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Application of ALE contact to composite shell finite element model for pneumatic tires

Joshua Herron
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A Thesis
entitled
Application of ALE contact to Composite Shell Finite
Element model for Pneumatic Tires.

by
Joshua Robert Herron

As partial fulfillment of the requirements for the
Masters of Science Degree in Mechanical Engineering

Advisor: Dr. Ric Mousseau

Graduate School

The University of Toledo
May 2005
An Abstract of

Application of ALE contact to Composite Shell Finite Element model for Pneumatic Tires.

Joshua Robert Herron

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To predict the spindle reaction forces and moments of a tire during rolling, engineers require a computationally efficient model. This thesis describes an efficient model for use in tire prototyping to generate force and moment curves for a rolling tire. Since the intent of this model is use in tire prototyping, it must be able to be built from tire engineering parameters and material properties. Furthermore, the model must be able to predict the forces and moments under several boundary conditions; including normal loads, inflation pressures, slip angles, and camber angles. To be practical, the model must also be able to solve for all these conditions under a day of CPU time.

Darnell [6] proposed a tire model for vehicle impact simulations that used a Galilean Transformation to predict the rolling contact. The Darnell model required the generation of a sidewall look-up table, and composite properties of the tread region. The contact model was implemented in the commercial finite element software ABAQUS as a user element.
The proposed model will negate the requirement of the sidewall look-up table by approximating the sidewall using finite elements. It also uses the same ABAQUS contact element developed by Darnell. The tire structure was approximated by the composite shell formulation available with ABAQUS. A rigid rim element, which accommodates the Galilean transformation, was developed and implemented as a user element within ABAQUS. This model was compared to a similar model that uses a fully Lagrangian contact. The models were tested to demonstrate the following:

- Accurate prediction of static force and moment results.
- Accurate prediction of force and moment behavior of a tire during steering.
- Computational efficiency. That is the ability to achieve a solution to rolling contact within a day of CPU time.
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Chapter 1

Introduction

Computer simulations have become the primary tools with which engineers develop and test many of their ideas in the tire industry. As computer power has improved, tire models have become increasingly complex, taking into account many properties such as hysteresis, viscoelasticity, and non-linear hyperelasticity. With all these complexities the solution times for a static analysis can be over 2 days. If the engineer would like to look at the performance of the tire under rolling conditions the solution times can exceed a month of CPU time just for one load, pressure, and slip condition. When it comes to developing force and moment plots to be used in vehicle dynamics packages, the extended run times make the simulation unfeasible in new product prototyping.

This thesis is organized into several chapters. The first few chapters review the history of rolling tire solution, tire construction basics, tire mechanics, and tire force and moment terminology. The thesis then moves into a statement of the rolling tire problem, followed by discussions on both the Lagrangian and Galilean solutions. The
next chapters discuss how the models were setup and the physical testing that was modeled. Results and a discussion on what assumptions were made in this work will be presented. A brief set of conclusion and recommendations for further research will close out this thesis.

1.1 Nomenclature

This thesis uses a mixed nomenclature to express mathematical quantities in equations. Vectors will be represented with overhead arrows, $\hat{V}$, and with a single letter subscript, $V_i$. Tensors will be represented by a bold capital letter, $T$, and with double letter subscripts, $T_{ij}$. When the subscripts are used they are assumed to range from 1 to 3 following the rules for tensors. A subscripted comma $f_i$ is used to represent partial differentiation with respect to position.

1.2 Rolling Tire Problem

As a tire rolls under some loading condition only a small region contacts the road surface. This area of contact is referred to as the contact path or footprint. Forces are generated within this contact path that balance the vehicles accelerations and loadings.

When the tire is steered, the contact patch has a particular direction of travel where as the wheel/hub assembly has a different direction, see figure 1-1. The angle between these two directions of travel is defined as the slip angle. As the tire rolls
the material that comes in to the contact patch must deform due to the stresses that are placed onto the material by the contact. The typical deformation at the tire centerline is shown in figure 1-2. As the material travels through the footprint it begins to deflect, it continues to deflect until it either exits the footprint area or the tangential stress exceeds the maximum amount of sustainable frictional force.

![Slip Angle Diagram](image)

Figure 1-1: Slip Angle in Tire Contact Patch (Adapted from Gillespie[8])

The main focus of all rolling tire problem is to predict the deflection of a material point on the tire as it rolls through the contact patch. The secondary focus is to determine how the rest of the tire structure behaves and transfers the loads to the vehicle axle as the material point moves through contact.
1.3 History of Rolling Tire Models

1.3.1 Simple Mechanical Models

Some of the first models for rolling tires were based upon approximating the tire’s footprint displacements as a simple mechanical device. Models of this category are typically the string [7], elastic foundation[7], and the beam models[7]. Each of these models build on the idea that the displacement within the tire’s footprint can be approximated by a structural member. The elastic foundation model assumes the deflection can be modeled by a group of parallel springs where each spring acts independently of its neighbor, as is shown in Figure 1-3. The string model build upon the idea of the elastic foundation model but also assumes that then tension from each element must be passed through its neighbor, as illustrated in Figure 1-4. The beam model is just an extension of the string model where not only are tensions passed through but the moments due to bending are also inter-related, as is shown in figure 1-5.

![Figure 1-2: Tread Element Displacement](image_url)
1.3.2 Empirical Tire Models

Empirical tire models use a mathematical function, derived from test data to approximate the tire’s response. Models of this category include the Magic Formula [4]. These models produce good results but require a large amount of test data to ensure a good fit of the functions. The empirical models can only predict responses within the limits of the test data that were used to derive the functions. This limits their usefulness in tire design and proto-typing.

Another version of the empirical model is the look-up table approach [7]. In this approach a table is generated that contains the displacements and rotations of the tire’s spindle along with the resulting reaction forces and moments at the spindle. The look-up table provides data quickly but is only as good as the input data and cannot accurately extrapolate outside the range of measured data.
1.3.3 Implicit FEA

Implicit FEA tire models have been used in tire design since the 1970’s. Generally these models are used in predictions of static tire behavior due to the large size of the models 100,000+ DOFs, and the long run times associated with solutions. Over the years several simplified models have been proposed, such as the ring elastic foundation (REF), model, [15]. This model approximated the tread as stiff ring that was built upon an elastic foundation, ie the sidewall. This modeling approach has undergone several refinements over the years. In 1994 Mousseau [13], demonstrated a curved elastic arch element to model the sidewall in the REF model which improved the accuracy of spindle force predictions.

1.3.4 Explicit FEA

Recently there have been several attempts to use Explicit FEA codes to solve the rolling tire contact problem. Explicit FEA model sizes are very similar to those models found in Implicit solvers. Solution times in Explicit FEA are dictated more by the wave propagation speed in the model, thus is tied directly to the smallest element in the model. Work by Wu [17], Kao [11], and Koishi [12] all describe tire
models that were solved using Explicit FEA.

1.3.5 Galilean Transformation

The Galilean Transformation was proposed by Padovan, [16] for tire contact in the mid-80s. This formulation approximates the rolling tire by convecting the tire material through an axi-symmetric deformable mesh. The Galilean Transformation is a form of the Arbitrary Lagrangian Eulerian (ALE) formulation of problems. Using this approach, Darnell [6] proposed a model that uses an elastic foundation model and the Galilean Transformation to approximate the displacement of the contact path during rolling. Darnell’s model also includes a sidewall element that uses a look-up table approach to approximate the reaction forces of the tire during rolling. Darnell demonstrated that this model had good potential for predicting the force and moment response of a tire in computationally efficient manner. However the look-up table limited its usefulness in actual engineering use.

