Impact of bilateral filter parameters on medical image noise reduction and edge preservation

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Impact of Bilateral Filter Parameters on Medical Image Noise Reduction and Edge Preservation

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In partial fulfillment of the requirements for the degree of Master of Science in Biomedical Sciences

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Impact of Bilateral Filter Parameters on Medical Image Noise Reduction and Edge Preservation

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Abstract

The objective of this paper is to examine the bilateral filter and its application to digital medical imaging. The bilateral filter is a non-linear adaptive smoothing filter which is capable of preserving the edges of image structures. Selection of the bilateral filter’s parameters greatly influence how well an image is smoothed and how well edges are preserved.

The parameters of the bilateral filter were analyzed individually. The noise reduction for each parameter is related to the much simpler box (averaging) filter. Edges were also analyzed for each parameter. Techniques for proper parameter selection are studied and general guidelines are suggested. After the properties of each parameter were examined, they were recombined and applied to CT images. Solutions addressing the limitations of this filter for high noise CT images are also considered.

It was found that the bilateral filter is capable of providing effective smoothing and at the same time preserves edges which are otherwise blurred by other filters. The viability of this filter greatly depends on the selection of its parameters.
Dedication

I dedicate this work to my daughter Elena.
Acknowledgements

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# Table of Contents

Introduction ........................................................................................................... 1

Literature .................................................................................................................. 19

Materials ................................................................................................................... 26

Methods .................................................................................................................... 29

Results ....................................................................................................................... 33

Summary of Results & Discussion ............................................................................. 95

Conclusions .............................................................................................................. 98

Bibliography ............................................................................................................ 100


Introduction

The number of photons used for radiographic images is relatively small. Photographic images utilize approximately $10^{10}$ photons/mm$^2$. In general radiographic images make use of about $10^5$ photons/mm$^2$, (Huda and Slone 2003).

Photon statistics follow a Poisson distribution which has a standard deviation given by $\sqrt{N}$, where N is the number of photons per resolution element or pixel. The image matrix formed by those photons will have pixels which may deviate from the mean intensity, i.e. some pixels will have grey levels which deviate from the mean grey level. This is the origin of image noise. As the number of photons incident on the detector increases, the relative noise level decreases.

Image noise can significantly reduce the quality of radiographic images. When noise is present, objects have reduced visibility and information regarding their size and shape is compromised. Thus it is often advantageous to apply a smoothing filter to remove noise.

Image smoothing reduces the varying photon intensities per pixel by averaging them together. This modifies the image so that adjacent pixels are more uniform in intensity. However, smoothing can significantly blur structures in an image. Like noise, pixels which represent structures will have a different intensity from their non-structure neighbors. In medical images significant anatomy and pathology can be blurred. Thus,
there is a need to develop filters which can accomplish smoothing yet preserve structural
detail.

In general the methods of image processing can be divided between linear and non-linear
operations. Linear filtering is commonly used for smoothing to reduce noise. Linear
processing is well understood and has predictable outcomes. The outcomes of non-linear
processing are more ambiguous and are not reversible, and therefore have limited
application, (Gonzalez & Woods 2008). However, non-linear filters can process images
in a way which allows them to adapt to the distinctive attributes of a neighborhood within
an image. Consequently non-linear processing can produce better results than linear
filters. A discussion of linear and non-linear operations and filters can be found in

Filtering can take place in the spatial or frequency domain. Processing in the spatial
domain is described mathematically by the sum of products or a convolution:

\[
\begin{align*}
\mathbf{w}(x,y) \ast \mathbf{f}(x,y) &= \sum_{s=-a}^{a} \sum_{t=-b}^{b} \mathbf{w}(s,t) \mathbf{f}(x-s,y-t)
\end{align*}
\]  (1)

Where \( \mathbf{w}(x,y) \) is the filter mask and \( \mathbf{f}(x,y) \) is the input image. The mask is of size
\( m \times n \) and \( a = (m-1)/2 \) and \( b = (n-1)/2 \) (Gonzalez et al. 2008).

The filter performs some operation, such as averaging the pixel values in the
neighborhood. A new pixel is formed from that operation with the same coordinates as
the center pixel of the neighborhood. The filter is passed over the entire image pixel by pixel and a new image is formed.

Some common spatial linear filters used to smooth noise from an image are the box (averaging) filter and the spatial Gaussian filter. These filters take the weighted average of the pixels in a neighborhood. An averaging filter, which has filter coefficients that are equal, is called a box function. Equation 2 shows how $f(x, y)$, the input image, is processed with a box filter. The processed image is $g(x, y)$ and $M \times N$ is the number of pixels being averaged:

$$g(x, y) = \sum_{i=1}^{m} \sum_{j=1}^{n} \left( \frac{1}{(M \times N)} f(x_i, y_j) \right)$$  \hspace{1cm} (2)$$

In this and subsequent equations the summation from 1 to m or from 1 to n represents stepping through all of the pixel values within the filter kernel. Filters can also perform a weighted average with varying coefficients. A common averaging filter with this quality is the spatial Gaussian filter. The spatial Gaussian weights the center value higher than any other pixel in the neighborhood. The other pixels are weighted based on the Euclidean distance from the center. To calculate the coefficients of the filter:

$$w(x, y) = \frac{1}{k} \sum_{i=1}^{m} \sum_{j=1}^{n} e^{-\frac{1}{2} \left( \frac{\sqrt{(x_i - \text{center}_x)^2 + (y_j - \text{center}_y)^2}}{\sigma_{\text{domain}}} \right)^2}$$  \hspace{1cm} (3)$$
Where $\sigma_d$ is the spatial standard deviation or domain value, $i$ and $j$ are the coordinates of $x$ and $y$, $m$ and $n$ are the dimensions of the kernel and $1/k$ is the normalization factor. The larger the standard deviation the broader the Gaussian distribution. A larger domain value will allow inclusion of more distant pixels and greater smoothing will occur.

A key property of non-linear filters is they are not mathematically reversible. A common non-linear filter is the median filter. It is a type of ordered statistic filter. The median filter works by sorting the pixels in the neighborhood. The median intensity pixel is found and that pixel is used for the new pixel in the filtered image. Median filters are particularly effective at reducing speckle or spike noise. Other order statistic filters are the maximum and minimum filters. These filters are commonly used to find bright and dark pixels in an image. Additional non-linear filters are described in the Literature section.

For completeness, frequency filtering is briefly discussed. Frequency domain filtering is sometimes a more suitable way to filter images. For example sinusoidal interference can be filtered in the frequency domain by designing an appropriate filter to remove that interference. To filter an image in the frequency domain it must be transformed into the frequency domain and then back to the spatial domain through the following process, (Gonzalez et al. 2008):

1.) The image is transformed to the frequency domain through the following Fourier transformation:
\[ T(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)e^{-j2\pi(ax/M+by/N)} \]  

(4)

Where \( T(u, v) \) is the Fourier transform, \( f(x, y) \) is the input image, \( M \) and \( N \) are the matrix dimensions and \( u \) and \( v \) are the frequency domain variables.

2.) An appropriate filter is applied that will enhance or depress the intensity at certain frequencies. That is: \( T'(u, v) = H(u, v) \times T(u, v) \)

3.) The image is transformed back to the spatial domain through the inverse Fourier transform:

\[ f(x, y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} T'(u, v)e^{j2\pi(ax/M+by/N)} \]  

(5)

Where \( f(x, y) \) is the filtered image. The transformation given in these equations are discrete Fourier transforms since the data is given as a series of discrete elements or pixels, rather than as a continuous mathematical function. Smoothing performed in by Fourier filtering is equivalent to the corresponding convolution filtering in the spatial domain.

