have been considered in which, a number of strands have been considered corroded. For each scenario, the ultimate moment capacity over cracking moment ratio have been calculated to figure out the exact number of corroded strands needed for the beam to experience brittle failure.

Cross-sectional analysis was performed by Response2000 software and the results were evaluated and later validated by hand calculations. Crack patterns throughout the length of the beam and the existence of a dominant macro-crack was also studied.
Chapter 2

Concrete Deterioration and Steel Corrosion

2.1. Introduction

This chapter focuses on causes of concrete deterioration and the effects that various deterioration mechanisms have on structural performance, behavior and life span of reinforced concrete members. Moreover, reasons behind corrosion initiation in steel reinforced concrete and corresponding affects have been discussed [2].

2.2. Concrete Deterioration

Deterioration in reinforced concrete typically happens when the structure is exposed to weather, water or other chemicals over an extended period of time [3]. However, high original concrete quality, proper construction and protection from these elements, affects the rate at which the pre-stressed concrete deteriorates so that it could last for decades with minimal annually maintenance [3]. However, many structures are exposed to harsh environmental conditions, such as multi-story parking garages, bridges and etc. [3]. Once the deterioration has begun, it is very hard to control and it will propagate to embedded steel reinforcement which could lead to reduction in ultimate strength of the structure. Therefore, it is essential for a concrete structure to be maintained properly [4].
2.2.1. Mixture Design of Concrete

The quality of mixture design of concrete plays a significant role in affecting the concrete deterioration rate. Low quality concrete results in significant cracking, which lets moisture and other harmful chemicals to penetrate and accelerate the deterioration process [3]. Concrete components and the chemical reactions between them are also considered to be one of the key contributors to its internal degradation. Shrinkage of concrete at early stages and thermal restraints can cause cracking of the concrete which ultimately would have impact on durability and lead to a reduced life span [3].

Moreover, undesired impurities in concrete as a result of chemical reactions may also be a severe cause of concrete deterioration. Therefore, quality control of concrete mix
and using a higher compressive strength, which would lead to higher tensile strength of the concrete, shall result in concrete cracking to occur at more severe loads and conditions which would extend the intended service life [4].

2.2.2. Carbonation

Carbonation of concrete is one of the main causes of corrosion in steel reinforcement. Carbon dioxide from the air penetrates into the concrete through the pores and reacts with the hydroxides in the concrete, such as calcium hydroxide and generates calcium carbonate. Following equation represents the carbonation of concrete in atmosphere [5].

\[
Ca(OH)_2 + CO_2 \rightarrow CaCO_3 + H_2O
\]

The precipitation of calcium carbonate as shown above, reduces the pH level of the concrete, which ultimately would enable the corrosion reaction of steel when the pH in the concrete falls below 9. The carbonation process starts from the surface of the concrete and slowly infiltrates into the concrete. However, carbonation will not occur in fully submerged concrete, though fully submerging concrete is not advised. Even though it prevents carbonation it causes moisture to penetrate and cause corrosion of steel reinforcement [3]. Moreover, carbonation increases the drying shrinkage in concrete, thereby initiating cracking, which leads to harmful chemicals and other substances to penetrate in concrete [2]. Low cement contents, high water-cement ratios, short curing periods, low concrete strengths and highly permeable paste are the factors that intensify the carbonation in concrete [4].
In order to verify if deterioration is caused by carbonation; a test is used in which the concrete surface is treated with a 1% phenolphthalein solution in ethanol. Phenolphthalein turns colorless in acidic solutions and pink in basic solutions [5]. In this case, if concrete turns pink, it means that the pH is within basic solution limitations, in other words, the concrete is not carbonated. However, if concrete in carbonated, it does not change color and illustrates carbonation. Using this method, it is also possible to check the depth of the carbonation of concrete.

2.2. Depth of Carbonation, [1]

2.3. Deterioration caused by carbon
2.2.3. Chloride Attack

Chloride attacks on concrete can occur in high chloride content environments, like seawater or de-icing salts [6]. Chlorides attack begins by penetrating into the concrete via pores and once reaches the reinforcement bars; it exposes the rods to corrosion by removing the protective iron oxide. Calcium chloride have been used as an accelerator in concrete mix especially in winters, however, use of chloride content accelerators have been discouraged lately as it speeds ups the corrosion rate of reinforcing steel. Chloride penetration into concrete depends on concrete quality, cover, cement type and exposure conditions. Periodic wet and dry exposure, like the splash zone, would severely accelerate the corrosion [2].

2.4. Corrosion caused by chloride attack, [6]
2.2.4. Sulfate attack

Sulphate ions may be found in external sources, such as water or in the ground, and they may also be in aggregates of concrete mix as impurities. Once the sulphate ions penetrate into concrete, they react with the calcium hydroxide and form gypsum with chemical formula of CaSO₄·2H₂O. After gypsum formation, the calcium silicate hydrates in the binder weakens which would lead the concrete to expand and crack. Expansion of concrete due to sulfate attack would cause loss of strength in concrete. Moreover, cracking and spalling of concrete will provide sulfate ions with easy access to reinforcing bar which causes pitting corrosion of reinforcement and accelerates the degradation process [2].

2.5. Leaching effect on a concrete surface, [6]
2.2.5. Mechanical Damage

Another factor in concrete deterioration may be mechanical damage caused by abrasion, impact, erosion or cavitation. Improving abrasion resistance could be achieved by reducing the water/cement ratio and increasing the compressive strength of the concrete [2].

As mentioned, impact due to mechanical elements could be another factor for concrete deterioration. Since concrete is a brittle material, severe damage can be caused if subjected to certain impact intensity; which would lead to loss of strength of concrete and deterioration. Using adequate reinforcement, would allow the concrete to be more ductile and distribute the energy absorbed by impact more evenly [6].

Erosion and cavitation are two of other mechanical factors that can cause concrete deterioration [6]. Cavitation occurs when high speed water flows in an irregular surface. In such situation, turbulence will be provoked with areas of low pressure and vortexes which would form bubbles. These bubbles would then implode and cause impact forces which would result in erosion and concrete deterioration. Cavitation could be avoided by using smooth surfaces along the water path [6].
2.2.6. **Fire Damage**

High temperatures can have catastrophic effects on concrete. Steel reinforcement can resist temperatures up to 932°F while concrete resists up to 1200°F. The thickness of the concrete around the reinforcement bars plays a significant role in protecting them against heat as it slows down heat propagation [6].

High temperatures may provoke other types of damage to concrete as well, even if the failure temperature of the reinforcement rods have not yet achieved. High temperatures would cause reinforcement bars volume to increase which would lead to building up stresses in concrete and parts of concrete to break off. Once the failure temperature of the reinforcement bars is achieved, their functionality would be lost and loss in tensile strength which could lead the entire structure to collapse would occur [6].

2.2.7. **Structural Overloading**

Every structure, during its life span, may undergo loading conditions which exceed their design criteria. These overloading conditions may occur as a result of change of use, severe impact, blast or natural forces. These forces can generate stresses which exceed the strength of the concrete as the structure has not been designed to resist such loadings. These stresses could result in localized or general failure of the structure. This type of damage is typically indicated by cracking or spalling of the concrete [2].
2.2.8. Freeze and Thaw

Freezing and thawing could be another cause of deterioration and damage to concrete. In fact, the volume of water increases by 9% when it turns to ice. In order to prevent freeze and thaw phenomena, concrete does not necessarily have to be completely dry. However, the level of humidity and water inside the concrete must be lower than “Critical Saturation”. This criteria illustrates that a certain amount of water could be present in porosity of concrete that even after turning to ice and expanding, still remains within the concrete pores and do not cause any damage to concrete. On the other hand, if water level exceeds the Critical Saturation level, when it freezes it will cause stresses and pressure and it will break the concrete and cause deterioration [6].

To avoid freeze and thaw, it is recommended to keep the water/cement ratio low, and increasing the level of porosity by using controlled air trapped concrete mixture [6].
2.3. Corrosion

Corrosion is a naturally occurring phenomenon commonly defined as the deterioration of a material (usually a metal) that results from a chemical or electrochemical reaction with its environment [7]. In other words, corrosion is the electrochemical oxidation of metals in reaction with an oxidant such as oxygen. Most metals, including steel are likely to corrode during their service life [5]. Extent of the corrosion depends mostly on the environmental conditions and steel properties. Corrosion of steel can be postponed or slowed down by applying corrosion protection methods such as coating and cathodic protection.

Corrosion of steel in reinforced concrete structures was not a big concern because of the high alkalinity inside the concrete and the protection provided by the concrete cover over reinforced bars. However, over time as concrete deterioration happens, the remaining cover would not be able to protect the steel reinforcement inside the concrete from environmental factors [6].
2.4. Corrosion in Steel Reinforced Concrete

With time passing and structures aging, corrosion of steel in concrete has become a severe problem worldwide. Based on Federal Highway Administration, the economic loss due to corrosion damage of highway bridges is estimated to be $90-150 billion only in USA. Moreover, these deteriorating structures need to be repaired or rehabilitated. Based on Transportation Research Board estimations, the annual cost required for repair of bridge deck, substructures and parking garages in USA is $200-450 million. Beyond the direct economic losses due to steel corrosion phenomena, irreparable damages and loss would be caused in structures that would lead to complete collapse and fatal injuries of occupants [5].

In order to avoid or at least delay this phenomena, we should be able to understand, detect and monitor the corrosion of steel; so that we can apply appropriate actions to achieve our goal and therefore, great amount of money would be saved in rehabilitation, and most importantly potential serious accidents that could cause fatal losses would also be avoided [5].

2.5. Types of Defects in Steel Members

The medium surrounding the steel plays a magnificent role in the basic processes of corrosion. Different mediums would result in different basic processes which then cause different forms of steel damage due to corrosion. Moreover, different concrete properties and environmental factors results in different forms of corrosion damage to reinforced concrete structures.
Different criteria could be used to classify corrosion types. Categorizing them could be according to corrosion mechanisms, environments that induce corrosion or final damage appearances. In this section, corrosion types have been classified as followed, mostly based on the corrosion mechanisms and damage forms [5].

2.5.1. Crevice Corrosion

Crevice corrosion is a localized attack on a metal surface at, or immediately adjacent to the gap or crevice between two joining surfaces exposed to corrosive environments. This intense localized corrosion is normally related to small volumes of stagnant solutions which are cause by holes, lap joints, gasket surfaces and crevices under bolts and rivet heads. Consequently, this kind of localized corrosion is known as crevice corrosion. In order for crevice corrosion to happen, a crevice must have certain width parameters. In order for liquid to penetrate through the crevice, the gap should be wide enough, but in the meantime, it should be sufficiently narrow to maintain a stagnant zone. Therefore, the openings dimensions in which the crevice corrosion potentially occurs should be a few tenths of millimeter in width and it rarely happens in grooves or slots greater than 3 millimeters. Crevice corrosion can be designed out of the system in order to prevent it from occurring [2].

Below are a few examples of the actions necessary to prevent crevice corrosion.