1.4 Research Problem

By combining the Galilean transformation contact developed by Darnell [6] and a laminate composite shell carcass model it is hypothesized that a very efficient tire model can be developed. This model will be able to provide as accurate a prediction of spindle forces as the current Lagrangian models, but in much less computational time. To verify this hypothesis I will modify the Darnell contact to work with a composite shell carcass model. This model will be termed ALE throughout this thesis. I will also
use a similar carcass model that uses the same materials, composite layups, lateral cuts as the ALE model with Lagrangian contact. Both models will be ran through a suite of simulations to measure their static lateral and longitudinal stiffness, along with their prediction of rolling lateral force and aligning moment as a function of slip angle input. The results of both models are both compared to a physical tire that was tested under the same conditions. These results will determine which model has the most accurate predictions. The ALE and Lagrangian model’s solution time will also be compared to determine which model solved the quickest.
Chapter 2

Pneumatic Tire Basics

This chapter provides, in the context of this thesis, a review on the basics of pneumatic tires. A pneumatic tire is a composite structure that must be able to generate reaction forces and moments to balance vehicle accelerations. The tire must also be able to absorb road impacts and features. The tire must be able to do all this without impacting the vehicle’s fuel economy or performance. This is achieved by using a combination of component design, steel wires, polymer cords, and rubber compounds.

2.1 Tire Construction

The general tire construction is illustrated in 2.1.1. A majority of the tires on the road today are belted radial in construction. This means that the plies run in a radial direction about the tire. The plies are responsible for suspending the vehicle from the belt ring. The plies also add stiffness to the tire carcass. The belts are placed
at opposing angles and help to flatten out the tread as it comes through the contact area also known as the tire’s footprint. This helps to increase the lateral and aligning stiffness of the tire. The bead anchors the tire to the rim.

The tread compound is formulated for good wear, traction under both dry and wet conditions. The sidewall of a tire is meant to protect the plies of the tire from side impacts such as curbs. The rim cushion helps to minimize any damage that could occur to the tire due to mounting.

2.1.1 Tire Cross-Section

A tire profile is a thin slice of the tire, taken through the radial direction. Each component of the tire is laid upon the previous component forming layers of different materials.

A tire is comprised of several different rubber layers. Each layer has a specific function within the tire. The tread cap is formulated for good traction properties and to wear, or last along tire. The belt compound is formulated to adhere to the metal cords, or belts, that are in the tire. The inner liner is specially formulated to hold air inside the tire. Several other compounds can be found in a tire to enhance performance, durability and aesthetic appeal.

There are three major areas in a tire that will contain a reinforcing cord or wire. The belt package is generally made up of two to four layers of a steel wire and rubber composite. These wires are generally made up of several smaller strands wound around in a tight twist. This allows for better adhesion to the rubber.
The plies are another group of components that are reinforced. The plies are reinforced by either polymer or steel cords. The polymer cords are generally made of twisting smaller strands of polyester, rayon and even Kevlar into a cord. Similar to the belts the twists allow for better adhesion to the rubber. The plies loop under the beads and form a “handle” that supports the load of the vehicle through the rim.

The bead is another area of reinforcement. The beads come in several configurations: ribbon, hex and cable. These configurations represent the different geometries that the wires of the bead take on in the profile. The bead helps to ensure that the tire mounts on to the rim and stays in the mounted position. It also anchors the plies to the rim.

A tire rim is a metal device that ties the tire into the vehicles hub. The tire is held onto the rim by inflation pressure and the tensioning of the bead. The tire and rim make contact along a small circumferential area. Since the rim is the only connection between the vehicle spindle and the tire all steering and drive torques are applied through the rim.

### 2.2 Tire Mechanics

#### 2.2.1 Footprint/Contact Patch

When a tire is pushed into the ground the area of contact is called the tire’s footprint or contact patch. The contact pressure is non-uniformly distributed over the contact patch. The distribution of the contact pressure is affected by tread pattern,
belt angles, tire inflation pressure, and body stiffness.

2.2.2  Ply-Steer

Ply steer is a phenomenon observed in all belted tires as a net lateral force bias when the tire is rolled without any steer. Figure 2-2 illustrates the sign of the lateral force bias changes when the tire is rolled forward and in reverse. This occurs due to inter-ply shear stresses that occur in the tire’s belts.

2.2.3  Cornering

When a driver wants to change the direction of a vehicle he or she input a steer angle into the tire. This steer angle change the direction of the wheel’s heading by rotating the rim about the steer axis. The contact patch of the tire is still traveling in the original direction of the vehicle. Since a tire is comprised of a flexible composite it takes approximately 1-1.5 revolutions of the tire to bring the direction of the wheel’s heading and the direction of the wheel’s travel together. The difference between these two directions is defined as the slip angle.

2.2.4  Slip Angle

The slip angle is defined as the angle between the direction of heading and the direction of travel. See Figure 1-1.
2.3 Tire Coordinate System

A reference coordinate system has been defined by the Society of Automotive
Engineers (SAE). This coordinate system is shown in Figure 2.3. The SAE coor-
dinate system has been adopted as the industry standard by both tire companies and
automotive manufacturers. Using this coordinate system one is able to define terms
to represent the directions of forces and motion of a tire. In this coordinate system,
the z axis points down, the x axis forward, and y axis to the right. Unless otherwise
noted this is the coordinate system that will be used throughout this thesis.

2.4 Tire Forces and Moment Terminology

The normal force of the tire is loading the vehicle places onto the tire during
operation. Generally it is the weight of the vehicle distributed to the tire by load
transfer. Tire stiffness values vary greatly on the normal force applied to the tire.

Lateral force is a measure of how well a tire can hold the vehicle onto the road
during cornering and side slip maneuvers. Since a tire’s cornering performance is
directly related to the lateral force, how well the tire develops and transfers this force
from the road to the vehicle is immensely important to tire engineers.

This force can be broken into two values depending on the direction of the force
and the direction of wheel travel. If both the wheel travel and longitudinal force are
acting in the same direction the force is considered a drive force. This is the force
that causes the vehicle to move forward. If the force and travel directions are opposite
then the force is referred to as a braking force. A braking force causes the vehicle to
decelerate and eventually stop.

The aligning moment is caused by the uneven distribution of shear stresses in the tire’s contact patch. These stresses cause a net lateral force that occurs behind the center of contact pressure. The moment arm formed is called the pneumatic trail. The aligning moment produces a feedback to the driver during steering that allows the driver to gage how much effort and steer has been placed into the tire.

The moment that acts about the y-axis, \( M_y \) can have several contributors. The first contribution is the rolling resistance moment. The rolling resistance moment results from the center of contact pressure distribution occurs behind the hub of the tire. The second contribution is a torque that is applied to the spindle from the drive train of the vehicle. When the torque results in an increase of velocity of the vehicle it is referred to as drive torque. When this torque results in a decrease in velocity it is referred to as braking torque. The drive torque results from moment caused by the longitudinal force acting at the loaded tire radius. When the rolling-resistance moment balances the drive torque, a free rolling condition is achieved. Thus the total moment about the y-axis is zero.

The overturning moment is the reaction moment that occurs due to a lateral force on the tire. The overturning moment occurs in a tire due to the center of contact pressure in the footprint does not remain in the centerline plane. This causes the net force to be offset by a length. The length becomes a moment arm, thus causing a reaction moment about the hub around the x-axis.
2.5  Tire Stiffness

Tire stiffness can be measured under two conditions: static and rolling. Both conditions are important in the design of tires. Static stiffness is measured by deflecting the tire in a direction and measuring the reaction force. The initial linear slope is defined to be the stiffness of the tire. For a rolling stiffness measure the tire must be rolled at a given speed. The tire is then subject to a sweep of slip angle inputs, the reaction forces are measured and plotted with respect to the slip angle. The stiffness values are defined to be the slope of the curve near a zero slip angle.

Tire stiffness is affected by many conditions: normal loading, inflation pressure, temperature, inclination angles, and tire aging. The two biggest influences on tire stiffness are the inflation pressure and the normal load. A few psi change in tire inflation pressure can result in several hundred pounds per inch change in the stiffness of the tire. Similarly a 50 pound change in the normal load of the tire can result in a hundred pound per inch change in stiffness. Capturing the influence of both inflation pressure and normal loading is important in measure tire stiffness values.

2.5.1  Static Stiffness

The static stiffness values are measured under a quasi-static loading conditions. All the rolling tire stiffness values must be measured while the tire is in motion. These stiffness values are functions of the slip angle, $\alpha$.

