Supply chains with bi-level demand: analyzing the impact of inventory policies

Parag R. Dhumal

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A Dissertation

Entitled

Supply Chains with Bi-level Demand: Analyzing the Impact of Inventory Policies

by

Parag R. Dhumal

Submitted as partial fulfillment of the requirements for
the Doctor of Philosophy Degree in
Manufacturing Management

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In retail stores, we often see that products are placed on sales promotion programs in a cyclical manner. Demand for the product increases during promotional period and returns to the regular level at the end of promotion. This cycle is then repeated with almost regular frequency. We refer to this demand pattern as bi-level demand and the supply chains operating in such environments are the main focus of this study. We study the problem of formulating an inventory policy in such supply chain. Because of the periodic steps in the demand function and the uncertainly of the demand, the problem is very challenging and the optimal solution is hard to obtain.

First we consider only the retailer, and narrow down focus to the development of the inventory policy for such retailer operating in bi-level deterministic demand environment. We define bi-level demand problem (BDP) as finding the order quantity (how much) and ordering time (when) given the holding and ordering costs. We develop
three heuristics for the BDP and a procedure for finding an optimal solution for a subset of BDP. We generated a large set of test problems using appropriate experimental design based on problem parameters and compare the performance of heuristics against lower bounds or optimal solution as the case may be. The heuristics are found to be efficient and provide close to optimal results in most cases.

Next, we model a generic version of supply chain consisting of three members—the retailer, distributor and manufacturer—and focus on the problem of formulation of inventory policy for members of such supply chain operating in stochastic (uncertain) bi-level demand environment. Considering the tradeoff between ordering/setup cost, holding cost, and shortage cost, we develop three ordering policies—Moving Average Policy (simple policy), Target Inventory Level Policy (sophisticated policy), and Complete Cooperation Policy (CCP). First two policies assume supply chain members do not share any information and order independently to minimize their costs, while in the last policy members cooperate to minimize total supply chain costs. We evaluate the performance of these policies by testing them on large set of problems using simulation. Results reinforce that cooperation results in lower supply chain costs. We also found that the major cost savings can be achieved by using more sophisticated policies than simple policies like moving average even if cooperation may not be possible.

In addition to the problem of formulating inventory policies, we also develop a game called ‘Cola-Game’ which instantiates the supply chain discussed above. The game involves simulating a supply chain and could be played in either independence or cooperation modes. This game has been field tested in engineering and business classes at the University of Toledo. We found that players developed an appreciation for
fluctuating demand and its impact on the costs and performance of a supply chain. They also learned the benefits and a monetary evaluation approach for cooperation. Our statistical analysis revealed that as the game progressed, the performance of the teams improved. Thus this game can be used as a tool to educate students and managers on the various issues in supply chain inventory management.
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CHAPTER 1: INTRODUCTION

In retail stores, we often see that products are placed on sales promotion programs in a cyclical manner in an effort to boost demand. Demand for the product increases during promotional period and returns to the regular level at the end of promotion. Promotions are usually repeated with almost regular frequency. Thus such promotional policies results in a demand pattern with two levels corresponding to promotional and regular periods. We refer this demand pattern as bi-level demand. Such demand poses lot of inventory management challenges for retailer because of periodic steps and uncertainty in demand. Information distortion and shipping lead times in supply chains further complicate these challenges for upstream members i.e. distributor and manufacture. In the current era of increasing competition and supply chain cooperation, it becomes essential for supply chain members to cooperate or at least use a strategic approach in formulating their inventory management policies.

Motivated by these challenges, we address the problem of finding appropriate inventory management policies for the supply chains operating in a bi-level demand environment. First we consider only the retailer, and focus on developing an inventory policy assuming bi-level deterministic demand. We define bi-level demand problem (BDP) as finding the order quantity (how much) and ordering time (when) given the
holding and ordering costs. We develop a procedure for finding an optimal solution for a subset of BDP and three heuristics for the BDP.

Next, we consider the problem of developing inventory policies for a complete supply chain consisting of the retailer, distributor and manufacture where retailer faces a stochastic (uncertain) bi-level demand. In any real-life supply chains, various approaches are used for designing ordering policy. The simplest include forecasting demand based on expert’s opinion or by calculating the average of past demand periods. Some supply chain members use more sophisticated approaches based on computer algorithms. Cooperation among players is desirable and some incorporate this information in their decision making, but it cannot always be mandated. Thus taking into considerations these approaches and tradeoffs between ordering/setup cost, holding cost, and shortage cost, we develop three ordering policies. First one is simple policy called Moving Average Policy (MAP). Second policy uses complex calculation and we call it as Target Inventory Level Policy (TILP). Both policies assume that supply chain members do not share any information and order independently to minimize their costs. Lastly, we develop Complete Cooperation Policy (CCP) which is a modification to an earlier policy in which members cooperate to minimize total supply chain costs.

In addition to the formulating inventory policies and evaluating performance using computer simulation, we wanted to see how students would perform if placed in similar challenging environment. Studying student behavior and evaluating need for educating them to manage such challenges, will further strengthen implications of our research. It has been long argued that we are facing scarcity of trained supply chain managers. Supply chain managers require a variety of skills such as planning, analysis
and modeling to perform their jobs effectively. To meet this requirement, substantial changes in supply chain education is necessary (Gammelgaard and Larson, 2001) which reiterates the need for innovative techniques to impart such education. Hence we develop ‘Cola-Game’ which is a generalized version of supply chain discussed above. The game involves role playing, decision making by the teams of three in either independence or cooperation modes. We conducted this game in engineering and business classes at the University of Toledo and report our finding in this research.

In the course of this research, we have done an adequate literature review. This has improved our understanding of contributions made by earlier studies in this area and also pointed out the need for undertaking this research. In the next section we summarize literature reviewed during this study.

1.1 Literature Review

Designing a suitable inventory system has always been a question of great interest for both practitioners and researchers. The classical EOQ model has been widely used in practice to determine replenishment policy. It has been the starting point for academic projects. The classical EOQ model, although widely generalizable is based on lot of restrictive assumptions. Thus there is room for improvement or in some cases need for better models especially in situations like bi-level demand considered here.

The EOQ model assumes that demand rate is constant for the infinite time horizon. This condition can be met for some time period during the product life cycle. But demand is constantly changing as product moves through the various stages of its life cycle. There are number of models designed to accommodate varying demand patterns.
Juneau and Coates (2001) have developed a model for products that are entering the growth phase of the product life cycle. During growth phase demand is increasing and can be formulated as an exponential function. Assuming instant replenishment of inventory and no shortages, they developed an inventory policy that will minimize total cost.

Various models have been formulated for linearly increasing or decreasing demand. Teng et al. (2005) focused on products in the high-tech industry where the unit cost of products declines significantly over short life cycles resulting into increasing demand for the products. Assuming that the demand and unit purchasing cost are functions of time, they develop an easy to use algorithm to find the optimal number of replenishments and a schedule. Friedman (1982) provided a procedure for optimal replacement schedule, i.e. number and timing of replenishments. Assuming no shortages, they considered items having demand and carrying costs as linear functions of time for a finite horizon. Their model is applicable in discounted or inflated cost problems and in the problem where demand and/or carrying cost functions are estimated through time series analysis. Hariga and Goyal (1995) developed a model for items having linear time dependent demand for finite horizon. They provide procedure for determining optimal replenishment policy assuming that shortages are allowed and items can be backordered. Resh et al. (1976) had previously studied a similar problem with deterministic linearly increasing demand for a limited time horizon. They determined number of replenishments $m$ and a unique vector containing $m$ time intervals that minimizes total cost. The cost of this vector results in a convex total cost function so that the optimal $m$
can be determined. Similarly Teng et al. (1996, 1997) have developed more inventory models with linear demand applicable for different situations.

Ray and Chaudhari (1997) have a developed EOQ model for finite horizon. Allowing shortage and taking into consideration internal (company) and external (general economy) inflation rates and possibility of shortage for on hand inventory dependent demand they provide procedure for finding an optimal solution. Kim et al. (2004) developed an (s, Q) inventory model for items with Erlang lead times and constant demand and provide a procedure to find a near optimal solution. Arcelus and Srinivasan (1987) extended the deterministic EOQ model especially for retailing, to evaluate inventory policies using three optimizing criteria: profits, return on investment and residual income. They assume demand as function of price using well known price elasticity of demand function.

Similar to order quantity models, various models have been developed to find production lot size for manufacturer. Sarker and Coates (1997) study the problem of identifying the economic lot size under varying lead time through a finite number of setup cost reduction opportunities corresponding to investment.

Though EOQ model was the starting point, in the past various studies have extended, modified or formulated totally new model. We observed that new models are essential in situations where product life cycle stages or their characteristics or operating policies results in different demand distributions that cannot be formulated in EOQ model where demand rate is assumed to be constant. In this research we are addressing an unique situation where periodic retailer promotional policies resulting in bi-level demand. Literature lacks inventory models based on this unique situation. Further it is shown in
the Chapter 2 that the inventory policies that we have developed here results in lower costs than using a simple EOQ model during high and low demand periods independently. Hence we address the situation of bi-level demand and develop retailer inventory policies in Chapter 2 to fill this void in the literature.

In formulating inventory policies, various researchers have restricted their work to the organizational level. Castro et al. (1997) developed a model with stochastic demand for a chocolate milk manufacturer, for a process having a constant defect rate and a capacity constraint, to develop superior production plans taking into consideration holding and back order cost. Hall (1996) examined the ways in which attributes of the distribution system affect inventory accounting and EOQ/Economic Production Quantity (EPQ) decisions. Roundy and Muckstadt (2000) study the problem of determining production quantities in each period of an infinite horizon for a single item produced in a capacity limited facility. Bollapragada and Morton (1999); Sethi, et al. (2003); and Cheng and Sethi (1999) also restrict their research to a single organizational level. Bollapragada and Morton (1999) developed a heuristic to identify an ordering policy in the context of non-stationary inventory with set-up costs, proportional ordering costs, and stochastic demands.

Few studies have looked at more than one organizational level but have not considered supply chain models with three or more stages. Moses and Seshandri (2000) studied the problem of determining both a review period and a stocking policy that are mutually beneficial to producer and supplier. In their model, the retailer faces stationary stochastic demand with independent increments from an item. He et al. (2002) examine several inventory replenishment policies for make to order inventory production system
consisting of two levels: warehouse and production workshop facing Poisson demand. Iida (2000) considers periodic review dynamic multi-echelon inventory problem with non-stationary demand and shows that myopic policies are sufficiently close to optimal. Although Lazenauer and Pilz-Glombik (2002) model the multi-stage supply chain optimizing the flows to support tactical decisions making, their research focuses on transportation decisions. There are few studies in the area of developing inventory models for supply chain but none of them have considered supply chains with stochastic bi-level demand. Hence, we develop an inventory model for three-member supply chain with retailer facing bi-level stochastic demand.

There are numerous approaches mentioned in the literature to solve supply chain optimization problems. These include genetic algorithms (Kimbrough et al., 2002), analytical modeling, and simulation. The genetic algorithm approach is more recent one and may not be suitable due to its mathematical complexity. Cheng and Sethi (1999) solve the finite horizon problem using dynamic programming. But in many cases, difficulties in obtaining closed form expressions for supply chain optimization problems discourage its use (Sterman, 1989; Metters, 1997; Machuca and Barajas, 2004). Intractability of the problem lends itself to the use of heuristics or approximations as a preferred approach to find solutions to these models and later use simulation or other tools for evaluating performance. Bollapragada and Morton (1999) developed a heuristic and compared it with an optimal dynamic solution procedure. Moses and Seshandri (2000) construct an algorithm for calculating optimal policy parameters for a supply chain, where as Roundy and Muckstadt (2000) use approximation to analyze their problem. Many researchers have explored simulation (Machuca and Barajas, 1997, 2004;
Hong-Minh et al., 2000; Sparling, 2002; Hieber and Harte, 2003). A simulation based approach is indicated as the preferred approach for the development of supply chain managers by academicians and management educators (Mangan and Christopher, 2005). Thus in this study we develop several inventory algorithms for supply chain members and use simulation methodology to evaluate its performance.

We also find variations in the inventory models developed for particular situations. Some of the research addresses the problem of different delivery options and promotional policies. Sethi, Yan, and Zhang (2003) addresses the problem of a periodic review inventory system with fast and slow delivery modes, fixed ordering cost and regular demand forecast update. Cheng and Sethi (1999) use the Markov Decision Process (MDP) to model the joint inventory promotion decision problem. They determined the threshold inventory level for each demand state such that if it exceeded, then it is desirable to promote the product. Roundy and Muckstadt (2000) studied the problem under the assumption that unfilled demand is backordered. Moses and Seshandri (2000) formulate the problem assuming excess customer demand to be lost. Most of the studies assume stationary stochastic demand as compared to the few studies (Iida 2000; Bollapragada and Morton 1999) dealing with non-stationary demand. Furthermore, the problem is formulated under different demand distribution assumptions. Some researchers have formulated the problem under a finite horizon assumption (Bollapragada and Morton 1999; Cheng and Sethi 1999) while others (Roundy and Muckstadt 2000; Iida 2000) have formulated the problem assuming an infinite horizon. In the past literature different models are developed for problems under different circumstances. We find the problem of supply chains with bi-level demand to be unique.
with almost no existing research literature. In Chapter 3, we address the problem of
developing inventory management policy for three member supply chain with retailer
facing bi-level stochastic demand. We have developed three policies for members of
supply chain and conduct simulation experiments reporting the performance of heuristics.

In the literature problem of lack of trained Supply Chain Management personals
in businesses and industry is discussed extensively. Mangan and Christopher (2005)
have noted that demand for experienced and qualified supply chain managers is
continuously growing. As observed by Gammelgaard and Larson (2001, p. 27), business
and industry will face a scarcity of trained supply chain managers. An appropriate
response to this concern is to develop programs to enhance managerial skills and
competencies by training and educating employees as well as students who are the future
managers of supply chains. Hence, we developed a game called Cola-Game to provide
toll for such training and education.

Mangan and Christopher (2005) investigated the knowledge areas, skills, and
competencies required by supply chain managers. They surveyed academicians,
management developers, graduate students and executives. Besides knowledge of
Operations and Supply Chain Management, analytical skills were identified as the most
important competency. The Cola-Game presented here is designed to impart knowledge,
improve analytical skills and decision-making competency, thus addressing this need.

The Beer-Game, developed at MIT (Forrester, 1958, 1961; Sterman, 1989), is a
well known supply chain simulation used to illustrate the fundamental challenges in
managing supply chains. It effectively demonstrates the Bullwhip Effect due to
information and material flow delays in the supply chain. This game is very simple,
which is a benefit as well as a pitfall (Hieber and Hartel, 2003). For example, it does not have demand parameters accessible to all members of the supply chain which an important input for overall supply chain cost minimization. It also does not consider ordering costs and is not designed for information sharing or to inculcate skills in supply chain optimization. It fails to provide the students with an opportunity to learn strategic decision-making (Sparling, 2002). New teaching methods to enhance such skills and techniques are required for effectively training the students and the Cola-Game can help students take such a strategic view. Specifically, the bi-level demand, the step function for the ordering cost, and the ability to cooperate with other players help them develop such a strategic view.

Sparling (2002) has presented a modified version of the Beer Game in which students play two rounds of the game. In the first round, students play the Beer Game using the traditional approach. Then, the students are given historical data to identify the demand trend and are encouraged to develop a game strategy before playing the final round of the game. However, even the modified version is not very realistic in that it omits ordering and setup costs except in a back handed way. Including a cost structure (ordering, shortage and holding) in the game encourages decision makers to think in terms of optimization and trade-offs. Further it helps them appreciate the impact of cooperation on costs as well as decisions pertaining to quantity and timing of orders. Thus Cola-Game fills this void.

One of the modifications to the beer game is reported by (Hieber and Harte, 2003), where one out of four players, say the retailer is allowed to play interactively, while the remaining three players (the wholesaler, distributor and manufacturer) are
assigned a strategy. Now, there is an opportunity to study the effect of retailer strategy on game performance since all other players use a given known strategy. They found that greater the variation in order quantities, greater the costs will be. Also their study confirms that more consistent strategies would result in better performance. The Cola-Game explicitly incorporates both these aspects of their finding and hence it is similar in spirit to this reported work.

The original Beer Game was introduced with the aim to highlight the Bullwhip effect which is caused by lack of information sharing, uncertainty, and variability in demand. As demonstrated by Lee et al. (1997), order batching and price fluctuations contribute to variability in demand. These two are realistic assumptions in retail supply chains. The supply chain members take advantage of savings in ordering cost by consolidating the future orders thus inducing large fluctuations and magnifying the Bullwhip Effect. Consequently ordering cost is an important parameter which the Beer Game and its modified versions have not considered. Further, cooperation will improve the supply chain performance where as independence will impede supply chain performance. All of these points which are made in diverse papers in the literature are brought together by way of the Cola-Game thus providing a platform for education and training of supply chain managers.

Emiliani (2005) has suggested that criticisms like “too many slides” can be overcome by introducing adult learning methods such as simulation and break out activities. Gaming is one such method. Thus there is a need to develop teaching methods that will demonstrate impact of demand variability on supply chain. The Cola-Game has been developed to fill this need.
1.2 Dissertation Organization

In this chapter, we briefly summarize the dissertation problem and described the existing the literature. In the Chapter 2, we begin with brief introduction to Bi-Level Demand Problem (BDP). We provide exact definition of terms used and notation used in derivations. We explain the difficulty in solving BDP problem with an illustrative example. In the next section we describe the characteristics of optimal solution based on various lemmas/theorems developed in this work. We find that optimal solution is attainable for some of the cases of Restricted BDP (RBDP) and describe this procedure with an illustrative example in the next subsection. In the following section we explain three types of heuristics that can be used for most of the BDP. In the next section, we present two methods for evaluating heuristics performance. Finally at the end of this chapter, we explain the experimental design used and the results obtained from simulation experiments.

In Chapter 3, we have modeled the three-member supply chain consisting of the retailer, distributor and manufacturer with retailer facing bi-level stochastic demand. We briefly introduce this problem, present the supply chain model, and describe three policies along with algorithms for members of supply chain. In the next section, we describe simulation experiments, present result, and discuss conclusions.

In Chapter 4, we provide brief introduction in the beginning explaining the need for developing teaching tools that will enhance decision making capabilities in area of supply chain inventory management. In the following section, we describe the “Cola-Game” developed by us and explain the procedure for conducting this game. In the following sub section we describe our experience of conducting this game at The
University of Toledo and report on our analysis to evaluate if students learned from this game.

Finally, in the last chapter, we provide concluding remarks describing opportunities for future research.
CHAPTER 2: BI-LEVEL DEMAND PROBLEM

2.1 Introduction to BDP

The Economic Order Quantity model is well known and widely cited in the literature. The model provides the solution to the problem of finding the optimal quantity to order when the demand for an item is known and constant, and the only relevant costs are the costs of ordering and holding, no shortages are allowed and no quantity discounts are available. The focus here is on determining an optimal ordering policy for a retailer facing a deterministic bi-level demand for the product.

First, in the next section the problem is defined. This problem is not easy to solve due to the discontinuous nature of the demand function. These difficulties are explained and the characteristics of an optimal solution are studied. Next, for restricted class of BDP, a procedure for obtaining the optimal solution is presented. For the general class of problems, heuristics have been developed. To compare performance of heuristics, lower bound solution is developed and the results are presented at the end of this chapter.

2.2 Definition of Terms and Notations

We use various terms with the following definitions. Also notations given below are frequently used throughout derivations.
**Bi-level Demand Problem (BDP)** —Demand for a product follows a repetitive alternating bi-level demand pattern, in which demand intensity is $d_1$ for a period of length $t_1$ and $d_2$ for a period of length $t_2$. Without loss of generality, assume $d_1 < d_2$. The problem is to find an ordering policy, which will minimize sum of holding and ordering costs per unit time. No shortages are allowed.

**Demand Stream:** It is a period during which demand intensity is constant.

**Non-Bridge Order:** A non-bridge order is an order that satisfies demand during only one demand stream.

**Bridge Order:** A bridge order is an order that satisfies demand of two demand streams. We are not considering the problems where an order satisfies demand during more than two demand streams. Later in Lemma 9, we have shown that Restricted Bi-level Demand Problem (RBDP), which represents the subset of BDP problems, cannot have orders satisfying demand during more than two demand streams.

**Notation:**

- $S$ a solution to the BDP problem
- $s$ A subset of $S$ containing one or more orders which are relevant for a comparison.
- $O_k$ Order $k$, $k \in N = \{1, 2, 3, \ldots\}$
- $Q_k$ Order quantity of Order $O_k$, $k \in N$
- $L_k$ Order cycle length of Order $O_k$, $k \in N$
\( \theta \) The set of demand streams, \( \theta \equiv \{ 1, 2 \} \)

\( d_i \) Demand intensity during demand stream \( i \in \theta \)

\( t_i \) Length of time period with demand intensity \( d_i, i \in \theta \)

\( U_i \) Set of possible number of non-bridge orders in \( t_i \)

\( n_i \) The number of non-bridge orders in \( t_i \) where \( n_i \in \hat{n}_i \)

\( n_{\text{min},i} \) Minimum in the set \( \hat{n}_i \)

\( n_{\text{max},i} \) Maximum in the set \( \hat{n}_i \)

\( H \) holding cost per unit per unit time

\( A \) is the ordering cost per order

\( Q_i^* \) Order Quantity corresponding to simple EOQ model for demand \( d_i \)

\( \lambda_i \) Order cycle length corresponding to simple EOQ model for demand \( d_i \)

\( n_i^* \) The number of orders in \( t_i \) corresponding to EOQ model

\( HC(s), RC(s), \) and \( TC(s) \) represent holding, ordering, and total cost of partial solutions \( s \), where \( s \) is subset of \( S \).

\( TCT(s) \) and \( TCU(s) \) represent total cost per unit time of partial solutions \( s \), where \( s \) is subset of \( S \).

RBDP represents the subset of BDP problems in which the length of demand stream satisfies (1). This restriction on time of each demand stream enables us to find lower bound and if problem satisfies one other condition, then it can be solved optimally.

\[ t_i > \sqrt{2} \lambda_i \]  \hspace{1cm} (1)
2.3 Difficulty in Solving BDP

The problem considered here is the same as one solved by the EOQ model except that the demand pattern follows two levels in alternating cycles. The following example illustrates that using simple EOQ model does not solve BDP. We illustrate this using the following example.