- Use welded butt joints instead of riveted or bolted joints in new equipment
- Eliminate crevices in existing lap joints by continuous welding or soldering
- Avoid creating stagnant conditions and ensure complete drainage in vessels
- Use solid, non-absorbent gaskets such as Teflon [8].
2.5.2. Pitting Corrosion

Pitting corrosion attack forms as highly localized cavities or holes that penetrate inside swiftly, whereas the rest of the surface still is intact. Pits are defined by their surface diameter being equal to, or less than their depth. Pitting is known as an insidious form of corrosion as a result of its localized and destructive nature. Its insidious behavior is due to the fact that pitting can cause complete perforation with only minor weight loss and the fact that pits are hard to detect by visual inspection; as a result, pits, often cause failure in structures without any prior warning which would increase fatal damages [2].

Pitting can be initiated by a localized chemical or mechanical damage to the protective oxide film, or by localized damage to, or poor application of a protective coating and by presence of non-uniformities in metal surface of the component [9].

A materials resistance to pitting corrosion is usually evaluated and ranked using the Critical Pitting Temperature (CPT), in accordance with ASTM G48-03. The critical pitting temperature is the minimum temperature (°C) to produce pitting corrosion and CPT is usually higher than the critical crevice temperature (CCT). Pitting corrosion can be prevented through:

- Proper selection of materials with known resistance to the service environment
- Control pH, chloride concentration and temperature
- Cathodic protection and/or Anodic Protection
- Use higher alloys (ASTM G48) for increased resistance to pitting corrosion [8].
2.5.3. Stress Corrosion Cracking

This kind of cracking is caused by simultaneous presence of tensile stresses and corrosive environment. Stress Corrosion Cracking (SCC) can be prevented or eliminated by removal or changes in any of the named factors. SCC does not attack metal on a massive surface area, however, when first crack occurs, during stress corrosion cracking, the cracks progress through the metal perpendicular to tensile stress. This kind of cracks can cause severe consequences for steel or reinforced concrete structures as it occurs at tensile stresses well below the yield stresses of steel. SCC is hard to be detected during visual inspections as there would be no visible evidence of such damage as the cracks can penetrate deeply into the metal with very little evidence of corrosion on surface [5].

In order for SCC to occur, sufficient magnitude of tensile stresses should exist in the presence of corrosive environment. The origin of these stresses may rise from applied loads, residual stresses due to welds or from manufacturing fault or actual service application. SCC can be controlled by choosing a material resistant to SCC and by applying quality control to make sure they are manufactured properly. The other method is to control the existing stresses by eliminating or at least reducing them below the threshold stress for SCC. This may not be possible for working stresses; however, it is conceivable to relieve the residual stresses by stress-relief annealing. Finally, controlling the environment by removing or replacing the components responsible for the problem; however, this procedure is relatively rare to apply as mostly we cannot control the environment and manipulate it for our good [10].
2.5.4. Corrosion Fatigue

Corrosion fatigue is a special case of Stress Corrosion Cracking (SCC) in which the applied stress is not constant as it experiences cyclic variations.

Fatigue is the weakening and tendency of a material to fracture under recurring cyclic loading and unloading which occurs at stress levels much below the yield strength limit of the material, also known as ultimate tensile strength limit. If the applied cyclic loads are greater than a certain threshold known as fatigue limit; which can be obtained from test results in data presented as plot of stress (S) against the number of cycles to failure (N), which is known as S-N curve; micro cracks will initiate where the stress concentration is maximum. After the initiation of crack and when it reaches to a critical size; it will propagate through the section at the right angles to the stress. When the crack initiates and it starts to propagate, the maximum stress concentration is always where the crack is propagating. Crack propagation will continue until the cross-sectional area of the
remaining metal is reduced to the point that its ultimate strength is exceeded; at this point, sudden failure of the component will occur. Fatigue life of steel becomes independent of stress below a certain stress level called the fatigue limit. This means that if the metal is stressed below this limit, it will endure an infinite number of cycles without failure [2].

Corrosion fatigue is the reduction in fatigue resistance of the component, due to the presence of a corrosive medium in harsh environments and it is more noticeable at low stress frequencies since this allows greater contact time between metal and corroder [2].
Chapter 3

Fracture Mechanics for Concrete

3.1. Introduction

Concrete is a composite material obtained by mixing water, cement, sand and gravel or other aggregates and admixtures that would be easily poured and formed into desired shapes. Different additives can be added to concrete to achieve higher strength and workability based on design requirements. Due to the accessibility and low cost of the materials used in concrete and its high compressive strength, concrete is widely used in different structures worldwide nowadays. Despite high compressive strength, concrete’s low tensile strength and fairly brittle behavior results in use of steel reinforcement bars in regions of concrete cross section subjected to tensile stresses. By reinforcing the concrete, we take advantage of concretes strength in compression and steels tensile strength. Moreover, the use of a concentric or eccentric pre-stressing force in the longitudinal direction of the member would reduce or prevent the cracks from occurring by either eliminating or extremely reducing the tensile stresses at critical sections. The idea of applying such compressive force in longitudinal direction is called pre-stressing [12].
Cracking of conventional reinforced concrete is almost inevitable due to its low tensile strength and low ductility. Once concrete is cracked, the steel reinforcement bars embedded inside the concrete are exposed to surrounding environment, which could lead to corrosion of the steel. Moreover, a sudden reduction in bending stiffness due to cracking of the member increases the deflection.

Concrete is an inelastic non-linear anisotropic heterogeneous composite material with lots of flaws. These flaws could be the origin of crack initiation and propagation of cracks when subjected to stress. Concrete typically experiences extensive cracks prior to failure and reach of maximum load. Therefore, the behavior of concrete after cracking and fracture mechanics becomes of interest. ACI Committee 446 defines fracture mechanics as a failure theory which firstly uses energy criteria in conjunction with strength criteria, and secondly, takes into account failure propagation throughout the whole structure. While methods like theory of plasticity and damage mechanics are used where displacement field and usually the strain field remain continuous everywhere, fracture mechanics is fundamentally formulated to deal with strong discontinuities, such as cracks, where both the displacement and strain fields are discontinuous across a crack surface [13]. ACI Committee 446 also discusses five arguments as reasons for need of fracture mechanics approach [14].

1. Although the reason for crack initiation should be recognized, however, the actual formation of crack and propagation of it should also be studied. To do so we must realize that crack growth consumes a certain amount of energy, which is called the
fracture energy. Hence, energy criteria should be used for studying the crack propagation.

2. Physical theories must be objective and the results must be independent on choices of coordinates, mesh refinement and etc. Rashid (1986) introduced smeared cracking, a powerful approach to finite element analysis of concrete cracking. This methodology limited the stress in finite element to the tensile strength of the material and after exceeding this limit, the stress in finite element should decrease as well. Initially a sudden vertical drop was practiced to take account for this deduction, however, it was later realized that better and more realistic results would achieve if stress is gradually decreased, also known as strain-softening concept. However, after implementing strain-softening in large finite element programs, convergence properties are incorrect and calculation results significantly change with change in mesh refinement, which makes this approach not to be objective and hence not reliable. To overcome such problem, fracture mechanics shall be used by specifying the energy dissipated by cracking per unit length of the crack and as a result, the overall energy dissipation is forced to be independent of the element subdivision and mesh refinement.

3. Two basic types of structural failure could be specified based on load-deflection diagrams: Plastic and brittle failure. A long-yield plateau exists in load-deflection diagrams of a material corresponds to plastic failure of the material and illustrates formation of plastic hinges until a single degree of freedom mechanism is developed and failure occurs. On the other hand, an absence of such yield plateau
implies the softening of the structure, which is a sign of propagation of failure zone throughout the structure and brittle failure.

4. The area under load-deflection diagrams is the product of force and displacement, which represents the energy absorbed by the structure during failure process. This energy also determines the ductility of the structure. Post-peak response and behavior is not considered in plastic limit analysis and hence, it cannot be used to indicate this energy and as a result, some sort of fracture mechanic becomes necessary.

5. Fracture mechanics may opposite to strength criterions predict the influence of the structural size on the failure load and ductility [14].

The stated above arguments explains the need of fracture mechanic theory in modeling and analyzing of concrete when cracking is of interest.

3.2. Fracture Development in Concrete

Three different levels of cracking in concrete have been suggested by Wittmann in 1987. These three different levels are as followed:

- Micro cracks: only visible by electron microscope
- Meso cracks: visible by conventional microscope
- Macro cracks: visible to naked eye

Wittmann then specifies that Micro-level cracks occur on the level of hydrated cement due to different coefficient of thermal dilatation of aggregate and hardened cement paste that would result in a complex state of internal stresses to be built up. Meso-level
cracks form in the bond between cement paste and coarse aggregates as in hardened concrete, the interface between hardened cement paste and aggregates remains weak for a long time. Macro-level cracks, however, form in the mortar between the aggregates or run through the mortar aggregate [15].

The fracture in concrete could origin either in compression or tension zone of the concrete element. In order to study the fracture development in compression, J.P. Ulfkjaer (1992) has divided the concrete stress-strain curve into four sections in terms of ultimate strength percentage. Micro-level cracks occur in concrete at the very early stages of concrete pouring in the cement paste, even prior to any loading, due to shrinkage, swelling and bleeding. In the range of 0-30% of the ultimate load, the curve is linear to some good extent and no growth of initial cracks are observed at this stage. When the stress range is between 30-50% of ultimate load, the cracks between cement paste and aggregates start to grow. The difference between modulus of elasticity of aggregates and cement paste increases the non-linearity of the stress-strain curve. It is only above 50% that the macro-level cracks begin to form and propagate between the aggregates and mortar parallel with the direction of the load. Beyond 75% of the ultimate load, the bonding cracks and mortar cracks merge together and form a more complex crack formation until complete failure occurs [16].

Tensile strength of the concrete is also dependent on the strength of each link in the cracking process, whether it is micro, meso or macro-cracks that occur respectively in cement paste, bond or in the mortar. J.P. Ulfkjaer describes this by using a concrete rod which is under tensile loading only. As described earlier, micro-level cracks occur in the
cement past prior to any loading; as a result, when the specimen is at approximately 80% of ultimate tensile load, the fracture development starts with growth of existing micro-level cracks. With load increase, the fracture procedure continues by formation of new cracks. At this stage, stress redistribution and presence of aggregates in crack pattern would cause a halt in formation of other cracks. Once the ultimate tensile load is achieved, a localized fracture zone will develop in which a macro-level crack will occur, which splits the concrete rod in two. The fracture zone develops in the weakest part of the concrete rod, i.e. existing flaw in the structure of the specimen [16].

In our case, a crack has initiated at mid-span of the beam, as a result of tendon breaks due to corrosion. At this point, the cracking moment of the beam and its ultimate flexural strength are equal. This results a sudden complete brittle failure of the beam with a macro dominant crack at mid-span. This brittle failure of the beam has been studied and verified by use of response2000 in this report.

3.3. Linear Elastic Fracture Mechanism

Fracture mechanics endeavors to characterize a material’s resistance to fracture, also called as toughness. Fracture toughness indicates the amount of required stress to make a pre-existing flaw propagate. It is normal to assume that some kind of flaw will be present in a few number of components and use the linear elastic fracture mechanics (LEFM) approach to design critical components. LEFM takes advantage of parameters such as flaw size, component geometry, loading conditions and fracture toughness to evaluate the fracture resistance of the component [17].
In 1913, Inglis et al. recognized that local stresses which exist around a corner or hole in a plate is many times higher than the average applied stresses. He then illustrated, by using electricity theory, that the radius of curvature of the hole is a major factor in the degree of stress magnification. Smaller radius of curvature would lead to higher stress concentration [18].