Radial stiffness of the tire is defined as the slope of the normal load versus normal deflection. This is important for both vehicle load transfer calculation and for vehicle
ride quality.

\[ K_v = \frac{F_z}{R_l - R_u} \]  \hspace{1cm} (2.1)

Where

- \( K_v \) – Radial Stiffness
- \( F_z \) – Normal Force
- \( R_l \) – Loaded, Inflated Tire Radius
- \( R_u \) – Unloaded, Inflated Tire Radius

The lateral stiffness of the tire is defined as the lateral force verses the lateral displacement during static loading. This stiffness is a measure of the vehicle’s reaction to lateral motion. This is also an important measure of how well the tire transmits lateral force to the vehicle.

\[ K_y = \frac{F_y}{\delta_y} \left|_{0}^{0.5\text{in}} \right. \]  \hspace{1cm} (2.2)

Where

- \( K_y \) – Lateral Stiffness
- \( F_y \) – Lateral Force
- \( \delta_y \) – Lateral Deflection

(2.3)

The longitudinal stiffness is defined as the slope of the longitudinal force verses
the longitudinal displacement of the tire during static loading.

\[ K_x = \frac{F_x}{\delta_x} \quad \text{with} \quad \delta_x \mid_0 = 0.5\text{in} \quad (2.4) \]

Where

- \( K_x \) – Longitudinal Stiffness
- \( F_x \) – Longitudinal Force
- \( \delta_x \) – Longitudinal Deflection

The torsional stiffness is a static measure of the amount of aligning torque generated by the tire when rotated about the steer axis. This value is used to determine how well the tire transmits aligning moments to the steering system during cornering.

\[ K_t = \frac{M_z}{\theta_z} \quad (2.6) \]

Where

- \( K_t \) – Torsional Stiffness
- \( M_z \) – Aligning Moment
- \( \theta_z \) – Angle of Rotation about x

The cornering stiffness of a tire is defined by SAE as “The negative of the rate of change of lateral force with respect to change in slip angle, usually defined at a zero
slip angle.” [5]. This is a measure of how well the tire generates lateral force due to a slip angle input.

\[ C_\alpha = - \frac{\partial F_y}{\partial \alpha} \bigg|_{\alpha=0^\circ} \]  
(2.8)

Where

\( F_y \) – Lateral Force \\
\( \alpha \) – Slip Angle

The aligning stiffness is defined by SAE as “The rate of change of the aligning torque with respect to change in slip angle, usually evaluated at a zero slip angle.” [5]

This is measure of how quickly the tire generates a feedback moment to the steering system. This is important for the driver to gage how much input has been input to the tire system.

\[ C_{Mz} = - \frac{\partial M_z}{\partial \alpha} \bigg|_{\alpha=0^\circ} \]  
(2.9)

Where

\( M_z \) – Aligning Moment \\
\( \alpha \) – Slip Angle
Figure 2-2: Ply Steer and Conicity
Figure 2-3: SAE Tire Coordinate System
Figure 2-4: Lateral Force vs. Slip Angle
Figure 2-5: Aligning Moment vs. Slip Angle
Chapter 3

Rolling Contact Problem

3.1 Problem Statement

Find the stiffness of a tire as it rolls, given the structural layup of the tire, and prescribed boundary conditions. The layup of the tire will include definition of the molded tire geometry, properties of all the materials within the tire, and definition of the rim geometry, location of the road surface, and friction between road and tread. The prescribed boundary conditions may include a normal load on the rim, inflation pressure on the tire, slip angle, camber angle, velocity of the rim, braking torque and driving torque.
3.2 Statement of Strong Form

The strong form of the rolling tire problem as was stated by Padovan [14] as:

\[(S_{jk}(\delta_{ik} + u_{i,k}))_j + f_i = \rho \frac{d^2 u_i}{dt^2} + 2\rho \epsilon_{imn} \Omega_m \frac{du_m}{dt} + \rho \epsilon_{irr} \epsilon_{mkn} \Omega_r \Omega_k u_m \] (3.1)

Where:

\(S_{ij}\) – Second Piola-Kirchoff Stress
\(u_i\) – displacement
\(f_i\) – body force
\(\Omega_i\) – rotation vector
\(\rho\) – initial Density
\(\delta_{ij}\) – Kronecker Delta Operator
\(\epsilon_{ijk}\) – Epsilon Operator
\(\frac{d}{dt}\) – co-moving time derivative

The work conjugate pair of the second Piola-Kirchoff stress and Lagrangian strain is used in order to account for large rotations and finite strains. The finite strains are important when dealing with any type of structure that is comprised of rubber.
3.2.1 Boundary Conditions

Padovan [14] states that the boundary conditions that are associated with 3.1 can be given by

\[ S_{jk} (\delta_{ik} + u_{i,k}) n_j = S_i \quad \text{for } x_i \in \partial R_\sigma \]  (3.2)

\[ u_i = U_i \quad \text{for } x_i \in \partial R_u \]  (3.3)

Equation 3.2 requires:

\[ \partial R = \partial R_u + \partial R_\sigma \]  (3.4)

Equation 3.4 states that the entire surface of the problem must either contain a traction boundary condition or a prescribed displacement boundary condition. A region that contains a tractive condition may not also include a prescribed displacement condition.

Example 3.1 A particle in space is given force in the x direction. According to 3.2 that particle can not also be constrained to have a 0 displacement in the x direction.

It may however have a displacement of 0 in the y direction and 0.5 inches in the z direction.

For a structure that must move, Padovan [14] defines the boundary loads as a function of the initial coordinates \( x \), the load trajectory \( c(t) \), and time \( t \). This shown in Equation 3.5.
\[ S_i = S_i(x_j - c_j(t), t) \]  
\hspace{1cm} (3.5)

The forces due to inflation pressure are given by equation 3.6

\[ F_p = \int_S p n_i ds \]  
\hspace{1cm} (3.6)

Where:

- \( F_p \) – Force due to pressure
- \( p \) – Inflation pressure
- \( n_i \) – normal of the surface the pressure is being applied to
- \( ds \) – differential surface area

The forces due to contact with the road are given by the equation 3.7

\[ F_c = \int_{FP} \tau_{ij} n_i ds \]  
\hspace{1cm} (3.7)

Where:

- \( F_c \) – Force due to contact
- \( \tau_{ij} \) – Shear Stress on surface
- \( n_i \) – normal of the surface the stress is being applied to
- \( ds \) – differential surface area
The forces due to a load at the hub are distributed through the rim contact as contact stress, therefore the force is given by 3.8

\[ F_{\text{hub}} = \int_{\text{rim}} \tau_{ij} n_i ds \]  \hspace{1cm} (3.8)

Where:

- \( F_{\text{hub}} \) – Force due to contact
- \( \tau_{ij} \) – Shear Stress on surface
- \( n_i \) – normal of the surface the stress is being applied to
- \( ds \) – differential surface area

### 3.3 Weak form of Rolling Contact Problem

To create a finite element solution to the rolling tire problem the equation 3.1, which represents the strong form, must be recast into the virtual work principle. This is also known as the weak form of the problem, [9]

The weak form of the rolling tire problem as given by Padovan [14] is given in equation 3.9
\[ \int_V \delta E_{ij} S_{ij} dV - \int_S \delta u_i p n_i ds + \int_{FP} \delta u_i \tau_{ij} n_i ds + \int_{rim} \delta u_i \tau_{ij}^{rim} n_i ds = \int_V \delta u_i \frac{d^2}{dt^2} u_i \]

(3.9)

Where:

\( E_{ij} \) – Lagrange Strain Tensor

\( S_{ij} \) – Piola-Kirchoff Stress

\( \delta u_i \) – Virtual Displacements

\( p \) – Inflation pressure

\( \tau_{ij} \) – Contact Stress as road

\( \tau_{ij}^{rim} \) – Contact Stress at rim

\( \frac{d (\cdot)}{dt} \) – co-moving time derivative

Subscript values

\( V \) – Tire Volume

\( S \) – Internal Tire surface

\( FP \) – Footprint contact surface

\( rim \) – Rim contact surface
Chapter 4

Lagrangian Formulation

A solution formulated under the Lagrangian approach uses a single reference frame for both the mesh and the material. This reference frame translates and rotates as the tire rolls. A material point tracked from this frame of reference is observed to have a trajectory as depicted in Figure 4-1. Even when a tire is rolling at a constant velocity, known as steady state, the local displacement of the point changes as a function of time.