Some common frequency domain filters are the Butterworth low-pass filter and the Gaussian low-pass filter. The Butterworth filter is mathematically characterized by:
\[ H(u,v) = \frac{1}{1 + \left[ \frac{D(u,v)}{D_0} \right]^{2n}} \quad (6) \]

where \( D(u,v) \) is the distance of point \((u,v)\) from the origin in the frequency domain and \( D_0 \) is the limiting radius or cutoff frequency of the filter. Using a smaller \( D_0 \) results in reducing the high frequency content of the filtered data. The order \( n \) determines how smooth the transition is from higher and lower frequencies. A smoother cutoff will result in less Gibb’s ringing artifact in the image. All frequencies past \( D_0 \) are filtered out of the image. Hence, the higher frequencies on the outskirts of the Fourier space are removed from the image. The Gaussian low-pass filter is mathematically characterized by:

\[ H(u,v) = e^{-D^2(u,v)/2D_0^2} \quad (7) \]

A larger value of \( D_0 \) translates into a smoother transition between the low and high frequencies. The effect of the Gaussian filter applied in the frequency domain is similar to the spatial Gaussian convolution filtering except a large \( \sigma \) (domain) maximizes smoothing and a large \( D_0 \) minimizes smoothing.

Using local statistics for image processing was suggested by Ketcham (1976) and Wallis (1976). It was extended to contrast and edge enhancements by Lee (1981). In his original paper Lee used this method for additive and multiplicative noise. His technique processed pixels independently. The assumption is that a sample mean and variance of a
pixel is equal to the mean and variance of all pixels in a neighborhood surrounding it (Lee 1980). The advantage of this method is that it is non-iterative and image modeling is not required as with the other techniques such as Wiener filtering. Lee’s papers are discussed further in the Literature chapter.

Wiener filtering requires that an estimate of the spectral properties of the uncorrupted image is known or assumed. More specifically:

\[ g(x, y) = h(x, y) \otimes f(x, y) + \eta(x, y) \quad (8) \]

Where \( g(x, y) \) is the corrupted image, \( h(x, y) \) the degradation function and \( \eta(x, y) \) the additive noise. The input image is convolved with the degradation function. The point of the Weiner filter is to minimize the mean square error between the estimate of the uncorrupted image (corrupted image \( \div \) degradation function) and the original uncorrupted image.

The bilateral filter was first introduced by Tomasi and Manduchi (1998). Similar to Lee’s technique, the bilateral filter does not require iteration or image modeling and employs the use of local statistics. The motivation for creating this filter was to remove image noise and preserve edges. The filter operates by examining both spatial and photometric characteristics (intensity differences) of the pixels in a neighborhood, (Tomasi et al. 1998).
As described in Tomasi et al. (1998), domain filtering is based on spatial location or closeness of neighborhood pixels. Range filtering is based on photometric similarity or similarity in gray levels. Domain filtering alone filters an image based only on the distance, of a given pixel, to the center pixel of the neighborhood. The assumption is that pixels close to the center are likely to be of the same structure. Domain filtering tends to blur edges because no consideration is given to pixel intensity differences or whether they are truly of the same structure. Range filtering examines the pixel intensities of a neighborhood with respect to the intensity of the center pixel.

The following derivation of the bilateral filter is from Tomasi et al. (1998):

\[ k^{-1}(x) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)c(\xi, x)s(f(\xi), f(x))d\xi \]  

The term \( c(\xi, x) \) is the weighting factor for the spatial component. This term is a standard spatial Gaussian filter:

\[ c(\xi, x) = e^{-\left( \frac{1}{2} \left( \frac{d(\xi, x)}{\sigma_d} \right)^2 \right)} \]  

The term \( d(\xi, x) \) is the distance between the center value of the neighborhood and one of the neighboring pixel, \( \xi \). The term \( s(f(\xi), f(x)) \) is the weighting factor for the
photometric similarity. It is also Gaussian in form and measures the intensities between the center pixel and a neighboring pixel:

\[
s(f(\xi), f(x)) = e^{-\frac{1}{2\sigma^2} \left( \frac{\delta(f(f(\xi)), f(x))}{\sigma} \right)^2}
\]  

(11)

The term \( k^{-1}(x) \) is the normalization and given by:

\[
k(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi, x)s(f(\xi), f(x))d\xi
\]  

(12)

The result is a new pixel at location \( x \) found by averaging the neighborhood values centered on \( x \) with pixels that are spatially close and photometrically close. The final equation is given by:

\[
h(x) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)c(\xi, x)s(f(\xi), f(x))d\xi}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi, x)s(f(\xi), f(x))d\xi}
\]  

(13)

The theoretical underpinnings of the bilateral filter and its relationship to well known iterative algorithms, such as anisotropic diffusion are discussed in Elad (2002) and Barash (2002).
The domain standard deviation, of the bilateral filter, will determine the amount of blurring. If large, more distant pixels will be used and edges will become blurred. The domain filter may be limited by specifying a limited kernel size, \( m \times n \), which effectively cuts the tails off or truncates the spatial Gaussian function. If the kernel is small and \( \sigma_d \) is large, the filter approaches a box filter. On the other hand if \( \sigma_r \) is relatively small there will be little smoothing.

The bilateral filter weighting factors, in the discrete case, are calculated by the following:

\[
1 \left( \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} e^{-\frac{1}{2} \left( \frac{|f(x_i, y_j) - f(\text{center})|}{\sigma_{\text{range}}} \right)^2 \left( \frac{\sqrt{(x_i - \text{center}_x)^2 + (y_j - \text{center}_y)^2}}{\sigma_{\text{domain}}} \right)^2}}{k} \right)
\]

Where \( m \) and \( n \) are the dimensions of the kernel and \( 1/k \) is the normalization term. The two terms of the exponential are the range and domain components respectively.

This paper will analyze how the bilateral filter performs on CT images. Thus a discussion of CT image reconstruction is introduced. As an example, a small round attenuating object is placed in the CT scanner. An x-ray is transmitted through the object and the attenuation across the x-ray path is measured by a detector opposite the source of that ray. Each element along the ray’s path contributes equally to the total attenuation and the ray is backprojected onto an empty image matrix.
More transmissions are generated at multiple angles and the result is a summed attenuation coefficient for each element. Each backprojection from each ray is added and areas where the rays intersect produce a greater intensity, due to the attenuation of the object. The point spread function of the object diminishes with $1/r$. Where $r$ is the distance from the point where the rays overlap.

Round Attenuating Object

![Figure 1](image)

As the image above indicates, there is blurring around the object where all of the rays converge.

In order to remove the blur a sharpening filter is applied before the rays are backprojected onto the matrix. In the spatial domain the projection data can be convolved with the de-blurring kernel as follows:

$$f'(x) = f(x) \otimes w(x) \quad (15)$$
$f''(x)$ is the filtered projection data which is backprojected to form the CT image. The filtering in Equation 15 can also be performed in the frequency domain as was shown by Equations 4 and 5. Once in the frequency domain the image data can be multiplied by the frequency domain kernel (de-blurring kernel) and transformed back to the spatial domain through the inverse Fourier transform. The Fourier transform pair of Equation 15 is given by:

$$f(x) \ast w(x) \Leftrightarrow H(\mu)F(\mu) \quad (16)$$

Equation 16 illustrates the Convolution Theorem that the convolution of two functions in the spatial domain can be expressed by the product of the Fourier transform of these two functions in the frequency domain.

The Lak filter, ramp filter, is given by $L(f) = f$ and depicted graphically in Figure 2 (Ramachandran and Lakshminarayanan, 1971).

The $1/r$ blurring due to the backprojection in the spatial domain corresponds to a $1/f$ degradation in the frequency domain.
When multiplying the image in the frequency domain by the Lak filter, the blurring is removed, \(1/f \cdot f = 1\), (Bushberg et al. 2002). The product of the image transform, \(H(\mu)\), and \(L(f)\) is then transferred back to the spatial domain through the inverse Fourier transform.

When noise is present it is beneficial to have some roll off at high frequencies. The Shepp-Logan filter (Shepp and Logan, 1974) has roll off at high frequencies and is depicted in Figure 3.

![Shepp-Logan Filter](image)

**Figure 3**

The Hamming filter can also be applied to give even more high frequency role off and shown in Figure 4.
The product of the Lak filter and the Hamming window is depicted in Figure 5.

![Diagram of Lak Filter × Hamming Window](image)

In CT, filtering a high projection value causes negative contributions to the adjacent filtered projection values. This appropriately reconstructs the image from the true signal data, when combined with other filtered data in the backprojection process. Noise, however, is random and the negative contribution to adjacent projection values causes correlation of the noise in the reconstructed images. Referring to the spatial filter in Figure 5, the neighbors of the central pixel are lower in intensity. If the central pixel is a noise spike its neighbors will be lower in value. Correlation of noise in CT images will influence the performance of spatial smoothing filters. This will be shown in the Results chapter.
As discussed the bilateral filter has both a domain and range component, both of which are represented by a Gaussian distribution in this paper. Analyzing them individually provided a better understanding of each component’s contribution to the filter’s overall performance. The domain portion of the bilateral filter was analyzed by comparing its noise reduction and edge preservation characteristics to the box filter. The box filter works well for smoothing over noise and its properties are straightforward. However, its limitations include poor edge preservation and smoothing over fine detail.