In 1921, Griffith did a theoretical analysis of fracture centered on minimum potential energy. Inglis’s theory illustrated that stress increase at the tip of a crack was only dependent on geometrical shape of the crack and not its size. This theory conflicted with the fact that larger cracks propagate more easily than smaller ones. Therefore, Griffith proposed that the reduction in strain energy due to formation of a crack must be equal or greater than the increase in surface energy required by the new crack faces [19].

According to Griffith, there are two conditions necessary for crack growth:

I. The bonds at the crack tip must be stressed to the point of failure. The stress at the crack tip is a function of the stress concentration factor, which depends on the ratio of its radius of curvature to its length.

II. For an increment of crack extension, the amount of strain energy released must be greater than or equal to that required for the surface energy of the two new crack faces [20].

Griffith worked on the influence of a sharp crack on a subjective body with a thickness of $t$ when loaded remotely with an arbitrary load of $F$ from the crack tip. The potential energy of the following body is given by:
\[ \Pi_c = \Pi - \Pi_e - \Pi_F - \Pi_K \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(\Pi_c)</td>
<td>Fracture potential</td>
</tr>
<tr>
<td>(\Pi)</td>
<td>Total potential energy</td>
</tr>
<tr>
<td>(\Pi_e)</td>
<td>Elastic energy in the body</td>
</tr>
<tr>
<td>(\Pi_F)</td>
<td>Potential of external forces</td>
</tr>
<tr>
<td>(\Pi_K)</td>
<td>Total kinetic energy in the system</td>
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</table>

The energy that releases during crack growth is the fracture potential, \(\Pi_c\). Griffith presented in 1921 the energy release rate, \(G\), a parameter to define the fracture criteria, in which “\(a\)“ is the length of the crack [19], [21].

\[ G = -\frac{\partial \Pi_c}{t \cdot \partial a} = R \]

\(R\) is the material’s fracture resistance and is constant in linear elastic fracture mechanics (LEFM). When a crack occurs, the total potential energy of a system increases as a new surface is created and that increases the fracture potential. Though, crack formation consumes energy in the form of surface and frictional energy, \(G\). Crack growth will become unstable, if the energy release rate is larger than the required energy for crack formation [12].

\[ \frac{\partial G}{\partial a} > \frac{\partial R}{\partial a} = 0 \]

Three different modes of fracture have been differentiated. In mode I fracture, the direction of crack plane and the applied load are perpendicular to each other. In mode II fracture, crack plane is parallel to the applied load direction. Finally, mode III fracture is a tearing mode and it only matters in three dimensional domain. Irwin illustrated that in a
linear elastic material, the stress variation at crack tip is a function of the distance to the crack tip [21, 22].

\[ \sigma = \frac{K}{\sqrt{2\pi r}} f_i(\theta) + \text{higher order terms} \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \sigma_{ij} )</td>
<td>Stress tensor</td>
</tr>
<tr>
<td>( K )</td>
<td>Stress Intensity Factor (SIF)</td>
</tr>
<tr>
<td>( \theta, r )</td>
<td>Polar coordinates at the crack tip</td>
</tr>
<tr>
<td>( f_{ij} )</td>
<td>A trigonometric function</td>
</tr>
</tbody>
</table>

As expected, there is a linear relationship between the stress tensor and the stress intensity factor which reflects the concept of theory of the linear elasticity. As stated by the above equation, at distances close to crack tip, when \( r \to 0 \); the stress tensor approaches infinity. As a result, defining a stress criteria for failure is not applicable. Irwin, developed a correlation between stress intensity factor, \( L \), and the energy release rate, \( G \) [21, 22].
\[ K = \sqrt{G. E} \]

And therefore, defining a fracture criteria became possible.

\[ K = K_c \]

\( K_c \) is the fracture toughness of the material and it is assumed to be constant in linear elastic fracture mechanism. As stated earlier, fracture toughness is an indication of the amount of stress required to propagate a pre-existing flaw i.e. crack.

The following equation for stress tensor in Linear Elastic Fracture Mechanism

\[ \sigma_{ij} = \frac{K}{\sqrt{2\pi r}} \cdot f_i(\theta) \]

implies that the stresses at the crack tip to be infinity, which violates the linear elasticity principle, which relates small strains to stresses through Hooke’s law. When crack is propagating, a fracture process zone exists ahead of the crack tip, in which, plastic deformation of the material occurs. In order to describe such highly non-linear phenomenon, especially for concrete being an inelastic non-linear anisotropic heterogeneous composite material with lots of flaws, a non-linear fracture mechanics must be implemented [21].

### 3.4. Non-Linear Fracture Mechanism

Dugdale (1960) and Barrenblatt (1959) were the first ones to endeavor plasticity at the crack tip in crack propagation analysis. They both introduced models in which closing forces were present at the crack tip, preventing the crack from further propagation. These
forces are also referred to as cohesive forces, and the zone, over which they act is called the cohesive zone. In this model, the stress infinity problem at the crack tip has been taken care of as the cohesive closure stress in assumed to be the yield strength of the material in the model proposed by Dugdale and the cohesive closure stress model proposed by Barrenblatt in a characteristic material molecular force of cohesion which has an unknown variation along the fracture process zone ahead of the crack tip [23, 24].

Later on, Hillerborg and Peterson introduced a fictitious crack in front of the real crack tip, based on Dugdale and Barrenblatt work, in order to improve the tractions acting in the fracture process zone. In this model, all nonlinear behavior of the materials is developed within fracture process zone on the crack surface. Interactions amongst aggregates and crack surfaces provide resistant against crack propagation [25]. The rest of the model is assumed to behave elastically.

3.2. The cohesive zone model by Barrenblatt

Later on, Hillerborg and Peterson introduced a fictitious crack in front of the real crack tip, based on Dugdale and Barrenblatt work, in order to improve the tractions acting in the fracture process zone. In this model, all nonlinear behavior of the materials is developed within fracture process zone on the crack surface. Interactions amongst aggregates and crack surfaces provide resistant against crack propagation [25]. The rest of the model is assumed to behave elastically.
Fictitious or mathematical crack tip, is a point with zero displacement ahead of the real physical crack tip [13]. In this model, the closure stress distribution in the fracture process zone has a maximum value equal to the tensile strength of the material, $f_t$ at the boundary of the fracture process zone and it yields to zero at the real crack tip. A softening law which relates stresses to crack opening displacements, $\omega$ is used to derive the stress variation between the boundary of the Fracture Process Zone (FPZ) and the real crack tip. The tension softening law functions in FPZ exactly as the Hooke’s law function in elastic materials, in another words, tension softening is the constitutive law in the fracture process zone. Tension softening law is used to justify the change that occurs between the continuous state of the material and the discontinuous state of it when crack propagates. A general load-displacement plot of concrete and fictitious crack ahead of the real crack is shown below. As illustrated below, the fracture process zone only extends over the length of the tension softening section of the plot, shown by BCD. As stated earlier, tension softening in FPZ’s constitutive law, a correlation between cohesive stress and crack opening displacement in FPZ. It is obvious that this relationship in non-linear, and Young’s modulus reduces gradually inside this fracture process zone as the slope of the load-displacement curve reduces [21, 26].
As discussed earlier, in order for the crack to propagate and forms new crack surfaces, energy should be absorbed. They refer to this amount of absorbed energy as fracture energy and they show it by $G_f$ which is equivalent to the area under the tension softening part of the load-displacement plot discussed earlier, BCD region of the plot.

3.3. Tensile load-displacement curve of a typical concrete

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3.4. Tension-softening
\[ G_f = \int_0^{\omega_c} \sigma(\omega) \ d(\omega) \]

The model presented by Hillerborg and Peterson, the fictitious crack at the tip of the real crack, fits in the class of discrete crack models. Abaqus uses the cohesive segments method which relies on the fictitious crack model [21].
Chapter 4

Cracking Moment and Ultimate Moment Calculations

4.1. Introduction

ACI 318-11 defines limits for reinforcement of flexural members. ACI 318-11 indicates in 18.8.2 that the “Total amount of pre-stressed and non-pre-stressed reinforcement in members with bonded pre-stressed reinforcement shall be adequate to develop a factored load at least 1.2 times the cracking load computed on the basis of the modulus of rupture $f_r$ specified in 9.5.2.3”. The modulus of rupture is calculated using the following equation suggested by ACI 318-11 [27].

$$f_r = 7.5 \sqrt{f_c'}$$

However, according to ACI 363R, this value is the lower limit of the modulus of rupture, therefore, it is too conservative. The value of modulus of rupture could vary between $7.5\sqrt{f_c'}$ and $12\sqrt{f_c'}$. It is assumed by the author that in the initial design of the beam the lower limit of modulus of rupture has been used. The $7.5\sqrt{f_c'}$ have been used as modulus of rupture in MathCAD calculations and Response2000 analysis. However, the
author has used $9.5\sqrt{f_c'}$ as modulus of rupture in section 5.4 to better capture and simulate the actual brittle failure of the beam. It will be then validated in section 5.4 by response2000 that, with the use of updated modulus of rupture, the brittle failure of the beam occurs exactly after 1.75 half-inch tendons have been broken. This number of broken tendons matches the exact number of broken tendons that was found upon inspection of the actual failed beam on-site.

In commentary section of ACI 318-11, R18.8.2, it specifies that this provision is to prevent abrupt flexural failure immediately after cracking. Based on ACI 318-11, a flexural member that is designed based on code provisions should endure considerable additional load beyond cracking load to reach its ultimate flexural strength. This would cause the flexural member to undergo considerable deflection, which could warn occupants to evacuate the structure. A structure with such behavior is a ductile structure. The purpose of ACI 318 code is to provide such provisions so that designed structural members have enough ductility so that they do not fail abruptly and without warning [27].

Based on cross sectional dimensions, pre-stressing strands layout, concrete and strands properties and use of principal pre-stressing equations and linear elastic method, ultimate moment capacity of the cross section was calculated and then it was compared to cracking moment of the cross section to investigate the ductility of the beam.

4.2. Beam Cross Sectional and Strand Layout

The pre-stressed deep spandrel parking garage beam is a simply supported beam with a span of 35 ft. 10 inches. The depth of the beam is 84 inches and strands are located
in 11 layers across the depth of the beam. Cross-sectional dimensions, strand layout and shear reinforcement distribution along the length of the beam can be found in the following figures.

4.1. Cross-sectional dimensions and strand layout of the beam
4.3. MathCAD Calculations

In order to calculate the ultimate moment capacity of the beam, the eccentricity of each strand layer was calculated to the center of gravity of the whole cross section. Afterwards, the depth of the neutral axis, $c$; was determined by trial and error so that the tension forces were equal to compression forces. In order to calculate the tension forces, the total strain at each level of strands were calculated. The total calculated strains were then used to calculate the corresponding stress of each strand at different layers, by using the plot and equations given in Design Aid 11.2.5 of PCI 6th Edition [28].

The corresponding stresses then were multiplied by cross sectional area of strands at each layer to calculate the tensile forces. The depth of the neutral axis was chosen so that the summation of tensile forces is equivalent to compression force acting over the compression zone of the beam.

Finally, the ultimate moment capacity of the section was calculated by multiplying tensile forces and their corresponding lever arms. Due to the volume of calculations and need of try and errors, MathCAD software has been used.