Figure 4-1: Material Point and Mesh Motion (Lagrangian)
4.1 Finite Element Approximation

Standard displacement finite elements can be used to approximate the solution to the weak form of the rolling tire problem. An iterative quasi-static solution method, which neglects the effects of mass and damping, is used to approximate the displacements throughout the time of rolling. This results in several hundred small iterations throughout the time history of the roll.

4.2 Composite Shell Element

The tire structure is modeled with a 4 node curved shell element that is available in ABAQUS. To improve convergence and solution speed the reduced integration formulation is implemented. To prevent hour-glassing in areas of high stiffness change, an enhanced integration scheme for calculating the hour-glass stiffness. The composite nature of the tire is modeled using a laminate comprised of layers containing rubber and reinforcement material. During the formulation of the element’s stiffness matrix the reference surface is taken to be the outer surface of the tire. This benefits modeling for several reasons:

1. The shell’s initial geometry is much easier to define.

2. The contact will not require an offset to account for the location of the nodes at the mid-plane.

3. The outer surfaces displacements are available directly from the solution, so tire shape is easier to determine.
The S4R four node finite strain element formulation is selected for use in this model. The element accounts for large rotations and finite strains. It also accounts for variable thickness from each node. The element uses a bi-linear shape function to approximate the nodal variables along the shell’s surface.

A single element is used to approximate the thickness of the tire. Individual layers are specified within the element formulation. The shell element is a thick formulation that accounts for inter-laminar shearing by numerical integration through the thickness. The shell is integrated through the thickness to determine the stiffness matrix. Each layer has a three gauss points located at the top, middle, and bottom. The gauss point locations are depicted in Figure 4-2

![Layer Integration Point](image)

Figure 4-2: Gauss Integration Points for Composite Shell

Along the shell surface a reduced integration scheme is used. This results in one gauss point along the shell surface. Since the reduced integration of the element exhibits hourglass mode shapes, a stiffness control must be defined. For further
formulation details please refer to the ABAQUS theory manual [2] and the ABAQUS User’s Manual Volume VI: Elements [3]. The composite layers are modeled as a section of rubber with a layer of reinforcements embedded within it. This is done with the *REBAR LAYER card in ABAQUS. A rebar is a single strand of reinforcement material. The rebar is only able to withstand tensile force, thus provides no additional bending stiffness. Perfect adhesion of the rubber to the cord is also assumed when using rebar to model reinforcements. The angle of the reinforcement is specified relative to a local element coordinate frame. Additional inputs are the spacing of the individual cords, the area of the rebar, and the material that the rebar is comprised of. Since the layer of reinforcement is being placed within a thick shell, an offset from the shell’s mid-plane is also specified to allow for layers at different thicknesses, refer to figure 4-3.

The result of the rebar reinforcement is a smeared layer of reinforcing material located at the specified depth. This additional layer is then accounted for in the numerical integration scheme noted above. The layer’s thickness is determined by the current configuration of the element, the spacing of the reinforcement, and the area of each individual reinforcement. The material layer is assumed to orthotropic with the major stiffness being aligned with the 1 direction of a coordinate frame that has been rotated the reinforcement angle from element 1 direction.
4.3 Rim Element

A finite element was developed that will constrain the tire node position to rotate about the hub at a fixed vector. The hub position vector is defined by:

$$x_i^{rim} = x_i^{hub} + X_j^{rim} C_{ji} (\phi)$$

(4.1)

The rim vector, $X_j^{rim}$, is the original coordinate of the material point on the rim. The rim vector is defined by the rim width $w^{rim}$, rim radius $r^{rim}$, rim reference angle $\theta$. The rim reference angle is calculated from the original tire nodes coordinate,

$$\theta = arg \left( \frac{X_3^{tire}}{X_1^{tire}} \right)$$
The rotation matrix $C_{ji}(\phi)$ transforms the current tire material point into the rim reference plane, 4.3. This allows for the rim to be defined by only two parameters regardless of the orientation of the tire. The rim reference vector is then defined as:

$$X^{rim}_i = \begin{cases} 
  r^{rim} \cos(\theta) \\
  u^{rim} \\
  r^{rim} \sin(\theta)
\end{cases}$$ (4.2)

The displacement of the rim can then be given by:

$$u^{rim}_i = u^{hub}_i + X^{rim}_j C_{ji}(\phi)$$ (4.3)

The tire nodes that contact the rim are then constrained to rim using a penalty function. This forces the node position to be a function of the hub node position, hub node rotation, rim width, rim radius, and the original position of the tire nodes. The penalty constraint is defined by:

$$\delta_i = u^{tire}_i - u^{rim}_i$$ (4.4)

$$F^{rim}_i = k^{rim} \delta_i$$ (4.5)
4.4 Contact Element

The contact element used is based on the work presented by Darnell [6]. This element is a streamline of nodes that uses a penalty function to constrain their displacements to a flat road surface. The element also uses a stick-slip approach to model the friction between the tire and the road.
Figure 4-5: Contact Element Penetration

The penalty function for contact is defined as:

\[ p^n = \begin{cases} 
-k^n \delta^n & \text{if node penetrates surface} \\
0 & \text{No Penetration} 
\end{cases} \quad (4.6) \]

4.4.1 Stick-Slip Contact

To correctly model the tire and road contact condition the in plane shear must be accounted for at the road/tread interface. This is accomplished by using a Stick-Slip contact model. The stick-slip model assumes that a contacting point lies on a frictional yield surface throughout its contacting life. At every point in time the tread material will exhibit one of three conditions:

1. The material will reach a maximum allowable displacement, \( \delta_{\text{slip}} \), and begin to slip.
2. Continue to stick on the surface.

3. Be lifted out of contact with the surface.

As described in Darnell [6], the following equations will constrain the in plane displacement of the tread point $\delta^\tau$.

\[
\Phi_1 = (\delta^\tau - \delta^{\text{slip}}) \cdot \dot{e}^\delta \leq 0 \quad (4.7)
\]

\[
\Phi_2 = \left(\delta^\tau - \dot{\delta}^{\text{slip}}\right) \cdot \dot{e}^\delta \quad (4.8)
\]

\[
\Phi_3 = \left(\delta^\tau - \dot{\delta}^{\text{stick}}\right) \cdot 
\]

\[
\hat{e}^\delta \quad (4.9)
\]

\[
\Phi_1 \Phi_3 = 0 \quad (4.10)
\]

\[
\Phi_2 \Phi_3 = 0 \quad (4.11)
\]

Where $\dot{\delta}^{\text{slip}}$ is the time rate of change of $\delta^\tau$ when slipping occurs and $\dot{\delta}^{\text{stick}}$ is the time rate of change when the point sticks. $e^\delta$ is a unit vector pointing in the direction of tread point displacement. $\delta^{\text{slip}}$ is the maximum displacement of the point prior to slipping.

\[
\delta^{\text{slip}} = e^\delta \frac{|p^\mu \mu|}{k^\tau} \quad (4.12)
\]

\[
\dot{\delta}^{\text{slip}} = \frac{\partial \delta^{\text{slip}}}{\partial t} \quad (4.13)
\]

\[
= \frac{\mu}{k^\tau} \left[ e^{\delta^\tau} ||f^n|| + \left\| \dot{f}^n \right\| e^{\delta^\tau} \right]
\]

\[
\delta^{\text{stick}} = \frac{\partial \delta^{\text{stick}}}{\partial t} \quad (4.14)
\]
When a material point is sticking the velocity of the point is zero, $\frac{\partial \xi_i}{\partial t}$. Thus equation 4.14 becomes:

$$\delta_{\text{stick}} = \frac{\delta_{\text{stick}}}{t}$$

(4.15)

The equation 4.7 requires that the point must remain on the frictional yield surface. Equation 4.8 states that if the material is in the interior of the yield surface then it must be sticking. Equations 4.7 and 4.11 enforce that the point must slip if the displacement would exceed the yield criteria.