Because the range component does not yield meaningful results by itself (Tomasi et al. 1998), it was combined with the box filter which defines the spatial neighborhood or kernel. How the range component influenced the box filter’s noise reduction performance provided insight into how the range performed in areas of uniformity and edges. After the domain and range components were analyzed individually they were combined in the form of the bilateral filter.

This study was performed in two parts. First the spatial box filter, spatial Gaussian filter and the bilateral filter were applied to images of uniform intensity and images with defined edges. Gaussian noise is chosen because of its mathematical tractability. It can provide a good approximation of Poisson noise and often used, (Wernick and Aarsvold 2004).
The second part is the application of the bilateral filtering to CT images. Both phantom and clinical CT images will be studied. The CT phantom images will include low contrast and resolution slices with varying mAs or noise levels. The slices will be reconstructed with a soft tissue and bone reconstruction kernels. The aim of the analysis is to produce higher quality images by reducing noise, preserve edges and structure detail.

The objectives of this paper are to investigate the bilateral filter and assess its utility for medical images, specifically CT brain images. This filter claims to produce images with decreased noise while preserving edges. These are highly sought goals of medical imaging. Thus, it is worthy to further examine this filter’s performance on medical images. Due to the generality of this filter its parameters will be examined with respect to noise reduction and edge preservation. As will be demonstrated, proper selection of the bilateral’s parameters can significantly change the response of the filter.

This paper will specifically study the parameters of the bilateral filter. The following propositions will be examined:

1. The bilateral filter can separated into range and domain components and studied individually.
2. Each parameter, with respect to noise reduction, can be linked to the box filter.
3. The noise dependency of the bilateral filter can be represented by the ratio
\( \sigma_{\text{range}} / \sigma_{\text{noise}} \). This ratio can then be linked to a box equivalent size for a bilateral
filter with a box domain.

4. The range value can be effectively determined by object contrast.

5. The techniques of using a pre-filter and iterating the bilateral filter will increase
noise reduction and preserve edges. They will allow using a low range value even
when \( \sigma_{\text{range}} / \sigma_{\text{noise}} \) is small.

To accomplish the objectives of this paper the bilateral filter will be tested by the
following methods:

1.) A computer generated phantom with various objects and varying gray levels will
be created and noise will be added. The bilateral’s general performance will be
tested in a Gaussian distributed noise environment. Gaussian noise offers the
most general and mathematically tractable types of noise.

2.) A CT phantom will be used to obtain images that are then post processed with the
bilateral filter. Noise reduction, edge preservation and low contrast will be
examined.

3.) Clinical CT brain images will be processed with the bilateral filter.
The tools that will be used to achieve the goals of this paper are MatLab and MatLab’s image processing tool box. An M-file, a user defined MatLab function, will be implemented to create the bilateral filter. MatLab’s predefined function ‘nlfilter’ will be used for the convolutions. All medical images will be in the dicom format and a standard dicom viewer will be used to access the processed images.
Literature Search

Adaptive filtering can be implemented in a variety of ways. However all adaptive techniques share the same general paradigm. That paradigm is to utilize the unique characteristics of a pixel neighborhood to create a new (filtered) image. Linear filtering does not take into account the unique characteristics of pixel neighborhoods. Therefore edges and fine detail can become lost with linear smoothing filters.

The first uses of local statistics in image processing can be found in Ketcham (1976) and Wallis (1976). The theoretical advantages and a description of general adaptive neighborhood processing are discussed in Debayle and Pinoli (2006). This paper relates the adaptive approach to mathematical morphology and demonstrates that a priori information about image structures can be derived from the local mean and variance. This diverged from previous methods which relied on a correlation model to find the mean and variance. The advantage is that contrast can be better preserved.

Utilizing the local variance and mean for image filtering was discussed by Lee (1980). In this paper Lee demonstrated that that the local mean and variance can be successfully applied to images with additive or multiplicative noise. The novelty of this paper is extending the use of local statistics to contrast enhancement. It is also demonstrated that the computational efficiency is superior to such techniques as frequency domain or
recursive processing. Thus Lee’s technique is better for real time processing because each pixel can be individually processed.

In a following paper by Lee a technique was developed to reduce noise near edges. Depending on the orientation of an edge, the local mean and variance are calculated from a reduced number of pixels (Lee 1981). Using local gradient information allows the mean and local variance to be calculated on one side of the edge. These early works by Lee demonstrated that improved image contrast can be obtained by use of local pixel statistics.

Rangayyan et al. (1998) proposed an adaptive neighborhood filter which grows from a single pixel for images with signal dependent noise. This filter works by estimating the noise and signal of a neighborhood. The neighborhoods are grown from a single pixel based on the type of noise and power of the noise. Only pixels that belong to the same object are included in the neighborhood.

Similar to Rangayyan’s approach Guis et al. (2003) suggested an adaptive neighborhood approach for mammographic phantom images. Local statistics around a given pixel are calculated. Based on those local statistics, the neighborhood will grow to include only those pixels which are similar to the reference pixel. This method uses a maximum growth size for the neighborhood. Growth stops if the maximum size is reached or the percentage of pixels in the neighborhood are over the pre-determined threshold by 60%.
Anisotropic diffusion as introduced by Perona and Malik (1990), uses the idea of scale-space filtering. Scale space filtering is based on hierarchical organization Perona et al. (1990). The process of scale-space filtering involves embedding the original image with a family of derived images with a one variable parameter, resolution. This can be accomplished through a diffusion process or a convolution with a Gaussian kernel. Koenderink (1984). The scale space design involves a convolution of an image with a Gaussian filter that has a scale space parameter t, that is:

\[ I(x, y, t) = I_0(x, y) \otimes G(x, y, t) \quad (17) \]

Where \( I_0 \) is the input image and \( G(x, y, t) \) is a Gaussian kernel. Larger values of t correspond to coarser resolutions. It is show in by Koenderink (1984) that the family of derived images can be viewed as the solution to the diffusion equation. Perona et al. (1990) extended scale based filtering through the use of a modified diffusion equation. The diffusion equation is given by:

\[ I_t = \text{div}(c(x, y, t) \nabla I) \quad (18) \]

where \( c(x, y, t) \) is the diffusion coefficient. The result \( I_t \) is a scalar field. The modification of the diffusion equation includes an edge stopping function:

\[ c(x, y, t) = g(\| \nabla I(x, y, t) \|) \quad (19) \]
Where the edge stopping function is zero in the interior of a region. This prevents smoothing outside of the region. Black et al. (1998) develops a relationship between anisotropic diffusion and robust statistics. The edge preserving property of this method considers edges between constant regions to be outliers, Black et al. (1998). The purpose of this method is to use robust statistics to analyze and design isotropic diffusion.

A connection between the bilateral filter and the anisotropic diffusion (non-linear diffusion equation) was established by Barash (2002). In his paper the bilateral filter and the diffusion equation are linked through adaptive smoothing. The diffusion coefficient and the kernel, in the bilateral filter, perform the same purpose. In addition there is global dependence on intensity for the kernel of the bilateral filter. The gradient (the diffusion coefficient) shows only local pixel dependence. Thus there is a need to perform several iterations with the diffusion equation.

The bilateral filter was first introduced by Tomasi et al. (1998). The filter was formally defined in the Introduction chapter. The bilateral filter utilizes both a spatial and a photometric (intensity) component. The spatial component is referred to as the domain and the photometric component is referred to as the range. The original paper uses a Gaussian distribution for both components. However, other kernels could be used.

Combining the domain and range components is essential to the filter. The range component alone simply modifies the gray map of the image. However when combined
with spatial constraints the remapping is local and dissimilar at different locations in the image (Tomasi et al. 1998).