In order to calculate the cracking moment of the cross section, Eq. 4.2.1.4 given in PCI 6th Edition has been used. for calculating the cracking moment, a 25% long term losses have been assumed, modulus of rupture have been calculated based on ACI 318-11 [27, 28]. Section modulus and cross sectional area have been calculated based on cross sectional dimensions. The concrete was assumed to have a 28th day compressive strength of 6000 psi and the pre-stressing strands were assumed to be 250 ksi, seven-wire strands.
Ultimate moment capacity and cracking moment calculations according to PCI 6th Edition and ACI 318-11 by MathCAD is provided below.
University of Toledo is interested in assessing the condition of the strands used in prestressed spandrel beams in their parking garages to evaluate the remaining strength. To do so, magnetic inspection of strands is proposed to assess the present condition, capacity and remaining life of these spandrel beams.

Below is the cross-sectional analysis of the spandrel beam done by University of Toledo team to evaluate the ultimate moment capacity and cracking moment to find out whether the initial design of the beam has enough ductility. According to ACI318, if Mu/Mci > 1.2 then the beam has enough ductility.

Section Properties of Spandrel Beam:

\[ h_1 = 9 \text{in} \quad b_1 = 18 \text{in} \quad h_b = 12 \text{in} \]

\[ y_b = \frac{\left( b_1 h_1 \left( \frac{h_1}{2} + 12 \text{in} \right) + b_b h_b \left( \frac{h_b}{2} \right) \right)}{b_1 h_1 + b_b h_b} = 37.5 \text{-in} \]

Centroid from bottom

\[ y_t = 84 \text{in} - y_b = 46.5 \text{-in} \]

Centroid from top

\[ I = 568295.9 \text{in}^4 \quad A = \left( b_1 h_1 \right) + \left( b_b h_b \right) = 864 \text{in}^2 \]

\[ r = 25 \text{in} \]

\[ S_{bot} = \frac{1}{y_b} = 15154.6 \text{-in}^3 \quad S_{top} = \frac{1}{y_t} = 12221.4 \text{-in}^3 \]

Properties of Strands:

\[ f_{pu} = 250 \text{ksi} \quad E_p = 28500 \text{ksi} \]

\[ f_{ps} = 0.9 f_{pu} = 225 \text{-ksi} \]

\[ f_{pi} = 0.7 f_{pu} = 175 \text{-ksi} \]

\[ f_{pa} = 0.75 f_{pi} = 131 \text{-ksi} \]

250 ksi strands

stress at release

Long Term stresses

25% loss
Properties of Concrete:

\[ f_c = 6000 \text{ psi} \]
\[ E_c = 57000\sqrt{f_c} = 4415 \text{ ksi} \]

Concrete comp. strength

\[ f_{cy} = 0.8f_c = 4800 \text{ psi} \]
\[ E_{cy} = 57000\sqrt{f_{cy}} = 3949 \text{ ksi} \]

Concrete comp. strength at release

\[ f_t = 7.5\sqrt{f_c} = 581 \text{ psi} \]

Modulus of rupture beam will crack if tensile stress exceed 581 psi

\[ a_{qhalf} = 0.153 \text{ in}^2 \]
\[ a_{quai} = 0.036 \text{ in}^2 \]

Area of 1/2" strand

\[ A_b = 5a_{qhalf} + 17a_{quai} = 1.377 \text{ in}^2 \]

Tensile strands area

\[ A_s' = 2a_{qhalf} = 0.306 \text{ in}^2 \]

Comp. strands area

\[ P_e = 2a_{quai}f_{pe} = 9.45 \text{ kip} \]

Center of Gravity of strands from top:

\[
\begin{align*}
y &:= \begin{bmatrix}
11 \\
20 \\
29 \\
38 \\
47 \\
56 \\
65 \\
74 \\
76 \\
82
\end{bmatrix} \text{ in}
\end{align*}
\]

Strand distances from top of the beam, excluding the first layer of strands which are in compression

\[
CG_1 := \frac{2a_{quai}(y_0 + y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7) + (2a_{qhalf} + a_{quai})y_8}{17a_{quai} + 5a_{qhalf}} = 64.3 \text{ in}
\]

Eccentricity of strands

\[ \varepsilon := CG_1 - y_8 = 17.644 \text{ in} \]
**Ultimate Moment Capacity, Mu:**

**Final Trial:**

- \( c = 8.1 \text{ in} \)

\[
\varepsilon_p = \frac{(c - 2\text{in})}{c} \cdot 0.003 = 0.002
\]

- Distance from extreme compression fiber to neutral axis

- Strain in comp strand

\[
C_1 = 0.85 \cdot f_y \cdot (0.85-c) \cdot b_1 = 316 \text{-kip}
\]

- \( C_2 = (f_y \cdot \varepsilon_p) \cdot A_s = 20 \text{-kip} \)

- Total compression force

\[
C_T = C_1 + C_2 = 336 \text{-kip}
\]

- Bar distances below
  
- \( c = 8.1 \text{ in} \) from Top
  
- For strand arrangement refer to drawing 1

\[
\begin{array}{c}
11 \\
20 \\
29 \\
38 \\
47 \\
56 \\
65 \\
72 \\
82 \\
\end{array}
\]

- \( d = \) in

\[
\varepsilon_1 = \frac{f_y}{E_p} = 0.0046
\]

- Eccentricity of strands

\[
\begin{array}{c|c}
0 & 0 \\
1 & 26 \\
2 & 17 \\
3 & 8 \\
4 & -1 \\
5 & -10 \\
6 & -19 \\
7 & -26 \\
8 & -28 \\
9 & -36 \\
\end{array}
\]
MathCAD calcs.  Parking Garage Beam  Calculated By: O. Haftroodi

\[ \varepsilon_2 = \frac{P_e}{A_eE_e} \left( 1 + \frac{\varepsilon_3^2}{r^2} \right) \]

decompress strain

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<td>4</td>
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\[ \varepsilon_3 := \left( \frac{d - \varepsilon}{r} \right) \times 0.003 \]

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\[ \varepsilon := \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \]

total strain in strands in tension at different layers

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<td>8</td>
<td>0.02902</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.03190</td>
<td></td>
</tr>
</tbody>
</table>
corresponding stresses due to strains:

PCI 8th Edition

Design Aid 11.2.5 (Curiously, this design aid does not appear in the PCI 7th. It has been replaced by Design Aid 5.14.3 which does not address 250 ksi strand or ultimate strain. 0.05 in./in. is the lower limit on ultimate strain.

\[ f_{ps} = \begin{cases} 
250 \text{ ksi} - \frac{0.04}{\varepsilon} \text{ ksi} & \text{if } 0.05 > \varepsilon \geq 0.0076 \\
250 \text{ ksi} - \varepsilon & \text{if } 0.0076 > \varepsilon \geq 0 \\
(-500 \text{ ksi}) & \text{otherwise}
\end{cases} \]

\[ f_{ps} = 162 \text{ ksi} \]

<table>
<thead>
<tr>
<th>( i )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{ps} )</td>
<td>162</td>
<td>235</td>
<td>243</td>
<td>246</td>
<td>247</td>
<td>247</td>
<td>248</td>
<td>248</td>
<td>248</td>
<td>248</td>
</tr>
</tbody>
</table>

Refer to Figure 2

Design Aid 11.2.5

PCI 8th edition

Note: Approximate strain at rupture is:

0.05 to 0.07 in./in.

0.032 in./in. which is much less than the lower limit on ultimate strain 0.05 in./in.

Looking at our ultimate strain in strands we notice that the maximum strain is

44
**Ultimate Moment Capacity:**

\[ c = 8\text{-in} \]
\[ a = 0.85c = 7\text{-in} \]

\[ M_0 := f_{p0} (2\cdot a_{qu}) \left( d_0 - \frac{a}{2} \right) = 7\text{-kip-ft} \]
\[ M_1 := \sum f_{p} \left( 2\cdot a_{qu} \right) \left( d_1 - \frac{a}{2} \right) = 23\text{-kip-ft} \]
\[ M_2 := \sum f_{p2} \left( 2\cdot a_{qu} \right) \left( d_2 - \frac{a}{2} \right) = 37\text{-kip-ft} \]
\[ M_3 := \sum f_{p3} \left( 2\cdot a_{qu} \right) \left( d_3 - \frac{a}{2} \right) = 51\text{-kip-ft} \]
\[ M_4 := \sum f_{p4} \left( 2\cdot a_{qu} \right) \left( d_4 - \frac{a}{2} \right) = 65\text{-kip-ft} \]
\[ M_5 := \sum f_{p5} \left( 2\cdot a_{qu} \right) \left( d_5 - \frac{a}{2} \right) = 78\text{-kip-ft} \]
\[ M_6 := \sum f_{p6} \left( 2\cdot a_{qu} \right) \left( d_6 - \frac{a}{2} \right) = 92\text{-kip-ft} \]
\[ M_7 := \sum f_{p7} \left( 2\cdot a_{qu} \right) \left( d_7 - \frac{a}{2} \right) = 102\text{-kip-ft} \]
\[ M_8 := \sum f_{p8} \left( 2\cdot a_{qu} \right) \left( d_8 - \frac{a}{2} \right) = 147\text{-kip-ft} \]
\[ M_9 := \sum f_{p9} \left( 3\cdot a_{qu} \right) \left( d_9 - \frac{a}{2} \right) = 805\text{-kip-ft} \]

\[ M_u := M_0 + M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7 + M_8 + M_9 = 1700.8\text{-kip-ft} \]

*check to see if \( T = C \), if not, \( c \) should be modified*

\[ T := 2\cdot a_{qu} \left( f_{p0} + f_{p1} + f_{p2} + f_{p3} + f_{p4} + f_{p5} + f_{p6} + f_{p7} \right) = 334\text{-kip} \]
\[ - \left[ f_{p8} \left( 2\cdot a_{qu} \right) + f_{p9} \left( 3\cdot a_{qu} \right) \right] = 334\text{-kip} \]

\[ T = C \]

\[ C_T := C_1 + C_2 = 336\text{-kip} \]
**Cracking Moment, Mcr:**

\[
f_{pu} = 250 \text{ksi} \\
\frac{f_{py}}{f_{pu}} = 0.9 \Rightarrow f_{py} = 225 \text{ksi} \\
\frac{f_{pcr}}{f_{pu}} = 0.75 \Rightarrow f_{pcr} = 131 \text{ ksi} \\
\]

25% of long term losses assumed

\[
T_0 := f_{pcr}(2-a_{qua}) = 9 \text{ kip} \\
T_2 := f_{pcr}(2-a_{qua}) = 9 \text{ kip} \\
T_4 := f_{pcr}(2-a_{qua}) = 9 \text{ kip} \\
T_6 := f_{pcr}(2-a_{qua}) = 9 \text{ kip} \\
T_8 := f_{pcr}(2-a_{half}) = 43 \text{ kip} \\
T_1 := f_{pcr}(2-a_{qua}) = 9 \text{ kip} \\
T_3 := f_{pcr}(2-a_{qua}) = 9 \text{ kip} \\
T_5 := f_{pcr}(2-a_{qua}) = 9 \text{ kip} \\
T_7 := f_{pcr}(2-a_{qua}) = 9 \text{ kip} \\
T_9 := f_{pcr}(\frac{a_{half} + a_{qua}}{2}) = 65 \text{ kip} \\
\]

\[
T_T := T_0 + T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 + T_8 + T_9 = 181 \text{ kip} \\
\]

\[
M_{cr} = \sqrt{A} \left[ \frac{C_2}{2} + \frac{T_T + C_2}{A} + \frac{T_T(C_1 - Y_1)}{A} + \frac{C_2(Y_1 - 2a)}{A} \right] = 1219.3 \text{ kip-ft} \\
M_u = 1707 \text{ kip-ft} \\
\]

\[
\frac{M_u}{M_{cr}} = 1.3958 \\
\frac{M_{cr}}{M_{cr}} = 1.2 \\
\text{beam has enough ductility} \\
\]

This indicates that the initial design of the beam with full tendons intact is ductile, however, the failed beam in parking garage had a brittle failure due to broken tendons and loss of cross sectional area of prestressing steels.