Example 1:

Consider a BDP with the following data.

\[ d_1 = 50 \text{ units/day}, \quad d_2 = 200 \text{ units/day}, \quad t_1 = 2 \text{ days}, \quad t_2 = 1 \text{ days}, \quad H = \$1 \text{ per unit per day}, \text{ and} \]

\[ A = \$100 \text{ per order} \]

Solution:

Applying simple EOQ model to BDP order quantity and cycle length during demand stream \( i \) is given by

\[
Q_i^* = \sqrt{\frac{2Ad_i}{H}}, \quad i \in \theta
\]

(2)

\[
\lambda_i = \sqrt{\frac{2A}{d_iH}}, \quad i \in \theta
\]

(3)

Using (2) and (3), we have —

\[ Q_1^* = 100, \quad Q_2^* = 200, \quad \lambda_1 = 2, \text{ and } \lambda_2 = 1. \]
The number of orders during time \( t_i \) is given by

\[
n^*_i = \frac{t_i}{\lambda_i}
\]  

(4)

Which leads to \( n^*_1 = 1 \) and \( n^*_2 = 1 \). One simple solution would be to place an order of 200 units at the beginning of demand stream 2, followed by an order of 100 units at the beginning of demand stream 1 and so on.

The total relevant cost for duration \( t_1 + t_2 \) of this solution is

\[
TC(s^*) = \frac{1}{2} Q^*_1 \lambda_1 n_1 H + \frac{1}{2} Q^*_2 \lambda_2 n_2 H + (n_1 + n_2)A = 400
\]  

(5)

Since EOQ quantities exactly meet demand during each demand cycle one might conjecture that it may be in fact the optimal solution to BDP. We verify this as given below. Suppose instead of placing orders \( Q^*_2 \) and \( Q^*_1 \), orders \( Q'_2 \) and \( Q'_1 \) as defined below are placed.

\[
Q'_2 = Q^*_2 + \Delta Q
\]

\[
Q'_1 = Q^*_1 - \Delta Q
\]

To check for optimality, we compute \( TC(s) \) for different values of \( \Delta Q \) and plot the results in Figure 2.3.1 and in Table 2.3.1.
Table 2.3.1 Total Cost comparison for selected solutions of Integer EOQ problem

<table>
<thead>
<tr>
<th>ΔQ</th>
<th>Q1</th>
<th>Q2</th>
<th>HC(s)</th>
<th>RC(s)</th>
<th>TC(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-200</td>
<td>0</td>
<td>300</td>
<td>$300.00</td>
<td>$100.00</td>
<td>$400.00</td>
</tr>
<tr>
<td>-100</td>
<td>100</td>
<td>200</td>
<td>$200.00</td>
<td>$200.00</td>
<td>$400.00</td>
</tr>
<tr>
<td>-50</td>
<td>150</td>
<td>150</td>
<td>$187.50</td>
<td>$200.00</td>
<td>$387.50</td>
</tr>
<tr>
<td>0</td>
<td>200</td>
<td>100</td>
<td>$200.00</td>
<td>$200.00</td>
<td>$400.00</td>
</tr>
<tr>
<td>25</td>
<td>225</td>
<td>75</td>
<td>$187.50</td>
<td>$200.00</td>
<td>$387.50</td>
</tr>
<tr>
<td>50</td>
<td>250</td>
<td>50</td>
<td>$200.00</td>
<td>$200.00</td>
<td>$400.00</td>
</tr>
<tr>
<td>100</td>
<td>300</td>
<td>0</td>
<td>$300.00</td>
<td>$100.00</td>
<td>$400.00</td>
</tr>
</tbody>
</table>

Figure 2.3.1 ΔQ Vs Total cost for integer EOQ problem

From Figure 2.3.1 it is clear that the EOQ solution can be improved and in this example total cost can be reduced from $400.00 to $387.50 either by increasing or decreasing quantity of first order. These solutions with lowest cost are highlighted in Table 3.2.1. It turns out that solutions to BDP require elaborate study and results show that some BDP lends themselves to finding an optimal solution, while others require the use of heuristics.
2.4 Characteristics of Optimal Solution

In this section, we develop several necessary conditions or lemmas to characterize the optimal solution of a BDP problem. We attempt to identify the optimal solution by applying these conditions on solution space.

**Lemma 1:** A necessary condition for the optimality of a solution to BDP is that, all the consecutive non-bridge orders in the same demand stream are equal.

**Proof:** The lemma is clearly true for the EOQ model. Here, this property is being extended to portions of the solution space where this condition should hold. Hence, it is only a necessary condition. By simple counter example, it can be shown that unequal order sizes would lead to suboptimality.

**Lemma 2:** A necessary condition for the optimality of a solution to BDP is that the order quantity of bridge and non-bridge orders placed during same demand stream will be equal.

**Proof:** Let $S$ be the optimal solution to an instance of BDP which contains a non-bridge order $O_k$ and an immediate successor bridge order $O_{k+1}$ both placed in same demand stream $i$. Let the combined order cycle length of the two orders be $w+y$, where $w$ is the period during demand stream $i$ and $y$ is the period during demand stream $j$ (immediate successor to $i$). From lemma 1, we know that non-bridge orders, if any placed during same demand stream will be equal and hence not considered here.
Let variable $x$ be such that by changing its value in the interval $(0, w)$ all the possible values for order quantities $Q_k$ and $Q_{k+1}$ can be obtained. Pictorially we have depicted the orders $O_k$ and $O_{k+1}$ in Figure 2.4.1 with $x > 0$, $w$, $x$, and $y$ hold the following relationships with order cycle lengths $L_k$ and $L_{k+1}$.

$$L_k = w - x$$

$$L_{k+1} = x + y$$

Since holding cost of all other orders, except $O_k$ & $O_{k+1}$ remain unaffected by the value of $x$, the relevant holding cost is given by

$$HC(s) = \frac{1}{2} (w - x)^2 d_i + \frac{1}{2} (x)^2 d_i + xyd_j + \frac{1}{2} (y)^2 d_j$$

Where $s=\{O_k, O_{k+1}\}$

**Figure 2.4.1 Inventory levels corresponding to orders $O_k$ and $O_{k+1}$ from Lemma 2**
We wish to show that holding cost function for \( S \) is convex and is minimized at \( Q_k = Q_{k+1} \). Differentiating \( HC(s) \) with respect to \( x \) and using the first order condition, we set

\[
\frac{dHC(s)}{dx} = -(w-x)d_i + xd_i + yd_j = 0
\]

Using the second order condition, we get

\[
\frac{d^2HC(s)}{dx^2} = 2d_i > 0
\]

Since \( d_i > 0 \), \( HC(s) \) is a convex function. The minimum value of holding cost can be obtained as follows.

\[-(w-x)d_i + xd_i + yd_j = 0\]

\[(w-x)d_i = xd_i + yd_j\]

\[Q_k = Q_{k+1}\]

QED.

**Lemma 3:** A necessary condition for the optimality of a solution to BDP is that the order cycle length of a bridge order and non-bridge order ending in same demand stream should be equal.

**Proof:** Let \( S \) be the optimal solution to an instance of BDP which contains a bridge order \( O_k \) and an immediate successor non-bridge order \( O_{k+1} \) both ending in demand stream \( j \). The combined order cycle lengths of the two orders is \( x+w \),
where $x$ is the time period during demand stream $i$ (immediate predecessor to $j$) and $w$ is the time period during demand stream $j$. From lemma 1, we know that the size of non-bridge orders, if any placed during demand stream $j$ will be equal to $Q_{k+1}$ and hence not considered here.

Let $y$ be a variable such that by changing its value in the interval $(0, w)$ all the possible combinations of order quantities $Q_k$ and $Q_{k+1}$ can be obtained.

Pictorially we have depicted the orders $O_k$ and $O_{k+1}$ in Figure 2.4.2. Symbols $x$, $y$, and was shown in Figure 2.4.2 hold the following relationships with order cycle lengths $L_k$ and $L_{k+1}$.

\[ L_k = x + y \]  \hspace{1cm} (6)

\[ L_{k+1} = w - y \]  \hspace{1cm} (7)

**Figure 2.4.2 Inventory levels corresponding to orders $O_k$ and $O_{k+1}$ from Lemma 3**

Since the holding cost of all other orders, except $O_k$ and $O_{k+1}$ remains unaffected by the value of $y$, the expression for relevant holding cost is
\[ HC(s) = \frac{1}{2} (x)^2 d_j + x y d_j + \frac{1}{2} (y)^2 d_j + \frac{1}{2} (w - y)^2 d_j \]

We wish to show that the holding cost function for \( S \) is convex and is minimized at \( L_k = L_{k+1} \). Differentiating \( HC(s) \) with respect to \( x \), and using the first order condition, we set

\[ \frac{dHC(s)}{dx} = xd_j + y d_j - (w - y)d_j = 0 \]

The second derivative of \( HC(s) \) with respect to \( x \) yields the following second order condition

\[ \frac{d^2HC(s)}{dx^2} = 2d_j > 0 \]

Thus \( HC(s) \) is a convex function. The minimum value of holding cost can be obtained as follows.

\[ xd_j + yd_j - (w - y)d_j = 0 \]

\[ x + y = (w - y) \]

From equations (6) & (7)

\[ L_k = L_{k+1} \]

QED.
Lemma 4: A necessary condition for optimality of a solution to BDP is that, if there are two consecutive bridge orders \((O_k \text{ and } O_{k+1})\), placed in consecutive demand streams \((i \text{ and } j)\) as shown in Figure 2.4.3, the following relation holds.

\[ Q_{k+1} = L_k \cdot d_j \]

Figure 2.4.3 Inventory levels corresponding to orders \(O_k \text{ and } O_{k+1}\) from Lemma 4

Proof: Let \(S\) be the optimal solution to an instance of BDP which contains a bridge order \(O_k\) placed in the demand stream \(i\) and an immediate successor bridge order \(O_{k+1}\) placed in immediate successor demand stream \(j\). Referring to Figure 2.4.3, let periods \(x\) and \(y\) be occur during demand \(d_i\) and \(w\) occur during demand \(d_j\). Let \(z\) be a variable such that order \(O_k\), satisfies demand during periods \(x\) and \(z\) and order \(O_{k+1}\) satisfies demand during periods \(w-z\) and \(y\).

Let \(z\) be the variable such that by changing its value in the interval \((0, w)\) all the possible combinations of order quantities \(Q_k\) and \(Q_{k+1}\) can be obtained and symbols \(x, w, z, \text{ and } y\), hold the following relationships.
\[ L_k = x + z \]  
\[ Q_k = xd_j + zd_j \]  
\[ L_{k+1} = (w - z) + y \]  
\[ Q_{k+1} = (w - z)d_j + yd_i \]  

Since \( O_k \) and \( O_{k+1} \) are the only affected orders, the expression for relevant holding cost is

\[ HC(s) = \frac{1}{2} (xd_j + zd_j) + \frac{1}{2} (z) = 2d_j > 0 \]

We wish to show that holding cost function for \( S \) is convex in \( z \) and minimize \( HC(s) \). The first and second order condition yields

\[ \frac{dHC(s)}{dz} = xd_j + zd_j - (w - z)d_j - yd_i \]

\[ \frac{d^2 HC(s)}{dz^2} = 2d_j > 0 \]

Since the second order condition is positive, the cost function is convex.

Minimum value of holding cost is obtained when:

\[ xd_j + zd_j - (w - z)d_j - yd_i = 0 \]

Or \( (w - z)d_j + yd_i = xd_j + zd_j \)  

From equations (8) and (11)

\[ Q_{k+1} = L_k * d_j \]

QED.
**Lemma 5:** A necessary condition for the optimality of a solution to BDP is that if a non-bridge order $O_k$ is an immediate predecessor to another non-bridge order $O_{k+1}$ placed in a different demand stream, then $Q_k \geq Q_{k+1}$.

**Proof:** Let $S$ be the optimal solution to an instance of BDP which contains two consecutive non-bridge orders $O_k$ and $O_{k+1}$ in demand streams $i$ and $j$ respectively. We show by contradiction that $S$ cannot be optimal if $Q_k > Q_{k+1}$.

By assumption $Q_k > Q_{k+1}$

Let $S'$ be a solution obtained from $S$ such that

$$Q_k' = Q_{k+1}' = Q' = \frac{Q_k + Q_{k+1}}{2}$$

And let all other orders in $S$ remain unchanged. Pictorially we have depicted $k^{th}$ and $(k+1)^{th}$ orders of $S$ and $S'$ in Figure 2.4.4. For simplicity symbols $x$, $y$, and $z$ are used to denote non-negative numbers with the following relationships.

$$x = L_k'$$

$$y = L_x - L_k'$$

$$z = L_{k+1}$$
Figure 2.4.4 Inventory levels corresponding to orders $O_k$ and $O_{k+1}$ from Lemma 5

Also note that

\[ Q' = xd_j = yd_i + zd_j \]

or \[ zd_j - xd_i = -yd_i < 0 \] (13)

The change in the total cost as we go from $S$ to $S'$

\[ \Delta TC = TC(S') - TC(s) \]

For $S$ to be optimal, we expect $\Delta TC \geq 0$. In solutions $S$ and $S'$ the only relevant cost is the holding costs of orders $Q_k$ and $Q_{k+1}$. From Figure 2.4.4, canceling all the costs that are identical we get

\[ \Delta TC = yzd_j - xyd_i \]

\[ = y(zd_j - xd_i) \]

From (13) it follows that the change in total cost is negative thus contradicting the optimality of $S$. 

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Lemma 6: A necessary condition for the optimality of a solution to BDP is that the order cycle length of a non-bridge order is greater than or equal to the order cycle length of immediate successor order in next demand stream if it is a non-bridge order.

Proof: Let $S$ be the optimal solution to an instance of BDP which two consecutive non-bridge orders $O_k$ and $O_{k+1}$ in demand streams $i$ and $j$ respectively. We show by contradiction that $S$ cannot be optimal if $L_k < L_{k+1}$.

By assumption $L_k < L_{k+1}$

Let $S'$ be the solution obtained from $S$ such that

$$L'_k = L'_{k+1} = L' = \frac{L_k + L_{k+1}}{2}$$

All other orders of $S$ remain unchanged. Pictorially we have depicted $k^{th}$ and $(k+1)^{th}$ orders of $S$ and $S'$ in Figure 2.4.5. For simplicity symbols $x$, $y$, and $z$ are used to denote non-negative numbers with the following relationships.

$x = L_k$

$y = L'_k - L_k$

$z = L'_{k+1}$
Also note that

\[ L' = x + y = z \]

Or \( x - z = -y < 0 \) \hspace{1cm} (14)

The change in the total cost as we go from \( S \) to \( S' \)

\[ \Delta TC = TC(s') - TC(s) \]

For \( S \) to be optimal, we expect \( \Delta TC \geq 0 \). In solutions \( S \) and \( S' \) the only relevant cost is the holding costs of orders \( O_k \) and \( O_{k+1} \). From Figure 2.4.5 canceling all the costs that are identical we get

\[ \Delta TC = xyd_j - yzd_j \]

\[ = y(x - z) \]

\[ = -y^2 < 0 \]

Thus contradicting the optimality of \( S \)
Lemma 7: A necessary condition for the optimality of a solution to BDP problem is that, the number of consecutive non-bridge orders \( n_i \) placed in period \( t_i \), must be equal to one of the values in the following set of integers given by \( U_i \)

\[
U_i = \left[ \frac{-5}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{1}{\lambda_i} \right)^2} \right], \quad \left[ \frac{-3}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{1}{\lambda_i} \right)^2} \right], \ldots, \quad \left[ \frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{1}{\lambda_i} \right)^2} \right] \tag{15}
\]

Proof: Let \( S \) be the optimal solution to an instance of BDP. As defined, \( n_i \) \((n_i \in U_i)\) be the number of consecutive non-bridge orders placed during demand stream \( i \).

Let \( t'_i \) be the period in \( t_i \) such that the demand during \( t'_i \) is completely satisfied by \( n'_i \) non-bridge orders. Let \( S' \) and \( S'' \) be solutions obtained from \( S \) that have \((n'_i + 1)\) and \((n'_i - 1)\) equal non-bridge orders demand during period \( t'_i \) and orders during other periods remained unchanged. Pictorially we depict these orders as shown in Figure 2.4.6. For simplicity symbols \( x, y, \) and \( z \) are used to denote non-negative order cycle lengths with the following relationships.

\[
x = t'_i / n'_i \tag{16}
\]

\[
y = t'_i / (n'_i + 1) \tag{17}
\]

\[
z = t'_i / (n'_i - 1) \tag{18}
\]
Figure 2.4.6 Inventory levels corresponding to relevant orders from Lemma 7

For $S$ to be optimal, the total cost of $S$ should not exceed cost of $S'$

$$TC(s') \geq TC(s)$$

For cost comparison of $S$ and $S'$, the relevant costs are the holding cost of orders during period $t'_i$ and the cost of placing one order. Using the property ordering cost is equal to holding cost from EOQ model, we can express an ordering cost in terms $\lambda_i$ as $\lambda_i^2 d_i H / 2$. The relevant costs in above equation are
\[
\frac{1}{2} y^2 d_i H(n'_i + 1) + \frac{1}{2} \lambda_i^2 d_i H \geq \frac{1}{2} x^2 d_i H(n'_i)
\]

Substituting the value of \(x \text{ and } y\) from equation (16) & (17) yields

\[
\left( \frac{t'_i}{(n'_i + 1)} \right)^2 (n'_i + 1) + \lambda_i^2 \geq \left( \frac{t'_i}{n'_i} \right)^2 (n'_i)
\]

\[
\lambda_i^2 n'_i^2 + \lambda_i^2 n'_i - t'_i^2 \geq 0
\]

(19)

Solving the above inequality at boundary condition yields \(n'_i\)

\[
n'_i = \frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{4}{\lambda_i} \left( \frac{t'_i}{\lambda_i} \right)^2} \quad \& \quad n'_i = \frac{1}{2} - \frac{1}{2} \sqrt{1 + \frac{4}{\lambda_i} \left( \frac{t'_i}{\lambda_i} \right)^2}
\]

Thus disregarding the negative value, an upper bound for optimal value of \(n'_i\) is,

\[
\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{4}{\lambda_i} \left( \frac{t'_i}{\lambda_i} \right)^2}
\]

(20)

Similarly, for \(S\) to be optimal the total cost of \(S\) should not exceed cost of \(S''\)

\[
TC(s'') \geq TC(s)
\]

For cost comparison of \(S\) and \(S''\), the relevant costs are

\[
\frac{1}{2} z^2 d_i H(n'_i - 1) - \frac{1}{2} \lambda_i^2 d_i H \geq \frac{1}{2} x^2 d_i H(n'_i)
\]
Substituting the value of $x$ and $z$ from equations (16) & (18) yields

$$
\left( \frac{t_i'}{(n_i' - 1)} \right)^2 (n_i' - 1) - \lambda^2 \geq \left( \frac{t_i'}{n_i'} \right)^2 (n_i')
$$

$$
- \lambda_i^2 n_i'^2 + \lambda_i^2 n_i' + t_i'^2 \geq 0
$$

$$
\lambda_i^2 n_i'^2 - \lambda_i^2 n_i' - t_i'^2 \leq 0
$$

Solving the above inequality at boundary condition yields $n_i'$

$$
n_i' = -\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{t_i'}{\lambda_i} \right)^2} \quad \& \quad n_i' = -\frac{1}{2} - \frac{1}{2} \sqrt{1 + 4 \left( \frac{t_i'}{\lambda_i} \right)^2}
$$

Thus disregarding the negative value, an upper bound for optimal value of $n_i'$ is,

$$
-\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{t_i'}{\lambda_i} \right)^2}
$$

(22)

From equations (20) & (22) we have,

$$
-\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{t_i'}{\lambda_i} \right)^2} \leq n_i' \leq \frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{t_i'}{\lambda_i} \right)^2}
$$

For $t_i = t_i'$, the number of non-bridge orders is given by

$$
-\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{t_i}{\lambda_i} \right)^2} \leq n_i \leq \frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{t_i}{\lambda_i} \right)^2}
$$
Since \( n_i \) is integer, the set of all possible values can be written as

\[
U_i = \left[ -\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{t_i}{\lambda_i} \right)^2} , \frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{t_i}{\lambda_i} \right)^2} \right]
\]  

(23)

The above yields almost candidate two values \( n_i \) under the assumption that there are no bridge orders satisfying the demand during period \( t_i \). But there can be two bridge orders: a bridge order placed in earlier demand stream satisfying demand during period \( t_i \) and bridge order placed during period \( t_i \) satisfying demand of next demand stream. In either case bridge order cannot satisfy more demand during demand stream \( i \) than a non-bridge order. This is clearly evident from Lemmas 2 and 3. Accounting for the possibility of bridge orders, the lower limit of the above interval should be decreased by 2. However upper limits remains unchanged, because there may be two non-bridge orders which satisfy only a miniscule amounts of the demand during demand stream \( i \). QED

**Lemma 8:** A necessary condition for the optimality of a solution to RBDP is that it will at least have one order placed in every demand stream and will not have any order satisfying demand during more than two demand streams.

**Proof:** Let \( S \) be the optimal solution to RBDP. By definition, the length of demand stream \( t_i > \sqrt{2\lambda_i} \). Substitute \( t_i = \sqrt{2\lambda_i} \) in equation (15) to obtain \( U_i \)

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\[ U_i = \left[ -\frac{5}{2} + \frac{1}{2} \sqrt{1 + \frac{\sqrt{2} i \lambda}{\lambda_i}} , \frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{\sqrt{2} i \lambda}{\lambda_i}} \right] \]

For \( t_i = \sqrt{2} \lambda_i \), \( n_{\text{min}} = -1 \). If \( t_i > \sqrt{2} \lambda_i \), then \( n_{\text{min}} > -1 \). This implies that the minimum number of non-bridge orders that satisfies demand during \( t_i \) can be 0 assuming that there are two bridge orders that satisfy some of the demand during period \( t_i \). Of these one bridge order must begin during \( t_i \). QED.

**Lemma 9:** A necessary condition for the optimality of a solution to RBDP is that it will have a bridge order placed in every high demand period.