In the presence of edges the bilateral filter works especially well due to the range component. For example, the kernel is at the border between a light and dark region and centered on the light region. The pixels on the light side of the kernel will be photometrically similar. Thus the range weighting factors for those pixels will be approximately one. The dark side of the kernel will have low range weighting factors. Those pixels will have little influence on the calculation of the new pixel.

A paper by Elad (2002) bridges the bilateral filter to more classical filtering approaches such as anisotropic diffusion. This paper demonstrates that bilateral filter emerges from a Bayesian approach. Speeding up the bilateral filter and increasing its smoothing is also proposed. A theoretical background for the origin of the bilateral filter is proposed in this paper.

Another paper by Durand and Dorsey (2002) uses the bilateral filter for the display of high dynamic range images on low dynamic range media. The image is decomposed into a base layer and a detail layer. The base layer is obtained with the bilateral filter and it is only this layer which has its contrast reduced. That is large scale features have their contrast reduced while detail is preserved. This paper demonstrates the versatility of the bilateral filter to other image processing demands.
Xie et al. (2008) discusses a technique to overcome the problem of choosing a kernel size for the bilateral filter. The kernel size is important when balancing edge preservation and noise reduction. Constant kernels are replaced by a function that decreases with boundary saliency. The idea is that uniform areas will have a broad kernel and at edges a sharper kernel. A pixel in an image plane has a saliency measure which is the saliency of the greatest edge across it. For example point p is located on a salient edge. If another point, q, is not located on the edge the similarity is computed using only a sub-region containing p and q. If q is located on the edge the entire filtering window is used for the similarity calculation. This technique has a strong dependence on proper calculation of the salient features.

Walker et al. (2006) used the bilateral to minimize the smoothing, typically performed in low signal fMRI images, in the presence of brain lesions. The purpose of this paper was to delineate boundaries between a brain lesion and normal brain tissue. Obtaining a good boundary allows minimal resection of normal functional brain tissue and maximal removal of the pathological brain tissue. Zhou et al. (2007) applied the bilateral filter to PET imaging.

Eibenberger et al. (2008) evaluated the bilateral filter, anisotropic diffusion and the wavelet approach for CT images. This paper analyzed the feasibility of each filtering technique for the segmentation of organs. It was concluded that the bilateral filter and anisotropic diffusion had to have their parameters adapted to each organ and for different noise levels for proper segmentation. The parameters for the wavelet approach had to
have its parameters adjusted only once for proper segmentation of different organs with
different noise levels. This can be addressed by selecting the range value based on the
lowest contrast object in the image and the image noise level. This will be discussed
further in the Results chapter.

The trilateral filter was proposed by Wong et al. (2004). Like the bilateral filter the
trilateral uses the spatial and range components. However it adds a third component,
which examines the local structural orientations between neighboring pixels. Orientation
tensors from eigen decomposition are used to identify the local structural information.
For medical imaging smaller kernel sizes are necessary so that detail is not smoothed
from the image. However in order to obtain good noise reduction more iterations are
required. Fewer iterations are required with the trilateral filter.

As reviewed in this chapter, the bilateral filter has been successfully used in medical
image processing and other areas. Improvements have also been proposed. Compared to
other filtering techniques, like anisotropic diffusion, the bilateral filter is fairly
straightforward in design. Though this filter is simple in concept its parameters need
thorough analysis as they greatly influence its total response. In addition those
parameters have many dependencies. As discussed in the Introduction, this paper will
examine the parameters of the bilateral filter. In doing so better predictions can be made
of how the bilateral will respond for different noise levels and types of noise.
Materials

A windows based computer was used in this study. *MatLab Image Processing Toolbox* was used for all image processing. A user defined *M-File* function was implemented for the bilateral filter. *MatLab’s nlfilter* was used for convolving the bilateral filter. *Sante Dicom Viewer* was used for viewing the CT clinical and phantom images. The image capture program, *Snagit* was used to capture images for this paper. *Open Office 3.1* spreadsheet program was used for the data analysis, graphs and tables.

A computer generated phantom was developed to evaluate the bilateral filter and is depicted in Figure 6. The phantom contains the following structures:

**Intensity Pyramid:** The top of the pyramid begins at intensity 0 (on the left side) and 1 (on the right side). Descending the pyramid, the intensities are incremented over 128 pixels gradually blending to 0.5 at the base of the pyramid. The purpose of the pyramid is to assess how each filter performed on edges with various levels of contrast. The intensity of the phantom’s background is 0.5.

**Low Contrast Boxes:** There are five rows and five boxes in each row. The rows increase by 5%, above background in intensity from the top row to the bottom. The sizes of the
boxes are: $3 \times 3$, $5 \times 5$, $7 \times 7$, $9 \times 9$ and $11 \times 11$. The low contrast boxes were used to assess how well each filter was able to maintain the visibility of small structures.

**Resolution Bars**: There are five sets with three bars for each set. The first set of bars are 15 pixels wide and 15 pixels apart. The subsequent bars are 10, 5, 3 and 2 pixels wide and separated by their width.

The phantom in Figure 6 was used to evaluate the bilateral filter under Gaussian noise conditions. The noise was generated using MatLab’s function `imnoise`.

The CT phantom used was the Phantom Laboratory’s Catphan 600. The low contrast, resolution and uniformity slices were used. The mAs levels were 900, 400, 200 and 100mAs. All, except the 900mAs, were 4mm slices. The 900mAs 16mm slice was used for the measuring the average CT numbers for the three Supra-Slice target groups. The nominal target contrast levels for the three Supra-Slice target groups were, 0.3%, 0.5% and 1.0%. The target group diameters ranged from 2.0mm to 15mm. The sub-slice
varying length rods were not examined since their contrast varied with the slice thickness.
The reconstruction filters used for this study were the bone and soft tissue filters.

CT data was obtained from a Toshiba Aquillion 16 including scans of a 20 cm diameter American College of Radiology CT Accreditation Phantom. An anonymized axial head scan was acquired under an exempt IRB protocol. CT phantom and head scans were reconstructed utilizing different reconstruction filters.
Methods

As discussed in the Introduction chapter, the bilateral filter is non-linear adaptive smoothing filter which can preserve edges. It combines both domain and range filtering. The amount of noise reduction and edge preservation is dependent on the kernel size, domain value, range value, and noise level.

This paper will analyze the impact the bilateral filter parameters have on smoothing and edge preservation. In analyzing the parameters a more concrete prediction can be made, for a given parameter combination, regarding noise reduction. It will demonstrate how for different range values, some image structures are better preserved. A link, with respect to noise reduction, will be made to the simpler box (averaging) filter. Finally a method for the selection of the range value will be evaluated.

The amount of noise reduction for the filters examined in this paper can be found through the standard deviation of a weighted mean. The standard deviation of a weighted mean is:

$$\sigma(\bar{x}) = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2} \quad (20)$$

In the case of uniform noise, $\sigma_i$ is the same for all samples. For a box filter all of the weighting values equal $1/N$ and the standard deviation of the mean is:
\[ \sigma(\bar{x}) = \sqrt{\sum_{i=1}^{n} \left( \frac{1}{N} \right)^2 \sigma_i^2} \]  \hspace{1cm} (21)

\[ \sigma(\bar{x})/\sigma = \frac{1}{\sqrt{N}} \]  \hspace{1cm} (22)

where \( N \) is the number of pixels in the neighborhood and \( \sigma(\bar{x}) \) is the noise of the processed image. For example if the box filter kernel size was \( 3 \times 3 \) then:

\[ \sigma(\bar{x})/\sigma = \frac{1}{\sqrt{9}} \]  \hspace{1cm} (23)

The reduction in noise is \( 1/3 \). For the spatial Gaussian filter the noise reduction is given by:

\[ \sigma(\bar{x})/\sigma = \frac{1}{k} \left( \sum_{i=1}^{m} \sum_{j=1}^{n} e^{-\frac{1}{2} \left( \frac{\left( x_i - \text{center}_i \right)^2 + (y_j - \text{center}_j)^2}{\sigma_d} \right)^2} \right)^{-2} \]  \hspace{1cm} (24)

Where \( m \) and \( n \) are the dimensions of the kernel and \( \sigma_d \) is the domain value and \( \sigma(\bar{x}) \) is the noise of the processed image. Noise reduction for the spatial Gaussian filter is dependent on the kernel size and the domain value. The noise reduction of the bilateral filter is given by:
\[
\sigma(\bar{x})/\sigma = \frac{1}{k} \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} e^{-\frac{1}{2} \left( \frac{f(x_i, y_j) - f(\text{center}, x_i, y_j)}{\sigma_{\text{range}}} \right)^2 \left( \frac{(x_i - \text{center}, x_i)^2 + (y_j - \text{center}, y_j)^2}{\sigma_{\text{domain}}} \right)^2}}
\]

Noise reduction for the bilateral filter depends on the range value and the same dependencies of the spatial Gaussian filter. Because the weighting factors of the bilateral are generated by comparing intensities (a neighborhood pixel is compared to the center intensity) the bilateral is also dependent on noise. For example should the kernel be centered on a noise spike the neighbors of that spike will receive low weighting values and the noise spike will be preserved.