Next step would be to study the behavior of the beam and identify the damage scenario in which the beam experiences a completely brittle failure.
Figure 1

Strand arrangements in beam
Figure 2:
PCI 6th, Design Aid 11.2.5

MATERIAL PROPERTIES
PRESTRESSING STEEL

Design Aid 11.2.5  Typical Stress-Strain Curve, 7-Wire Low-Relaxation Prestressing Strand

Note: approximate strain at rupture is 0.05 to 0.07 in./ft.

These curves can be approximated by the following equations:

270 ksi strand:
\[ \varepsilon_y \leq 0.0075: \quad f_p = 28,000 \varepsilon_y \text{ (ksi)} \]
\[ \varepsilon_y > 0.0075: \quad f_p = 250 - 0.04 \left( \frac{\varepsilon_y - 0.0075}{0.0004} \right) \text{ (ksi)} \]

230 ksi strand:
\[ \varepsilon_y \leq 0.0086: \quad f_p = 28,000 \varepsilon_y \text{ (ksi)} \]
\[ \varepsilon_y > 0.0086: \quad f_p = 270 - 0.04 \left( \frac{\varepsilon_y - 0.0086}{0.0007} \right) \text{ (ksi)} \]
4.4 Analysis Results

By comparing the ratio of ultimate moment capacity over cracking moment of the initial design of the cross section, it was illustrated that this ratio exceeds 1.2; which is the ACI 318-11 and PCI 6th Edition requirement for a structure to have enough ductility to undergo considerable deflection before ultimate failure happens. Based on hand calculations, under normal circumstances, i.e. no damage to structural element such as tendons, concrete, etc. the primary design of the beam seems to be ductile enough and as a result, no abrupt failure should occur after cracking happens. Therefore the beam should tolerate considerable additional load before ultimate failure occurs and the beam fails.

Although calculations illustrate that the beam should have enough ductility, the pre-stressed concrete spandrel L beam of the parking garage had a brittle failure with a dominant crack at mid-span, initiated from bottom of the beam and propagated through the depth of the beam. Upon visiting and inspecting the failed beam on-site, signs of severe corrosion were found in steel strands embedded in the beam. The hypothesis examined in the following chapters is the effect of cross-sectional area loss of steel strands due to severe corrosion on ultimate flexural strength of the beam, the cracking moment and the corresponding ratio. The author believes that almost two of the tendons were initially broken due to severe corrosion, resulting in a significant decrease in ultimate moment capacity of the beam. Therefore, the gap between ultimate moment capacity and cracking moment of the section also decreased, causing the ratio to drop below 1. It was only at this time that the structure experienced a brittle failure and one dominant macro-crack at mid-span propagated throughout the whole depth of the beam.
Chapter 5

Crack Analysis of the Beam by Response2000

5.1. Introduction

Initially the ultimate moment strength and cracking moment was calculated by use of MathCAD. Afterwards, the ductility of the beam was investigated according to ACI 318-11 provisions, in which the ratio of ultimate moment strength to cracking moment is suggested to exceed 1.2. It was illustrated that the original design of the beam had enough ductility as the ratio surpassed the lower limit of 1.2. Moreover, upon visiting the beam on site, it was observed that almost two half inch, seven-wire pre-stressing strands at the bottom layer of the beam were entirely corroded away.

Parking garages of Northwest Ohio are relatively old structures and they have experienced harsh environmental conditions throughout their life span. This has led to concrete deterioration and corrosion in strands, which ultimately decreased the ratio of ultimate flexural strength of the beam over cracking moment. With loss of flexural strength of the beam, the ductility of the beam, its ability to undergo considerable deflection and tolerate loads after cracking significantly decreased.
In this chapter, Response2000 software is initially used to calculate the ultimate moment strength and the cracking moment of the beam with all strands fully intact and validate them with results obtained by hand calculations. Furthermore, after the results were validated, then Response2000 will be primarily used to investigate the beam’s ductility by calculating the ultimate moment strength and cracking moment of the beam under different strand conditions setups. The ultimate goal is to figure out the exact number of broken tendons required for the structure at which the behavior of the beam switches from ductile to brittle.

5.2. Response2000 Software

Response2000 is a user friendly software which can be used for non-linear sectional analysis of conventional and prestressed concrete sections to calculate the strength and ductility of reinforced concrete sections subjected to shear, moment and axial load. Response2000 provides load-deformation response by applying the Modified Compression-Field Theory. This program was developed at the University of Toronto by Evan Bentz in a project supervised by Professor Michael P. Collins [29].

Modified Compression Field Theory (MCFT) was developed by Vecchio and Collins for predicting the load-deformation response of reinforced concrete elements subjected to in-plane shear and normal stresses. The proposed model considers cracked concrete as a new material which has its own stress-strain characteristics. Moreover, average stresses and average strains are used in formulating the equilibrium, compatibility and stress-strain relationships. The average strains are derived by taking account of the combined effects of local strains at cracks, strains between cracks, bond slip and crack slip.
Moreover, average stresses are also calculated by implicitly including stresses between cracks, stresses at cracks, interface shear on cracks and the dowel action [29].

Response2000 considers tension stiffening of the concrete. Tension stiffening is related to the interaction between embedded steel reinforcement and concrete and significantly effects the deflection of the reinforced concrete. When a reinforced concrete section cracks, it is assumed that the stiffness of the section reduces remarkably based on the assumption that concrete does not tolerate tension anymore after it cracks, so it does not contribute to the overall behavior of the member. However, in reality, the intact concrete between adjacent cracks can still carry tensile stresses even after cracking happens in reinforced concrete members. This phenomenon is referred to as tension stiffening and is primary generated due to the bond between reinforcing bars and surrounding concrete. Response2000 will provide us with more accurate, realistic results, especially regarding the deformation of the pre-stressed beam by considering tension stiffening phenomenon of concrete in its material properties [30].

5.2.1. Modelling of the Beam

In order to model the pre-stressed concrete beam in Response2000, “Quick Define” module of the software can be used. Initially the concrete cylinder strength, conventional steel yield strength and pre-stressed steel type are defined. Afterwards, a cross sectional shape is selected and the corresponding dimensions are defined. However, in this case, a pre-existing standard L-shaped cross section did not exist in the module. As a result, the cross section was modelled as an inverted T-shaped cross section with use of “User Defined” module. The use of an inverted T-shaped cross section instead of L-shaped is
justified due to the fact that the variables that are a factor in calculating the ultimate flexural strength and cracking moment are equivalent in a T-shaped in comparison with an L-shaped. Some of these variables are the cross-sectional area, moment of inertia, center of gravity of the section, eccentricity of the strands, modulus of section, radius of gyration and etc. Under such loading, the actual L-shaped beam experiences torsion, which is not the case in an inverted T-shaped cross-section. The fact that torsion is small at the location of the crack and that we are performing a cross-sectional analysis with no interest in torsional behavior of the beam justifies the use of a symmetrical inverted T-shaped instead of a non-symmetrical L-shaped.

Next step, after modelling the cross section of the beam is to define the material properties of both the concrete and pre-stressing strands. Concrete’s cylinder strength was assumed as 6000 psi. Modulus of rupture of the concrete was defined under “Tension Strength” tab in Response2000 and it was calculated according to PCI and ACI equivalent to \(7.5\sqrt{f_{c'}^'} = 581\) psi. In order to define the seven-wire, pre-stressing strands properties, an ultimate strength of 250 ksi with an Elastic modulus of 28500 ksi was defined.

In order to define the tendons layout across the depth of the section, “Define Tendon Layers” module has been used. The pre-stressed concrete spandrel beam tendons are located in 11 layers through the depth of the beam. Each layer has been defined and added to the concrete cross section of the beam. Number of strands, strand area cross-section, distant from bottom and slope of each tendon layers has been defined. Moreover, the jacking force of each tendon layer has been defined by specifying the pre-strain of each
layer to 6140 microstrain, which is essentially an initial jacking force of 70 percent of the ultimate tensile strength of the stress-relieved tendons.

\[ f_{pl} = 0.7 f_{pu} = 0.7 \times 250 ksi = 175 ksi \]

\[ \varepsilon = \frac{f_{pl}}{E} = \frac{0.7 \times 250 ksi}{28500 ksi} = 0.00614 = 6140 \text{ microstrain} \]

Finally, after defining the cross-sectional dimensions, material properties of concrete and pre-stressing tendons and strand layouts, it is time to define the loading conditions under the full member properties module, such as length of the beam and support conditions. In this case, the pre-stressed concrete spandrel beam is modelled as a simply supported beam with a span of 35 foot, 10 inches under a uniform distributed load with its maximum moment occurring at mid-span where shear is zero. Response2000 does the analysis only on the half-length of the beam. We are able to define loads under “Define Loading” tab in Response2000. Axial load, Moment and shear can be applied to the member simultaneously both as constant or increments. In our case, we are only interested in the flexural behavior of the beam. Therefore, moment loading has been defined by 1 kip.ft increments. Response2000 performs a step by step analysis on the beam, and at the end of each step the moment loading increases by 1 kip.ft. This procedure continues until failure of the member occurs. The failure moment load is referred to as the ultimate moment load, or flexural capacity of the member. Performing a step-by-step analysis with incremental increase in loading, Response2000 also provides load-deformation plots, which would provide us with great information regarding behavior of the member.
The ultimate moment strength of the beam will be illustrated in the sectional response solution of the Response2000 and will be compared with results from hand calculations done by MathCAD. Moreover, to determine the cracking moment of the beam, the moment-curvature plot provided by Response2000 was used. The slope of the moment-curvature illustrates the bending stiffness of the member, $EI$. Prior to occurrence of cracking, the member behaves elastically and the slope of the moment-curvature remains constant and linear. However, when the member cracks, there is a sudden reduction in stiffness of the member, resulting the slope of the moment-curvature diagram not to be linear anymore and to decrease. The point where the moment-curvature diagram stops acting linearly is the moment at which the first crack has occurred at the member and it is referred to as the cracking moment.

5.3. Analysis Results

5.3.1. Pre-stressed Beam with Full Tendons

In order to primary use Response2000 as the software for calculating the pre-stressed beam’s ultimate moment strength and cracking moment and investigate its behavior and ductility under different tendon break damage scenarios; Response2000’s results needed to be validated. For validation purposes, Response2000’s results were compared with the hand calculation results obtained by MathCAD and use of PCI 6th design handbook and ACI 318-11.