**Proof:** Let \( S \) be the optimal solution to a RBDP. We show by contradiction that placing a bridge order is necessary in high demand period for optimality of solution \( S \).

Suppose \( S \) does not have a bridge order placed in one of the high demand streams. Hence there should be two consecutive orders \( O_k \) and \( O_{k+1} \) such that \( O_k \) ends at the end of demand stream 2 (high) and \( O_{k+1} \) starts at the beginning of demand stream 1 (low).
Since $S$ is optimal, it follows from Lemma 8 that, at least one order should be placed during time period $t_i$. Hence $O_k$ should be non-bridge order placed in demand stream 2 and satisfying demand till the end of stream 2.

Lemma 8 results are based on the assumption that there are two bridge orders. If there is no bridge order ending within demand stream 2, then it can be easily shown that two orders need to be placed in stream 2, implying that $O_{k+1}$ is a non-bridge order.

$O_{k+1}$ and $O_k$ being non-bridge orders placed in consecutive demand streams, results of Lemma 5 can be applied and hence for $S$ to be optimal,

$$Q_k \leq Q_{k+1} \quad (24)$$

Since $d_2 > d_1$

$$Q_k/d_2 < Q_{k+1}/d_1$$

Or $L_k < L_{k+1}$

From lemma 6, $L_k$ should be greater than or equal to $L_{k+1}$, thus contradicting the optimality of $S$.

Violating (24) will immediately contradict the optimality of $S$. Thus $O_k$ must be a bridge order and there cannot be any solution to RBDP without a bridge order being placed during every high demand period.
Lemma 10: A necessary condition for the optimality of a solution to RBDP is that it cannot have a bridge order placed during the low demand period \( t_i \), if the following inequality holds:

\[
\frac{d_2}{d_1} > \frac{2(n_{\min,j} + 1)}{n_{\min,j}}
\]  

(25)

Proof: Let \( S \) be the optimal solution to an instance of RBDP. Following the optimality of \( S \), from lemma 7, the set \( U_i \) for the possible number of non-bridge orders \( n_I \) \((n_I \in U_i)\) can be computed from (15). We denote these \( n_I \) non-bridge orders as \( O_k, O_{k+1}, \ldots, O_{k+n_i-1} \). From Lemma 1:

\[
L_k = L_{k+1} = \ldots = L_{k+n_i-1} = L
\]

\[
Q_k = Q_{k+1} = \ldots = Q_{k+n_i-1} = Q
\]
We will show by contradiction that it is not optimal to place a bridge order during the low demand period if condition (25) is satisfied. Suppose \( O_{k+n_i} \) is a bridge order placed during \( t_J \). Let demand during period \( w \), which is in demand stream 1, be satisfied by orders \( O_k \) to \( O_{k+n_i-1} \) and part of order \( O_{k+n_i} \).

From Lemma 8, we assume that there is at least one order say \( O_{k+n_i+1} \), placed during the next demand stream 2. For simplicity, we assume \( O_{k+n_i+1} \) is a non-bridge order. But the results are also valid, as shown later, in case of it being a bridge order. Let demand during period \( v \), which is in demand stream 2, be satisfied by order \( O_{k+n_i+1} \) and part of order \( O_{k+n_i} \) that is in demand stream 2. (Refer to Figure 2.4.8 a).

Let \( S' \) be a solution obtained from \( S \) such that, \((n_J+1)\) equal non-bridge orders are placed during period \( w \) and one order is placed during \( v \). (Refer to Figure 2.4.8 b). From Lemma 1: all the \( n_J+1 \) orders in time period \( w \) are equal which implies

\[
Q'_k = Q'_{k+1} = \ldots = Q'_{k+n_i} = Q'
\]

And

\[
L'_k = L'_{k+1} = \ldots = L'_{k+n_i} = L'
\]
Figure 2.4.8 Inventory levels corresponding to solutions $S$ and $S'$ from Lemma 10.

(a) Orders $O_k$ to $O_{k+n+1}$ in Solution $S$

Orders: $O_k \quad O_{k+1} \quad \ldots \quad O_{k+n-1} \quad O_{k+n} \quad O_{k+n+1}$

Orders: $O'_k \quad O'_{k+1} \quad \ldots \quad O'_{k+n-1} \quad O'_{k+n} \quad O'_{k+n+1}$

(b) Orders $O'_k$ to $O'_{k+n+1}$ in Solution $S'$

Area of large rectangle = $(n_1 z)(xd_1)$
Area of small rectangle = $(z)(zd_1)$
Number of small rectangles
= Sum of numbers up to $(n_1 - 1)$
= \[ \frac{n_1(n_1 - 1)}{2} = \frac{n_1^2 - n_1}{2} \]
Increase in holding cost
= \[ (n_1 z)(xd_1) + \left( \frac{n_1^2 - n_1}{2} \right)(z)(zd_1) \]

(c) Enlarged view of Order $O'_{k+n}$ in Solution $S'$
For simplicity symbols \( m, x, y, \) and \( z \) are used to denote non-negative numbers with the following relationships.

\[
x = w - n_1 L \tag{26}
\]

\[
y = v - L_{k+n_1+1} \tag{27}
\]

\[
m = L \tag{28}
\]

\[
z = L - L \tag{29}
\]

From equation (28) & (29) we have

\[
L = m + z \tag{30}
\]

From equation (26) and (30) we can express \( w \) as

\[
w = (m + z)n_1 + x \tag{31}
\]

Similarly as shown in Figure 2.4.8, time period \( w \) contains \( (n_1 + 1) \) non-bridge orders of cycle length given by equation (28) which implies

\[
w = (n_1 + 1)m \tag{32}
\]

Combining equations (31) and (32) to express \( m \) in terms of \( x, z \) and \( n_1 \) yields

\[
m = n_1 z + x \tag{33}
\]

Also from (30) and (33) \( L \) can be expressed as

\[
L = (n_1 + 1)z + x \tag{34}
\]
Also it should be noted that

\[ L_{k+n} = x + y \]

Following the optimality of \( S \) from Lemma 3, order cycle length of bridge order is equal to order cycle length of succeeding non-bridge order, which implies

\[ L_{k+n+1} = L_{k+n} \]

Hence,

\[ L_{k+n+1} = x + y \]

Following the optimality of \( S \) from Lemma 2, the order size of a non-bridge order should be equal to that of the succeeding bridge order, which implies

\[ Q = Q_{k+n} \]

\[ Ld_1 = xd_1 + yd_2 \]

From equation (34)

\[ ((n_1 + 1)z + x)d_1 = xd_1 + yd_2 \]

\[ y = (n_1 + 1)z \frac{d_1}{d_2} \]  \hspace{1cm} (35)

For \( S \) to be optimal we expect that the change in the total cost as we go from \( S \) to \( S' \) should be positive. This change in cost is given by
\[ \Delta TC = TC(s') - TC(s) \]

In both solutions—\( S \) and \( S' \), the total number of orders remains unchanged. Hence the only relevant cost is the holding costs of orders \( Q_k \) to \( Q_{k+n+1} \). From Figure 2.4.8 canceling all the costs that are identical yields

\[
\Delta TC = (n_1 z)(xd_i) + \left( \frac{n_i^2 - n_1}{2} \right)(z)(zd_i) + (x + y)d_2(y) - (n_1)(m)(zd_i) - (x)(yd_2)
\]

In the above equation, the only term \( (x + y)d_2(y) \) is related to order \( O_{k+n_i+1} \) and represents increase in holding cost. In case of it being bridge order, increase in holding cost will be \( (Q_{k+n_i+1} \cap y) \). In that case Lemma 4 can be applied which yield \( Q_{k+n_i+1} = L_{k+n_i+1}d_2 = (x + y)d_2 \). Thus the above equation remains unaffected even in case of \( O_{k+n_i+1} \) being bridge order.

Substituting the value of \( m \) from (33) in above equation

\[
\Delta TC = n_1 zxd_i + \left( \frac{n_i^2 - n_1}{2} \right)(z^2d_i) + y^2d_2 - (n_1)(n_1 z + x)(zd_i)
\]

\[
= \left( \frac{n_i^2 - n_1}{2} \right)(z^2d_i) + y^2d_2 - n_1^2 z^2d_i
\]

\[
= y^2d_2 - \left( \frac{n_i^2 + n_1}{2} \right)(z^2d_i)
\]

Substituting the value of \( y \) from (35) yields
\[ \Delta TC = (n_1 + 1)z \frac{d_1}{d_2} d_2 - \left( \frac{n_1^2 + n_1}{2} \right) (z^2 d_1) \]

\[ = \left( n_1 + 1 \right)^2 z \frac{d_1^2}{d_2} - \left( \frac{n_1(n_1 + 1)}{2} \right) (z^2 d_1) \]

\[ = \left( \frac{d_1}{d_2} - \frac{n_1}{2(n_1 + 1)} \right) \left( z^2 d_1 \right) \]

Thus \( \Delta TC \) will be negative when

\[ \frac{d_2}{d_1} > \frac{2(n_1 + 1)}{n_1} \]

This gives the condition under which a bridge order will not exist in an optimal solution. If this condition is not satisfied then a bridge order may exist.

The above inequality is function of \( n_1 \) and is most restrictive for lowest possible value \( n_1 \), i.e. \( n_{\text{min}} \), which would make \( \Delta TC < 0 \) for all candidate values \( n_1 \). QED.

### 2.5 Efficient Enumerative Procedure for Optimal Solution

In this section the procedure to find the optimal solution to the set of RBDP problems that satisfies (25) is described. Satisfying this condition implies there will not be a bridge order placed in period \( t_1 \). Also from Lemma 9, a bridge order will always be
placed in period \( t_2 \). Hence the infinite horizon problem can be broken down into finite horizon problem of length \( t_2 + t_1 \).

Since there is only one bridge order in \( t_1 \) the set \( U_i \) for number of possible non-bridge orders \( n_i (n_i \in U_i) \) given by (15) can be modified as

\[
U_i = \left[ -\frac{3}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{t_i}{\lambda_i} \right)^2}, \frac{1}{2} + \frac{1}{2} \sqrt{1 + 4 \left( \frac{t_i}{\lambda_i} \right)^2} \right]
\]

We can restate the range for \( \hat{n}_i \) in terms of maximum number of orders \( n_{\text{max},i} \), as follows

\[
U_i = [n_{\text{max},i} - 1, n_{\text{max},i}]
\]  

(36)

We can have four possible solutions during \( t_2 + t_1 \) for RBDP say \( S_j, j \in \{1,2,3,4\} \), for different combinations of \( n_1 \) and \( n_2 \). (It is theoretically possible for lower and upper bounds given by (15) to be integers. In that case we would have to compute maximum of 9 candidate optimal solutions and choose the best among them. However in an elaborate experimentation, we never encounter integer limits given by (15). Hence we decided to omit elaborations of that scenario.)

In Figure 2.5.1 we have shown relevant orders from \( O_{k+i} \) to \( O_{k+n_i+n_{j+1}} \) of solution \( S_j \) in time period \( t_2 + t_1 \). Symbols \( l_2, l_i, l_p, q_2, q_b, \) and \( q_j \), are used with following relationships.
\[ L_{k+1} = \ldots = L_{k+n_2} = l_2 \]
\[ L_{k+n_2+1} = l_1 \]
\[ Q_{k+1} = \ldots = Q_{k+n_2} = q_2 = l_2 d_2 \quad (37) \]
\[ Q_{k+n_2+2} = \ldots = Q_{k+n_2+n_1+1} = q_1 = l_1 d_1 \quad (38) \]
\[ L_{k+n_2+1} = l_b = x + y \quad (39) \]
\[ Q_{k+n_2+1} = q_b = x d_2 + y d_1 \quad (40) \]

It should be noted that, the following relationships also exist.

\[ t_2 = l_2 n_2 + x \quad (41) \]
\[ t_1 = l_1 n_1 + y \quad (42) \]

From Lemma 3, order cycle length of a bridge order should be equal to order cycle lengths of non-bridge order in period \( t_1 \).

\[ l_1 = x + y \quad (43) \]

From lemma 2, the quantity of a bridge order should be equal to quantity of non-bridge order in period \( t_2 \).

\[ l_2 d_2 = x d_2 + y d_1 \quad (44) \]
Solving equations (41) to (44) simultaneously for variables $l_2$, $l_1$, $x$, and $y$ yields

$$l_2 = -t_1 \frac{d_1/d_2 - (n_1(d_1/d_2 - 1) - 1)t_2}{n_1n_2 d_1/d_2 - (n_1 + 1)(n_2 + 1)}$$

(45)

$$l_1 = -\frac{(n_2(d_1/d_2 - 1) - 1)t_1 - t_2}{n_1n_2 d_1/d_2 - (n_1 + 1)(n_2 + 1)}$$

(46)

$$x = \frac{n_2t_1 d_1/d_2 - (n_1 + 1)t_2}{n_1n_2 d_1/d_2 - (n_1 + 1)(n_2 + 1)}$$

(47)

$$y = \frac{-(n_2 + 1)t_1 + n_1t_2}{n_1n_2 d_1/d_2 - (n_1 + 1)(n_2 + 1)}$$

(48)

The order cycle lengths of non-bridge orders can be calculated from (45) and (46).

The order cycle lengths of the bridge order can be calculated by substituting (47) and (48) in (43). Order quantities can be obtained from equations (37), (38), and (40).

The relevant holding, ordering and total cost of solution $S_j$ is:
The Optimal solution to RBDP is obtained by evaluating $TC(s_j)$ for $s_j \in S_j$ and selecting the lowest cost solution. Algorithm 1 provides the steps required to obtain the optimal solution.

Algorithm 1:

**Step 1:** Compute $\lambda_i$ using (3) and check whether the BDP satisfies (1) which makes it an RBDP. If it is not an RBDP, Algorithm 1 cannot be applied.

**Step 2:** Compute $n_{\text{min},i}$ from $U_i$ given by (15) and check whether (25) is satisfied. If not satisfied, Algorithm 1 cannot be applied.

**Step 3:** Compute $n_{\text{max},i}$ from $U_i$ given by (15). Compute the four possible solutions $S_j$, in period $(t_2+t_1)$ considering different combinations of $n_1$ and $n_2$ obtained from (36).

**Step 4:** For all solutions $S_j$, compute bridge order cycle length using (47), (48), and (39). Compute lengths of non-bridge orders using (45) and (46). Compute holding, ordering and total cost for relevant orders of $S_j$ using (49) to (51).
Step 5: Optimal solution to RBDP is obtained by repeating $S_j$ corresponding to the lowest cost among all $s_j$.

Example 2:

Find optimal ordering policy for BDP with following data.

$d_1 = 930 \text{ units/day}, \ d_2 = 3780 \text{ units/day}, \ t_1 = 50 \text{ days}, \ t_2 = 50 \text{ days}, \ H=\$1/\text{unit/day}, \text{ and} \ A = \$50,000/\text{order}$

Solution:

1. $\lambda_1 = \sqrt{\frac{2A}{Hd_1}} = 10.3695$
   
   $\lambda_2 = \sqrt{\frac{2A}{Hd_2}} = 5.1434$
   
   $t_1 = 50 > \sqrt{2}\lambda_1 = \sqrt{2} * 10.3695 = 14.6649$
   
   $t_2 = 50 > \sqrt{2}\lambda_2 = \sqrt{2} * 5.1434 = 7.2739$
   
   Hence the problems is RBDP

2. From (15), the possible range for number of non-bridge orders during $t_1$ is in the set [3, 4, 5] and during $t_2$ is in the set [8, 9, 10]. The minimum number of non-bridge orders during $t_1$ is 3. Using equation (25):

   \[
   \frac{d_2}{d_1} = \frac{3780}{930} = 4.064 > \frac{2(n_{\min} + 1)}{n_{\min}} = \frac{8}{3} = 2.67
   \]

3. The maximum number of non-bridge orders during $t_1$ and during $t_2$ as given by (15) is 5 and 10 respectively. Hence the possible values of $n_2$ and $n_1$ using (36) are [9, 10] and [4, 5] respectively.
4. The order cycle lengths of bridge and non-bridge orders and holding, ordering and total cost of these relevant orders of the four possible solutions $S_j$ is given in Table 2.5.1. For solution with $n_1 = 4$ and $n_2 = 9$ the total cost is lowest and optimal solution is obtained by repeating these orders during each demand cycle.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>Order Cycle length</th>
<th>Order Quantities</th>
<th>Holding Cost</th>
<th>Ordering Cost</th>
<th>$TC(S_j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>9</td>
<td>10.68 5.18 10.68</td>
<td>9929.68 19578.13 19578.13</td>
<td>787695.00</td>
<td>650000</td>
<td>1437695</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>10</td>
<td>10.56 4.72 10.56</td>
<td>9823.02 17837.08 17837.08</td>
<td>741503.00</td>
<td>700000</td>
<td>1441503</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>9</td>
<td>8.98 5.13 8.98</td>
<td>8349.63 19375.18 19375.18</td>
<td>743120.00</td>
<td>700000</td>
<td>1443120</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>10</td>
<td>8.88 4.67 8.88</td>
<td>8260.86 17654.15 17654.15</td>
<td>697875.00</td>
<td>750000</td>
<td>1447875</td>
</tr>
</tbody>
</table>

5. The cost per unit time of optimal solution is

$$TCT(S_1) = \frac{TC(S_1)}{(t_1 + t_2)} = \frac{1437695}{50 + 50} = $14376.95/day$$

2.6 Heuristics Development

Based upon the Economic Order Quantity (EOQ) model we have developed heuristics for BDP using the following objectives.

a) Minimizing total cost per unit time

b) Minimizing total cost per unit

c) Setting holding cost per order equal to ordering cost

These three objectives are achieved for each order in the EOQ model. However in BDP, only one of the three can be achieved at any time. In sections 4.1 to 4.3, three
heuristics are described, one for each of the above objective. In section 4.4, Heuristics 4a and 4b are presented which describe two modifications of Heuristic 1. Wherever necessary, heuristics logic is supported with an example.

Our heuristics work well for a wide range of BDP. However these heuristics cannot be applied in the cases where a bridge order satisfies the demand during more than two demand streams. This possibility can be excluded if length $t_i$ of a demand stream is large enough than possible length of bridge order. As per our heuristics, the length of bridge order in period $i$ will not exceed $\lambda_i$. For $d_2 > d_1$, from (3) $\lambda_1 > \lambda_2$. Hence the length of bridge order in any period $i$ will always be less than $\lambda_1$. We are focusing on the problems with $t_i \geq t_2$. Hence, verifying the following condition is sufficient for analyzing feasibility for applying heuristics.

$$t_2 \geq \lambda_1$$  \hspace{1cm} (52)

First three heuristics presented here are forward looking, implying that past orders cannot be changed based on future circumstances. In Heuristics 4a and 4b, the orders resulting from Heuristic 1 is revisited and modified. The following variables are used in describing the heuristics.

- $S$ is the solution to the BDP problem determined by Heuristics.
- $TC(S)$ is the total cost of solution $S$.
- $TCT(S)$ is the total cost per unit time of solution $S$.
- $T$ is the finite time horizon for which the heuristic is applied
- $T_c$ is the variable used to keep track of current time.
\( i \) and \( j \) are the index for demand streams such that \( i, j \in \Theta \) and \( i \) denotes the current demand stream.

\( x \) is the remaining time in the current demand stream.

\( y \) is the portion of the order cycle length of an order placed in demand stream \( i \) that lies in demand stream \( j \). \( y \) is the variable of interest in all the heuristics.

Also it should be noted that \( d_2 > d_1 \) without loss of generality.

### 2.6.1 Heuristic 1: Minimizing Total Cost per Unit Time

In this heuristic we start with the high demand period \( t_2 \). Orders are placed using the EOQ model as long as the order quantity can be consumed within the time remaining in the current demand stream. Otherwise using the principle of minimizing total cost per unit time, order cycle length is computed and bridge or non-bridge order is placed as described in step 3 of Algorithm 2. The derivation for order cycle length computation is given in Appendix A1. This procedure is repeated until total time exceeds \( T \). The last order is adjusted to match time horizon length \( T \). Algorithm 2 gives the detailed implementation of Heuristic 1.

**Algorithm 2:**

**Step 0:** Set \( i = 2 \); \( x = t_2 \); \( T_c = 0 \); \( TC(S) = 0 \); and \( j = i(mod \ 2) + 1 \)

Compute \( \lambda_i \) & \( \lambda_j \) from equation (3) and \( Q'_i \) & \( Q'_j \) from equation (2)

If \( t_2 \geq \lambda_i \) go to Step 1 else Algorithm 2 cannot be applied.

**Step 1:** If \( \lambda_i < x \), go to Step 2. Else, go to Step 3.

**Step 2:** If \( T > T_c + \lambda_i \) then
Place an order of quantity $Q_i$ at $T_c$

Set $TC(S)$ to $TC(S) + \frac{1}{2} * Q_i * \lambda_i * H + A$

Set $x$ to $x - \lambda_i$ and $T_c$ to $T_c + \lambda_i$

Go to Step 1.

Otherwise

Place an order of quantity $xd_i$

Set $TC(S)$ to $TC(S) + \frac{1}{2} x^2 d_i H + A$

Go to Step 4.

**Step 3**: Compute order cycle length of bridge order given by

$$L = x + y$$

Where,

$$y = -x + \sqrt{\frac{x^2 (d_i - d_j) H + 2A}{d_j H}}$$  \hspace{1cm} (53)$$

Set $y$ to 0 if the above equation results in a negative or imaginary value.

If $T > T_c + x + y$ then

Place order for quantity $Q = xd_i + yd_j$ at $T_c$

Set $TC(S)$ to $TC(S) + \left(\frac{1}{2} x^2 d_i + x y d_j + \frac{1}{2} y^2 d_j\right) H + A$

Set $T_c$ to $T_c + x + y$

Set $i$ to $i \bmod 2 + 1$ and $j$ to $j \bmod 2 + 1$

Set the value of $x$ to $t_i - y$
Go to Step 1.