Once the current displacement is calculated it must be evaluated to determine if the point is sticking or slipping. If the point is slipping then the maximum allowable slip is used to adjust the point’s displacement back onto the frictional yield surface. This is enforced by equation 4.16. Darnell assumes an isotropic Coulomb friction model (i.e., symmetric friction coefficient and contact stiffness in the plane of contact) to calculate the contact behavior of the material point.

$$\delta = \begin{cases} 
\delta_{\text{slip}} & \text{If } \| \delta^\tau \| > \frac{|p^\tau p|}{k^\tau} \\
\delta_{\text{stick}} & \text{Otherwise}
\end{cases}$$

(4.16)

The frictional shear stress can now be calculated by:

$$\sigma^\tau = -k^\tau \delta^\tau$$

(4.17)
Chapter 5

Galilean Formulation

The Galilean formulation is an Arbitrary Lagrangian-Eulerian approach to the tire kinematics. The tire’s mesh is viewed as a control volume that the tire material is convected through. The mesh is allowed to deform and translate, and rotate about two directions, but is constrained in such a way so that it does not rotate about the spindle axis. The material is allowed to convect through the tire mesh thus simulating rolling, while keeping the mesh in constant contact with the road surface. This allows for a greater refinement of the mesh in the contact patch without over-refinement of the rest of the tire model.

In the Galilean formulation the observer of motion is attached to a frame of reference that is located on the tire’s hub. This frame of reference is allowed to both translate with and rotate about this point. During a tire rolling the observer only notes local deformation of the tire. As an illustration, this type of observer would be fixed on the hub and would only observe what was occurring at a certain location on the tire. Points would flow past the observation point, but the only the localized
deformation would be observed. The material point exhibits the same motion as the Lagrangian formulation, but the mesh does not rotate about the axle. The mesh still deforms, translates and rotates about all other axis. This motion is depicted in Figure 5-1.

![Material Point and Mesh Motion (ALE)](image)

**Figure 5-1: Material Point and Mesh Motion (ALE)**

### 5.1 Reference Frames

The Galilean formulation uses two frames of reference to describe the motion of a material point. The first frame of reference is constrained to move with the finite element mesh, this frame is refereed to as the mesh frame. The second frame, refereed to as the material frame, are constrained to only rotate about the axle of the tire. All other motions of the material frame are governed by the mesh frame. Thus all translations are according to the mesh frame and all rotations not about the tire axle are in the mesh frame. Therefore the only motion that does not require a change within the mesh frame is an axi-symmetric spin of the material about the tire axle. The relationship between the coordinate frames is depicted in Figure 5-2.
Figure 5-2: Material and Mesh Reference Frames

5.2 Model Changes

There are several changes that must be made to the Lagrangian model in order to use it with the Galilean formulation. An additional node must be defined at the hub and axial positions. The hub node is used to drive the model, the material reference node is used to monitor the relative rotation of the hub about the axle. Three constraint must be applied to the model to keep the mesh from rotating about the axle. Also the contact and rim elements formulations must be updated.
5.3 Tire Structure/Hub

As proved by Darnall [6] it is possible to use standard finite elements in a Galilean formulation. The same finite element formulation that was used in the Lagrangian models will also be used in the Galilean solution of rolling contact. The rim element must be reformulated. The element is comprised of five nodes: the hub node, the tire node, two tire reference nodes, and the mesh node. The difference between the Lagrangian formulation and the Galilean formulation of the rim element is that the constraint forces are calculated according to the position of the mesh node, but the stiffness and force are applied at the hub node. This allows for a decoupling of the mesh and tire material.

5.4 Contact Element

The contact element is reformulated according to Darnell [6]. The material displacements during contact are recast into the Eulerian reference. Using the material derivative we calculate both $\dot{\delta}_{\text{slip}}$ and $\dot{\delta}_{\text{stick}}$.

\[
\dot{\delta}_{\text{slip}} = \frac{\partial \delta_{\text{slip}}}{\partial t} + \frac{\partial \delta_{\tau}}{\partial x_{i}} \frac{\partial x_{i}}{\partial t} = e^{\delta_{\tau}} \frac{p^\nu \mu}{k^\mu} + e^{\delta_{\tau}} \dot{p}^\nu \mu - e^{\delta_{\tau}} \frac{p^\nu \mu}{(k^\nu)^2} k^\nu
\]

(5.1)

\[
\dot{\delta}_{\text{stick}} = \frac{\partial \delta_{\text{stick}}}{\partial t} + \frac{\partial \delta_{\tau}}{\partial x_{i}} \frac{\partial x_{i}}{\partial t}
\]

(5.2)

Assuming that $r$ is a vector that points from the center of the hub to the material point being evaluated, $\dot{\delta}_{\text{stick}}$ is restated as equation 5.3.
\[ \dot{\delta}^{stick} = \dot{r} + \frac{\partial \delta^{tau}}{\partial \theta} \dot{\theta} \]  

(5.3)

Where:

\( \dot{\delta}^{stick} \) – Velocity of the tread point in the stuck condition

\( \dot{r} \) – The change in radius with respect to time

\( \frac{\partial \delta^{tau}}{\partial \theta} \) – The change in deflection with respect to rotation

\( \dot{\theta} \) – Material rotation velocity
The time rate of change for \( r \) was stated by Darnell [6] as:

\[
\dot{r} = \dot{n} + \dot{\theta} R_0 \gamma e_s \quad (5.4)
\]

Where:

\( \dot{r} \) – The change in radius with respect to time

\( \dot{n} \) – The change in position with respect to time

\( \dot{\theta} \) – Material rotation velocity

\( R_0 \) – Undeformed radius of the point

\( \gamma \) – stretch ratio in the direction of mass flow

\( e_s \) – unit vector in the direction of mass flow

By applying finite differences at a node \( J \), the contact displacement can be ap-
proximated by equation 5.5.

\[
\delta_j^r = \frac{b\delta_j^r + \Delta n_J + + R_0 e^\gamma_j J \Delta \theta + \frac{R_0 \delta_{J-1}^r \Delta \theta}{l_{h-1}}}{1 + \frac{R_0 \Delta \theta}{l_{h-1}}} \quad (5.5)
\]

5.5 Mesh Rotation Constraint

Three constrain equations are used to tie the different reference frames together. The mesh reference frame is tied to a reference node that is called mesh. The material reference frame is tied to a reference node referred to as hub. The relative rotation between the two reference frames is measured by the third node called material relative.

The first constraint is a standard ABAQUS REVOLUTE MPC. The revolute multi-point constraint constrains the rotation of one node to equal the rotation of a master node plus a scalar rotation about the axle defined by the third node. This constraint is defined in equation 5.6, [2]
\[ C \left( \phi_{\text{Mesh}} \right) = C \left( \phi_{\text{Hub}} \right) \cdot C \left( \phi_{\text{MatRev}} \bar{a} \right) \] (5.6)

Where:

- \( C (\cdot) \) – Rotation Matrix Operator
- \( \phi_{\text{Mesh}} \) – Mesh Node Rotation Vector
- \( \phi_{\text{Material}} \) – Hub Node Rotation Vector
- \( \phi_{\text{MatRev}} \) – Material Rotation about the axle
- \( \bar{a} \) – Axle Vector

The rotation matrix operator \( C (\cdot) \) transform the rotation vector into a rotation matrix by the equation 5.7.

\[ C \left( \phi \right) = C_{ij} = \cos \phi \delta_{ij} + (1 - \cos \phi) \phi_i \phi_j + \sin \phi \epsilon_{ijk} \phi_k \] (5.7)

Where:

- \( C_{ij} \) – Rotation Matrix
- \( \phi \) – Length of the rotation vector
- \( \phi_i \) – Rotation Vector
- \( \delta_{ij} \) – Kronecker Delta
- \( \epsilon_{ijk} \) – Alternator Tensor

(5.8)
The second constraint used is a standard ABAQUS PIN MPC. This constraint forces the displacement of the mesh to equal the displacement of the material coordinate frame, this is defined in equation 5.9.

\[
\vec{u}^{\text{Mesh}} = \vec{u}^{\text{Material}} \quad (5.9)
\]

Where:

\[
\vec{u}^{\text{Mesh}} - \text{Displacement of the mesh}
\]

\[
\vec{u}^{\text{Material}} - \text{Displacement of the hub}
\]