The pre-stressed concrete spandrel beam was modelled according to available construction drawings in Response2000 by considering all the pre-stressing tendons to be healthy and operating at their full capacity.
Geometric properties, a schematic of beam’s cross-section and pre-stressing tendons with stress-strain diagrams of concrete and pre-stressing tendons are illustrated in the following figure, which is a snapshot from Reponse2000. It must be indicated that, the bottom layer of the beam consists of three half-inches and one quarter of an inch pre-stressing tendons. However, defining tendons with different cross-sectional area in one layer is not possible with Reponse2000. Therefore, the total cross-sectional area of three half inch tendons and one quarter of an inch tendon were calculated and the average was determined. Finally, four tendons with the average cross-sectional area of 0.124 in² were used as bottom layer tendons of the beam in Response2000. Moreover, as it is illustrated in the following figure, Reponse2000 has calculated both the gross and transformed cross-sectional properties.

<table>
<thead>
<tr>
<th>Geometric Properties</th>
<th>Gross Conc.</th>
<th>Trans (m=9.84)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area (in²)</td>
<td>864.0</td>
<td>878.9</td>
</tr>
<tr>
<td>Inertia (in⁴)</td>
<td>568295.9</td>
<td>583388.4</td>
</tr>
<tr>
<td>Y₁ (in)</td>
<td>46.5</td>
<td>46.6</td>
</tr>
<tr>
<td>X₀ (in)</td>
<td>37.5</td>
<td>37.4</td>
</tr>
<tr>
<td>S₁ (in²)</td>
<td>12221.4</td>
<td>12519.1</td>
</tr>
<tr>
<td>S₀ (in²)</td>
<td>15154.6</td>
<td>15598.6</td>
</tr>
</tbody>
</table>

5.1. Properties of beam with full tendons
For first comparison, the gross cross-sectional properties such as area, moment of inertia, center of gravity, and modulus of sections calculated by Reponse2000 is compared with MathCAD calculations and as it is illustrated in the following table, they are equivalent.

5.1. Section Properties by MathCAD and Response2000

<table>
<thead>
<tr>
<th></th>
<th>Area (in²)</th>
<th>Moment of Inertia (in⁴)</th>
<th>C.G. (in)</th>
<th>S (in³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MathCAD</td>
<td>864</td>
<td>568295.9</td>
<td>37.5</td>
<td>15154.6</td>
</tr>
<tr>
<td>RESPONSE2000</td>
<td>864</td>
<td>568295.9</td>
<td>37.5</td>
<td>15154.6</td>
</tr>
</tbody>
</table>

Next step is to perform the cross-sectional analysis of the beam to calculate the corresponding ultimate moment capacity. By clicking the “Sectional Response” under “Solve” tab in Response2000, the ultimate moment capacity of the beam is calculated. Moreover, Moment-curvature diagrams of the beam is also provided which would allow us to find the cracking moment as discussed earlier.
Comparing the ultimate moment capacity, calculated by Reponse2000 to be 1706.2 kip.ft with ultimate moment capacity of the section by hand calculations, 1706.8 kip.ft, the difference is less than 0.03%, which validates the Response2000 results.

In order to calculate the cracking moment of the section by Response2000, we take advantage of the Moment-Curvature plot provided by Reponse2000. By looking at the moment-curvature plot fig. 19, the point at which there is a sudden difference in slope of the plot is determined. This point illustrates the loading at which the first cracks have initiated in the section. This is due to the fact that after initiation of the first cracks, there is a sudden reduction in the stiffness of the member, and since the slope of the moment-curvature plot demonstrates the bending stiffness of the member, sudden change in slope...
means initiation of cracks and non-linear behavior of the section. The corresponding moment at the point illustrated by red-cross, is the cracking moment of the section which is 1334.7 kip.ft. Comparing it with the cracking moment calculated in chapter 4 by MathCAD, 1219.3 kip.ft., there is an 8% error, which is reasonable acceptable considering concrete being a non-homogenous material with highly non-linear behavior.

5.2. Ultimate Moment and Cracking Moment, beam with full tendons

<table>
<thead>
<tr>
<th></th>
<th>( M_u ) (kip.ft)</th>
<th>( M_{cr} ) (kip.ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MathCAD</td>
<td>1706.8</td>
<td>1219.3</td>
</tr>
<tr>
<td>RESPONSE2000</td>
<td>1706.2</td>
<td>1334.7</td>
</tr>
<tr>
<td>Difference (%)</td>
<td>0.3 %</td>
<td>8 %</td>
</tr>
</tbody>
</table>

5.3. Beam with full tendons, Moment Curvature curve
Next step is to calculate the ratio of ultimate moment capacity and the cracking moment and compare it with ACI 318 provisions for ductile members.

\[
\frac{M_{tu}}{M_{cr}} = \frac{1706.8 \text{ kip.ft}}{1334.7 \text{ kip.ft}} = 1.27 > 1.2
\]

Based on above calculations and ACI 318 provisions, the ratio is greater than 1.2, therefore, it is expected from the member to have enough ductility and undergo large deformations before failure. Moreover, equally distributed crack patterns is expected to occur in a ductile member before complete failure of the member in comparison with a brittle member. Furthermore, Reponse2000 provides member crack diagram plots when considering the full member properties by taking the length of the beam into account. These plots provide us with information regarding the total behavior and performance of the beam under loading. The crack diagram is illustrated in the following picture.

5.4. Beam with full tendons, Crack Diagram

The above picture, is only half-length of the beam and as it was expected, the presence of equivalently distributed straight cracks is observed. This was expected, due to the fact that the beam is under only bending loading by a uniformly distributed load. There is no sign of diagonal 45 degree shear cracks occurring at or near supports.
Moreover, it is interesting to know if the failure of the beam is due to bending or shear. Response2000 provides us with a Moment-shear interaction diagram of the beam, which is essentially the capacity of the beam, shown by blue color. Response2000 then determines the largest envelope that would fit into the diagram, shown by red color. It can be seen from the following figure that the loading envelope touches the failure envelope on the right, indicating a flexural failure. If the loading envelope had touched the failure envelope at top, it would have indicated a shear failure, which was not expected in this beam and it was validated both by crack diagram and moment-shear interaction diagram.

5.5. Beam with full tendons, Moment-shear interaction diagram
Response2000 also provides the load-displacement plot of the beam. It is evident from the following plot that the beam with full tendons is ductile as it experiences strain-hardening and undergoes significant deflection prior to failure.

5.3.2. Pre-stressed Beam with One Broken Tendon

To simulate the tendon effects on the strength and behavior of the beam, half-inch tendons have been removed from the bottom layer one at a time and the beam has been analyzed under the same exact loading and boundary conditions. This approach will let us study and investigate the effects of tendons condition on overall strength and ductility of the beam.

The gross cross-sectional properties of the beam remains constant as expected, however, the transformed cross-sectional properties of the beam decreases as the steel area has been reduced. As illustrated in the following figure, transformed area of the section, moment of inertia, centroid and modulus of section are the parameters that have been noticeably reduced.
5.7. Properties of beam with one broken tendon

As it is illustrated in the following pictures, by removing one half-inch tendon from bottom layer of the beam, the ultimate capacity of the beam decreased to 1463.0 kip.ft

\[ N+M \]
\[ M: 1463.0 \text{ ft-kips} \]
\[ N: -0.1 \text{ kips} \]

5.8. Beam with one broken tendon results
The cracking moment of the section is determined by the moment-curvature plot given in analysis results of Response2000. The cracking moment is determined to be 1230 kip.ft shown by the red-cross on the plot.

Next step is to calculate the ratio of the ultimate moment capacity over cracking moment of the section and compare it with the ACI 318-11 recommendation ratio of 1.2 for ductile behavior.

\[
\frac{M_u}{M_{cr}} = \frac{1463 \text{ kip.} \text{ft}}{1230 \text{ kip.} \text{ft}} = 1.18 < 1.2
\]

Based on above calculations the ratio has just fallen below the 1.2 threshold for ductile behavior of members. In another words, after removal of one half-inch tendon (to
account for a fully corroded tendon) from bottom layer of the initial beam design, the ratio gets smaller than 1.2.

This means that the cross-section with one half-inch tendon removed acts less ductile and more brittle than the cross-section with all its tendons fully attached and not corroded. It is expected to see less equivalently distributed cracks prior to complete failure of the beam. Crack patterns are shown graphically in the following picture. Comparing the crack patterns of this case to the beam with full tendons it is obvious that less cracks have been occurred through the length of the beam and they are even less equivalently distributed. Moreover, looking at the width of the cracks, it is obvious that these cracks seem to be more severe than the case with full tendons.

![Member Crack Diagram](image)

5.10. Beam with one broken tendon, Crack Diagram

Investigating the moment-shear interaction diagram of the beam, indicated that the beam undergoes a flexural failure as the loading envelope has touched the failure envelope on the right. However, the failure moment is much less than the case with full tendons.
5.3.3. Pre-stressed Beam with Two Broken Tendons

In this section, the behavior of the pre-stressed spandrel parking garage beam is studied assuming two of the half-inch tendons at the bottom layer of the beam as broken. This would cause in another deduction in pre-stressing force applied to the bottom layer of the beam. More loss of moment flexural strength is expected in comparison with the latter two case scenarios. Below is a screen shot of the geometric properties and strand layouts of the beam with two broken tendons captured from Response2000. It is obvious that the transformed section properties have decreased due to loss of tendons at the bottom layer.
5.12. Properties of beam with two broken tendons

The boundary conditions at supports and loading have not been changed from previous analysis. As it was expected, and it is illustrated in the following figures, the ultimate moment capacity of the beam has decreased to 1221.5 kip.ft.

5.13. Beam with two broken tendons results
Furthermore, cracking moment was derived from the moment-curvature plot and is equal to 1050.3 kip.ft, as illustrated in the following figure.

\[
\frac{M_u}{M_{cr}} = \frac{1221.5 \text{ kip. ft}}{1050.3 \text{ kip. ft}} = 1.163 < 1.2
\]

As shown above, the ratio has decreased even more with two tendon breaks and it is expected that this beam shows a less ductile behavior in comparison with beams analyzed under earlier damage scenarios.

5.14. Beam with two broken tendons, Moment Curvature curve
Due to the fact that the beam is expected to show a much brittle behavior with two broken tendons, it is expected from the crack patterns to illustrate less equivalently distributed flexure cracks. Moreover, at this stage, the presence of one dominant macro crack should be visible. Provided below, is the crack pattern for the beam with two broken tendons.

As it was expected, less flexural cracks with less widths are visible at this stage and a well-straight dominant crack initiating from the bottom of the beam and propagating through the whole depth of the beam is evident. This validates the earlier brittle behavior assumption of the beam based on the calculated ratio for this case. As it is illustrated in the above figure, the crack width of the dominant crack has increased, however, at the same time, the width of other cracks have decreased.

However, even at this stage, after removal of two tendons from the bottom layer, as it is illustrated in the following P-Δ plot, the failure of the beam is ductile as the beam experiences significant deformation before complete failure.

5.15. Beam with two broken tendons, Crack Diagram
Next step of the study would be to find the conditions under which the beam experiences total brittle failure without any extensive deformation prior to complete failure.