Otherwise

Set $L = x + y - (T_e - T)$

If $(T_e - T) \geq y$ then

Place an order of quantity equal to $L/d_i$

Set $TC(S)$ to $TC(S) + \frac{1}{2} L^2 d_i H + A$

Otherwise

Place an order of quantity equal to $x/d_i + (L-x)/d_j$ at $T_e$

Set $TC(S)$ to $TC(S) + \left(\frac{1}{2} x^2 d_i + x(L-x)d_j + \frac{1}{2} (L-x)^2 d_j\right)H + A$

Go to Step 4

**Step 4:** Compute total cost per unit time using following equation.

$$TCT(S) = \frac{TC(S)}{T}$$

**Example 3:**

Apply Heuristic 1 to solve following BDP.

$d_1 = 930 \text{ units/day}, d_2 = 3780 \text{ units/day}, t_1 = 50 \text{ days}, t_2 = 50 \text{ days}, H=\$1/\text{unit/day}, \text{ and } A = \$50,000 /\text{order}, T = 100 \text{ days}$

**Solution:**

1. Using (3), the cycle lengths of orders using simple EOQ during $t_2$ and $t_1$ are

   $\mathcal{\lambda}_2 = 5.1434 \text{ and } \mathcal{\lambda}_1 = 10.3695$. Since $t_2 \geq \mathcal{\lambda}_1$ Heuristic 1 can be applied.
2. Start with demand stream $t_2$. Initialize $x = t_2 = 50$. Since $\lambda_2 < x$, place an order of size $Q_2^* = 19442.22$ and $\lambda_2 = 5.1434$. The total cost of this order is $100,000. The current time $T_c = 5.1434$ and $x = 50 - 5.1434 = 44.8566$.

Repeating this procedure, 8 more orders could be placed for this problem. At the end of $9^{th}$ order, $x = 3.7094$, $T_c = 46.2906$, and cumulative total cost will be equal to $900,000$. Since $\lambda_2 > x$ go to 3.

3. Place an order of $y = 8.5255$, $L = 12.2349$, and $Q = 21948.79$. Total cost for this order is $139,214.60$ resulting in cumulative total cost of $1,039,214.60$. At the end of this order $T_c = 54.8161$, the current demand stream is switched to $t_1$, and $x = 45.1839$.

4. Repeating the procedure described in 2, 3 more orders are placed with $\lambda_1 = 10.3695$, and $Q_1^* = 9643.65$. The total cost of this order is $100,000. At the end of $13^{th}$ order, $x = 10.3660$, $T_c = 89.6340$, and cumulative total cost is equal to $1,339,214.60$. Since $\lambda_1 > x$ go to 5.

5. Repeating the procedure described in 3, place an order of $y = 0$, $L = 10.3660$, and $Q = 9640.38$. The total cost for this order is $99,966.09$ resulting in cumulative total cost of $1,439,180.60$. At the end of this order $T_c = 100$, the current demand stream is switched to $t_2$, and $x = 50$.

6. Since $T_c$ exactly matches with $T$, stop at this point without making any changes in last order. The total cost per unit time using Heuristic 1 is $14,391.80$. 


2.6.2 Heuristic 2: Minimizing the Total Cost per Unit

In this heuristic we start with the high demand period $t_2$. Orders are placed using the EOQ model as long as the order quantity can be consumed within the time remaining in the current demand stream. Otherwise using the principle of minimizing total cost per unit, order cycle length is computed and bridge or non-bridge order is placed as described in step 3 of Algorithm 3. Derivation for order cycle length computation is given in Appendix A2. This procedure is repeated till total time exceeds $T$. Last order is adjusted to match time horizon length $T$. Algorithm 3 gives the detailed implementation of Heuristic 2.

Algorithm 3:

**Step 0:** Set $i=2$; $x = t_2$; $T_c = 0$; $TC(S) = 0$; and $j = i(mod \ 2) + 1$

1. Compute $\lambda_i$ & $\lambda_j$ from equation (3) and $Q_i^*$ & $Q_j^*$ from equation (2)

2. If $t_2 \geq \lambda_i$ go to Step 1 else Algorithm 2 cannot be applied.

**Step 1:** If $\lambda_j < x$, go to Step 2. Else, go to Step 3.

**Step 2:** If $T > T_c + \lambda_i$ then

1. Place an order of quantity $Q_i^*$ at $T_c$

2. Set $TC(S)$ to $TC(S) + \frac{1}{2} * Q_i^* * \lambda_i * H + A$

3. Set $x$ to $x - \lambda_i$ and $T_c$ to $T_c + \lambda_i$

4. Go to Step 1.

Otherwise

1. Place an order of quantity $xd_i$
Set $TC(S)$ to $TC(S) + \frac{1}{2}x^2d_iH + A$

Go to Step 4.

**Step 3**: Compute order cycle length of bridge order given by

$$L = x + y$$

Where,

$$y = -x\left(d_i/d_j\right)\pm \sqrt{x^2\left(d_i/d_j\right)^2\left(d_i - d_j\right)H + 2A}$$

(54)

Set $y$ to 0 if the above equation results in both negative and imaginary values.

If $T > T_c + x + y$ then

Place order for quantity $Q = xd_i + yd_j$ at $T_c$

Set $TC(S)$ to $TC(S) + \left(\frac{1}{2}x^2d_i + xyd_j + \frac{1}{2}y^2d_j\right)H + A$

Set $T_c$ to $T_c + x + y$

Set $i$ to $i(mod\ 2)+1$ and $j$ to $i(mod\ 2)+1$

Set the value of $x$ to $t_i-y$

Go to Step 1.

Otherwise

Set $L = x + y - (T_c - T)$

If $(T_c - T) \geq y$ then

Place an order of quantity equal to $L/d_i$

Set $TC(S)$ to $TC(S) + \frac{1}{2}L^2d_iH + A$
Otherwise

Place an order of quantity equal to \( x/d_i + (L-x)/d_j \) at \( T_c \)

Set \( TC(S) \) to

\[
TC(S) + \left( \frac{1}{2} x^2 d_i + x(L-x) d_j + \frac{1}{2} (L-x)^2 d_j \right) H + A
\]

Go to Step 4

**Step 4:** Compute total cost per unit time using following equation.

\[
TCT(S) = TC(S)/T
\]

**Example 4:**

Apply Heuristic 2 to solve following BDP.

\( d_1 = 930 \text{ units/days}, d_2 = 3780 \text{ units/day}, \ t_1 = 50 \text{ days}, \ t_2 = 50 \text{ days}, \ H = \$1/\text{unit/day}, \) and \( A = \$50,000/\text{order}, T = 100 \text{ days} \)

**Solution:**

1. Using (3), the cycle lengths of orders using simple EOQ during \( t_2 \) and \( t_1 \) are

   \( \lambda_2 = 5.1434 \) and \( \lambda_1 = 10.3695 \). Since \( t_2 \geq \lambda_1 \) Heuristic 2 can be applied.

2. Start with demand stream \( t_2 \). Initialize \( x = t_2 = 50 \). Since \( \lambda_2 < x \), place an order of size \( Q_2^* = 19442.22 \) and \( \lambda_2 = 5.1434 \). The total cost of this order is $100,000. The current time \( T_c = 5.1434 \) and \( x = 50 - 5.1434 = 44.8566 \).

   Repeating this procedure, 8 more orders could be placed for this problem. At the end of 9th order, \( x = 3.7094, \ T_c = 46.2906, \) and cumulative total cost will be equal to $900,000. Since \( \lambda_2 > x \) go to 3.
3. Place an bridge order of \( y = 1.6238 \), \( L = 5.3332 \), and \( Q = 15531.67 \). The total cost for this order is $82,833.50 resulting in cumulative total cost of $982,833.50. At the end of this order, the current demand stream is switched to \( t_f \), and \( x = 48.3762 \).

4. Repeating the procedure described in 2, 4 more orders are placed with

\[
\lambda_i = 10.3695, \quad \text{and} \quad Q^*_i = 9643.65.
\]

The total cost of this order is $100,000. At the end of the 14th order, \( x = 6.8982 \), \( T_c = 93.1018 \), and cumulative total cost is equal to $1,382,833.50. Since \( \lambda_i > x \),

5. Repeating the procedure described in 3, place a bridge order of \( y = 2.5158 \) and \( L = 9.4140 \). If this order is placed then \( T_c = 102.5158 \). Since \( T_c > T \), the length of last bridge order is modified so that to \( T_c = 100 \) resulting in \( L = 6.8982 \).

This new modified is a non-bridge order. The total cost for this order is $77,127.10 resulting in cumulative total cost equal to $1,454,960.60.

6. The total cost per unit time using Heuristic 2 is $14,549.60.

**2.6.3 Heuristic 3: Setting Holding Cost per order Equal to Ordering Cost**

In this heuristic we start with period \( t_2 \). Order is placed using simple EOQ model until the need for the first bridge order the current demand stream. The cycle length of the bridge order is computed by setting holding cost equal to ordering cost as given below.

The order cycle length of the bridge order (See Figure 2.6.3.1) is given by

\[ L = x + y \]
The \( x \) is the time remaining in the current demand stream and is known. The value of \( y \) that will make holding ordering cost equal to ordering cost can be determined as shown below.

\[
\left( \frac{1}{2} x^2 d_i + xy d_j + \frac{1}{2} y^2 d_j \right) H = A
\]

Solving the above equation for \( y \) yields

\[
y = -x \pm \sqrt{x^2 - \frac{x^2 d_i}{d_j} + \frac{2A}{d_j H}}
\]

Discarding the negative root the only positive root of \( y \) will be

\[
y = -x + \sqrt{x^2 - \frac{x^2 d_i}{d_j} + \frac{2A}{d_j H}}
\]

From which bridge order cycle length is determined.

**Figure 2.6.3.1 Inventory Level for the Bridge order corresponding to Heuristics**
This overall procedure is repeated to place the rest of the orders till total time exceed $T$. The last order is adjusted to match the time horizon length $T$. Algorithm 4 gives the detailed implementation of Heuristics 3.

**Algorithm 4:**

**Step 0:** Set $i=2; \ x = t_2; \ T_c=0; \ TC(S) =0; \ and \ j = i \mod 2 + 1$

Compute $\lambda_i$ & $\lambda_j$ from equation (3) and $Q^*_i$ & $Q^*_j$ from equation (2)

If $t_2 \geq \lambda_i$ go to Step 1 else Algorithm 2 cannot be applied.

**Step 1:** If $\lambda_i < x$, go to Step 2. Else, go to Step 3.

**Step 2:** If $T > T_c + \lambda_i$ then

Place an order of quantity $Q^*_i$ at $T_c$

Set $TC(S)$ to $TC(S) + \frac{1}{2} \times Q^*_i \times \lambda_i \times H + A$

Set $x$ to $x - \lambda_j$ and $T_c$ to $T_c + \lambda_i$

Go to Step 1.

Otherwise

Place an order of quantity $xd_i$

Set $TC(S)$ to $TC(S) + \frac{1}{2} \times x^2d_iH + A$

Go to Step 4.

**Step 3:** Compute order cycle length of bridge order given by

$$L = x + y$$

Where,
\[ y = -x + \sqrt{x^2 - \frac{x^2 d_i}{d_j} + \frac{2A}{d_j H}} \] 

(55)

If \( T > T_c + x + y \) then

Place order for quantity \( Q = xd_i + yd_j \) at \( T_c \)

Set \( TC(S) \) to \( TC(S) + \left( \frac{1}{2} x^2 d_i + xyd_j + \frac{1}{2} y^2 d_j \right) H + A \)

Set \( T_c \) to \( T_c + x + y \)

Set \( i \) to \( i (\text{mod} 2) + 1 \) and then \( j \) to \( i (\text{mod} 2) + 1 \)

Set the value of \( x \) to \( t_i - y \)

Go to Step 1.

Otherwise

Set \( L = x + y - (T_c - T) \)

If \( (T_c - T) \geq y \) then

Place an order of quantity equal to \( L/d_i \)

Set \( TC(S) \) to \( TC(S) + \frac{1}{2} L^2 d_i H + A \)

Otherwise

Place an order of quantity equal to \( x/d_i + (L-x)/d_j \) at \( T_c \)

Set \( TC(S) \) to \( TC(S) + \left( \frac{1}{2} x^2 d_i + x(L-x)d_j + \frac{1}{2} (L-x)^2 d_j \right) H + A \)

Go to Step 4

**Step 4:** Compute total cost per unit time using following equation.

\[ TCT(S) = TC(S) / T \]
Example 5:

Apply Heuristic 3 to solve following BDP.

\[ d_1 = 930 \text{ units/days}, \; d_2 = 3780 \text{ units/day}, \; t_1 = 50 \text{ days}, \; t_2 = 50 \text{ days}, \; H = \$1/\text{unit/day}, \text{ and} \]

\[ A = \$50,000/\text{order}, \; T = 100 \text{ days} \]

Solution:

1. Using (3), the cycle lengths of orders using simple EOQ during \( t_2 \) and \( t_1 \) are
   \[ \lambda_2 = 5.1434 \text{ and } \lambda_1 = 10.3695. \text{ Since } t_2 \geq \lambda_1 \text{ Heuristic 2 can be applied.} \]

2. Start with demand stream \( t_2 \). Initialize \( x = t_2 = 50 \). Since \( \lambda_2 < x \), place an order of \( Q_2^* = 19442.22 \) and \( \lambda_2 = 5.1434 \). The total cost of this order is \$100,000. The current time \( T_c = 5.1434 \text{ and } x = 50 - 5.1434 = 44.8566 \).

   Repeating this procedure, 8 more orders could be placed for this problem. At the end of 9\(^{th} \) order, \( x = 3.7094, \; T_c = 46.2906, \) and cumulative total cost will be equal to \$900,000. Since \( \lambda_2 > x \) go to 3.

3. Place a bridge order of \( y = 4.3752, \; L = 8.0846, \) and \( Q = 18090.44 \).The total cost for this order this order is \$100,000 resulting in cumulative total cost of \$1,000,000. At the end of this order \( T_c = 54.3752 \), the current demand stream is switched to \( t_1 \), and \( x = 45.6248 \).

4. Repeating the procedure described in 2, 4 more orders are placed with \( \lambda_1 = 10.3695 \), and \( Q_1^* = 9643.65 \). The total cost of this order is \$100,000. At the end of 14\(^{th} \) order, \( x = 4.1468, \; T_c = 95.8532, \) and cumulative total cost of is equal to \$1,400,000. Since \( \lambda_1 > x \) go to 5.
5. Repeating the procedure described in 3, place a bridge order of \( y = 2.1318 \) and \( L = 6.8982 \). If this order is placed then \( T_c = 102.1318 \). Since \( T_c > T \), the length of last bridge order is modified so that \( T_c = 100 \) resulting in \( L = 4.1468 \). This new modified order is a non-bridge order. The total cost for this order this order is $57,996.12 resulting in cumulative total cost equal to $1,457,996.12.

6. The total cost per unit time using Heuristic 3 is equal to $14,579.96.

In this section, so far we have described three heuristics to solve BDP. In Section 5, performance measurements of heuristics are described. Section 6 begins with experimental design to test these heuristics. We apply these heuristics to large set of problem instances and results are described in later in Section 6. From the analysis of results we found Heuristic 1 to be best performer. Hence we try to improve Heuristic 1 by making some subtle adjustments to quantity and lengths of each order by looking back at these values after applying Heuristics 1. These adjustments are described in next sub section as Heuristics 4a and 4b.

2.6.4 Heuristic 4a and 4b: Quantity and Length Adjustments to Heuristic 1

The performance of Heuristic 1 can be improved by making some adjustments to the solution obtained. From Lemma 2 we know that in an optimal solution to BDP the order quantity of a bridge and a non-bridge order placed in the same demand stream must be equal. Also from Lemma 3 the cycle length of a bridge order and a non-bridge order ending within a demand stream will be equal. Based on these properties, we propose two adjustments—Quantity Adjustment and Length Adjustment.
Let us consider demand stream \( t_i \) which is part of the solution obtained from Heuristics 1. This demand stream contains \( n_i \) non-bridge from \( O_{k+1} \) to \( O_{k+n_i} \) and two bridge orders—\( O_k \) and \( O_{k+n_i+1} \) as shown in Figure 2.6.4.1. From the characteristics of heuristic we know that

\[
L_{k+1} = \ldots = L_{k+n_i} = L
\]

\[
Q_{k+1} = \ldots = Q_{k+n_i} = Q
\]

In order to improve the solution, we propose two adjustments described below.

**Figure 2.6.4.1 Relevant Orders during Period \( t_i \) for Heuristics 4a and 4b**

![Figure showing relevant orders during period \( t_i \) for Heuristics 4a and 4b](image)

### 2.6.4.1 Heuristic 4a: Quantity Adjustment to Heuristic 1

In Quantity Adjustment we consider all the orders placed during period \( t_i \) (from \( O_{k+1} \) to \( O_{k+n_i+1} \)) and equalize the quantity as follows.

\[
Q'_{k+1} = \ldots = Q'_{k+n_i} = Q'_{k+n_i+1} = Q' = \frac{n_iQ + Q_{k+n_i+1}}{n_i + 1}
\]
Using standard equations, the cycle lengths of these orders can be determined from demand rate and order quantities. This procedure is repeated for all demand streams.

2.6.4.2 Heuristic 4b: Length Adjustment to Heuristic 1

In Length Adjustment we consider all the orders ending during \( t_i \) (from \( O_k \) to \( O_{k+n_i} \)) and equalize the cycle lengths as follows.

\[
L'_k = L'_{k+1} = \ldots = L'_{k+n_i} = \frac{L_k + n_i L}{n_i + 1}
\]

Using standard equations, order quantities of these orders can be determined from demand rate and cycle lengths. This procedure is repeated for all demand streams.

We applied the length and quantity adjustments to the solutions obtained from Heuristic 1. However in some of the problems it was observed that Length Adjustment results in first order \( O_k \) (See Figure 2.6.4.1) changed to non-bridge and successor order changed to bridge order. For few problems, in case of Quantity Adjustment, the last order \( O_{k+n_i+1} \) changed from bridge to non-bridge and predecessor order changed from non-bridge to bridge order. For our elaborate experimentation and results for such problems adjustments did not necessarily result into improvements. Also the resulting solutions violated the lemmas on which adjustments were based. Hence we recommend to apply these adjustments only when the order types are not changed.
2.7 Performance Measurement of the Heuristics

We have developed an optimal solution procedure which is applicable to a subset of BDP. The performance of the heuristics for these problems can be easily established. In this section we present two methods to assess the performance of heuristics when they are applied to problems where the optimal solution is not known.

i) Weighted average EOQ cost

ii) Lower bound

2.7.1 Weighted Average EOQ Model Cost

Average EOQ model cost is the cost one would have incurred if orders were placed using the EOQ model for each demand period. In this model the cost per unit time is constant as long as demand is constant. Since BDP has two demand streams with demand \( d_i \) during \( t_i \), the weighted average cost per unit time is calculated using the procedure given below in Algorithm 5. However, this solution will underestimate actual cost, since it could require placing orders smaller than EOQ quantity and incurring only partial ordering costs. However, it does provide a useful benchmark for heuristics comparison.

Algorithm 5:

Step 1: For both demand streams, compute the value of \( Q_i^*, \lambda_i, \) and \( n_i^* \) using (2), (3), and (4).
Step 2: Let $S^*$ be the solution to BDP problem using Weighted average EOQ model. Let $s^*$ relevant part of solution $S^*$ for analysis purposes. Using (5), calculate $TC(s)$.

Step 3: The total cost per unit time of the solution $S^*$ and $s^*$ will be same.

Calculate $TCT(S^*)$ using following.

$$TCT(S^*) = \frac{TC(s^*)}{t_1 + t_2}$$

2.7.2 Lower bound for RBDP

Lower bound is the lowest total cost per unit time for a given problem instance. Here each demand stream, a lower bound is calculated separately.

To find possible solutions, first the set $U_i$ for possible number of non-bridge orders $n_i (n_i \in U_i)$ is computed using (15) for each demand stream. We restate the range for $U_i$ in terms of maximum number of orders $n_{\text{max},i}$ as follows

$$\hat{n}_i = [n_{\text{max},i} - 2, n_{\text{max},i} - 1, n_{\text{max},i}]$$

(For most of the cases set of $U_i$ will have three integers. But it is theoretically possible for lower and upper bounds in (15) to be integers. In this case we would have to compute four candidate solutions and choose the lowest cost solution. However in an elaborate experimentation, we never encounter integer limits in (15). Hence, we decided to omit elaborations of that scenario.)
Let $S_{n_i}$ be the solution with $n_i$ non-bridge orders in $t_i$. For holding cost calculations, only the part of the order cycle length of a bridge orders that is in period $t_i$ is considered. The lowest holding cost for $n_i$ non-bridge and two bridge orders in $t_i$ period can be obtained under following both conditions.

i) Both bridge orders satisfy demand for the same length of time, say $x$ in $t_i$.

ii) The difference in order cycle lengths of non-bridge orders and $x$ is as small as possible, ideally 0.

Since $x$ should be less than or equal to the order cycle length of non-bridge orders, the lower bound on order cycle length of a non-bridge order, say $l_{\text{min},i}$, should be found. It should be noted that if $n_i = 0$ then $l_{\text{min},i} = 0$ and the value of $x$ should be obtained using (58).