To examine how the domain and range values affect the response of the filter, they were separated and studied individually. The spatial Gaussian filter was used to evaluate the domain component. Noise reduction for a given kernel size and domain value, of the spatial Gaussian filter, was linked to the noise reduction of the box filter. In doing so the dependence of noise reduction is effectively reduced to only the kernel size. It will also be shown that the spatial Gaussian filter preserves edges better than the box filter for equivalent noise reduction.

The range component was combined with a spatial box filter and the amount of noise reduction was linked to the box filter. By linking the noise reduction of the bilateral (Gaussian range/box domain) filter to the box filter, a better prediction could be made of the bilateral’s noise reduction in a uniform area. In addition generalizations can made
regarding the maximum amount of noise reduction for a given range value. For object contrast the range value will be selected based on what objects are to be delineated.

The bilateral filter was evaluated first with Gaussian noise. The computer generated phantom from Figure 6 was used for this part of the analysis. Edge preservation was evaluated by an examination of the low contrast boxes and intensity pyramid. The bilateral filter was then applied to CT phantom images. Some general conclusions are drawn for using the bilateral filter for bone and soft tissue reconstructed images. The techniques of iterating the bilateral and applying a pre-filter prior to the bilateral are also studied. These techniques are useful when the noise value is high compared to the desired range value.

The terminology used throughout this paper follows the original paper by Tomasi et al. (1998) and summarized below:

1. Domain Value or $\sigma_d$ : Refers to the spatial spread of the bilateral filter. It also references the standard deviation for the Gaussian convolution kernel.
2. Range Value or $\sigma_r$ : Refers to the photometric spread of the bilateral filter.
3. Noise level or $\sigma_{\text{noise}}$ : Refers to the spread of the noise in an image scaled [0,1].
Results

As discussed in the methods section, the parameters of the bilateral (domain and range variables) filter are separated and studied individually. This paper uses a Gaussian distribution to determine the weighting factors for both the range and domain components. If the range component of the bilateral filter is removed, the filter effectively becomes a Gaussian spatial filter. Domain values can be analyzed, independent of the range component, via the spatial Gaussian filter. Since the box filter has very straightforward noise reduction properties, its noise reduction will be related to the Gaussian spatial filter.

The range component will be analyzed by combining it with a spatial box domain. The noise reduction properties of the bilateral filter (Gaussian range/box domain) will also be related to the box filter. The dependence on noise will be generalized by using a range/noise ratio. A method for selecting the range value based on object contrast will also be examined.

The Results chapter will conclude with an examination of CT phantom and CT clinical images for bone and soft tissue reconstruction kernels. The pre-filter and iterative techniques will be analyzed. These techniques will be essential when the range/noise ratio is low.
The Results section is outlined as follows:

1) **Computer Generated Phantom with Gaussian Noise**
   
a) Domain Component Analysis
b) Range Component Analysis
   
i) Method for Selecting the Range Value

2) **CT Images**
   
a) Soft Tissue Kernel
b) Bone Kernel
c) High Resolution Slice
d) Clinical CT Images

1) **Computer Generated Phantom with Gaussian Noise**
   
a) *Domain Component Analysis*
   
Figure 7 displays the observed and theoretical noise reduction vs. kernel size for the box filter. The theoretical matches very closely to what was observed for a 512×512 image of uniform intensity.
Noise reduction for the Gaussian spatial filter is dependent on the geometric spread, $\sigma_d$, and the kernel size. The value of $\sigma_d$ influences the effective size of the kernel. Figure 8.A-C displays the un-normalized weighting values for $3 \times 3$, $5 \times 5$, $7 \times 7$ and $9 \times 9$ kernel for various domain values.
Figure 8.A-C shows that an increasing domain value broadens the Gaussian weighting value which will be truncated by the limits of the kernel size. For example when $\sigma_d=2.0$ the $3 \times 3$ kernel is effectively a $3 \times 3$ box filter. The figures also show that when $\sigma_d=2.0$ the $3 \times 3$ kernel is effectively a $3 \times 3$ box filter. The figures also show that when
the domain value is small relative to the kernel size, the weighting values fall off to zero with distance from the center. For example, when \( \sigma_d = 1.0 \) and the kernel size is 7×7 the result is an un-truncated Gaussian distribution, see Figure 2.B.

Noise reduction of the spatial Gaussian filter and the box filter can be related by solving Equations 22 and 24 for neighborhood size \( N \). The result is:

\[
N(\sigma_d) = \left( \frac{1}{\sum_{i=1}^{m} \sum_{j=1}^{n} e^{-\frac{1}{2} \left( \frac{(x_i - \text{center}_i)^2 + (y_j - \text{center}_j)^2}{\sigma_d} \right)^2}} \right)^{\frac{1}{2}}
\]  

(26)

Figure 9 plots effective box kernel sizes vs. domain values of the spatial Gaussian filter.
Recall that noise reduction for the box filter is simply $1/\sqrt{n}$. By relating the spatial Gaussian filter to the box filter, through Equations 22 and 24, the influence of the domain value and kernel size on noise reduction can be simplified. When working with the bilateral filter, selection of the domain value and kernel size can be thought of in terms of the box filter through this relationship.

The advantage of the spatial Gaussian over the box filter is that consideration can be given to pixel distance. The box filter weights all of the pixels in a neighborhood with the same value. Through the selection of the domain value, the Gaussian weights pixels based on their distance from the center pixel. Thus, it is expected that the spatial Gaussian’s performs better for edge preservation. A computer generated phantom was created to test how well a filter preserves edges. The phantom described in the Materials chapter is displayed in Figure 10.
A 5×5 spatial Gaussian filter with $\sigma_d=0.86$, produces the same noise reduction (0.34) as a 3×3 box filter. As an example, a 5×5 spatial Gaussian filter with $\sigma_d=0.86$ will be compared to a 3×3 box filter. The filters are applied to the phantom in Figure 10. Figure 11 A-D shows the result.

**Figure 11**

(A) **Highest Contrast Row, First Three Boxes**

3×3 Box Filtered, Low Contrast Boxes

(B) **Highest Contrast Row, First Three Boxes**

5×5 Spatial Gaussian Filtered, $\sigma_d=0.86$

(C) **3×3 Lowest Intensity Contrast Box**

3×3 Box Filtered

(D) **5×5 Spatial Gaussian, $\sigma_d=0.86$**
As can be seen in Figure 11 the spatial Gaussian filter preserved more of the low contrast boxes than the box filter.

Figure 12 shows results for the left side of the intensity pyramid after being filtered. As described in the materials section, the two sides of the intensity pyramid gradually blend to an intensity of 0.5 over 128 pixels. The $5 \times 5$ Gaussian kernel with $\sigma_d = 0.86$ preserved the edge better than the $3 \times 3$ box filter.

![Intensity Pyramid Descending Edge](image)

Figure 12: Intensity Measurements for the left side of intensity pyramid.

The point where the plots merge is when the two filters begin to produce similar results with respect to edge preservation.

The Gaussian’s use of weighting factors reduces the blurring of edges. This is because pixels on the periphery of the kernel are weighted less than the center. The previous
example demonstrated that for equivalent smoothing, the spatial Gaussian filter preserves edges better than the box filter. Thus, the spatial Gaussian filter is a reasonable choice for the domain component of the bilateral filter.