### 5.4. Complete Brittle Failure Conditions

The purpose of this chapter is to find the conditions under which the beam experiences total brittle failure. Upon visiting the beam on-site and inspecting the failed beam, it was observed that approximately 1.75 of the half-inch tendons from the bottom layer were corroded away. One tendon was completely corroded and one had been severely corroded. The modulus of rupture equation for concrete, suggested by ACI 318 is $7.5\sqrt{f'_c}$. However, as indicated in ACI 363R, modulus of rupture of both normal and light-weight concrete is in the range of $7.5\sqrt{f'_c}$ to $12.5\sqrt{f'_c}$ based on various researchers works. [31] It is obvious that value for modulus of rupture offered by ACI 318 is simply the lower limit of the above range to be conservative.
In order to better simulate the complete brittle failure of the beam, author chose to use a modulus of rupture equal to $9.5\sqrt{f_c'} = 9.5\sqrt{6000} = 735 \text{ psi}$, which falls almost in the middle of the range suggested by ACI 363R. Moreover, a total number of 1.75 half-inch tendons were removed from the bottom layer of the beam to better simulate and capture the failure of the beam in reality. This was done by manipulating the area of tendons in Response2000.

5.17. Properties of beam with complete brittle failure
5.18. Complete Brittle failure of beam with two broken tendons results

As it is illustrated in the above figures, the ultimate flexural capacity of the moment is 1319.8 kip.ft in this condition. Below is a screen shot of the moment-curvature plot taken directly from Response2000. The cracking moment calculated by response2000 is 1319.8 kip.ft, shown by red-cross in the moment-curvature plot. With the updated modulus of rupture and number of broken tendons, the ratio of ultimate moment over cracking moment has become exactly 1. It is expected from the beam to experience a completely brittle failure at these circumstances.
Response2000 was unable to provide the member crack diagram, therefore, it was not possible to observe the one dominant macro-crack at mid-span. However, response2000 provides a load-deflection plot as well shown below. Looking at figure 36, it is obvious that the plot in linear until it reaches the failure load, at which it suddenly fails with no further deformation. This is a complete brittle failure, in which the ultimate moment capacity and the cracking moment are exactly equivalent. As a result, when the beam reaches its ultimate capacity, it has reached its cracking moment as well.

In the below figures, the load-deflection plot of a complete brittle failure has been compared with a ductile failure side to side. As it is observed in the right figure, the load-deflection is linear until it reaches its cracking moment. Due to the fact that the ultimate
moment capacity of that beam is greater than its cracking moment, the beam is able to take more loading, until it reaches its ultimate moment capacity. During this phase, the beam is experiencing significant deformation before the total failure occurs. This is a ductile behavior from the beam and its failure is called ductile failure.

5.5. **Reponse2000 Results Conclusion**

In each section of this chapter, one of the half-inch tendons was considered to be broken by keeping all other factors such as boundary conditions, supports and loadings constant. The beam was then analyzed by Reponse2000 to calculate the ultimate moment capacity and cracking moment of each damage scenario. Moreover, the ratio of ultimate moment capacity over cracking moment was also calculated and it was observed that by
each tendon breaking in the beam, the ratio decreased, resulting in a less ductile, more brittle beam.

These were also validated by observing the crack patterns along the length of the beam. With each tendon breaking, less equivalently distributed cracks with less widths were observed throughout the length of the beam, however, the presence of a dominant macro-crack at mid span of the beam became evident.

Table below provides a brief summary of the effects of tendon removals from the bottom layer of the beam, one at a time, on ultimate moment capacity, cracking moment, corresponding ratio and the behavior of the beam.

5.3. Summary of results

<table>
<thead>
<tr>
<th>kip.ft</th>
<th>*Full Tendons</th>
<th>*One Broken Tendon</th>
<th>*Two Broken Tendons</th>
<th>**1.75 Broken Tendons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate Moment Capacity</td>
<td>1706.2</td>
<td>1463</td>
<td>1221.5</td>
<td>1319.8</td>
</tr>
<tr>
<td>Cracking Moment</td>
<td>1334.7</td>
<td>1230</td>
<td>1050.3</td>
<td>1319.8</td>
</tr>
<tr>
<td>Mu / Mcr</td>
<td>1.27</td>
<td>1.18</td>
<td>1.163</td>
<td>1</td>
</tr>
<tr>
<td>Failure</td>
<td>Ductile</td>
<td>Ductile / Brittle</td>
<td>Almost Brittle</td>
<td>Brittle</td>
</tr>
</tbody>
</table>

*7.5\sqrt{f_c'} as modulus of rupture **9.5\sqrt{f_c'} as modulus of rupture

Moreover, to allow for ease of comparison, crack pattern diagrams of the beams with full tendons, one broken tendon and two broken tendons have been provided below respectively. It is more convenient to study the crack patterns and how they become less equivalently distributed along the length of the beam as beam’s behavior switches from ductile to brittle as tendons are removed one by one from the bottom layer of the beam.
Furthermore, the crack widths of distributed cracks tends to decrease, however, at the same time, the width of one dominant macro-crack at mid-span increases by each tendon removal from the beam.

5.22. Crack Patterns, (a) Beam with full tendons, (b) Beam with one broken tendon, (c) Beam with two broken tendons

Unfortunately, Response2000 is not able to provide the crack pattern for the completely brittle failure mode of the beam. However, studying the crack patterns in above figures, illustrate that by each removal of the tendon from the beam, the beam acts more brittle and less ductile, and the existence of one dominant macro-crack at mid-span becomes more evident.
Chapter 6

 Crack Propagation Simulation Using ABAQUS

6.1. Introduction

In this chapter the results obtained from crack propagation analysis of the three-dimensional model of the parking garage pre-stressed spandrel beam have been illustrated. Cross-sectional dimensions, boundary conditions, concrete and pre-stressing strands material properties have been discussed. Moreover, the modelling procedure and the method used by ABAQUS for performing the crack propagation analysis have been discussed in detail. In order to investigate the strands effect on crack propagation and behavior of the beam, two bottom-layer strands have been modelled as broken strands - to simulate existing condition of the beam in UT parking garage - which do not contribute to the behavior of the beam and the results have been compared with a complete model of the beam with all the strands to be fully bonded and embedded in the beam.
6.2. General Description of Finite Element Method

In this chapter, the modelling process of a pre-stressed reinforced concrete beam using ABAQUS software is explained. Boundary conditions of the beam, material properties defined in ABAQUS for concrete and steel strands have also been discussed. Moreover, the required parameters to simulate the crack propagation and concrete behavior such as fracture energy have been discussed.

The three-dimensional model of the spandrel beam was created by SolidWorks and then the model was imported into ABAQUS. SolidWorks was used as it was more convenient to model the strand locations along the depth of the beam based on their coordinates. However, it was possible to model the beam from scratch by abaqus as well.

6.1. Three-dimensional model of the beam with full tendons
6.2.1. Finite Element Model Setup

In this section, a brief description on the modelling procedure of a pre-stressed reinforced concrete beam by ABAQUS is given. This chapter was inspired by the challenges that the author faced during modelling the three-dimensional finite element model of the pre-stressed beam as a first time ABAQUS user. There are not many step-by-step tutorials and guidelines available regarding modelling and analyzing of pre-stressed concrete structures by ABAQUS.

As previously mentioned the three-dimensional model of the beam was created by SolidWorks and then imported to ABAQUS, mostly due to SolidWorks modelling features which would allow us to model the beam in a faster and easier fashion.

ABAQUS software has different modules that would provide us with the required tools to model our structure. The imported file from SolidWorks appears in the “Parts” module of abaqus. “Parts” module of abaqus is essentially the initial module where you start your modelling process of any structure. In this module you can define each individual part of your model. The L-shaped spandrel pre-stressed concrete parking garage beam has been modelled by defining three different individual parts. Firstly the concrete has been modelled as a whole “3D deformable extrusion solid” part. For Creating the tendons, “3D deformable planar wire” part have been used for drawing purposes. In our case, as we are interested in crack propagation of the beam, another separate part has also been defined to act as the initial crack of the beam. The crack was modelled using “3D deformable extrusion shell” with 8 inches of extrusion. Later on this section, it will be explained how these different parts are assembled together to shape the complete beam.
The “Property” module of abaqus is where you can define all the material properties of your structure. For example, in our case it would be defining the material properties of concrete and steel such as Modulus of Elasticity, Poisson’s ratio, Density and even more complex properties necessary for crack propagation analysis which are explained in great detail on the “Material properties and crack characteristics” section of this chapter. Moreover, “Property” module of abaqus is the place to define the sections and assign them to corresponding parts. The section created and assigned for concrete is a “Homogeneous solid” section with concrete material properties. As for tendons, “Truss Beam” sections have been defined with steel being selected as the corresponding material. The reason behind selecting truss beam sections and assigning them to tendons is the fact that truss elements are rods that can carry only axial loads as tension or compression. They almost have no resistance to bending (EI=0) as the moment of inertia of these elements are small enough that is negligible. On the other hand, their cross-sectional area becomes of great importance as the axial stiffness (EA) of truss elements is the parameter that directly affects their behavior. Two different sections have been defined in abaqus for this project; one with a cross-sectional area of 0.153 in² for half-inch tendons and the other one with a cross-sectional area of 0.036in² for quarter-inch tendons. The pre-stressing tendons have been modelled as three-dimensional truss elements as in reality the cross-sectional area and axial stiffness of these members are of great importance regarding their contribution in pre-stressed concrete beam.

After creating different individual parts and defining the corresponding material and sectional properties of the parts, assembling the whole model by use of the “Assembly” module is the next step. In this step, instances are created from parts to build the complete
model of the pre-stressed concrete beam which consists of seven half-inch tendons, seventeen quarter-inch tendons, the solid concrete and the shell crack. All these instances are created and added to the model one by one. After each instance is created, it is translated to the desired location.

### 6.2.2. Boundary Conditions and Loading

The spandrel beam is a simply supported beam with a span of 35 ft. 10 inches. In order to realistically model the support conditions in the three-dimensional model of the beam, restrictions for boundary conditions have been applied along two lines parallel to x-axis, rather than one point so that no excess stress concentrations would occur at supports.

One side of the beam was modelled as a pinned support with restrictions in movement in every direction, $U_1=U_2=U_3=0$. The other side of the beam was modelled as a roller support, restricting only movements along X and Z, $U_1=U_2=0$. The $U_1$, $U_2$ and $U_3$ correspond to X, Y and Z major axis of the model. The beam orientation is shown in the following picture. Moreover, two nodes at either end of the beam were defined as torsional constraints to restrain the lateral displacements of the beam.
The boundary conditions of the beam are shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th>BC 1 (Fixed Support)</th>
<th>BC 2 (Roller Support)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1 = X</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>U2 = Y</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>U3 = Z</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

For loading, a uniform pressure load of 150 psi was applied to the beam ledge in order to get maximum moment at mid-span of the beam as we are interested in the flexural behavior of the beam. The actual parking garage beam failed at mid-span, where the moment is maximum. Therefore, the applied loading will cause maximum moment at mid-span and makes the vulnerable at that point.
In order to consider the pre-stressing effect of the tendons in the reinforced concrete beam, concentrated axial loads have been applied to both ends of each strands embedded in concrete as compression forces. The magnitudes of forces applied to each end of the strands were calculated by effective stress and considering a 25% long term stress losses. Moreover, the yield stress of the tendons was considered to be equivalent to 90% of the ultimate stress and the initial jacking stress applied was assumed to be 70% of the yield stress. All these assumptions are based on PCI and are consistent with MathCAD hand calculations. Finally, the force in each strand was calculated by multiplying the effective stress in tendons and strand area. The strands in this beam were either half-inch or quarter-inch strands with corresponding areas of 0.153 in² and 0.036 in² respectively.

\[
\begin{align*}
    f_{pu} &= 250 \text{ ksi} \quad \text{Ultimate Stress} \\
    f_{py} &= 0.9 f_{pu} = 225 \text{ ksi} \quad \text{Yield Stress (90\% of Ultimate Stress)} \\
    f_{pi} &= 0.7 f_{py} = 175 \text{ ksi} \quad \text{Initial Jacking Stress (70\% of Yield Stress)} \\
    f_{pe} &= 0.75 f_{pe} = 131 \text{ ksi} \quad \text{Effective Stress (assuming 25\% long term losses)}
\end{align*}
\]

\[
\begin{align*}
    f_{\text{half tendon}} &= 131 \text{ ksi} \times 0.153 \text{ in}^2 = 20.043 \text{ lb} \\
    f_{\text{quarter tendon}} &= 131 \text{ ksi} \times 0.036 \text{ in}^2 = 4.716 \text{ lb}
\end{align*}
\]

Based on above calculations, concentrated forces of 20.043 lb. and 4.716 lb. have been applied to both ends of the half-inch and quarter-inch tendons respectively, as compression. The applied pre-stressing forces and uniform pressure load applied to beam ledge has been shown in the following picture.
6.3. Pre-stressing forces applied to tendons as compression

It is important to model and define the interaction between tendons and concrete in ABAQUS. The tendons were embedded into the concrete by defining an embedded interaction between them.