Equation (21), from Lemma 7 can be used to calculate this bound as given below.

$$-\lambda_i^2 n_i^2 + \lambda_i^2 n' + t_i^2 \geq 0$$

$$\left(\frac{t_i'}{n_i'}\right)^2 \geq \lambda_i^2 \left(1 - \frac{1}{n_i'}\right)$$

In the derivation of this equation from Lemma 7, $t_i'$ represents the period in $t_i$ such that the demand during $t_i'$ is completely satisfied by $n_i'$ non-bridge orders. Hence in the current context, $l_{\text{min},i} = t_i'/n_i'$.

$$\left(\frac{l_{\text{min},i}}{n_i}\right)^2 = \lambda_i^2 \left(n_i - 1\right)/n_i$$

Discarding the negative root
\[ l_{\text{min},i} = \lambda_i \sqrt{(n_i - 1)/n_i} \] (57)

The value of \( x \) can be computed as

\[ x = \frac{(t_i - n_i l_{\text{min},i})}{2} \] (58)

Thus relevant lowest possible holding cost during \( t_i \) with \( n_i \) non-bridge orders is

\[ HC(s_{n_i}) = \left( \frac{1}{2} t_{\text{min},i}^2 d_i n_i + x^2 d_i \right) H \] (59)

Ordering cost is calculated based on number of non-bridge orders and required bridge orders placed during \( t_i \). There are different formulations for computing ordering cost depending on demand stream. From Lemma 9, one bridge order will be always placed during \( t_2 \). Hence the total orders placed will be \((n_2 + 1)\) during \( t_2 \) and corresponding cost is given by

\[ RC(s_{n_2}) = (n_2 + 1)A, \quad n_2 \in \left[ n_{\text{max},2} - 2, n_{\text{max},2} - 1, n_{\text{max},2} \right] \] (60)

During \( t_1 \) nothing can be said with certainty about the number of bridge orders placed unless it is a case of \( n_{\text{max},1} - 2 \) non-bridge orders which is possible only when one bridge order is placed. This makes the ordering cost of \( n_{\text{max},1} - 1 \) case equal to \( n_{\text{max},1} - 2 \) case and later can be excluded as corresponding holding cost will definitely be greater than earlier. Hence the relevant ordering cost during \( t_1 \) with \( n_1 \) non-bridge orders is given by

\[ RC(s_{n_1}) = n_1 A, \quad n_1 \in \left[ n_{\text{max},1} - 1, n_{\text{max},1} \right] \] (61)
Thus the corresponding relevant total cost during high and low demand periods is given by

\[
TC(s_{n_2}) = \left(1/2\right)l_{\min,2}^2 d_{n_2} + x^2 d_2 H + (n_2 + 1)A, \quad n_2 \in \left[n_{\max,2} - 2, n_{\max,2} - 1, n_{\max,2}\right]
\] (62)

\[
TC(s_{n_1}) = \left(1/2\right)l_{\min,1}^2 d_{n_1} + x^2 d_1 H + n_1 A, \quad n_1 \in \left[n_{\max,1} - 1, n_{\max,1}\right]
\] (63)

The lower bound for period \(t_i\) is the solution with lowest \(TC(s_{n_i})\), denoted by \(TC(s^*_{n_i})\), obtained from above equations as the case may be. Let \(S^*\) represents combination of \(S^*_{n_1}\) and \(S^*_{n_2}\). The total cost per unit for the lower bound solution \(S^*\) is given by

\[
TCT(s^*) = \frac{TC(s^*_{n_1}) + TC(s^*_{n_2})}{t_1 + t_2}
\] (64)

This developed lower bound has some limitations and cannot be applied to all BDP. To calculate ordering cost, Lemma 9 is used which is applicable only to RBDP. Also if the length of period \(t_i\) is not large enough to have one non-bridge order (RBDP) then assuming two bridge orders in period \(i\) for holding cost calculations would underestimate the holding cost. Also the ordering cost will be zero which is very unrealistic and would generate a loose bound. Hence the lower bound developed here is applicable only to RBDP. Algorithm 6 given below describes the implementation of the lower bound for RBDP.
Algorithm 6:

Step 1: Compute $\lambda_i$ using (3) for both demand streams and check whether the BDP satisfies (1) which makes it an RBDP. If it is not a RBDP, Algorithm 6 cannot be applied.

Step 2: For both demand streams compute the set $U_i$ using (23) and express it in terms of $n_{\text{max},i}$ as given by (56) to generate possible solutions $S_{n_i}$ with $n_i$ non-bridge orders during demand stream $t_i$.

Step 3: For all solutions $S_{n_i}$, set $l_{\text{min},i} = 0$ if $n_i = 0$ otherwise compute $l_{\text{min},i}$ from (57) and then $x$ using (58). Compute the total cost for demand stream $t_2$ using (62) and for demand stream $t_1$ using (63).

Step 4: Find $S_{n_1}^*$ and $S_{n_2}^*$ which in turn represents lowest cost $S_{n_i}$ solutions. Compute total cost per unit time for lower bound solution using (64).

2.8 Results

2.8.1 Design of Experiment

We generated a large set of problems using the range of values for various parameters and tested the performance of heuristics. The four parameter considered are—ratio of lengths of time period $t_1/t_2$, ratio of mean demand intensities $d_2/d_1$, ideal numbers of orders $c$ in time $T$, and number of demand cycles $k$ in time $T$ where demand cycle length is equal to $t_1 + t_2$. The parameters considered, the number of levels and the corresponding values for experiment is described in Table 2.8.1.1.
Table 2.8.1.1 Problem parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notations</th>
<th>Levels</th>
<th>Values corresponding to Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of lengths of time period</td>
<td>( t_1/t_2 )</td>
<td>3</td>
<td>1, 2, &amp; 3</td>
</tr>
<tr>
<td>Ratio of mean demand intensities</td>
<td>( d_2/d_1 )</td>
<td>3</td>
<td>2, 4, &amp; 8</td>
</tr>
<tr>
<td>Number of orders if order quantity equals to simple EOQ</td>
<td>( c )</td>
<td>3</td>
<td>150, 250, &amp; 350</td>
</tr>
<tr>
<td>Number of demand cycles in time ( T )</td>
<td>( k )</td>
<td>4</td>
<td>10, 20, 30, &amp; 40</td>
</tr>
</tbody>
</table>

A total of 108 cells (3*3*3*4) are formed in this experiment. The length of demand cycle is given by

\[
t_1 + t_2 = \frac{T}{k}
\]  

(65)

In each cell the values of \( t_1 \) and \( t_2 \) and are obtained by simultaneously satisfying (65) and the ratio constraint \( (t_1/t_2) \) corresponding to that cell. Holding cost is set to 1. Ordering cost is calculated by holding the values of \( c \) and \( k \) for each cell level using the method described below. The motivation for this approach of determining \( A \), rather than using a fixed value for \( A \), is to keep the problem diversity among all cells to depend on ideal number of orders rather than vary sporadically due to interaction between \( A \) and \( H \).

From (3) cycle length of an order placed using simple EOQ in demand \( d_i \) is

\[
\lambda_i = \sqrt{\frac{2A}{d_iH}}, \quad i \in \Theta
\]
Hence the total number of orders \( n_i^* \) in time period \( t_i \) is

\[
n_i^* = \frac{t_i}{\lambda_i}
\]

\[
n_i^* = t_i \sqrt{\frac{d_i H}{2A}}
\]

The total number of orders in time period \( t_1 + t_2 = t_1 \sqrt{\frac{d_1 H}{2A}} + t_2 \sqrt{\frac{d_2 H}{2A}} \)

\[
= \sqrt{\frac{H}{2A}} (t_1 \sqrt{d_1} + t_2 \sqrt{d_2})
\]

Hence using (65),

\[
c = k \sqrt{\frac{H}{2A}} (t_1 \sqrt{d_1} + t_2 \sqrt{d_2})
\]

By transforming the above equation, the value of ordering cost corresponding to that cell can be obtained as follows.

\[
A = \frac{k^2 H}{2c^2} (t_1 \sqrt{d_1} + t_2 \sqrt{d_2})^2
\]

By assumption \( H=1 \)

\[
A = \frac{k^2}{2c^2} (t_1 \sqrt{d_1} + t_2 \sqrt{d_2})^2
\]  

(66)
In each cell we generated 10 replications using the uniform distribution \(UN(d_i \pm 0.1d_i)\). The value of \(d_i\) is set to 1000 and \(d_2\) is computed using \(d_2/d_1\). The ordering cost \(A\) is calculated as above using the mean value of \(d_i\).

Table 2.8.1.2 gives the parameter values for all the cells of the experiment. In addition, maximum \(\lambda_i\), defined as order cycle length corresponds to minimum demand, and \(n_{min}\), defined as minimum number of non-bridge orders required during \(t_1\), are also given. First the cell conditions are checked to see if the problem can be classified as RBDP and if it also satisfies (25). If both hold then the optimal solution can be computed using Algorithm 1. Each cell in Table 2.8.1.2 is classified accordingly. Next, applicability of Heuristics 1-3 is verified by testing whether (52) holds. It is duly noted in the table corresponding to each cell. Lower bounds for all RBDP can be formed using Algorithm 6. It is also noted against each cell. The analysis is conducted at cell level rather than at individual replication level and a conservative approach is used by selecting the extreme demand values for analysis so as to satisfy all the possible cases in a given cell.

**Table 2.8.1.2 Cell parameters and applicability of corresponding solution techniques**

<table>
<thead>
<tr>
<th>Cell</th>
<th>(t_1)</th>
<th>(t_2)</th>
<th>(A)</th>
<th>(d_2/d_1)</th>
<th>(N)</th>
<th>Max (\hat{\lambda}_1)</th>
<th>Max (\hat{\lambda}_2)</th>
<th>(n_{min}) for (t_1)</th>
<th>Applicable?</th>
<th>Opt.</th>
<th>Heur.</th>
<th>L. B.</th>
</tr>
</thead>
<tbody>
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Table 2.8.1.1 depicts the various paths that can be used for BDP and results of further analysis. Analysis indicates that for 51 cells optimal solution can be derived, for 99 cells heuristics can be used, and 104 cells fall in category of RBDP implying lower bound can be calculated. Out of 108 cells, only for 97 cells heuristics and developed lower bound are applicable and hence are considered for problem generation. Highlighted cells are excluded from consideration.

**Figure 2.8.1.1 Solution strategy flow diagram**

- BDP (108 cells)
- **RBDP?**
  - Yes: Lower Bound exists (104 cells)
    - Satisfies Equation (25)?
      - Yes: Optimal Solution exists (51 cells)
      - No: Alg. 1 does not apply
  - No: Heuristics can be applied (99 cells)
    - Satisfies Equation (52)?
      - Yes: Cells with LB and Heuristics applicability (97 cells)
      - No: No further analysis
2.8.2 Heuristics Performance and Average EOQ model Cost

As described earlier, 970 problems were generated in 97 cells. The procedure for the heuristics, optimal solution procedure and lower bound procedure were coded using VB.net. Since different problems use different cost and demand parameters, performance across the problems is compared by expressing it as a percent of the cost of the EOQ model cost. The results are described in Table 2.8.2.1.

Heuristics 1 and 3 is clearly perform better than Heuristic 2. They also yield costs nearly the same as that for the simple EOQ model. In case of Heuristics 1, it is between 97.02% and 103.04 % with average of 99.86%. For heuristics 3, it is between 97.03% and 102.99% with average of 100.27%. It is found that for 477 problems out of 970 costs lower than simple EOQ model can be obtained by using one of the three heuristics. We have ranked the heuristics based on costs and found that for 626 problems Heuristic 1 is superior to others whereas for 333 problems Heuristic 3 is superior to others. For 11 problems Heuristics 2 is superior to others.

<table>
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<td>Heuristics Cost as % of EOQ Model</td>
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<tr>
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<table>
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To statistically validate the observed differences between heuristics performance, a one way ANOVA test is conducted followed by a post-hoc test. Results are shown in Table 2.8.2.2 and Table 2.8.2.3. The ANOVA test confirms that the differences between
the performance of the heuristics is significant ($p$ value < 0.000). Since homogeneity of variance could not be assumed, Tamhane multiple comparison tests is conducted. The differences between all the pairs of heuristic performance are found to be significant ($p$ value < 0.000). Performance of Heuristic 1 is found to be better than others. Also Heuristic 3 is found to perform better than Heuristic 2.

### Table 2.8.2.2 ANOVA test for heuristic performances

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### Table 2.8.2.3 Tamhane test for pair-wise differences between heuristics performance

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<th>(J) Heuristic</th>
<th>Mean Difference (I-J)</th>
<th>Std. Error</th>
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<td>.06451%</td>
<td>.000</td>
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Results across cell are analyzed to find, if any, clear patterns are exhibited by heuristics. Results of Heuristic 4a and 4b are not included as they are merely improvements on Heuristic 1. Table 2.8.2.4 describes the distribution of heuristics across cells for number of problems it is superior over others. For example, cell value 11 under Heuristics 1 and number of problems 10 indicates that it is best for all the 10 problems generated in 11 cells out of 97 in the total experiment.
For 53 cells Heuristic 1 performs best for at least 7 out of 10 problems and based on this criterion we define it as dominant heuristic for these cells. To highlight these cells are shaded in Table 2.8.2.4. Similarly Heuristic 3 is dominant for 14 cells and for none of the cells Heuristic 2 is dominant. Also there are only twelve cells with only one heuristics is superior for all the problems.

**Table 2.8.2.4 Performance of heuristic across cells**

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<thead>
<tr>
<th># of Problems</th>
<th>Number of cells with following heuristics performs best for number of problems mentioned</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heuristics 1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

To analyze the effect of time horizon $T$ on the performance, we picked one problem from each cell and extended the $T$ in stepwise increment of $0.1(t_2+t_1)$ and applied all the three heuristics. Thus an additional 10 replications of each problem are obtained making a total of 11 in each cell.

Table 2.8.2.5 describes the performance of these across cells for number of replications it is superior over others. For example, cell value 55 under Heuristics 1 and number of replications 11 indicates that Heuristic 1 has ranked best for all 11 replications.
generated in 55 cells out of 97. We also found that for 60 problems out of 97, Heuristic 1 is better than others for at least 9 replications. Similarly, Heuristic 3 has dominated 32 problems while Heuristic 2 did not dominate in any cell. There are only 7 cells in which we observed a switch between heuristics with no clear pattern. Thus, the value of T does not seem to impact the performance of the heuristics.

Table 2.8.2.5 Performance of heuristics over extended time horizon

<table>
<thead>
<tr>
<th># of replications</th>
<th>Number of cells with following heuristics performs best for number of problems mentioned</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heuristics 1</td>
</tr>
<tr>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>55</td>
</tr>
</tbody>
</table>

2.8.3 Heuristics Performance and Optimal Solution

The optimal solution is computed for 510 problems in 51 cells satisfying (25). The performance of the heuristics is measured by calculating the difference between heuristic and optimal solution and reported it as percent of the optimal solution in the Table 2.8.3.1. All the heuristics perform very well notably Huristics 1 and Heuristic 3. They are on an average 1.31% and 1.79% above optimal solution respectively.
Quantity (Heuristic 4a) and Length (Heuristic 4b) adjustments were also applied to improve the solutions obtained from Heuristic 1. From the results, Heuristic 4a seems to perform better than Heuristic 4b with on average 0.46% higher than optimal solution. Also it is applicable to more problems (356 problems) compared to Heuristic 4b (225 problems). Hence it is preferred over Heuristic 4b.

### Table 2.8.3.1 Performance of heuristics compared to optimal solution

<table>
<thead>
<tr>
<th>Heuristics</th>
<th>Percent above optimal solution</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>St. Dev.</td>
<td>Minimum</td>
<td>Maximum</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.32%</td>
<td>1.03%</td>
<td>0.05%</td>
<td>6.38%</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4.17%</td>
<td>2.71%</td>
<td>0.56%</td>
<td>15.94%</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.83%</td>
<td>1.00%</td>
<td>0.13%</td>
<td>5.32%</td>
<td></td>
</tr>
<tr>
<td>4a</td>
<td>0.46%</td>
<td>0.47%</td>
<td>0.03%</td>
<td>2.98%</td>
<td></td>
</tr>
<tr>
<td>4b</td>
<td>1.50%</td>
<td>0.99%</td>
<td>0.26%</td>
<td>6.35%</td>
<td></td>
</tr>
</tbody>
</table>

#### 2.8.4 Heuristics Performance and Lower Bound

Where an optimal solution could not be found, a lower bound can be calculated to evaluate the performance of the heuristics. Performance is measured by calculating the difference between heuristic and lower bound solutions and reported it as percent of lower bound solution in Table 2.8.4.1. All the three heuristics are applied to all 970 problems. Heuristics 4a and 4b could be applied to only 641 and 466 problems respectively. Results show that performance of Heuristic 1 is better than Heuristics 2 and 3 and after applying adjustments, more improvement is observed in the case of Quantity Adjustments (Heuristics 4a) than in the case of Length adjustments (Heuristics 4b). On average cost of all heuristics solutions is between 13.10% and 16.53% above lower bound. But for some problems cost of heuristics solutions are as high as 40% above lower
bound. Also large variation is an indicative of lower bound solution is loose bound for some of the problems. To understand if these numbers are reasonable, problems were divided into two groups—Group 1: Problems with optimal solution (510 problems) and Group 2: Problems without optimal solution (460 problems). This is reported in Table 2.8.4.1. 356 of 641 problems solved using Heuristic 4a fall in Group 1 where as 225 of 466 problems solved using Heuristic 4b fall in Group 2. Results indicate that the cost of the optimal solution is on average on 10.32% above lower bound. The heuristics seem to have performed in the same order of magnitude. Thus lower bound seems to be loose bound with heuristics solution costs close to optimal solution.

Table 2.8.4.1 Performance of heuristics compared to optimal solution

<table>
<thead>
<tr>
<th>Category</th>
<th>Method</th>
<th>Percent Increase Compared to Lower Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Average</td>
</tr>
<tr>
<td>All Problems</td>
<td>Heuristic 1</td>
<td>13.99%</td>
</tr>
<tr>
<td></td>
<td>Heuristic 2</td>
<td>16.53%</td>
</tr>
<tr>
<td></td>
<td>Heuristic 3</td>
<td>14.49%</td>
</tr>
<tr>
<td></td>
<td>Heuristic 4a*</td>
<td>13.10%</td>
</tr>
<tr>
<td></td>
<td>Heuristic 4b**</td>
<td>14.45%</td>
</tr>
<tr>
<td>Group 1 (51 cells)</td>
<td>Heuristic 1</td>
<td>11.81%</td>
</tr>
<tr>
<td></td>
<td>Heuristic 2</td>
<td>15.01%</td>
</tr>
<tr>
<td></td>
<td>Heuristic 3</td>
<td>12.38%</td>
</tr>
<tr>
<td></td>
<td>Heuristic 4a*</td>
<td>10.66%</td>
</tr>
<tr>
<td></td>
<td>Heuristic 4b**</td>
<td>12.57%</td>
</tr>
<tr>
<td></td>
<td>Optimal</td>
<td>10.32%</td>
</tr>
<tr>
<td>Group 2 (46 cells)</td>
<td>Heuristic 1</td>
<td>16.42%</td>
</tr>
<tr>
<td></td>
<td>Heuristic 2</td>
<td>18.21%</td>
</tr>
<tr>
<td></td>
<td>Heuristic 3</td>
<td>16.83%</td>
</tr>
<tr>
<td></td>
<td>Heuristic 4a*</td>
<td>16.14%</td>
</tr>
<tr>
<td></td>
<td>Heuristic 4b**</td>
<td>16.22%</td>
</tr>
</tbody>
</table>

Group 1 includes problem with optimal solution and Group 2 contains problems without optimal solution.

* 641 problems were feasible using Heuristics 4a of which for 356 problems falls in category of problems with optimal solution.

** 466 problems were feasible using Heuristics 4b of which for 225 problems falls in category of problems with optimal solution.
Results also indicate that the performance of heuristics in Group 1 is better than in Group 2. To compare mean differences across groups, independent sample t-test is conducted for all the heuristics and results are reported in Table 2.8.4.2, Levene’s test for all heuristics was found to be significant (p value .000) indicating that homogeneity of variance cannot be assumed across groups. Results show that the t-test is significant (p value .000) with negative mean difference for all heuristics indicating performance of heuristics in Group 1 is better than Group 2. This is likely because of the fact that tighter lower bound will be obtained for problems with more orders per demand cycle and higher $d_2/d_1$ ratio. Also the problems with higher values on these two factors will satisfy (25), required to calculate optimal solution and thus are more likely to fall in Group 1.

Table 2.8.4.2 t-test for differences between heuristics performance

<table>
<thead>
<tr>
<th>Heuristic</th>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
<th>Mean Difference</th>
<th>Std. Error Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-9.82</td>
<td>799.89</td>
<td>.000</td>
<td>-4.61%</td>
<td>0.47%</td>
</tr>
<tr>
<td>2</td>
<td>-5.93</td>
<td>895.51</td>
<td>.000</td>
<td>-3.20%</td>
<td>0.54%</td>
</tr>
<tr>
<td>3</td>
<td>-9.31</td>
<td>804.48</td>
<td>.000</td>
<td>-4.45%</td>
<td>0.48%</td>
</tr>
<tr>
<td>4a</td>
<td>-9.17</td>
<td>441.76</td>
<td>.000</td>
<td>-5.48%</td>
<td>0.60%</td>
</tr>
<tr>
<td>4b</td>
<td>-5.36</td>
<td>430.36</td>
<td>.000</td>
<td>-3.65%</td>
<td>0.68%</td>
</tr>
</tbody>
</table>
CHAPTER 3: SIMULATION APPROACH TO SUPPLY CHAIN MANAGEMENT

3.1 Introduction

In this chapter, we have built a simulation model of three member supply chain consisting of manufacturer, distributor and a retailer. Our goal is to develop ordering policies for each member of the supply chain where retailer faces bi-level stochastic demand distribution.

In any real-life supply chain, various approaches have been used for designing an ordering policy. The simplest policy is to order each period and let the order quantity be equal to demand forecasted by an expert or using simple moving average forecast. Some supply chain members use more sophisticated approaches based on computer algorithms which balances various costs. Cooperation among players is desirable and some incorporate this information in their decision making, but it cannot always be mandated. Thus taking into considerations these approaches, we have developed three ordering policies. First one is a simple policy called Moving Average Policy (MAP). Second policy uses involved calculations and we call it as Target Inventory Level Policy (TILP). Both policies assume supply chain members do not share any information and order independently to minimize their own costs. Lastly, we develop a policy called Complete Cooperation Policy (CCP) in which members cooperate to minimize total supply chain costs. Our policies fall under two extremes modes—Total Independence and Complete
Cooperation. In practice there can be many shades of cooperation between the two poles leading to different policies.

The problem of determining optimal ordering policy in a stochastic bi-level demand environment is complicated and is not solved easily by analytical methods. Hence we developed heuristics and evaluate their performance using simulation. Our findings are reported in the last section of this chapter.

3.2 Supply Chain Model

Our Supply Chain model consists of three members – the manufacturer, distributor, and retailer. The focus of our current study is on consumer goods supply chains with a bi-level demand pattern. To operationalize the bi-level demand, we assume that there are three periods without promotion followed by one period of product promotion. During non-promotion periods demand is UN(400, 600) and during promotion demand is UN(1200, 1800). This cycle of four periods, which constitute a demand cycle, is repeated.