The third constraint is a user defined constraint. A user constrain equation has to be employed to keep the mesh from rotating with the material as the model rolls. The constraint equation uses the rotation of the material reference frame, as measured by the hub node, and calculates the required amount of rotation to keep the mesh reference frame from rotating. The rotation angle is given by:

\[
\theta = \arg \left( \frac{\hat{e}^{\text{material}}_2 \cdot \hat{e}^{\text{mesh}}_3}{\hat{e}^{\text{material}}_2 \cdot \hat{e}^{\text{material}}_2} \right) \quad (5.10)
\]

Where:

\[
\theta - \text{Rotation between the mesh and material reference frames}
\]

\[
\hat{e}^{\text{material}} - \text{Material Reference Frame}
\]

\[
\hat{e}^{\text{mesh}} - \text{Mesh Reference Frame}
\]

The negative of that angle is then applied to the material reference node, to ensure
that the mesh does not rotate but the material is free to rotate.
Chapter 6

Model Parameters

This chapter describes the input values needed to create a finite element models with the ALE contact and ALE rim element. The chapter also details the proper method of determining penalty values for the ALE contact and rim element. The model was developed to be solved using ABAQUS/standard finite element program. The model is comprised of composite shell elements that are available in ABAQUS coupled with user defined contact and rim elements.

6.1 Initial model geometry

The initial model geometry was created from a cured tire layout. A cured tire layout describes the geometry and component locations of tire that has curred in a tire mold. A profile shape is created by assuming that the center of the tire is origin of a coordinate system. The X direction is pointing radial toward the centerline, the Y direction is pointed in the lateral direction, this the profile coordinate system. The
length along the outside of the carcass, referred to as the mold line, was parametrized so that 10 parametrically equally spaced elements were created. The coordinate of each point was then taken with reference to the profile coordinate system. The total gage is measured by striking a line normal to the outside of the carcass at each coordinate, and measuring the distance along the normal to the inside of the tire carcass. The coordinates of these profile points were used as node points in the finite element model, called profile nodes.

Three additional nodes are also defined. The first two nodes are located at the origin of the tire model, one is a hub node, the other the mesh node. The mesh node is used as the origin of the mesh reference frame. The hub node is used as the origin of the material reference frame. The third node is used to track the relative rotation between the two reference frames, this node is referred to as the material reference node. The material reference node is located 1.0 in along the axle in the global Y direction.

The total gage is used to define the shell thicknesses in a piece-wise linear fashion, using *NODAL THICKNESS [1] and the NODAL THICKNESS parameter on the *SHELL SECTION [1] cards in ABAQUS.

The profile nodes were located on the global XY plane of the tire model, with X being the radial coordinate, see Figure 6-1. The nodes were then revolved around the Y axis to form a three-dimensional grid. Linear Shell element were then used to connect the nodal points together. Each group of elements at a given angle are referred to as a sector.

The Lagrangian model was generated with 72 sectors spanning 5° each. The size
of the Lagrangian mesh has a large impact on the rolling results. If the mesh is too coarse the tire’s response will suffer from aliasing. Aliasing will show up in the results as flat line segments where a peak or valley should appear. The mesh density also has a large affect on solution time, the more elements the greater the solution time. The 72 sectors were chosen to provide reasonable results in a timely manner. The Lagrangian mesh is shown in Figure 6-2

The ALE tire model was generated with 10 sectors spanning $5^\circ$ each in the footprint. Outside of the footprint 30 sectors spanning $10^\circ$ each make up the remaining
mesh. The ALE formulation only requires that a few elements be located outside of the footprint area. The 30 sectors were chosen to provide good convergence without a large impact on solution time. The 10 sectors in the footprint were chosen to be consistent with the span of the elements in the Lagrangian model. The ALE mesh is shown in Figure 6-3.
6.2 Definition of Shell Layers

The shell layer data maybe obtained from a virtual tire model, or from a physical tire mode. The layer thicknesses, type of material, and if the layer is reinforced must be noted at the mid point of each profile element. If the layer contains reinforcement it must also be noted the reinforcement distance form the middle of the section, reinforcement angle, reinforcement material, area of reinforcement, and the distance between each cord end.
The layup information is entered into the ABAQUS *SHELL SECTION card starting from the inside of the tire profile out. Each line represents a layer of the tire. Reinforcements are entered under the *REBAR LAYER card directly after the last layer. The formats for both *SHELL SECTION and *REBAR LAYER are given in the ABAQUS/Standard User’s Manual [1]

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### 6.3 Definition of Penalty Stiffness

The penalty values were chosen by running a sweep of static stiffness ALE models. The rim penalty stiffness was defined as 5.0 and increased by a magnitude of 10 for each run up to 5.0E+10. The contact penalty stiffness was also defined in a similar manner, but the initial value was 2.5 and the final value of 2.5E+10. The radial,
lateral, and longitudinal stiffness was calculated for each run. These values were then entered into a matrix of the penalty values. These test matrices are plotted as surface plots in Figures 6-4, 6-5 and, 6-6.

The penalty values for both the rim and the contact were then selected from the region that was observed to have a constant static stiffness for all three types of loading. One has to be careful not to select a penalty stiffness that is too large. If the penalty stiffness is much larger than the norm of the stiffness matrix the solution will become swamped by the penalty functions and the model may not converge. Observe in the penalty stiffness Figures 6-4, 6-5 and, 6-6 the sharp increase in stiffness at penalty values of 10E10. This due to the swamping of the solution by the penalty...
Figure 6-5: Lateral Stiffness for Penalty Sweep
Figure 6-6: Longitudinal Stiffness for Penalty Sweep
6.3.1 ALE Rim Element

The ALE rim element is implemented as an ABAQUS user element. The node list defines the wedge segment of the rim element and attaches to tire and both the mesh and hub nodes.

The property values for the ALE rim element are: rim radius, rim width, and rim penalty stiffness. The rim radius is half the rim diameter. The rim width is half the total width of the rim. The rim penalty is set to $2.5 \times 10^5$ as described in section 6.3. The units for the rim property values are given in table 6.1.
6.3.2 ALE Contact Element

The contact element requires defining a node streamline that runs in the direction of material convection. The contact element also requires that the first and last node in the streamline will not contact the road surface.

The property values for the ALE contact element consist of: road location, element width, coefficient of friction, contact penalty stiffness, and tread block stiffness. The road location is given according to the location of the road plane in the global +Z coordinate direction. The element width is the undeformed width of the contact streamline. The contact penalty is set at $2.5 \times 10^5$ by the procedure documented in
road location in
element width in
coefficient of friction unitless
contact penalty lbs/in³
tread block stiffness lbs/in³

Table 6.2: Units for ALE Contact Element (English)

section 6.3. The tread block stiffness is the modulus of the tread material in shear.

The units for the contact properties are given in table 6.2

6.4 Loading Conditions

The simulation of the tire’s performance is carried out in several steps depending on the goal of the analysis. The first three steps are common regardless of what type of stiffness values are being determined.

Step 1 Mount the tire and apply inflation pressure.

Step 2 Push the tire into the road with a displacement control.

Step 3 Apply final load with a force control.

Depending on if this is to be a static stiffness simulation or a rolling stiffness simulation determines the number and types of the steps. For a static stiffness calculation in either the lateral or longitudinal direction Step 4 pushes the tire with a displacement control in the direction of required stiffness.

For a rolling simulation step 4 begins with a straight ahead rolling by applying a longitudinal velocity boundary condition to the hub node. Note that this boundary condition must be in in/s since the model dimensions are in lbs, in, s. In the next
step a slip angle condition is simulated by applying a lateral velocity. The velocities are given by equation 6.1.

\[ V_x = V_{rolling} \cos \alpha \]  
\[ V_y = V_{rolling} \sin \alpha \]
Chapter 7

Testing Procedures

A physical tire was tested to provide a base line comparison of the finite element models. The tire was tested for lateral and longitudinal stiffness through a quasi-static load/deflection test. The footprint development was measured using a flat plank tester with a force transducer array embedded in the road surface. The lateral force and aligning moments were measured as the tire rolled through the transducer arrays.