The point of thoroughly examining the Gaussian spatial filter is that the range filter has no notion of space, Tomasi et al. (1998). Therefore, the range filter must be combined with a domain filter. When using a spatial Gaussian for the domain, the kernel size and $\sigma_d$ will determine how large of a spatial area the photometric spread, $\sigma_r$, will extend.

b) Range Component Analysis

The range component of the bilateral filter is represented graphically in Figure 13. The graph displays how weighting factors are assigned when the kernel is centered on intensity 0.5. The y-axis is the assigned relative weighting factor for a given intensity. The intensities within the curve are weighted based on their similarity from the center. The width of the curve Gaussian curve of Figure 13 is controlled by $\sigma_r$, the photometric spread. If the $\sigma_r$ is broader than the intensity spread, in a particular neighborhood, the response of the filter will be similar to a domain filter only. That is the domain component dominates the filter.
The bilateral filter is dependent on the noise level of an image. This is because the weighting factors for the range component are found by examining the intensities of pixels in a neighborhood in reference to the center pixel. The range value with respect to noise will influence the noise reduction of the filter. Figure 14 is a plot of the effective box kernel size vs. $\sigma_r/\sigma_{\text{noise}}$ for a uniform area.
In Figure 14, a box filter was chosen as the domain component so an equivalent box size could be found. Recall, for noise reduction in a uniform area, the spatial Gaussian, for a given domain value and kernel size, could be related to an equivalent box size. Therefore if a spatial Gaussian were used as the domain component, the domain value and kernel size could be derived from Figure 9 for a given $\sigma_r/\sigma_{noise}$.

When the plots on the graph of Figure 14 reach $dN/(d \sigma_r/\sigma_{noise}) \approx 0$, the bilateral filter effectively becomes a box filter, that is it averages all values within the kernel. The only limiting factor of the bilateral filter, when using a box as the spatial component, is $\sigma_r$ and the kernel size. The noise reduction of the bilateral filter with a Gaussian spatial component is represented by Equation 27.

$$\frac{\sigma(\bar{x})}{\sigma} = \frac{1}{k} \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} e^{-\frac{1}{2} \left[ \frac{(f(x_i,y_j) - f(\text{center}_{i,j}))}{\sigma_{\text{range}}} \right]^2 \left[ \frac{(x_i - \text{center}_x)^2}{\sigma_{\text{domain}}} + \frac{(y_j - \text{center}_y)^2}{\sigma_{\text{domain}}} \right]^2} \right]^{\frac{1}{2}}$$

(27)

Note from the equation that if $\sigma_d$ or $\sigma_r$ is large, the respective term approaches zero. When this happens the response of the filter becomes either a domain or range dominated filter.

For noise reduction analysis a Gaussian range and a box domain component was used. In doing so the bilateral could be linked to an effective box kernel, for a given $\sigma_r/\sigma_{noise}$. If
a spatial Gaussian was chosen for the domain component the kernel size and domain value, of the spatial Gaussian, could be derived from the effective box kernel size for a given $\sigma_r/\sigma_{noise}$.

To compare edge preservation, the spatial Gaussian filter and bilateral filter were matched with respect to noise reduction. A $5 \times 5$ spatial Gaussian filter with a $\sigma_d$ value of 0.65 produced fractional smoothing (uniform area) of 0.380. The bilateral with a $\sigma_d$ value of 1.2 and $\sigma_r$ of 0.195 produced fractional smoothing (uniform area) of 0.384.

Figure 15 A & B display the result of filtering the phantom with the above mentioned parameters.
5×5 Bilateral Filtered $\sigma_d = 0.65$

5×5 Bilateral Filtered $\sigma_d = 1.2$, $\sigma_r = 0.195$

Figure 15
Note from Figure 15 that the spatial Gaussian filtered image blurred more than the bilateral filtered image. This is especially noticeable for the high contrast resolution bars as shown in Figure 16.

![High Contrast Bars](image1)

<table>
<thead>
<tr>
<th>High Contrast Bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 5$ Bilateral Filtered $\sigma_d=1.2$, $\sigma_r=0.195$</td>
</tr>
<tr>
<td><img src="image2" alt="A" /></td>
</tr>
</tbody>
</table>

| $5 \times 5$ Spatial Gaussian Filtered $\sigma_d=0.65$ |
| ![B](image3) |

Figure 16

With regard to the low contrast boxes, Table I shows how the range weighting factor changed for each row of the low contrast boxes.

![Low Contrast Boxes](image4)

<table>
<thead>
<tr>
<th>Low Contrast Boxes</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5" alt="Table I" /></td>
</tr>
</tbody>
</table>

Table I

Table I shows that the low contrast boxes in row five will have the best edge preservation. The importance of choosing an appropriate value of $\sigma_r$ is critical for edge preservation. The weighting factors in Table I are large enough that when the kernel is
centered on a low contrast box and part of the kernel includes background pixels, those background pixels will be assigned fairly large range weighting factors. This will cause blurring of the low contrast boxes.

Figure 17 are plots of the center of the left side of the intensity pyramid.

![Bilateal vs. Gaussian Spatial Filter](image)

<table>
<thead>
<tr>
<th>Intensity Pyramid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pixel Location</td>
</tr>
<tr>
<td>0.00</td>
</tr>
<tr>
<td>0.10</td>
</tr>
<tr>
<td>0.20</td>
</tr>
<tr>
<td>0.30</td>
</tr>
<tr>
<td>0.40</td>
</tr>
<tr>
<td>0.50</td>
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<td>100</td>
</tr>
<tr>
<td>110</td>
</tr>
<tr>
<td>120</td>
</tr>
</tbody>
</table>

What is observed in Figure 17 is that not until approximately pixel seventy does the bilateral filter begin blur the edge. At pixel seventy the left side of the pyramid has an intensity of 0.24 and the right side 0.76. Thus, for the given range and domain values, the bilateral would preserve edges that have an intensity percent difference of up to 68% to their neighbors. After which the bilateral gradually responds more like a spatial Gaussian filter. The domain and range values of the previous example were chosen to compare the bilateral to the spatial Gaussian filter for equivalent noise reduction. A more precise method for choosing the range value will be discussed later.
The performance of the bilateral was next analyzed with noise of $\sigma = 0.1$ added to the phantom image. As before, the bilateral parameters were: a kernel size of $5 \times 5$, domain of 1.2 and range of 0.195. For the spatial Gaussian filter the kernel size was $5 \times 5$ and the domain value 0.65. Recall those parameters produced the same noise reduction in a uniform area. The results for the low contrast boxes are displayed in Figure 18 A-D.
The bilateral filter preserved the low contrast box edges better than the other filters. Also note that some noise outliers were preserved by the bilateral filter. This occurs when the kernel is centered on an outlier. Low weighting values will be assigned to the rest of the pixels in the neighborhood. This will cause some outliers to be preserved.
Figure 19 shows the largest (11×11) contrast box with the highest intensity (0.67) for the unfiltered and filtered images.

The median filtered image, especially the 5×5, is more uniform than the others. This is because the median filter selects the median of the neighborhood. Thus few outliers will be preserved.
b.i) *Method for Selecting the Range Value*

The range value in the previous example was chosen only to compare the bilateral filter to the spatial Gaussian filter for equivalent noise reduction. A method for selecting the range value based on object contrast is now examined.

When the kernel is centered on an object, a linear filter will use the pixels in the neighborhood without regard to their intensity similarity to the center. If those pixels surrounding the center are dissimilar to the center they will destroy the center pixel’s uniqueness. A linear filter may extend past the object and pull in pixels that can be very different in intensity. If a small object is filtered with a filter larger than the object, that object can be blurred from the image. An adaptive filter, such as the bilateral, excludes or includes pixels based on the unique intensity characteristics of a neighborhood. The range value will evaluate pixels in the neighborhood based on their intensity similarity to the center of the kernel. If a pixel is spatially distant from the center but similar in intensity it will receive a higher range weighting factor.

Examining the equation for the range component, of the bilateral filter, provides insight into the selection of the range value.

\[
W(\sigma_r) = \frac{1}{k} \left( \sum_{i=1}^{m} \sum_{j=1}^{n} e^{\frac{1}{2} \left( \frac{f(x_i, y_j) - f(\text{center}_{x,y})}{\sigma_r} \right)^2} \right)
\]  

(28)
If $\sigma_r = f(x,y)_{object} - f(\text{center}_{x,y})$, then the range weighting factor is approximately 0.606, $(w(\sigma_r) = e^{-1/2} = 0.606)$.