We assume discrete time model in which members make ordering decisions and conduct series of activities only once in each period in a predetermined sequence. At the beginning of the period, each supply chain member must decide whether and how much to order or produce. It may be noted that the retailer, while placing the current period’s order has to plan for an unknown demand for the current as well as the next period. However, the distributor and the manufacturer have the luxury of making their decisions after knowing the demand of their immediate customer for the current period, though the uncertainty of the next period’s demand needs to be taken into consideration. Thus, the
timing of the decision becomes important because of its effect on available information prior to decision making. The timing is captured by the sequence of events described in Table 3.2.1

**Table 3.2.1 Sequence of Events in each Period**

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Retailer sends order to Distributor</td>
</tr>
<tr>
<td>2</td>
<td>Distributor sends order to Manufacturer</td>
</tr>
<tr>
<td>3</td>
<td>Distributor ships materials to Retailer. The quantity shipped is either the quantity ordered by Retailer or the inventory on hand, whichever is smaller.</td>
</tr>
<tr>
<td>4</td>
<td>Manufacturer ships materials to Distributor. The quantity shipped is either the quantity ordered by Distributor or the inventory on hand, whichever is smaller.</td>
</tr>
<tr>
<td>5</td>
<td>Manufacturer decides whether to produce, and if so how much to produce.</td>
</tr>
<tr>
<td>6</td>
<td>Manufacturer completes production</td>
</tr>
<tr>
<td>7</td>
<td>Distributor receives shipment from Manufacturer</td>
</tr>
<tr>
<td>8</td>
<td>Simulated consumer demand for the period is revealed to Retailer. (Quantity sold will be equal to minimum of on hand inventory and actual demand.)</td>
</tr>
<tr>
<td>9</td>
<td>Retailer receives shipment from Distributor. (It can only be used in the next period.)</td>
</tr>
<tr>
<td>10</td>
<td>All members compute the costs and inventory levels at the end of the period</td>
</tr>
</tbody>
</table>

There are three types of costs—holding cost, ordering/setup cost, and shortage cost—incurred by each supply chain member. It is assumed that the retailer and distributor incur holding costs on finished goods inventory and incoming pipeline inventory. The manufacturer incurs holding costs on finished goods inventory. The retailer faces continuous demand, while the distributor and manufacturer face lumpy demand. This affects the average inventory held by each player.
A bottle of pop at a manufacturing site is less valuable than the same bottle at a distributor site (since it has moved closer to the ultimate consuming location), which in turn is less valuable than that at a retailer location. Hence, we assume that the cost of holding increases as the inventory moves downstream.

The retailer’s ordering cost is assumed to be independent of batch size. In addition to the typical costs such as material receiving cost at the retailer’s end and cost of clearing the order for payment, it includes the incremental delivery cost. We assume that the retailer’s orders will trigger only an additional stop in the route of the transporter. The orders will never be large enough to require additional transporter. The distributor incurs an ordering cost for each batch of size X or less (X may be interpreted as the truck capacity and here X is assumed to be 1000). We assume that a distributor will service a number of retailers. Therefore, a distributor order will be large, sometimes larger than a truck load. The distributor ordering costs includes fixed portion of shipment cost from the manufacturer to the distributor, cost of communication and paper work. The manufacturer incurs a set-up cost for each batch of size X or less (X may be interpreted as the maximum manufacturing batch size and in this case X is also assumed to be 1000).

Finally, a shortage cost per unit is incurred (no backorder is allowed) if a supply chain member is unable to satisfy the demand. Shortage cost is the highest for the retailer as it will lead to loss of goodwill and loss of sales. Shortage cost for the manufacturer is assumed to be higher than that of the distributor because of longer lead time generally required for manufacturing.

We assume all the three members use same general policy in each simulation run. In the first two policies members do not share any information and only information that
is known to retailer is the demand distribution. All the members try to minimize their own costs. In the last policy, supply chain members cooperate and share all the available information. Their goal is optimize total supply chain cost which is the sum of holding cost, ordering/setup cost, and shortage cost incurred by all the three members.

3.3 Moving Average Policy

Moving Average is a simple ordering policy in which demand for a period is forecasted as moving average of same type of periods from last \( w \) demand cycles. Order quantity for the current period is determined to satisfy complete demand of next period taking into consideration the opening inventory of current period. Factor \( w \) should be determined based on variation in demand. Algorithm 7 describes the Moving Average ordering policy. We have used following notations in describing three policies.

Notations and definition

\( \theta \) \hspace{1cm} \text{The set of supply chain members; } \theta \equiv \{ 1: \text{Retailer}, 2: \text{Distributor}, 3: \text{Manufacturer} \} \\
\eta \hspace{1cm} \text{The set of periods; } \eta \equiv \{1, 2, \ldots, N\} \\
\phi \hspace{1cm} \equiv \{1, 2, \ldots, m\} \text{ where } m \text{ is the minimum number of periods required to define the demand pattern (after } m \text{ periods, the demand repeats itself). Periods } i \text{ and } i+m \text{ are said to be corresponding periods since they have a same demand pattern.} \\
w \hspace{1cm} \text{Number of demand cycles used for moving average calculation (chosen by user)}
\( N \) Number of periods used for simulation

\( P \) Number of periods used to generate probability profile

\( H_i \) Holding cost per unit per period for \( i \in \Theta \)

\( R_i \) Ordering/Set up cost per batch for \( i \in \Theta \)

\( S_i \) Shortage cost per unit for \( i \in \Theta \) (Demand is not carried forward)

\( B_i \) Batch size for \( i \in \Theta \) (Note that \( B_1 \) is unconstrained)

\( D_j \) Actual demand realized by the retailer for period \( j \in \eta \)

\( I_{ij} \) Ending inventory for the supply chain member \( i \in \Theta \) in period \( j \in \eta \)

\( E_{ij} \) Expected ending inventory after satisfying demand in period \( j \in \eta \) before receipts for member \( i \in \Theta \)

\( T_{ik} \) Target inventory level of supply chain member \( i \in \Theta \) in period type \( k \in \phi \)

\( O_{ij} \) Quantity ordered by supply chain member \( i \in \Theta \) in period \( j \in \eta \)

\( Q_{ij} \) Preliminary order quantity of supply chain member \( i \in \Theta \) in period \( j \in \eta \)

\( q_{ij} \) Partial batch size corresponding to the preliminary order quantity \( Q_{ij} \) of supply chain member \( i \in \Theta \) in period \( j \in \eta \)

\( G_{ij} \) The shortage faced by supply chain member \( i \in \Theta \) in period \( j \in \eta \)

**Algorithm 7:**

**Step 0:** Set current period \( j = 1 \).

Set \( I_{i0} \) = Mean demand of Period 1 + Mean demand of Period 2 for all supply chain members \( i \in \Theta \).
Select appropriate value of $w$

*(Step 1 describes the logic for determining order quantity in period $j$ for retailer)*

**Step 1:** Determine $E_{ij}$ using following equation which is equivalent to stating that expected ending inventory is equal to beginning inventory less average demand over last $w$ corresponding periods, where $w$ is chosen by user.

$$E_{ij} = \text{Max} \left\{ I_{i(j-1)} - \frac{\sum_{l=0}^{w-1} D_{j-lm}}{w}, 0 \right\}$$

Determine $O_{ij}$ using following equation. (Intuitively $O_{ij}$ represents the difference between average demands for $w$ corresponding periods and the expected ending inventory.)

$$O_{ij} = \text{Max} \left\{ \frac{\sum_{l=0}^{w-1} D_{j+1-lm}}{w} - E_{ij}, 0 \right\}$$

Set $i = 2$

*(Step 2 describes the logic for determining order quantity in period $j$ for distributor and manufacturer)*

**Step 2:** Determine $E_{ij}$ using following equation.

$$E_{ij} = \text{Max} \left\{ I_{i(j-1)} - \frac{\sum_{l=0}^{w-1} I_{(i-1)(j-lm)}}{w}, 0 \right\}$$

Determine $O_{ij}$ using following equation.

$$O_{ij} = \text{Max} \left\{ \frac{\sum_{l=0}^{w-1} I_{(i-1)(j+1-lm)}}{w} - E_{ij}, 0 \right\}$$
Step 3: If i=2 then set i=3 and go to Step 2. Else go to Step 4.

Step 4: Calculate closing inventory and shortage for period $j$

$$G_{1j} = \max(D_j - I_{1(j-1)}, 0)$$

$$G_{2j} = \max(O_{1j} - I_{2(j-1)}, 0)$$

$$G_{3j} = \max(O_{2j} - I_{3(j-1)}, 0)$$

$$I_{1j} = I_{1(j-1)} - (D_j - G_{1j}) + (O_{1j} - G_{2j})$$

$$I_{2j} = I_{2(j-1)} - (O_{1j} - G_{2j}) + (O_{2j} - G_{3j})$$

$$I_{3j} = I_{3(j-1)} - (O_{2j} - G_{3j}) + O_{3j}$$

Step 5: Using standard equations, calculate ordering, holding, and shortage costs for all $i \in \emptyset$ for period $j$

If $j < N$ then Set $j = j+1$ and go to Step 1

3.4 Target Inventory Level Policy

The retailer faces known demand distribution in this problem. However, the manufacturer and distributor face lumpy demand. As a first step, we find the demand profile faced by each supply chain member. Clearly, demand profile for the retailer is given, while that of distributor and manufacturer will depend on the policies used by the downstream decision makers. We make the assumption that all decision makers will use our policy to make decisions. Using this as the basis, we first derive demand profiles for the distributor corresponding to each period in demand cycle (demand cycle is $m$ periods long after which demand pattern repeats) by running retailer policy independently.
(assuming all the retailer orders will be satisfied by distributor). Similarly we generate demand profiles for the manufacturer by running distributor policy independently.

Description of the policy is as follows. In this process, it is assumed that the all the members use the well-known ordering policy of ordering up to Target Inventory Level (TIL). To determine TIL, first we calculate critical ratio which is the ratio of shortage cost by sum of holding and shortage costs. Considering the distribution of demand, TIL is that inventory level which will provide service level equal to critical ratio (Chopra and Meindl, 2007). Incorporating this concept into our policy, first initial order quantities are determined for the current and next period which is difference between expected closing inventory of period \(j\) and TIL of period \(j+1\). Now considering the ordering cost, various decision alternatives are considered. These include setting over quantity to 0, or requirements for next period or requirements for next two periods. For the distributor and manufacturer, policies are slightly different as they have to take into consideration that ordering cost is a step function of batch size. Hence three alternative decisions discussed for made only for partial batch sizes and full batches as required are ordered. Algorithm 8 given below describes the Target Inventory Level ordering policy.

**Algorithm 8:**

**Step 0:** Set current period \(j = 1\)

Set \(I_{10} = \text{Mean demand of Period 1} + \text{Mean demand of Period 2}\) for each supply chain members \(i \in \theta\)
For given the distribution of demand from immediate customer, calculate the target inventory \( T_{ik} \) \( i \in \emptyset \) and \( k \in \emptyset \) such that it is the minimum value satisfying the following inequality.

\[
CDF(T_{ik}) \geq \frac{S_i}{H_i + S_i}
\]

(Step 1 to 3 describe the logic for determining order quantity in period \( j \) for retailer)

**Step 1:** Given \( I_{i(j-1)} \) determine \( E_{ij} \) for period \( j \) using the parameters of appropriate demand distribution.

Similarly determine \( E_{i(j+1)} \) given projected opening inventory for period \( j+1 \) is equal to \( T_{i(j+1)} \).

**Step 2:** Determine preliminary order quantity for period \( j \) and \( j+1 \) using following equations

\[
Q_{1j} = T_{i(j+1)} - E_{1j}
\]

\[
Q_{i(j+1)} = T_{i(j+2)} - E_{1(j+1)}
\]

**Step 3:** Evaluate three ordering options for the retailer, namely order \( Q_{1j} \), or

\[Q_{1j} + Q_{i(j+1)}\] or 0 as follows. Calculate three quantities (\( Z_1, Z_2, Z_3 \)), namely \( Z_1 \) which is set to equal to the \( R_i \), \( Z_2 \) which is the expected incremental cost of holding and shortage in period \( j+1 \) for ordering \( Q_{1j} + Q_{i(j+1)} \) Vs. \( Q_{1j} \), and

\( Z_3 \) which is set equal to the expected incremental cost of holding and shortage in period \( j+1 \) for ordering 0 Vs \( Q_{1j} \). If \( Z_1 \) is the minimum, then order quantity
is \( O_{ij} = Q_{ij} \), if \( Z_2 \) is the minimum then \( O_{ij} = Q_{ij} + Q_{i(j+1)} \) and if \( Z_3 \) is the minimum, set \( O_{ij} = 0 \)

Set \( i = 2 \)

_S(step 4 to 6 describe the logic for determining order quantity in period j for distributor and manufacturer)_

**Step 4:** Determine closing inventory of current period \( j \) \( (E_{ij}) \) using following.

\[
E_{ij} = \text{Max}(I_{i(j-1)} - O_{(i-1)j}, 0)
\]

Determine preliminary order quantity \( (Q_{ij}) \) and corresponding partial batch quantity \( (q_{ij}) \) of period \( j \) assuming expected closing inventory of \( E_{ij} \)

\[
Q_{ij} = \text{Max}(T_{i(j+1)} - E_{ij}, 0)
\]

\[
q_{ij} = Q_{ij} \mod B_i
\]

**Step 5:** Determine \( E_{i(j+1)} \) given projected opening inventory of period \( j+1 \) is \( T_{i(j+1)} \) using the parameters of appropriate order distribution of supply chain member \( i-1 \).

Determine \( Q_{i(j+1)} \) which is equal to \( T_{i(j+2)} - E_{i(j+1)} \) and \( q_{i(j+1)} \) which is equal to \( Q_{i(j+1)} \mod B_i \)

**Step 6:** Evaluate ordering options for the given member \( i \). All full batch sizes for a given period given by \( Q_{ij} - q_{ij} \) will be ordered. The logic that follows determines how the partial batches represented by \( q_{ij} \) and \( q_{i(j+1)} \) should be treated. There are three options considered. Calculate three quantities \( (Z_i, Z_2, \) ...
& Z_3), namely Z_1 which is set to equal to the R_i, Z_2 which is the expected incremental cost of holding and shortage in period j+1 for ordering q_j + q_{i(j+1)} Vs. q_j, and Z_3 which is set equal to the expected incremental cost of holding and shortage in period j+1 for ordering 0 Vs q_j. If Z_1 is the minimum, then O_j = Q_j. If Z_2 is the minimum, then r O_j = Q_j + q_{i(j+1)} and if Z_3 is the minimum, then O_j = Q_j - q_j.

Step 7: If i=2 then set i=3 and go to Step 4. Else go to Step 8.

Step 8: Calculate closing inventory and shortage for period j using equations given in Step 4 of Algorithm 7.

Step 9: Using standard equations, calculate ordering, holding, and shortage costs for all i ∈ Θ for period j

If j < N then Set j= j+1 and go to Step 1

3.5 Complete Cooperation Policy

In this policy, supply chain members share all of the available information and place orders using policy that will minimize supply chain costs. Since there is a complete cooperation in the supply chain, real shortage for all members is the shortage faced by retailer and not the internal shortage between members. To operationalize this and make comparison among policies compatible, we assume that the shortage cost of retailer is sum of shortage costs of all the three members. Based on this shortage cost, we calculate the retailer target inventory levels for all periods in the demand cycle. Initial retailer order quantities are determined for the current and next three periods which is the difference between expected closing inventory of period j and TIL of period j+1. Based on the
opening inventory of period \( j \), we then calculate initial distributor order quantities for current and next two periods. Applying similar logic, manufacturer’s initial production quantities and corresponding partial batches for current and next period are determined. We assume that complete batches in the initial production quantity for the current period will be produced. In addition, production decision about various combinations of current and next period’s partial batches will be made considering tradeoffs between holding, ordering, and shortage costs. In each period, manufacture and distributor will ship (push) all the opening inventory to downstream supply chain members. Retailer will ship the quantity demanded by customer. We describe Complete Cooperation policy using Algorithm 9.

**Algorithm 9:**

**Step 0:** Set \( I_{i0} = \text{Mean demand of Period 1} + \text{Mean demand of Period 2} \) for all supply chain members \( i \in \Theta \)

For given the distribution of demand from immediate customer, calculate the retailer target inventory level \( T_{ik}, k \in \phi \) using following equation.

\[
CDF(T_{ik}) \geq \frac{S_1 + S_2 + S_3}{H_j + S_1 + S_2 + S_3}
\]

Set current period \( j = 1 \)

*(Step 1 and 2 describe the logic for determining preliminary order quantities for retailer)*

**Step 1:** Given \( I_{i(j-1)} \), determine \( E_{ij} \) for period \( j \) using the parameters of appropriate demand distribution.
Similarly determine $E_{(j+1)}$, $E_{(j+2)}$, and $E_{(j+3)}$, given projected opening inventory for periods $j+1$, $j+2$, and $j+3$ is equal to $T_{(j+1)}$, $T_{(j+2)}$, and $T_{(j+3)}$ respectively.

**Step 2:** Determine preliminary retailer order quantity for periods $j$, $j+1$, $j+2$, and $j+3$ using following equation

$$Q_{l} = T_{l(j+1)} - E_{l} , \ l \in j, (j+1), (j+2), \text{and} \ (j+3)$$

(Step 3 describes the logic for determining preliminary order quantities for distributor)

**Step 3:** Determine expected closing inventory and preliminary order quantity of distributor for period $j$ using following.

$$E_{2j} = \max (I_{2(j-1)} - Q_{2j}, 0)$$

$$Q_{2j} = \max (Q_{2(j+1)} - E_{2j}, 0)$$

Determine expected closing inventory and preliminary order quantity of distributor for periods $l$ ($l \in j+1, j+2$) using following.

$$E_{2l} = E_{2(l-1)} + Q_{2(l-1)} - Q_{l}$$

$$Q_{2l} = \max (Q_{2(l+1)} - E_{2l}, 0)$$

(Steps 4 to 6 describe the logic for determining Manufacturer order quantity for period $j$)

**Step 4:** Determine expected closing inventory and preliminary order quantity of Manufacturer for period $j$ using following.

$$E_{3j} = \max (I_{3(j-1)} - Q_{2j}, 0)$$
\[ Q_{3j} = \text{Max}(Q_{2(j+1)} - E_{3j}, 0) \]

Determine expected closing inventory and preliminary order quantity of Manufacturer for period \( j+1 \) using following.

\[ E_{3(j+1)} = E_{3j} + Q_{3j} - Q_{2(j+1)} \]

\[ Q_{2(j+1)} = \text{Max}(Q_{3(j+2)} - E_{3(j+1)}, 0) \]

**Step 5:** Compute partial batch sizes for period \( j \) and \( j+1 \) using following.

\[ q_{3l} = Q_{l} \text{Mod} B_{3}, \ l \in j, j+1 \]

**Step 6:** Evaluate ordering options for Manufacturer in period \( j \). All full batch sizes for a given period given by \( Q_{3j} - q_{3j} \) will be ordered. The logic that follows determines how the partial batches represented by \( q_{3j} \) and \( q_{3(j+1)} \) should be treated. There are three options considered. Calculate three quantities \( (Z_{1}, Z_{2}, \ & Z_{3}) \), namely \( Z_{1} \) which is set to equal to the \( R_{1} + R_{2} + R_{3} \) or \( R_{2} + R_{3} \) as appropriate, \( Z_{2} \) which is the expected incremental cost of holding and shortage in period \( j+1 \) for ordering \( q_{3j} \ + q_{3(j+1)} \) Vs. \( q_{3j} \), and \( Z_{3} \) which is set equal to the expected incremental cost of holding and shortage in period \( j+1 \) for ordering \( 0 \) Vs \( q_{3j} \). If \( Z_{1} \) is the minimum, then \( O_{3j} = Q_{3j} \). If \( Z_{2} \) is the minimum, then \( O_{3j} = Q_{3j} + q_{3(j+1)} \) and if \( Z_{3} \) is the minimum, then \( O_{3j} = Q_{3j} - q_{3j} \).

**Step 7:** Calculate closing inventory for all supply chain members and shortage for retailer during period \( j \) using equations.

\[ G_{1j} = \text{Max}(D_{j} - I_{1(j-1)}, 0) \]
\[ I_{1j} = I_{1j-1} - \left( D_j - G_{ij} \right) + I_{2(j-1)} \]

\[ I_{2j} = I_{3(j-1)} \]

\[ I_{3j} = O_{3,j} \]

**Step 8:** Using standard equations, calculate ordering, and holding costs for all \( i \in \emptyset \) for period \( j \)

Using standard equations, calculate shortage cost for retailer for period \( j \) by setting cost per unit short equal to \( S_1 + S_2 + S_3 \)

If \( j < N \) then Set \( j = j+1 \) and go to Step 1

### 3.6 Design of Experiments

We have generated a large set of problems considering various parameters and tested the performance of three ordering policies we had developed and presented in algorithm 7-9. We chose various parameters based on common understandings of the procedures and with the some input from industry. The demand distributions and costs are more general to explore various combinations of demand and cost structures. We consider demand cycle of four periods consisting of three regular periods followed by a period of promotion. Demand is assumed to follow \( \text{UN}(400, 600) \) during regular periods and \( \text{UN}(1200, 1800) \) during promotions. We selected the initial cost matrix with the holding, ordering, and shortage cost for the retailer equal to $1.30/unit/period, $600/order, and $10/unit short respectively. For the distributor, the holding, ordering, and shortage costs are $1.25/unit/period, $700/batch of 1000, and $4/unit short respectively. For the manufacturer the holding, ordering, and shortage costs are $1.00/unit/period, $1000/batch of 1000, and $10/unit short respectively.
We generated four more cost matrices by increasing shortage cost in proportions of -40%, -20%, 20%, and 40%. Thus total five cells are formed. In each cells, 30 replications are generated using uniformly distributed demand. Thus total 150 problems are solved using three policies and simulation is run for 200 periods. Results of our experiments are described in the next section.