The tire tested was a smooth molded 205/65R15 touring passenger tire. The tire is comprised of a single ply and two belts along with several rubber components. The tire was smooth molded so that the physical tire and the model could be compared without the influence of a tread pattern.

The physical tire testing was done at Smither’s Scientific Testing in Ravenna, Ohio, [10]. All tests were performed on a Flat Surface Tire Dynamics Machine (FSTDM) test rig. The test rig is shown in Figure 7-1.

The base loading conditions for the tire are summarized in table 7.1.
Table 7.1: Physical Tire Testing Conditions

7.1 Radial, Longitudinal, and Lateral Stiffness

For the radial stiffness test the tire was inflated to the prescribed inflation pressure, capped, and loaded to a prescribed load on to the FSTDM surface. The force verses deflection characteristics are then measured by a transducer that is located in the hub of the rim. Note that during this test the wheel is in a locked position thus the tire is not allowed to roll. Once the tire has achieved the final normal load, the tire is longitudinally displaced three inches by the test rig. The force verses deflection is then measured. The procedure is repeated four times at each 90° section and once in each direction. The transducer measures forces and moments every 0.05 inches. The data is averaged at each displacement point for all sixteen runs. In a lateral stiffness test the tire is mounted and loaded similar to the radial stiffness test. Once the tire has achieved the final normal load, the tire is laterally displaced three inches by the test rig. The force verses deflection is then measured. The procedure is repeated four times at each 90° section and once in each direction. The transducer measures forces and moments every 0.05 inches. The data is averaged at each displacement point for all sixteen runs.
7.2 Rolling Tire Force Development

The rolling tire force values were obtained by loading the tire onto FSTDM with a prescribed slip angle. The tire was then rolled at 3 in/s and the forces and moments were recorded at the wheel hub. The forces and moments change until the tire reaches a steady state. The value at steady state is recorded as the force and moment at that slip angle. After the tire has rolled 120 in at a prescribed slip angle, the test rig is unloaded, tire replaced back at the starting position. A new slip angle is put into the tire and the tire is rolled again. The slow speed at which the tire is rolled was chosen so that the transient response of the tire could be accurately captured by the test.
Figure 7-1: Flat Surface Tire Dynamics Machine
Chapter 8

Results

The results of a comparison between a physical tire, and two finite element models are compared in this chapter. The first model uses the Lagrangian formulation to predict the model’s behavior during rolling. The second model uses the ALE formulation to predict the rolling behavior of the model. The model’s static stiffness values are compared to the physical tire, along with the lateral force and aligning moment development during rolling.

Finally model run times are compared between each model. These results will be used to prove that the ALE approach can be applied to tire models, the ALE approach can provide results similar to a full Lagrangian method, and the results can be achieved in a much shorter run times using ALE.
The stiffness results for both the models and the physical tire are summarized in table 8.1. Table 8.2 contains a summary of lateral force and aligning moment as the tire rolls and approaches a fully developed footprint. The force verses displacement is plotted for radial, lateral, and longitudinal directions. Lateral force and aligning moment are also plotted verses distance tire rolled with various slip angle inputs. The percentages of errors are calculated following equation 8.1.

\[
\% \text{Error} = 100 \times \frac{\text{Actual} - \text{Predicted}}{\text{Actual}}
\] (8.1)
8.2 Radial Stiffness

The physical tire’s radial stiffness was unavailable for the comparison. Both model predicted a similar radial stiffness value. The FAST tire model predicted a radial stiffness 1.9% lower than the lagrangian model. The radial force versus radial deflections are shown in Figure 8-1.

8.3 Longitudinal Stiffness

Both model predicted a higher longitudinal stiffness than the physical tire. The FAST tire model predicted a 6.05% higher longitudinal stiffness where as the lagrangian model’s predicted stiffness was 14.75% higher. Neither of the model was able to obtain the peak longitudinal force of 1861 lbs. The inability of the models to achieve peak longitudinal force may be a caused by using a Coulomb friction of 1.0 for the entire contact region. Increasing the coefficient of friction should allow the model to converge to the higher load.

The longitudinal load verses the longitudinal deflection are plotted in Figure 8-2.

8.4 Lateral Stiffness

The lateral stiffness for both models were predicted to be higher than the physical tire. The FAST tire model predicted a lateral stiffness of 7.3% higher than the physical tire. The Lagrangian model predicted a lateral stiffness of 11.8% higher
Figure 8-1: Radial Force
Figure 8-2: Longitudinal Force
that the physical tire. Neither model's peak lateral force was as high as the physical
tire. The FAST tire model reached its maximum lateral force according to Coulomb
friction with a coefficient of 1.0. The Lagrangian model has convergence issues once
the model began to slip at approximately 1400.0 lbs. This is a combination of friction
and contact method. Figure 8-3 shows the lateral force verses lateral deflection for
the physical tire, FAST model, and the Lagrangian model.

8.5 Lateral Force development during rolling

The lateral force was plotted verses the longitudinal distance rolled to obtain the
lateral force transients. The errors in force between the physical tire and the models
are given in table 8.5. Both the ALE and Lagrangian models under predict the lateral
force of the actual tire. This is especially true at lower slip angles. This error may be
a result of using an elastic foundation formulation to predict the deflection of stream
lines in the tire footprint region.

At low slip angles there is little to no slip of the tread material point, thus the
lateral force response is influenced by the stiffness and displacement nature. As the
tire reaches higher slip angles the rear of the footprint begins to slip. For slip to occur
on the tread block the lateral force must exceed the available friction force. Since
portion of the footprint approach and exceed the frictional force, the global lateral
force response is dominated by the frictional force. The friction force is dictated by
the normal road surface force and the coefficient of friction at the contact point.

The elastic foundation model assumes that the tread block reaction force is a func-
Figure 8-3: Lateral Force
tion of the tread material deflection and the material stiffness. This model neglects any affects of neighboring material points on reaction force. By using a string or beam model for the force calculation affects of neighboring particles can be captured.

<table>
<thead>
<tr>
<th>Slip Angle</th>
<th>FAST</th>
<th>Lagrange</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>-170.20</td>
<td>-68.57</td>
</tr>
<tr>
<td>0.5°</td>
<td>49.15</td>
<td>21.94</td>
</tr>
<tr>
<td>1°</td>
<td>32.89</td>
<td>15.28</td>
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<tr>
<td>3°</td>
<td>17.65</td>
<td>-23.38</td>
</tr>
<tr>
<td>6°</td>
<td>18.50</td>
<td>-1.73</td>
</tr>
</tbody>
</table>

Table 8.3: Percent Error in Steady State Lateral Force

8.6 Aligning Moment development during rolling

The aligning moment was plotted verses the rolling distance to provide the aligning moment transients. The steady state aligning moment was measured once a fully developed footprint was achieved. The percent error between the physical tire and the models are shown in table 8.4. For a majority of the slip angles the models over predict the aligning moment of the physical tire. The physical tire data is suspect though since it does not follow the trends that should be seen in the aligning moment as it rolls.