For example, when the center of the kernel is placed outside one of the phantom low contrast boxes, pixels of the neighborhood which happen to be inside the low contrast box should be weighted the least. When $\sigma_r = (1/4)(f(x,y)_{object} - f(\text{center}_{x,y}))$ the weighting factor for pixels inside a low contrast box approach zero. Thus, to eliminate the background from being blurred into the lowest contrast box (box intensity = 0.526, background intensity=0.5) a $\sigma_r = (1/4)(0.526 - 0.5)$, would separate this low intensity contrast box from background. Figure 20 shows approximate weighting factor for different values of $\sigma_r$.

![Bilateral Filter Range Weighting Factors](image)
The abscissa, of Figure 20 is the photometric difference \( (k \cdot (f(x,y)_{\text{object}} - f(\text{center}_{x,y}))) \).

For example when \( \sigma_r = f(x,y)_{\text{object}} - f(\text{center}_{x,y}) \), a pixel whose difference with the center pixel of the neighborhood is \( \sigma_r \), that pixel will receive a range weighting value of approximately 0.6. If the difference with the center pixel was \( 2 \cdot \sigma_r \), that pixel would receive a range weighing factor of approximately 0.15.

Figure 21 illustrates how using the general range values in Figure 20 will effect the response of the filter for noiseless data. The 11×11 low contrast box with intensity of 0.76 and for 0.55 on a background of 0.5 is used as an example.

Response of the Bilateral Filter

Figure 21
When the range value is high \(4 \cdot \Delta(I_{\text{Box}}, I_{\text{Background}})\) the smoothing result is similar to the spatial Gaussian filter. When the range value is lower \(\Delta(I_{\text{Box}}, I_{\text{Background}})/4\) the \(11 \times 11\) low contrast box is preserved completely. For reference, the \(11 \times 11\) low contrast box with an intensity of 0.55 (lower right image of Figure 21) will be blurred when the range value is set using the 0.76 intensity box. This demonstrates how some objects in an image may be filtered in a different way than others.

The method for selecting the range described previously has limitations. It was shown earlier in Figures 9 and 14 that the ratio \(\sigma_r/\sigma_{\text{noise}}\) should be relatively high for acceptable noise reduction. In order to use low range values with respect to noise, a pre-filter and/or the bilateral should be iterated.

The bilateral filter can be used as a pre-filter. When \(\sigma_r/\sigma_{\text{noise}}\) results in little or no smoothing, the ratio can be increased with a higher range value for the bilateral pre-filter. The range value then can the be lowered for subsequent iterations after noise level has been reduced by the pre-filter.

The spatial Gaussian could also be used for the domain component. The domain value in Figure 9 and \(\sigma_r/\sigma_{\text{noise}}\) value in Figure 14 can be combined into the bilateral filter for a certain equivalent box size. Using a spatial Gaussian for the domain component instead of the box filter is advantageous given the better edge preservation of the spatial
Gaussian. Table II shows the change in noise reduction when the domain value from Figure 9 and $\sigma_r/\sigma_{\text{noise}}$ values from Figure 14 are combined.

Table II

<table>
<thead>
<tr>
<th>Bilateral: Gaussian Range/Box Domain</th>
<th>$\frac{\sigma_r}{\sigma_{\text{noise}}}$ (For Column 1)</th>
<th>Spatial Gaussian</th>
<th>$\sigma_d$ (For Column 2)</th>
<th>Box Equivalent Size (For columns 1 &amp; 3)</th>
<th>Adjusted Box Equivalent Bilateral Filter (Columns 2 &amp; 4)</th>
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<td>8.33×8.33</td>
<td>8.09×8.09</td>
</tr>
</tbody>
</table>

Values in Table II obtained from a uniform area.

Table II is found by merging Figure 9 and 14. Columns one and two are the values for the bilateral filter that produced the noise reduction in column five. Columns three and four are the values for the spatial Gaussian filter which produced the noise reduction also in column five. Column six is the box equivalent size for the bilateral filter using the values of column two and four. The range value from column two can be extracted by solving for $\sigma_r$. The purpose of Table II is to link noise reduction of the spatial Gaussian filter and bilateral filter (with a box domain) through the box filter. Table II shows the
effect the range value has on noise reduction with a Gaussian range and Gaussian
domain.

As an example, noise with \( \sigma = 0.04 \) is added to the phantom. The lowest contrast boxes
in the phantom have an intensity of 0.56. Recall the background intensity is 0.5. Thus a
range value of 0.015 (\( 0.56 - 0.5 = 0.06 \) and \( 0.06/4 = 0.015 \)) would be the optimum range
value to preserve all of the low contrast boxes in the phantom. The value of
\( \sigma_r/\sigma_{noise} \) (0.015/0.04) is 0.375. Thus the noise reduction is approximately zero, see
Figure 14.

In order to use a range value of 0.015, the noise must be reduced. A 3.48×3.48 box filter
will reduce the noise by approximately by 1/3, see Table II column six. With a noise
level of 0.04, the noise should be reduced to about 0.01. This is below the desired range
value of 0.015. The \( \sigma_r/\sigma_{noise} \) for a 3.48×3.48 box filter is 2.7, see Table II. Thus a range
value of 0.108 will be used for the pre-filter (\( \sigma_r/0.04 = 2.7 \) and \( \sigma_r = 0.108 \)) with a
domain value of 1.39, see Table II.

The measured noise, after the bilateral pre-filter, is 0.0112. The unfiltered and filtered
images are shown in Figure 22 A and B.
A range value of 0.015 can be used now that the $\frac{\sigma_r}{\sigma_{noise}}$ has been increased, 0.015/0.011=1.25.

In summary the range value can have a global edge preservation if it is determined by using the lowest contrast object in the image. In doing so all objects higher in contrast will be preserved. If however, $\frac{\sigma_r}{\sigma_{noise}}$ is low, a pre-filter may need to be applied.

A link for noise reduction was established between the spatial Gaussian and bilateral (box domain) filter through the box filter. This allowed a better prediction of the noise reduction for a given value of $\sigma_d$ and $\frac{\sigma_r}{\sigma_{noise}}$. A method was analyzed for determining the range value based on object contrast. It was shown that finding the difference between two intensities and then dividing that difference by four roughly provides the maximum edge delineation between those to intensities.
When $\sigma_r/\sigma_{\text{noise}}$ is relatively low it was shown that noise reduction may be minimal and a pre-filter could be used. The bilateral filter can work as a pre-filter by raising the range value. After an image is pre-filtered and the noise is reduced to an acceptable level, the range value can then be lowered. The median and averaging filter can also be used as pre-filters. They will be analyzed as pre-filters with CT images.

2) CT Images

Noise in CT images is correlated (see Introduction for a description of CT noise). In CT, images are smoothed as part of the reconstruction process. The soft tissue reconstruction kernel smoothes an image more than the bone reconstruction kernel. This is because the soft tissue kernel has more high frequency roll off. The bone kernel will produce higher resolution images and the soft tissue will have less image noise. Table III lists the noise for each mAs level and reconstruction kernel type.

<table>
<thead>
<tr>
<th>mAs</th>
<th>Soft Tissue $\sigma_{\text{noise}}$ (HU)</th>
<th>Noise Bone $\sigma_{\text{noise}}$ (HU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>3.25</td>
<td>12.7</td>
</tr>
<tr>
<td>400</td>
<td>4.9</td>
<td>18.95</td>
</tr>
<tr>
<td>200</td>
<td>6.9</td>
<td>26.73</td>
</tr>
<tr>
<td>100</td>
<td>9.8</td>
<td>37.97</td>
</tr>
</tbody>
</table>

Table III
Table IV lists the data for the largest target of the low contrast slice for each mAs and reconstruction kernel. The purpose of the table is to ensure that the data is consistent for all the CT groups.