3.7 Results

For the 150 generated problems, we compared the results obtained by using Moving Average Policy (MAP), Target Inventory Level Policy (TILP), and Complete Cooperation Policy (CCP). We developed two parameters to compare these policies. First, we compare percent difference in the various costs between TILP and MAP. This parameter is calculated as cost difference between two policies (MAP-TILP) divided by cost obtained by using MAP and the summary of results of 150 problems is presented in Table 3.7.1. Similarly, second parameter was developed to compare CCP and TILP. This parameter is calculated as cost difference between two policies (TILP - CCP) divided by cost obtained by using CCP and the summary statistics of the results for 150 problems is presented in Table 3.7.2. Since in the CCP a unit short at retailer is conceptualized as a unit short for all the three members, for comparison purposes only shortage at retailer location is compared. Also for comparing total costs, shortage costs of distributor and manufacturer is excluded from the total cost.
Table 3.7.1 Percentage Cost Difference between MAP and TILP

<table>
<thead>
<tr>
<th>Retailer</th>
<th>Distributor</th>
<th>Manufacturer</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg.</td>
<td>25.53%</td>
<td>-5.95%</td>
<td>10.36%</td>
</tr>
<tr>
<td>St Dev</td>
<td>0.16%</td>
<td>1.66%</td>
<td>23.22%</td>
</tr>
<tr>
<td>Min</td>
<td>25.13%</td>
<td>-9.93%</td>
<td>-73.45%</td>
</tr>
<tr>
<td>Max</td>
<td>26.00%</td>
<td>-1.81%</td>
<td>48.00%</td>
</tr>
</tbody>
</table>

Table 3.7.2 Percentage Cost Difference between TILP and CCP

<table>
<thead>
<tr>
<th>Retailer</th>
<th>Distributor</th>
<th>Manufacturer</th>
<th>Total Cost*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg.</td>
<td>45.85%</td>
<td>-10.94%</td>
<td>10.82%</td>
</tr>
<tr>
<td>St Dev</td>
<td>2.43%</td>
<td>1.27%</td>
<td>21.61%</td>
</tr>
<tr>
<td>Min</td>
<td>36.70%</td>
<td>-14.36%</td>
<td>-20.68%</td>
</tr>
<tr>
<td>Max</td>
<td>47.52%</td>
<td>-8.45%</td>
<td>84.13%</td>
</tr>
</tbody>
</table>

*Distributor and manufacturer shortage costs are excluded from Total cost.

The results presented in Tables 3.7.1 and 3.7.2 shows that on average total supply chain cost is 16.24% lower for TILP than MAP and 4.30% lower for CCP than TILP. We also conducted pairwise t-test to analyze if there is difference between the total cost obtained by MAP and TILP. The difference between two policies is significant (p value 0.000) showing that total cost obtained using TILP is lower than MAP. Similar t-test between TILP and CCP is also significant (p value 0.000) showing that total cost obtained using CCP is lower than TILP. Thus results reinforce that cooperation results in lower supply chain costs. Also it is important to note that major cost savings can be achieved by using more sophisticated policies than simple policies like moving average even if cooperation may not be possible.
To analyze if there is any difference in cost differences of TILP and CCP across five cells, we conducted one-way ANOVA. Results of ANOVA test are significant ($p$ value 0.000) implying cost differences for different values shortage costs are different. We further analyze if these differences are increasing or decreasing with increasing values of shortage cost by comparing average differences between the cells. We found that no such relationship exists. Similar ANOVA test to analyze any difference in cost differences of MAP and TILP across five cells was insignificant at 5% significance level ($p$ value 0.062).
CHAPTER 4: TEACHING INVENTORY MANAGEMENT IN A SUPPLY CHAIN USING “COLA-GAME”

4.1 Introduction to Cola-Game

One of the major challenges of the decade is the expected scarcity of trained supply chain managers. Supply chain managers require a variety of skills such as planning, analysis and modeling to perform their jobs effectively. To meet this requirement, substantial changes in supply chain education are necessary (Gammelgaard and Larson, 2001) which reiterates the need for innovative techniques to impart such education.

One of the main challenges in managing a supply chain is the formulation of an ordering policy throughout the supply chain. This gets accentuated in a retail supply chain with frequent price based promotions. Typically, these are price sensitive products with a high mean demand during the promotion period, returning to its normal level of demand at other times. A popular example of such a product is soda-pop, hence the name Cola-Game. Such a demand pattern is denoted here as bi-level demand. We found that this pattern produces a substantial inventory management challenge for materials managers with almost no optimal solution techniques available in the literature.

In supply chain management education, a variety of teaching methods including lecture, game playing, computer simulation, site visits, and role playing have been
employed (Mangan and Christopher, 2005). A well-known example of game playing is the Beer Game. It requires the student to determine the quantity to order in each period, and is used to introduce students to the challenges of managing supply chains. While it effectively demonstrates the importance of information sharing, it fails to provide an opportunity for the students to formulate a supply chain strategy through information sharing which would address the management challenges posed by the supply chain (Sparling, 2002). These challenges become magnified in an environment characterized by periodic changes in demand level, which is typical of real-life scenarios. In the Cola-Game, we model this using bi-level (high and low) probabilistic demand. Players know the distribution of customer demand at the retail level during each period. However, the actual demand is revealed following their ordering decisions for the period.

We have used this game in a variety of classes with students from different backgrounds, academic preparation, and experience. The academic background varied from executive M.B.A. students to operations management specialists to advanced industrial engineering majors. There were also differences in the level of preparation prior to the actual game playing. We also employed two modes of playing: Complete Cooperation (information sharing) and Total Independence (no information sharing). In general, we had positive feedback from the students about the game and their learning experience. It was found that the performance of the teams improved as they gained more experience in playing this game. As one would expect, the role of information sharing was found to be critical to supply chain performance.

We also present an integer linear program (ILP) that can be used to find the lowest cost solution (set of orders) for a given demand set for the supply chain considered
in this game. Performance achieved by teams can be measured with respect to the solution obtained by using this model. Since the ILP model assumes that the exact demand set is known in advance, it is a lower bound. We also provide the parameters of the distribution of the lower bound solution. This is based on the lower bound solutions obtained for 1800 randomly generated demand sets. This distribution can be used to evaluate the performance of the teams without actually using the ILP.

In the next section, we describe the Cola-Game and the salient decisions in each period. In the following section the details for effectively conducting the Cola-Game in a classroom environment is described. Our experience with conducting the game is described in the next section. The distribution of the lower bound for used cost parameters is described after this. The results of the experiments as well as an analysis of the performance of the teams are presented next. We discuss self-reported strategies followed by different teams in the following section. In the last subsection of this chapter, we make concluding remarks and opportunities for future research.

4.2 Description of the game

Based on the supply chain model described in Chapter 3, we develop Cola-Game. Keeping demand pattern, cost structure, and other parameters same, we studied how student teams managed the challenges in supply chains and how playing this game helped them in learning the principles of supply chain optimization. Here we briefly describe the supply chain members, demand structures, costs, and other game parameters.
4.2.1 Supply Chain Members

We present the details of the Cola-Game which consists of three members – the manufacturer, distributor, and retailer as shown in Figure 4.2.1.1. Although consumers are an integral part of any supply chain (SC), only the upstream members of an SC deal with ordering and inventory issues.

![Figure 4.2.1.1 Cola-Game Supply Chain](image)

4.2.2 Demand

The focus of our current study is on consumer goods supply chains with a bi-level demand pattern. At the retailer end of such supply chains, items are placed on a sales promotion program in a cyclical manner. These are assumed to be price sensitive products with a high mean demand during the promotion period, returning to its normal level of demand at other times.

To operationalize the bi-level demand in this game, we assume that there are three periods without promotions followed by one period of promotion. Demand follows the Uniform Distribution for all periods. Demand is between 400 and 600 during the non-promotion periods and is between 1200 and 1800 during the promotion period. This cycle of four periods is repeated.
4.2.3 Costs

In the context of game playing, the actual value of the costs are less important than the fact that such costs exist and need to be considered in decision making. Clearly, ordering or manufacturing decisions should be made by examining the trade-offs among ordering, holding and shortage cost.

There are three types of costs—ordering/setup, holding, and shortage cost—faced by supply chain members. The complete cost parameters are provided in Table 4.2.3.1. The exact values of costs are not critical, and the game user may use some other combination of costs.

<table>
<thead>
<tr>
<th>Table 4.2.3.1 Supply Chain Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holding Cost ($ per unit per time)</td>
</tr>
<tr>
<td>----------------------------------</td>
</tr>
<tr>
<td>Retailer</td>
</tr>
<tr>
<td>Distributor</td>
</tr>
<tr>
<td>Manufacturer</td>
</tr>
</tbody>
</table>

* Retailer has no limit on the batch size

4.2.4 Overview of Decisions

Each cycle of the game covers one period of activities. At the beginning of the period, each supply chain member must decide whether and how much to order or produce. It may be noted that the retailer, while placing the current period’s order has to plan for an unknown demand for the current as well as the next period. However, the distributor and the manufacturer have the luxury of making their decisions after knowing the demand of their immediate customer for the current period, though the uncertainty of the next period’s demand remains. Thus, the timing of the decision becomes important.
because of its effect on available information prior to decision making. The timing is captured by the sequence of events described in Table 3.2.1.

4.2.5 Information and Shipping Lead Time

Once the orders are placed, they are received immediately by the upstream member and hence information lead time is zero. Shipments, if any, are made from opening stock for the period and are received by the downstream member at the end of the period. Therefore units ordered in any period are available for satisfying demand during the following period.

4.2.6 Modes of Play for the Cola-Game

The Cola-Game may be conducted in two modes: Complete Cooperation (CC) and Total Independence (TI). In CC mode, the objective is to minimize total supply chain cost. Supply chain members share all information on actual demand, order quantities and inventory levels. In the TI mode, the objective for each player is to minimize his/her own cost and the only information known to all members is the demand distribution. In addition, each member receives demand information only from their immediate customer.

4.3 Conducting the Cola-Game

4.3.1 Components of the Cola-Game

The Cola-Game has two components: the game instrument and the Demand and Order cards. The game instrument consists of brief description of the game, policies and
decisions to be made as well as data entry forms for each actor. The description of the game outlines of structure of the supply chain, the demand pattern, and the objective of the game. The policy and decision part describes the sequence in which the decisions are to be made, the associated costs, and the formulas to be used to compute their total costs. The data entry pages provide a convenient format for collecting game data and computing costs.

Another component of the game is the Demand and Order cards. The Demand Card is defined only for the retailer. It specifies Period #, Team #, and the simulated demand. There are two Order Cards, one for the retailer and the other for the distributor. They specify Team # and Period #. The downstream member (retailer or distributor) enters the quantity they wish to order and passes it on to the upstream member (distributor or manufacturer). The upstream member then enters the quantity that can be shipped and returns it to the downstream member.

4.3.2 Student Preparation for the Cola-Game

The game instrument should be distributed at least one class prior to the actual playing of the game. Students should be divided into teams of 3 and each team should have designated the manufacturer, distributor and retailer. The mode of play (CC or TI) should be chosen or assigned to each team as well as the length of the game run should be decided. They should be encouraged to think about strategy for their roles, practice cost calculations, get familiar with the event sequence such as order placement, receipt of goods, ending inventory and timing. Students may even complete some trial rounds.
4.3.3 Playing the Cola-Game

At the beginning of the game, opening inventory at each location in the supply chain is 1000 units which is twice the mean demand for Period 1. This is in line with similar assumptions made in the context of game playing (Sparling, 2002). The chronological sequence of activities that lead to completion of all events connected in a period is given in Table 3.2.1. This sequence is repeated for the desired number of periods to complete the Cola-Game. At the end of Cola-Game all the supply chain members are expected to have a closing inventory 1000 units and any shortfall is penalized as shortage. Refer to Appendix D for a sample set of demands, decisions made by one student group and a corresponding ILP solution value.

4.3.4 Game Playing Strategy Report

At the end of the game, teams in CC mode should be asked to submit a common report (300-400 words) describing their overall strategy along with the cost calculations. When the game is played in TI mode, each individual player should be asked to submit his/her own strategy. These reports can form the basis of class discussions. We used a standard format to capture the strategy. This helps us to compare strategies across teams in a formal fashion. A copy of our survey is in Appendix B and our analysis of the collected data appears in the Discussion section.
4.4 Summary of our Classroom Experience

4.4.1 Overview

We conducted six different runs of the Cola-Game in engineering, business, and executive MBA classes at a mid-western State University. In one of the classes (OM-669), the game was played in two modes: at the beginning of the semester under TI and at the end of the semester with CC. In the remaining classes the game was played only in the CC mode. We generated five sets of simulated demand. Table 4.4.1.1 lists the Course Name, Number of Supply Chain Teams and the Demand Set used. In each case, the game was played for 12 periods.

<table>
<thead>
<tr>
<th>Course Number</th>
<th>Course Name</th>
<th>Number of Teams</th>
<th>Demand Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>IE-551</td>
<td>Production Planning and Control</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>OM-552</td>
<td>Analysis of Manufacturing and Services</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>BU-660</td>
<td>Supply Chain Management</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>OM-669</td>
<td>Manufacturing Resource Management</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>MBA-631</td>
<td>Managing Global Supply Chains</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

4.4.2 Student Preparation and Prior Knowledge

The guidelines provided in earlier sections were followed in all the classes. However, in BU-660 class, students were prepared more rigorously for the game. Specifically, students practiced mock rounds under supervision a week before the actual
game. This also gave the students a chance to get their cost and inventory calculations verified by the instructor. These students have had exposure to inventory models in general, but not specifically tailored material for game playing. All the groups in this class were classified as “well-prepared” for the purpose of the analysis reported in the results section. A lot of emphasis was placed on the game and strategy report, which comprised of nearly 10% of the final grade. In other classes, students were encouraged to practice mock rounds and check their calculations prior to the actual game playing session, but it was not made mandatory. Decision to prepare these students somewhat extensively was made on the basis of instructor interest and class time availability. All students had some exposure to inventory management principles such as continuous and periodic review as well as EOQ. They were also familiar with the general concepts of supply chain management. They were not specifically prepared for the game except the BU660 students described earlier.

4.5 Lower Bounds on the Solution of Cola-Game Problem

For a stochastic problem, the lower bound can be interpreted as a bound on the average solution. In the context of the Cola-Game, the retailer’s demand is revealed only after all the ordering decisions for the period have been made. This, combined with the distributor and manufacturer decisions, the supply chain cost minimization problem becomes complicated. Therefore, we develop an integer programming approach which can be used as a guide in evaluating the solutions produced by the players. It may also be used as a loose target by the players for self correction. The details of the integer programming model are provided in Appendix C.
4.5.1 Characteristics of the Lower Bound Solution

Using ILP model to obtain the lower bound for a randomly generated demand set can become tedious and cumbersome. To provide a frame of reference for evaluation and to provide feedback to the teams, we found the solutions of 1800 randomly generated problem instances using the costs parameters described previously. The average total cost was found to be $65,153.77 with a standard deviation of $2,024.78. Using the Kolmogov-Smirnov test, we tested the null hypothesis that the total cost for the supply chain is normally distributed and found the $p$-value of 0.37. Similarly, using Kolmogov-Smirnov test, the retailer’s cost was found to follow the Normal distribution ($p$-value 0.34) with mean $26,137.82 and standard deviation $980.47. For the distributor, the mean and standard deviation of cost was found to be $19,851.37 and $796.53 and for the manufacturer the mean and standard deviation were found to be $19,164.56 and $1,093.48125. The distributor and manufacturer cost do not seem to follow any well known distribution. For problems with different cost structures, we recommend using the model in Appendix C to generate such costs for evaluation and feedback purposes.

4.6 Results

4.6.1 Supply Chain Team Performance: Complete Cooperation Mode

In order to give the readers an idea of the data and a typical decisions made by student teams, we present in Appendix D, a sample set of demand data, a typical set of team decisions, and the corresponding integer linear programming solution along with costs. For each supply chain team, the total cost is compared to the lower bound and is
reported in Table 4.6.1.1 as Percentage Above the Lower Bound (PALB). The supply chain games were conducted in five graduate classes from three different programs—Engineering, Business and Executive MBA. The students differed in educational background, prior relevant experience, and exposure to problem solving techniques. First, we use one factor ANOVA to investigate whether performance among classes differ. (Note that BU660/3 and BU660/4 are the same class). We failed to reject the null hypothesis (p-value 0.552) that there is no difference in the performance of the teams from different classes. Similarly, a t-test was conducted to determine if performance of Business students differed from that of Engineering students (IE-551). We again failed to reject the null hypothesis (p-value 0.525) and conclude that the mean performance of the two groups is not different. It may be noted that the results of one class (OM-669) is omitted from this analysis because they played the game twice using two different modes.

Table 4.6.1.1 Supply Chain Team Performance with Cooperation

<table>
<thead>
<tr>
<th>Course No./Demand Set</th>
<th>Percent Above Lower Bound for SC Teams (PALB)</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>IE-551 / 1</td>
<td>24.92, 30.79, 36.15, 43.10, 49.26, 63.44, &amp; 72.37</td>
<td>45.72</td>
<td>17.28</td>
</tr>
<tr>
<td>OM-552 / 2</td>
<td>29.68, 37.99, 38.48, 38.97, 45.66, 50.02, &amp; 72.89</td>
<td>44.82</td>
<td>13.94</td>
</tr>
<tr>
<td>BU-660 / 3</td>
<td>24.62, 30.78, 31.04, 45.74, 47.43, &amp; 87.64</td>
<td>44.54</td>
<td>22.96</td>
</tr>
<tr>
<td>BU-660 / 4</td>
<td>13.67, 21.20, 21.64, 25.56, 28.00, &amp; 54.75</td>
<td>27.47</td>
<td>14.23</td>
</tr>
<tr>
<td>BU-660 / 3 &amp; 4</td>
<td>13.67, 21.20, 21.64, 24.62, 25.56, 28.00, 30.78, 31.04, 45.74, 47.43, 54.75, &amp; 87.64</td>
<td>36.00</td>
<td>20.28</td>
</tr>
<tr>
<td>MBA-631 / 5</td>
<td>30.59, 30.85, 59.46, &amp; 70.24</td>
<td>47.49</td>
<td>20.19</td>
</tr>
</tbody>
</table>

*SC teams are arranged in increasing order of PALB

Next, we wanted to determine if the demand set had an effect on the performance of a group. We would not expect to reject the null hypothesis because the specific demand came from the same probability distribution. We failed to reject the null
hypothesis \( (p\text{-value} \ 0.308) \) and conclude that the demand set has no impact on group performance.

Finally, we wanted to examine the effect of preparation on the performance of the teams. As described in the sub-section Student Preparation under the section on Conducting the Cola-game, all groups in BU-660 were classified as “well-prepared”. The rest were classified as “others”. We conducted a t-test and found that the mean performance of BU-660 (well-prepared) is superior \( (p\text{-value} \ 0.073) \) compared to the rest.

4.6.2 Learning Effect on Performance

It is well documented that students or workers learn from doing. Here the game was played for 12 periods, involving a series of decisions in each period. We wondered whether the students exploited the learning opportunity provided by the initial periods to improve their performance in later periods. The twelve period horizon was divided into three parts – first four periods, middle four periods, and last four periods. As before, total cost for each partial period was divided by the lower bound obtained from the integer linear program and reported as a PALB. Based on this measure, the performance of the 30 student teams from the four classes (IE-551, OM-552, BU-660, & MBA-631) is presented in the first five columns of Table 4.6.2.1. A paired t-test with the null hypothesis that there is no improvement from first four periods to second four periods was rejected \( (p\text{-value} \ 0.007) \). A similar test for the second four periods to last four periods was also rejected \( (p\text{-value} \ 0.000) \). Of the 30 teams, 18 teams exhibited consistent improvement in performance. The twelve teams that did not show consistent improvement are highlighted. Of these, three teams showed improvement from the first
four periods to the second four periods, but failed to further improve. Eight of the teams showed improvement only from the second four periods to the last four periods. Only one team progressively got worse.

Table 4.6.2.1 Effect of Learning

<table>
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<tr>
<th>Class</th>
<th>Team #</th>
<th>First 4 Periods</th>
<th>Middle 4 Periods</th>
<th>Last 4 Periods</th>
<th>First 8 Periods</th>
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</tr>
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<td>32.97</td>
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<td>22.82</td>
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<td>66.87</td>
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<td>40.81</td>
<td>11.41</td>
<td>42.83</td>
<td>11.41</td>
</tr>
</tbody>
</table>

*Total cost is expressed as percentage above lower bound solution for respective periods.
It is well-known that learning rates may differ in different team settings. Next, we combined the performance in the first 8 weeks and compared that to the last four weeks. The result of the paired t-test for this comparison is statistically significant ($p$-value 0.000). These results are shown in the last two columns of Table 4.6.2.1. Of the 30 teams, 28 teams performed better in last four periods.

Therefore, almost all teams improved their performance while playing the game. Some teams showed improvements after four periods. While others took a longer time to improve their performance.

We found that 11 of the 12 teams in the BU-660 class demonstrated consistent improvement in performance. Whereas only 7 out of 18 teams in other three classes made consistent performance improvements. This may be because the students in BU-660 were trained more extensively for the game as compared to the students in other classes. This indicates that the game will be more effective if the students are better prepared.

4.6.3 Impact of Complete Cooperation

In the OM-669 class, the Cola-Game was conducted twice. First, the teams played in the TI mode where they attempted to minimize their individual costs without sharing information. After several class periods the same teams were assembled and the same roles were assigned. They played the game in CC mode. They were asked to formulate a common strategy that will reduce total supply chain cost. During both rounds of the game the same demand set was used. Table 4.6.3.1 presents the percent change in performance from the TI mode to the CC mode.
Table 4.6.3.1 Percent Change in Cost due to Information Sharing

<table>
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<tr>
<th>Team #</th>
<th>Retailer</th>
<th>Distributor</th>
<th>Manufacturer</th>
<th>Total SC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-13.29</td>
<td>-25.02</td>
<td>31.75</td>
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<td>29.99</td>
<td>23.32</td>
<td>19.21</td>
</tr>
</tbody>
</table>

* Negative numbers indicate an increase in cost

From the results, three out of four teams made improvements in total SC cost. Most of the improvement is due to gains made by the manufacturer of these teams. The manufacturer’s cost is reduced by as much as 31.75%. This is probably because the impact of the bullwhip effect is felt most severely at the end of supply chain. In the absence of information sharing, a slight variation in demand is amplified in upstream direction in supply chain. Thus cooperation has provided opportunities for the manufacturer to make the most improvement in cost. Our results suggest that the cooperation in supply chain may significantly improve performance.