Last step in modeling the beam would be to create seeds and mesh the entire model in abaqus. Below is a screen shot of the meshed three dimensional model of the beam in abaqus.
6.2.3. Material Properties and Crack Characteristics

In this section, the properties of the materials used to model the beam are described. The material properties for concrete and pre-stressing strands have been defined in “Define material” module of abaqus and assigned to corresponding elements.

Material properties for concrete

\[ f_{c'} = 6000 \text{ psi} \quad \text{Compressive Strength of Concrete at 28}\text{th Day} \]

\[ E_c = 4415 \text{ ksi} \quad \text{Modulus of Elasticity of Concrete} \]

\[ \nu = 0.2 \quad \text{Poisson’s Ratio of Concrete} \]

Material properties for steel

\[ f_{pu} = 250 \text{ ksi} \quad \text{Ultimate Yield Strength of Pre-stressing Steel} \]

\[ E_{ps} = 28500 \text{ ksi} \quad \text{Modulus of Elasticity of Pre-stressing Steel} \]

\[ \nu = 0.3 \quad \text{Poisson’s Ratio of Steel} \]
The above material properties are essential for a regular analysis of the pre-stressed concrete beam. However, in order to perform a crack propagation analysis we need to define some sort of damage criteria in the material mode for concrete. As discussed earlier, cohesive segment approach in abaqus is used to model the crack. Concrete material is assumed to be linear elastic and we define initiation of failure at crack tip once the maximum principal stress (Maxps) at that point reaches a critical value which will be defined in abaqus. This is an energy-based damage evolution criteria.

The “Maxps Damage” can be defined in “property” module of abaqus by browsing through “Mechanical”, “Damage for Traction Separation Laws” and then “Maxps Damage”. The Max principal stress defined is equal to the modulus of rupture of concrete, 735 psi, which has been calculated earlier.

6.5. Define Maxps Damage
As mentioned above, we are using an energy-based damage evolution. Therefore, we define the type of Damage Evolution in abaqus as Energy, and we define a Fracture Energy of 10 lb/in. [32] Fracture energy is the required energy to form a unit area of crack. The 10 lb/in. defined for fracture energy is higher than fracture energy for plain concrete (7 lb/in.) due to the influence of reinforcement. [32]

6.6. Define fracture energy of reinforced concrete

Fracture affects the response of a structure making it nonlinear and non-smooth. As a result, numerical methods will face converging issues. In this analysis, newton method is used for convergence. Defining a small “Damage Stabilization cohesive” value would regularize the analysis and helps with convergence with minimal effect on the response of the structure. [33]

6.7. Define Damage Stabilization Cohesive
In order to visualize how the crack propagates in the beam, the corresponding output variables should be added in abaqus analysis output variables. The required outputs are PHILSM, PSILSM and STATUSXFEM, which can be found under Failure/Fracture and State/Field/User/Time category in output variables of abaqus.

It is now time to define and initiate the XFEM crack at mid-span of the beam. In order to define a propagating XFEM crack, a frictionless interaction property should be created for the crack surfaces by use of “Interaction Property” module of abaqus. Do not forget to select the “Allow Crack Growth” option when defining the XFEM crack.

### 6.3. Crack Propagation Analysis of Beam with two half-inch broken Tendons

In this section, two half-inch tendons from the bottom layer of the beam are modelled as broken. This would result in the beam not to take advantage of the pre-stressing force applied by tendons as compression into concrete. The following figure is the complete three-dimensional model of the beam including the embedded pre-stressing tendons into concrete. The surface at bottom of the beam at mid-span is the initial crack surface modelled by author. Based on Response200 results and field observations, it was concluded that one dominant macro-crack will initiate and propagate at mid-span of the beam. The purpose of this chapter is to simulate and study the crack growth through the depth of the beam by assuming two half-inch broken tendons from the bottom layer of the beam.
As illustrated in the following figure, the crack initiated at mid-span has started to propagate throughout the depth of the beam. This corresponds to “Step 16” of the analysis at which %55.74 of the 150 psi load is applied to the beam ledge. Based on loading at this step and calculations, the moment caused at this step is the cracking moment of the beam and is equal to 1288.29 kip.ft. The cracking moment calculated by Response2000 in previous chapter is equal to 1319.8 kip.ft. The difference between the cracking moment calculated by Response2000 and Abaqus is only %2.38.
6.9. Initiation of Crack Propagation in Beam

The crack propagation of the beam at few stages before complete failure is presented in the figure below. It can be observed that at this stage, the crack has propagated through the depth of the beam and the crack tip is at the compression zone of the beam.
6.10. Crack propagation of beam, few steps prior to failure

Abaqus increases the load until the whole structure fails completely and becomes unstable. The following pictures display the stress contours and crack propagation of the beam at two continuous steps. First two one corresponds to one step before the dramatic failure of the beam and the second couple of figures corresponds to the failure step of the beam.
6.11. Crack propagation, one step before failure, front view

6.12. Crack propagation, one step before failure, isometric view
6.13. Crack propagation at failure, front

6.14. Crack propagation at failure,
For ease of comparison, a picture of the actual failed beam is provided below. It can be seen that the three-dimensional finite element crack propagation simulation done by ABAQUS is a great match to the crack growth in the actual failed beam.

![Crack propagation in the actual beam](image)

6.15. Crack propagation in the actual beam

The following pictures illustrate the deformation of the beam in Y-axis at the same two continuous steps as above. The first couple illustrates a maximum deflection of -1.982 inches. The second couple displays a deflection of -6.254 inches, more than three times the deflection of the previous step with only a 0.01% increase in the applied load.
6.16. Deflection in Y-axis, one step before failure, front

6.17. Deflection in Y-axis, one step before failure, isometric
6.18. Deflection in Y-axis at failure, front view

6.19. Deflection in Y-axis at failure, isometric view

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Crack surfaces for the mentioned two continuous steps are displayed in the following pictures.

6.20. Crack surface, one step before failure, front view

6.21. Crack surface, one step before failure, isometric view
6.22. Crack surface at failure, front view

6.23. Crack surface at failure, isometric view
Chapter 7

Conclusions

The purpose of this research was to determine the cause of such an abrupt complete brittle failure of the pre-stressing parking garage of Northwest Ohio. Severe signs of steel corrosion, concrete deterioration and tendon breaks were inspected upon examining the failed beam on-site. Based on observations, the theory that the brittle failure of the beam was due to sectional loss of corroded pre-stressing strands needed to be studied and verified.

In this regard, the ultimate flexural strength and cracking moment of the initial design of the beam was investigated. Based on ACI-318 provisions, as expected, it was concluded that the beam, initially was designed ductile.

Secondly, due to the fact that severe corrosion of pre-stressing strands and sectional loss of them were observed in the failed beam, the effects of tendon on behavior and failure of the beam were also investigated. This was done by considering different damage scenarios of tendon breaks in the beam. It was concluded that by every removal of the tendons from the bottom layer, the beam behaves less ductile. The ratio of ultimate moment
capacity over cracking moment was decreased by removal of each tendon. Moreover, the crack patterns illustrated less equivalently well distributed cracks. Furthermore, the occurrence of one dominant macro-crack at mid-span became more and more evident by each tendon removal. However, even after removal of two half-inch tendons, the expected complete brittle failure of the beam was not achieved.

Moreover, it was of interest to find out the exact conditions upon which the beam experiences an abrupt complete brittle failure. To do so, the very conservative modulus of rupture recommended by ACI-318 was increased to $9.5\sqrt{f_c}$, in order to get more realistic results and better capture the real life behavior of the beam. The aim was to find out the exact number of broken tendons which result in ultimate moment capacity and cracking moment to be equal, $\frac{M_{u}}{M_{cr}} = 1$. It was concluded that at exactly 1.75 half-inch tendon breaks from the bottom of the beam, the ultimate flexural strength and cracking moment of the beam are equal. The calculated number of required tendon breaks to achieve a complete brittle failure was exactly equal to the number of broken tendons inspected on-site upon visiting the failed beam.
The fact that ACI 318 suggests a conservative modulus of rupture results in the cracking moment to be calculated less. This cracking moment will then be used to calculate the ratio of ultimate flexural strength over cracking moment of the beam, which would result in a much higher number and will then be compared to 1.2. The use of this conservative modulus of rupture suggested by ACI 318 overestimates the ductility of the beam and decreased the factor of safety considered in designs. As a result, the loss of one or two pre-stressing strands will result in a switch from ductile behavior of the beam to brittle.

Finally, abaqus was used to simulate the propagation of the crack through the depth of the beam, stress distribution, deflection and total behavior of the beam by use of a three-dimensional finite element model. Two-half inch tendons were removed from the bottom layer of the model used in abaqus, as it was concluded from Response2000 results and inspection of the failed beam that this damage scenario is more likely to result in a brittle
failure of the beam. The cracking moment was then calculated by use of abaqus results and compared with Response2000’s cracking moment, a total difference of 2.38%.
Chapter 8

Future Studies

For future studies, the author suggests the use of XTRACT which is a cross-sectional analysis software with the capability to illustrate crack patterns along the member length. It would be helpful to compare the results from XTRACT with Response2000 and MathCAD hand calculations.

Furthermore, it would be interesting to use concrete plasticity damage to define the concrete properties in abaqus. However, with use of CPD, many convergence issues would occur while running the analysis in abaqus. Use of much smaller increments and higher maximum number of increments would help the convergence, but at the same time, this will make the analysis highly time-consuming.

Moreover, the author believes that the modulus of rupture equation recommended by ACI 318 should be revised. Using the lower limit magnitude for modulus of rupture recommended by ACI 318 results in calculation of a much smaller cracking moment than real life. This causes an overestimation in calculating the ratio of ultimate moment capacity over cracking moment, resulting a bigger number and considering the element to be ductile,
which would not necessarily mean a bigger factor of safety in real life. This work has established the foundation and basic tools for further studies of crack propagation of the beam. It would be ideal to review the forensic reports and update the material properties and actual strength of the concrete and tendons used for initial design of the beam and re-do the analysis and compare the results.
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