This may be explained by the fact that this is a smooth molded tire. When a smooth molded tire is placed into a press the belts may shift in a manner that is not normally seen in a treaded tire. This will cause a distorted footprint pressure, and will also result in uncharacteristic aligning moments. Even though the magnitudes of the aligning moments do not match up the results of both models trend correctly.
Figure 8-4: Lateral Forces Development, 0° Slip Angle
Figure 8-5: Lateral Forces Development, 0.5° Slip Angle
Figure 8-6: Lateral Forces Development, 1° Slip Angle
Figure 8-7: Lateral Forces Development, 3° Slip Angle
Figure 8-8: Lateral Forces Development, 6° Slip Angle
Figure 8-9: Development Aligning Moments, 0° Slip Angle
Figure 8-10: Development Aligning Moments, 0.5° Slip Angle
Figure 8-11: Development Aligning Moments, 1\(^\circ\) Slip Angle
Figure 8-12: Development Aligning Moments, 3° Slip Angle
Figure 8-13: Development Aligning Moments, 6° Slip Angle
<table>
<thead>
<tr>
<th>Slip Angle</th>
<th>FAST</th>
<th>Lagrange</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>-67.75</td>
<td>-304.29</td>
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<tr>
<td>0.5°</td>
<td>11.44</td>
<td>-117.31</td>
</tr>
<tr>
<td>1°</td>
<td>8.05</td>
<td>-139.29</td>
</tr>
<tr>
<td>3°</td>
<td>-19.67</td>
<td>-24.70</td>
</tr>
<tr>
<td>6°</td>
<td>-49.77</td>
<td>153.60</td>
</tr>
</tbody>
</table>

Table 8.4: Percent Error in Steady State Aligning Moment

<table>
<thead>
<tr>
<th>Simulation Type</th>
<th>Steps</th>
<th>Time Lagrange</th>
<th>Time FAST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral Stiffness</td>
<td>5</td>
<td>1473.0</td>
<td>1133.70</td>
</tr>
<tr>
<td>Longitudinal Stiffness</td>
<td>5</td>
<td>1592.2</td>
<td>713.35</td>
</tr>
<tr>
<td>Straight Roll</td>
<td>5</td>
<td>34516.0</td>
<td>788.89</td>
</tr>
<tr>
<td>0.5° Slip</td>
<td>5</td>
<td>28951.0</td>
<td>688.86</td>
</tr>
<tr>
<td>1° Slip</td>
<td>5</td>
<td>30549.0</td>
<td>604.52</td>
</tr>
<tr>
<td>3° Slip</td>
<td>5</td>
<td>23481.0*</td>
<td>661.94</td>
</tr>
<tr>
<td>6° Slip</td>
<td>5</td>
<td>20059.0*</td>
<td>692.02</td>
</tr>
</tbody>
</table>

* Did not converge to full 100 sec run

Table 8.5: Model Run Times

8.7 Model Run Times

The solution speeds of the FAST tire model and the Lagrangian tire model were compared. The Lagrangian model has 26784 DOF the FAST tire model has 9846 DOF. The table notes the number of steps necessary for each simulation and the total CPU time in seconds needed for solution. Both models were run on Itanium 64bit machine’s with 2CPUs, 8GB memory, and HP-UX11. The models were executed using only one CPU at a time.

According the the table 8.5 there is a significant speed increase, a factor of 4 during the rolling simulations, when running a simulation using the FAST model. The same increase in execution speed is observed in the static simulations. The faster solution times during rolling can be attributed to the ALE contact method, which keeps the contacting nodes concentrated in the footprint. This eliminates
additional looping in the solution to find the next nodes to contact the road surface. Also only the deformation due to straining the material must calculated in the ALE solution. This also cuts down on number of increments necessary to find a solution. Therefore a solution can be obtained much quicker using the FAST tire model than the Lagrangian model.
Chapter 9

Discussion of Assumptions

Some of the major assumptions made in this model are examined in this chapter. Several assumptions were made to create a model that could predict the behavior of the tire during rolling. Some of these assumptions may have impacted the accuracy of the model’s prediction, while other assumptions could be made knowing that they would have little to no impact on the end condition.

Modeling Assumptions

1. Tire Sectioning

2. Linear vs. Nonlinear Material Models

3. Quasi-Static vs. Dynamic Rolling

4. Shell Elements

5. Mesh refinement

6. Material Models
7. Friction

8. Contact Model

9. Neglecting the Bead

10. Rim contact

9.1 Tire Sectioning

To defined the composite lay-up of the shell elements sections of the tire are taken and measured, either from a model or on a physical tire. Since it is impossible to section the tire into infinite pieces there will be some error associated with the sectioning of the tire. This error is analogous to the error one sees with element convergence. The more sections that are taken the better the approximation is. When sectioning the tire it is best to ensure the sections encompass the endings of all major components: belts, plies, and bead filler. These components make up a majority of the structures stiffness, thus the better they are captured the better the approximation will be.

9.2 Mesh Refinement

Since a finite element model approximates a structures displacement using node points and element domains, the mesh can be a major source of model errors. This model uses a standard ABAQUS SR4 shell element for a majority of its structure. The more elements used in the model the more the model converges to the solution.
The trade-off is that the more elements the bigger the model, thus the greater the solution time. The rate of convergence of most FEA models will plateau at a certain number of elements. That is, no further mesh refinement will improve the results. The analyst uses this criteria to determine the required number of elements.

9.3 Shell Elements

The tire body was approximated with composite linear shell elements which solve only in plane displacement fields. Shells were used since only the global stiffness of the structure was required. Local stresses and strains, which require a knowledge of transverse displacement fields, were not to be considered by this model.

9.4 Linear vs Nonlinear Material Models

Accurately model the behavior of the materials has a large effect on the error of the model. Since the focus of this model was solution speed and global stiffnesses, a linear elastic constitute model was used for all materials. A linear elastic model assumes that the material’s stress-strain behavior is proportional and is defined by a single parameter, Young’s Modulus. The assumption also requires a definition of the volumetric change of the material, this is given as the Poisson’s ratio. For all rubber materials a Poisson’s ratio of 0.499 was used. This makes the model behave as nearly incompressible. A Poisson’s ratio of 0.5 cannot be used in linear elastics due to the calculation of the bulk modulus ($K$) from the Young’s elastic modulus using equation
9.1. A Poisson’s ratio of 0.5 causes a divide by zero. Thus 0.4999 is used. For all steel wire the Poisson’s ratio of 0.333 was used.

\[ K = \frac{E}{3(1 - 2\nu)} \]  

(9.1)

9.5 Quasi-static vs. Dynamic Rolling

Rolling of any body is generally assumed to be a dynamic process. But when the speeds are sufficiently slow enough dynamics effects such as acceleration, Coriolis effects, etc can be neglected and a quasi-static simulation approach can be used to solve the problem. Since the tire was rolled at only 3 in/s it was safe to assume that the dynamics influences would be small. If the speed of the tire were to increase the one must use an implicit dynamic solver with the models. Both elements were formulated with this ability.

9.6 Contact Model

The contact element assumes a linear elastic foundation model for the calculation of the shear stresses. The linear elastic foundation model is used to calculate the tread material displacement. As it has shown in Dixon [7] there are several better approximations for tread displacements: string and beam models. The contact element could be reformulated to accommodate both of these models. This would allow for a better approximation of the tread material displacement inside the footprint.
The current formulation also assumes that the material is isotropic. A better approach may be to use an orthotropic material. Material stiffness can then be assigned for a local coordinate system. This allows for more flexibility in defining the inter-facial stiffness of the tread material with the road surface. This may also allow for the approximation of tread patterns.

### 9.7 Friction

This model uses a coefficient of friction to define the relationship between normal contact force and tangential contact forces. This coefficient is assumed to isotropic and constant. Obtaining an accurate measure of the coefficient is difficult when it comes to rubber/road interactions. Factors such as temperature, rate of shear, morphology of the rubber, and surface aspirates in the road surface all can have an effect on the coefficient. A small change in the coefficient of friction has a large effect on the model’s reaction forces and convergence. In order to ensure a good model an accurate measure of friction must be made.
Chapter 10

Conclusions

10.1 Conclusion

A model proposed by Darnell [6] was adapted to be used in tire prototyping. The objective of the model was to provide a fast and accurate measure of both static and transient stiffnesses. It has been shown that by accurately modeling the sidewall, and creating a rim element one can obtain reasonable results for both static stiffnesses and rolling tire transients. Though the model provides a reasonable results in a quick time frame, there are additional improvements to be made. The static models were unable to obtain the peak forces of the physical tire model, and the tire steady state value of the lateral force and aligning moment were significantly different when compare to the physical model.
10.2 Recommendation for Further Research

- The contact model should be modified to allow for an anisotropic tread block material.

- The contact model should be modified to use either the string or the beam theory for tread block deflection.

- The contact formulation should be modified to account for tread pattern effects.

- The model should be verified to predict similar results for a wider range of tire types, including Radial Medium Truck tires and High performance Passenger tires.

- The model may benefit by using hyperelastic materials for both the rubber and the polyester cords. A study needs to determine if using these constitutive laws give more accurate results with too much of a time penalty.

- This model should be tested to determine if the accuracy holds at higher speeds.

- The model needs to be verified to determine how well it predicts all forces and moments during rolling.
Bibliography