We also observed that in the CC mode, individual costs of some of the supply chain members increased. Only Team 4 has shown improvement in the cost of all members of the supply chain. This emphasizes the practical difficulties in implementing information sharing as benefits achieved are uneven and that some of members may face cost increases. The above table is helpful in demonstrating this concept and challenges involved in cooperation.

Team 3 was the only team to have an increase in total SC cost when it moved to CC mode. To understand this, we analyzed their decisions and found two possible explanations. First, in their performance under TI (43.16 PALB) is about the same as the
average performance of the 30 teams in CC mode (41.90 PALB). This left little room for improvement. Second, during the CC mode, they forgot about the high demand during period 12, and suffered high shortage costs. Hence, we can conclude that this may be due to their exceptional performance while playing in a TI mode. Therefore, we should be able to use the game to impress upon the students the importance of thinking in terms of supply chains rather than stand alone units.

4.7 Discussion

We observed that students made use of various concepts learned in the class. Playing this game helped them in understanding and formulating their supply chain strategies. After analyzing the strategy reports submitted by students we identified some underlying policies. We present these policies in Table 4.7.1. We found that the teams in BU-660 were able to state their strategy most clearly, perhaps because of the training they received. It seems that to obtain better information about game strategy, we must either impart extensive training or use a survey such as the one presented in Appendix B.

The most popular approach was to establish some sort of target inventory level. To do this, some students used marginal cost analysis, while others used a more conservative approach by ordering enough to ensure that a stock-out does not occur. Some teams used judgment to determine order quantity that will maintain high inventory level to avoid stock-outs. They did not seem to have any analytical explanation. Another approach used an Economic Order Quantity type of formulation so that the order quantity was standardized for the two types of demand. In some teams, the distributors as well as manufacturers ordered or produced in multiples of the given batch size (1000) in an
attempt to minimize the ordering or setup cost per unit. Even though it was not necessary, in one team the retailer ordered in multiples of the batch size (1000) to standardize the order size through the supply chain. While all the teams emphasized the importance of communication, not all of them have effectively used the information in formulating policies. For example, only some of the retailers and distributors made sure that their immediate supplier has enough inventories before placing orders. Such cooperation would have avoided shortage costs for the supply chain. Some teams combined the orders for two or more periods considering the tradeoff between ordering and holding cost. In one team, the distributor and the manufacturer ordered or produced exactly the same quantity as retailer’s order.

We used these strategy statements to guide the post-game discussion in class. Students were able to understand what works, and what does not. The availability of the strategy documents allowed for easy sharing of the information among the students and resulted in a rich discussion. Based upon the strategy statements from this class, we developed a survey to capture the approach used by future participants. The survey also allows participants to describe a strategy that does not fit the given options.
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<th>Team 3</th>
<th>Team 4</th>
<th>Team 5</th>
<th>Team 6</th>
<th>Team 7</th>
<th>Team 8</th>
<th>Team 9</th>
<th>Team 10</th>
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<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Maximum Demand for the Period</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
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</tr>
<tr>
<td>Produce by Judgment</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
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<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Produce Multiples of Given Batch Size</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
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<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Produce Same Quantity as Retailer’s Order</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
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<td>✔</td>
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</tr>
</tbody>
</table>
CHAPTER 5: CONCLUSION & FUTURE RESEARCH

In this research, we have developed optimal solution to RBDP. This demand pattern is prevalent in retail industries and hence the models would be very useful. Solutions to this class of problems will provide very useful insights and perhaps guidelines to the retailers, who offer sales promotion on products in a cyclic manner. We have also provided heuristics that can be used in cases when the optimal solution is not known. From our results, we found that performance of the heuristics to be very good and near optimal.

But the above model has some limitations which may restrict its practical implementation. We have considered only two costs, holding and ordering cost, which are relevant in deterministic demand case. However, when demand is uncertain, service level becomes an issue and shortage costs must be considered to account for current as well as future losses in sales. Generally the shortage cost is many times higher than holding cost. Thus it prompts retailer in two ways. First, to hold more units as holding cost is lower than shortage. Second, to order more frequently as demand in the near future can be predicted more accurately than distant future.

We have modeled a generalized version of the supply chain consisting of retailer, distributor, and manufacturer where retailer faces stochastic bi-level demand. This type of demand pattern is caused by the factors such as promotional policies which are widely
observed in consumer products such as beverages. For the supply chains operating in such an environment, we have developed three ordering policies. First one is a simple policy called Moving Average Policy (MAP), second one is more sophisticated policy called Target Inventory Level Policy (TILP), and last one is idealistic, denoted as Complete Cooperation Policy. These policies are designed based on approaches (simple, sophisticated, & cooperation) practiced in real life supply chains. Results reinforce that cooperation results in lower supply chain costs. Also it is important to note that major cost savings can be achieved by using more sophisticated policies than simple policies like moving average even if cooperation may not be prevalent. This type of demand pattern is very common and thus policies and implications from results will be very useful for decision making in supply chains. In situations where individual member is unable to influence the complete supply chain, sophisticated policies such as TILP can be useful to achieve individual optimization.

Finally, Cola-Game developed here has great potential as a pedagogical tool for teaching efficient management of supply chains. It can be played in two modes: with Complete Cooperation or Total Independence. Studying the difference in the performance of the teams in the two modes helps reiterate the advantages as well as the challenges in eliciting cooperation even in simulated situations. The game helps the students think in terms of long term strategy and problem solving and discourages short-term thinking.

We also have developed methods for comparing team performance, both relative to other teams as well as in an absolute sense. To compare relative performance, we have developed the PALB measure. Using this we can compare the performance of a specific
team with that of another team even if they have different demand sets. At an absolute level, we can compare the cost performance of a specific team with the distribution of the optimal costs. Rather than using the lower bound solution for the simulated demand set, the mean solution presented here can also be used. This makes it somewhat easier to evaluate the supply chain’s performance.

We have also shown that the game presents an evidence for a significant learning effect. The players demonstrated significant improvement in performance in the second and third trimester of the game. A formal capture of the mechanics and details of this learning will help them to transport this ability to other situations. The game could also be used as a motivational tool for illustrating theoretical models of inventory management which uses the tradeoff between ordering cost, shortage cost and holding cost.

The game has been played using players from varying backgrounds like engineering, business and executive education. In all settings, it has been successful and results have reinforced our findings. Hence, it seems to be a robust teaching tool.

Though the game has several benefits, it has a few drawbacks. First, the availability of time can hinder the applicability of this game. For conducting the complete game it takes around 3 class periods of approximately 75 minutes each. It includes one class period for explaining the game and conducting trial rounds. One class period is required for playing the game followed by a class period for discussions which may also be used for reviewing relevant inventory models. From our experience and results the game will be most effective if thorough training is imparted. Secondly, the cost and inventory calculations involved are tedious. This has resulted in some of the teams
requiring extra time to complete the calculations for each period. We also found incorrect
calculations. We overcame this problem by providing a laptop to each team with an excel
worksheet calculator developed for the Cola-Game. Finally, this game is an obvious
simplification of real supply chains so it may not give players a feel for the more complex
and networked real life supply chains.

The game has many potential extensions which might spawn the interest of other
researchers. The demand distribution used here is UN(400,600) for all periods where n
mod 4 equal to 1,2, or 3 and it is UN(1200,1800) for periods with n mod 4 equaling 0,
where n is the number of the period. One could use other distributions and/or parameters.
A motivation for such an extension would be to accommodate products with different
price elasticities. This would also lend some flexibility to represent many types of
demand in the Cola-Game. Similar to the extension of the beer game by Hieber and
Hartel (2003), one could dictate the strategies for the manufacturer and distributor and
allow the retailer to develop a strategy. This would be useful if the game is primarily
conducted for training retailers or for evaluating retailer strategy against a backdrop of
fixed distributor and wholesaler strategies.
REFERENCES


Forrester, J. (1958), "Industrial dynamics, a major breakthrough for decision makers,"

Forrester, J. (1961), Industrial Dynamics, MIT Press (and John Wiley), Cambridge, MA


Teng, Jinn-Tsair; Chern, Maw-Sheng; Chan, & Ya-Lan (2005), “Deterministic Inventory Lot-Size Models with Shortages for Fluctuating Demand and Unit Purchase Cost,” International Transactions in Operational Research, 2005, 12(1), 83-100.
**APPENDIX A: BRIDGE ORDER DERIVATIONS**

**A1 Heuristic 1—Motivation and Derivation to compute Bridge Order**

This step is motivated by myopic minimization of total cost per unit time during bridge order. Let us assume that the order cycle length of bridge order is given by 

\[ L = x + y \]  

(See Figure 2.6.3.1) where \( x \) is partial cycle length in demand stream \( d_i \) and \( y \) is partial cycle length in demand stream \( d_j \). The total relevant cost of solution \( S \) containing this bridge order is

\[
TC = \left( \frac{1}{2} x^2 d_i + xy d_j + \frac{1}{2} y^2 d_j \right) H + A
\]

(67)

The total relevant cost per unit time of this solution is given by

\[
TCT(s) = \frac{\left( \frac{1}{2} x^2 d_i + xy d_j + \frac{1}{2} y^2 d_j \right) H + A}{x + y}
\]

The value of \( x \) is known and we want to determine value of \( y \) that will minimize total cost per unit time which can be easily computed if \( TCT(y) \) is convex. Taking first derivative yields

\[
\frac{dTCT(s)}{dy} = \frac{\left( 2xd_jHy + Hd_jy^2 - x^2(d_i - 2d_j)H - 2A \right)}{2(x + y)^2}
\]
Second derivative of $TCT$ is

$$
\frac{d^2 TCT(s)}{dy^2} = \frac{x^2 (d_i - d_j)H + 2A}{(x + y)^3}
$$

Thus $TCT$ is convex function if $x^2 (d_i - d_j)H + 2A > 0$. The value of $y$ at which total cost per unit time is minimum can be obtained by setting the value of first derivative to 0.

$$
\left(2xd_jHy + Hd_jy^2 - x^2 (d_i - 2d_j)H - 2A \right) = 0
$$

Solving the above equation for $y$ yields

$$
y = -x \pm \sqrt{\frac{x^2 (d_i - d_j)H + 2A}{d_jH}}
$$

Discarding negative root

$$
y = -x + \sqrt{\frac{x^2 (d_i - d_j)H + 2A}{d_jH}}
$$

If both the roots of $y$ given in are negative then the value of $y$ is set to 0. Also in the case where $x^2 (d_i - d_j)H + 2A < 0$, the total cost per unit time will increases for any positive value of $y$ and hence $y$ is set to 0. Order cycle length of bridge order can be computed by summing the values of $x$ and $y$. 

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In the cases where \( d_i = d_j \), indicating demand is constant, the length of bridge order \((x + y)\) obtained from above formula equals \( \sqrt{2A/d_H} \). This value is same as time between two consecutive orders given EOQ formula which is optimal solution for cases \( d_i = d_j \).

### A2: Heuristic 2—Motivation and Derivation to compute Bridge Order

This step is motivated by myopic minimization of total cost per unit during bridge order. Let us assume that the order cycle length of bridge order is given by \( L = x + y \) (See Figure 2.6.3.1) where \( x \) is partial cycle length in demand stream \( d_i \) and \( y \) is partial cycle length in demand stream \( d_j \). The total relevant cost of solution \( S \) containing this bridge order is given by \( \left( \frac{1}{2} x^2 d_i + xy d_j + \frac{1}{2} y^2 d_j \right) H + A \). The total relevant cost per unit of this solution is given by

\[
TCU(s) = \frac{\left( \frac{1}{2} x^2 d_i + xy d_j + \frac{1}{2} y^2 d_j \right) H + A}{xd_i + yd_j}
\]

The value of \( x \) is known and we want to determine value of \( y \) that will minimize total cost per unit which can be easily computed if \( TCU(y) \) is convex. Taking first derivative yields

\[
\frac{dTCT(s)}{dy} = \frac{d_j \left( 2xd_j H + d_j Hy^2 + x^2 d_i H - 2A \right)}{2(xd_i + yd_j)^2}
\]

Second derivative of TCT is
Thus TCU is convex function if \( x^2 \left( \frac{d_i}{d_j} \right) (d_i - d_j) H + 2A > 0 \). The value of \( y \) at which total cost per unit time is minimum can be obtained by setting the value of first derivative to 0.

\[
\frac{d^2TCU(x)}{dy^2} = \left( d_j \right) \left( x^2 \left( \frac{d_i}{d_j} \right) (d_i - d_j) H + 2A \right) \over (xd_i + yd_j)^3
\]

Solving the above equation for \( y \) yields

\[
y = -x \left( \frac{d_i}{d_j} \right) \pm \sqrt{\frac{x^2 \left( \frac{d_i}{d_j} \right) (d_i - d_j) H + 2A}{d_j H}}
\]

Discarding negative root

\[
y = -x \left( \frac{d_i}{d_j} \right) \pm \sqrt{\frac{x^2 \left( \frac{d_i}{d_j} \right) (d_i - d_j) H + 2A}{d_j H}}
\]

If both the roots of \( y \) are negative then the value of \( y \) is set to 0. Also in the case where \( x^2 \left( \frac{d_i}{d_j} \right) (d_i - d_j) H + 2A < 0 \), the total cost per unit will increases for any positive value of \( y \) and hence \( y \) is set to 0. Order cycle length of bridge order can be computed by summing the values of \( x \) and \( y \).
Please write your name, supply chain team number and role you are playing the game.

Name: ________________________

Team #: __________________

Role: ____________________

Based upon the strategy you have used for playing the game please check the corresponding box if you answer yes to the following question/s.

<table>
<thead>
<tr>
<th>Index</th>
<th>Respondents</th>
<th>Statements</th>
<th>Check if yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>Are your ordering/production decisions based on maintaining Target Inventory Level (TIL)? If yes explain below how you determine TIL.</td>
<td>☐</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>Are your ordering/production decisions based on Economic Order/Production Quantity (EOQ/EPQ)? If yes explain below how you determine this quantity.</td>
<td>☐</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>Have you leveled your order/production quantities? If yes explain below how you determine this quantity.</td>
<td>☐</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
<td>Have you combined orders for two or more periods?</td>
<td>☐</td>
</tr>
<tr>
<td>5</td>
<td>R</td>
<td>Have you always ordered in the multiples of complete batch size of distributor/manufacturer?</td>
<td>☐</td>
</tr>
<tr>
<td>6</td>
<td>D, M</td>
<td>Have you always ordered in the multiples of complete batches (1000’s)?</td>
<td>☐</td>
</tr>
<tr>
<td>7</td>
<td>R, D</td>
<td>Have you limited your order quantity to avoid upstream shortage?</td>
<td>☐</td>
</tr>
<tr>
<td>8</td>
<td>D, M</td>
<td>Is your order quantity the same as that of your upstream members?</td>
<td>☐</td>
</tr>
</tbody>
</table>

* R stands for Retailer, D stands for Distributor, M stands for Manufacture and A stands for all supply chain members. Instead of having respondents column, customized survey can be designed for each supply chain members.

**Comments:**

______________________________________________

______________________________________________

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APPENDIX C: INTEGER PROGRAMMING MODEL

Introduction:

Suppose that the game is played in CC mode and that the retailer demand for the twelve periods is known at the beginning of the game, we seek the least cost solution for the supply chain using an integer programming model.

Notation:

\[ \theta \] The set of supply chain members; \( \theta \equiv \{1: \text{Retailer}, 2: \text{Distributor}, 3: \text{Manufacturer} \} \)

\[ \eta \] The set of periods in the Cola-Game; \( \eta \equiv \{1, 2, \ldots, N\} \)

\[ H_i \] Holding cost per unit per period for \( i \in \theta \)

\[ R_i \] Ordering/Set up cost per batch for \( i \in \theta \)

\[ S_i \] Shortage cost per unit for \( i \in \theta \) (Demand is not carried forward)

\[ B_i \] Batch size for \( i \in \theta \) (Note that \( B_1 \) is set as large positive number since the retailer ordering cost is independent of batch size. \( B_2 = B_3 = 1000 \))

\[ D_j \]: Demand faced by the retailer for period \( j \in \eta \)

Decision Variables:

\[ I_{ij} \] Ending inventory for the supply chain member \( i \in \theta \) in period \( j \in \eta \)

\[ O_{ij} \] Quantity ordered by supply chain member \( i \in \theta \) in period \( j \in \eta \)

\[ G_{ij} \] The shortage faced by supply chain member \( i \in \theta \) in period \( j \in \eta \)

\[ Y_{ij} \] Indicator integer variable to capture the corresponding ordering cost for each supply chain member \( i \in \theta \) in period \( j \in \eta \)
Objective Function Z:

\[ \min Z = R_1 \left( \sum_{i=1}^{N} Y_{ij} \right) + S_1 \left( \sum_{i=1}^{N} G_{i1j} \right) + H_1 \left( \sum_{i=1}^{N} I_{1j} + 0.5 \sum_{i=1}^{N} (D_j - G_{i1j}) \right) \]
\[ + R_2 \left( \sum_{i=1}^{N} Y_{2j} \right) + S_2 \left( \sum_{i=1}^{N} G_{2j} \right) + H_2 \left( \sum_{i=1}^{N} I_{2j} \right) \]
\[ + R_3 \left( \sum_{i=1}^{N} Y_{3j} \right) + S_3 \left( \sum_{i=1}^{N} G_{3j} \right) + H_3 \left( \sum_{i=1}^{N} I_{3j} \right) \]

(68)

Constraints:

Constraints for Ending Inventory:

\[ I_{1j} = I_{1j,1} + O_{1j} - G_{2j} + G_{i1j} - D_j \quad \text{for } j \in \eta \]
\[ (69) \]
\[ I_{2j} = I_{2j,1} + O_{2j} - G_{3j} + G_{2j} - O_{1j} \quad \text{for } j \in \eta \]
\[ (70) \]
\[ I_{3j} = I_{3j,1} + O_{3j} + G_{3j} - O_{2j} \quad \text{for } j \in \eta \]
\[ (71) \]
\[ I_{10} = 1000 \quad \text{for } i \in \theta \]
\[ (72) \]
\[ I_{1N} = 1000 \quad \text{for } i \in \theta \]
\[ (73) \]

Constraints for computing shortages:

\[ G_{1j} \geq D_j - I_{1j,1} \quad \text{for } j \in \eta \]
\[ (74) \]
\[ G_{2j} \geq O_{1j} - I_{2j,1} \quad \text{for } j \in \eta \]
\[ (75) \]
\[ G_{3j} \geq O_{2j} - I_{3j,1} \quad \text{for } j \in \eta \]
\[ (76) \]

Constraint for Number of Batches:

\[ O_{ij} \leq (B_i) (Y_{ij}) \quad \text{for } i \in \theta, \ j \in \eta \]
\[ (77) \]

All variables are non-negative.

\( Y_{ij} \) is integer for all \( i \in \theta \) in period \( j \in \eta \).
**Explanation of Model:**

Equation (68) represents objective function of the model. The first three terms of (68) corresponds to the retailer’s ordering, shortage, and holding cost respectively. The next three terms refers to the distributor’s cost (ordering, shortage, and holding) and the last three terms refers to manufacturer’s costs (setup, shortage, and holding). The downstream supply chain members are responsible for pipeline inventory during transportation or work-in-process inventory during production and are reflected in holding cost terms. The retailer’s demand occurs uniformly throughout the period giving rise to an additional average inventory term \(0.5 \sum_{i=1}^{N} (D_j - G_{1j})\).

Constraints (69), (70), and (71) are standard inventory flow constraints. Constrains (72) and (73) are beginning and ending inventory conditions. Constraints (74), (75), and (76) along with non-negativity constraints will ensure that shortage is either 0 or exactly equal to the units short. It may be noted that demand for the current period can only be met by ending inventory in the previous period. Constraint (77) through variable \(Y_{1j}\), ensures that the ordering/setup cost is incurred for every full or partial batch shipped or manufactured. Variable \(Y_{ij}\) captures the number of full or partial batches ordered or manufactured. Since the retailers ordering cost is independent of batch size, for \(O_{1j} > 0\), \(Y_{1j}\) should be equal to 1. This is ensured by making \(B_1\) arbitrarily large.
APPENDIX D: DEMAND SET, STUDENT DECISIONS, & ILP SOLUTION

In this appendix we have included an example of a demand set used (set # 3), decisions made by a student team, and corresponding solution obtained from the integer linear programming.

Demand Set, Student Decisions, & ILP Solution

<table>
<thead>
<tr>
<th>Period</th>
<th>Demand</th>
<th>Decisions made by a student teams</th>
<th>Solution obtained from ILP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ret. Order</td>
<td>Dist. Order</td>
</tr>
<tr>
<td>1</td>
<td>565</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>587</td>
<td>565</td>
<td>1565</td>
</tr>
<tr>
<td>3</td>
<td>423</td>
<td>1787</td>
<td>587</td>
</tr>
<tr>
<td>4</td>
<td>1285</td>
<td>423</td>
<td>423</td>
</tr>
<tr>
<td>5</td>
<td>408</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>447</td>
<td>493</td>
<td>1593</td>
</tr>
<tr>
<td>7</td>
<td>503</td>
<td>1647</td>
<td>447</td>
</tr>
<tr>
<td>8</td>
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</tr>
<tr>
<td>9</td>
<td>423</td>
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<td>588</td>
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<tr>
<td>10</td>
<td>522</td>
<td>423</td>
<td>1623</td>
</tr>
<tr>
<td>11</td>
<td>424</td>
<td>1722</td>
<td>522</td>
</tr>
<tr>
<td>12</td>
<td>1349</td>
<td>824</td>
<td>1000</td>
</tr>
</tbody>
</table>

Supply Chain Cost of a Student Team & corresponding ILP Solution

<table>
<thead>
<tr>
<th></th>
<th>Retailer’s Cost</th>
<th>Distributor’s Cost</th>
<th>Manufacturer’s Cost</th>
<th>Supply Chain Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Team</td>
<td>$29,127.70</td>
<td>$25,071.00</td>
<td>$29,115.00</td>
<td>$83,313.70</td>
</tr>
<tr>
<td>ILP Solution</td>
<td>$24,609.40</td>
<td>$19,231.25</td>
<td>$19,864.00</td>
<td>$63,704.65</td>
</tr>
</tbody>
</table>