<table>
<thead>
<tr>
<th>mAs</th>
<th>Large Target Avg. CT #</th>
<th>Background Avg. CT #</th>
<th>% Contrast</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>58.4</td>
<td>47.04</td>
<td>19.45</td>
</tr>
<tr>
<td>400</td>
<td>57.81</td>
<td>46.35</td>
<td>19.82</td>
</tr>
<tr>
<td>200</td>
<td>57.77</td>
<td>46.41</td>
<td>19.66</td>
</tr>
<tr>
<td>100</td>
<td>58.27</td>
<td>45.9</td>
<td>21.23</td>
</tr>
</tbody>
</table>

**Table IV**

### Soft Tissue Kernel

### Bone Kernel

The CT phantom used for the following analysis is described in the materials and methods section. Noise in CT is multiplicative and due to the reconstruction process, correlated. The goal of this section is to study the bilateral’s performance with CT noise. The CT phantom was scanned using the soft tissue and bone kernels. The slices were 4mm and the mAs levels were: 400, 200 and 100.
a) Soft Tissue Kernel

Earlier it was found that noise reduction for the spatial Gaussian filter could be related to the spatial box filter, for a given kernel size and domain value, through the following equation:

\[
\text{Area}(\sigma_d) = \frac{1}{\sum_{i=1}^{m} \sum_{j=1}^{n} e^{\frac{1}{2} \left( \frac{\sqrt{(x_i - \text{center}_x)^2 + (y_j - \text{center}_y)^2}}{\sigma_d} \right)^2}} \quad (29)
\]

Noise reduction for the box filter, for the soft tissue reconstruction kernel over several mAs levels was found to be approximately:

\[
\sigma(\bar{x})/\sigma = 2 \cdot \frac{1}{\sqrt{N}} \quad (30)
\]

This relationship was verified within 12% for images reconstructed with the soft tissue kernel. The adjustment of the noise reduction equation for the box filter is because images are smoothed from the soft tissue reconstruction kernel.

Figure 23 are plots of the effective box kernel size vs. \( \sigma / \sigma_{\text{noise}} \) of bilateral box domain filter.
In Figure 23 the least amount of noise reduction occurs when the range value is low with respect to the noise. Also note that the equivalent box size values in Figure 23 are about half of those in Figure 14.

In CT a high projection value causes negative contributions to the adjacent filtered projection values. This appropriately reconstructs the image from the true signal data, when combined with other filtered data in the backprojection process. However, noise is random and the negative contribution to adjacent projection values causes correlation of the noise in the reconstructed images. Due to the correlation, when the high noise projection value occupies the center of a neighborhood, the neighbors will receive a low range weighting factor. This may tend to preserve the noise with bilateral filtering. A
potential solution is to use a median filter prior to using the bilateral filter. In doing so the median of the neighborhood, rather than the noise spike will be used.

Noise will ultimately limit what range value can be used. If the range value is low with respect to the noise there will be little noise reduction, see Figure 23. A solution is to remove some noise from the image prior to applying the bilateral filter. This can be done with a median filter. The median filter has good edge preserving properties and is more likely to be representative of the neighborhood. The bilateral filter can also be iterated several times after the median filter is applied. This will reduce the noise further and preserve edges.

As described, if the range value is \( \Delta(I_{\text{target}}, I_{\text{background}}) \), and the kernel is centered on a target pixel, the background pixels will receive a weighting factor of approximately 0.6. The approximate point at which the bilateral filter will respond as a spatial filter only (the range component effectively becomes one) is when the range value is large i.e. range = \( 4 \times \Delta(I_{\text{target}}, I_{\text{background}}) \). When the range = \( \Delta(I_{\text{target}}, I_{\text{background}}) / 4 \) the range weighting factor will assign roughly zero to the background pixels. The strategy listed above is a rule of thumb and information about the contrast levels of the image must be known. Distinguishing between CT numbers can be done through this method.

Figure 24 is the 900mAs 16mm soft tissue reconstructed low contrast slice used to measure the target CT numbers.
The largest target in the group of targets was measured to obtain the average CT value. The groups are labeled one through six. Target groups four through six are rods of constant attenuation and variable lengths. Their average CT number is similar to target group one.

Table V displays the average CT# for each target and background.

<table>
<thead>
<tr>
<th>Target Group</th>
<th>Average (HU)</th>
<th>Background (HU)</th>
<th>Difference (HU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>58.35</td>
<td>47.26</td>
<td>11.09</td>
</tr>
<tr>
<td>2</td>
<td>53.9</td>
<td>47.95</td>
<td>5.95</td>
</tr>
<tr>
<td>3</td>
<td>52.43</td>
<td>48.2</td>
<td>4.23</td>
</tr>
</tbody>
</table>
Example 1: 400mAs/4mm Soft Tissue Reconstructed Low Contrast Slice

If the difference of target group three and background were selected as the range value and the kernel is centered on a target in group three, a typical background pixel within the neighborhood will receive a weighting value of approximately 0.6. Table VI displays the weighting factors for the background when the kernel is centered on a target group when the range value is 4.23HU.

<table>
<thead>
<tr>
<th>Target Group</th>
<th>Weighting Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05462</td>
</tr>
<tr>
<td>2</td>
<td>0.41631</td>
</tr>
<tr>
<td>3</td>
<td>0.61083</td>
</tr>
</tbody>
</table>

\[ \sigma_r = 4.23HU \]

The values in Table VI are dependent on how close the measured average intensity of each target is to the actual intensity of the target. It also depends on the how close the background is to the actual background. Thus, these weighting factors are approximate.

A range value of 4.23HU assigns a relatively high range value to background pixels. Therefore, the range value is decreased to 2.5HU which is \( \Delta(I_{Group}, I_{Background})/4 \). This range value will assign approximate range weighting factors listed in Table VII.
Table VII

<table>
<thead>
<tr>
<th>Target Group</th>
<th>Background Weighting Factor, $\sigma_r = 2.5\text{HU}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.03</td>
</tr>
<tr>
<td>3</td>
<td>0.12</td>
</tr>
</tbody>
</table>

$\sigma_r = 2.5\text{HU}$

Although these are rough values, they provide a general idea on how the bilateral filter will range weight background pixels or target pixels when the kernel is centered on a target pixel or background pixel. The measured noise for the 400mAs 4mm soft tissue reconstructed slice is 4.42HU. If a range value of 2.5HU were to be used $\sigma_r/\sigma_{\text{noise}} = 0.57$. Referring to Figure 23, the noise reduction would be approximately zero for the bilateral box domain filter.

A pre-filter will be necessary given the lower range value. A $5 \times 5$ median filter is chosen. It was found that noise reduction for the box filter and median filter were similar, see Figure 25.
However, edge preservation should be better with the median filter. The noise will be reduced to approximately 2.21HU with a $5 \times 5$ median pre-filter.

The images in Figure 26 A and B demonstrate that applying a median pre-filter followed by the bilateral does not negatively impact edge preservation.
There was little change in edge preservation between A and B. There was a noticeable decrease in noise for Figure 26.B. This demonstrates that applying a median pre-filter prior to the bilateral does not degrade edges any more than the median filter alone.

The iterative approach was next applied to the 400mAs soft tissue reconstructed image. The image was first processed with a $5 \times 5$ median filter and then with three iterations of the bilateral filter. The result is displayed in Figure 27 A-E.
Iterative vs. Non-Iterative

A
5x5 Median 9x9 Bilateral
D/R Values: 2, 2.5HU
Noise: 1.54HU

B
5x5 Median 9x9 Bilateral
3 Iterations D/R Values: 2, 2.5HU
Noise: 0.98HU

Target I Group I

Non-Iterated Bilateral
Iterated

C
D
Target I Group II
Non-Iterated Bilateral  Iterated

Target I Group III
Non-Iterated Bilateral  Iterated

Target 1 Group 1
Spatial Gaussian Filtered, $11 \times 11$, $\sigma_d = 3$
Noise Level: 1.1HU

Figure 27
From Figure 27 the iterated bilateral had significant noise reduction over the non-iterated bilateral. Figure 28 shows the subtracted image (iterated bilateral – non-iterated) for the first target of group one.

Target Group I Targets I & II
(5x5 Median 9x9 Bilateral 3 Iterations $\sigma_d$ & $\sigma_r$ Values: 2, 2.5HU) – (5x5 Median 9x9 Bilateral $\sigma_d$ & $\sigma_r$ Values: 2, 2.5HU)

![Figure 28](image)

The first and second targets of group one showed some difference between the two images. The slightly higher intensity inside the first two targets of group one means that the non-iterated bilateral was lower in intensity inside the target. The lower intensity ring on the outside of the target means the non-iterated bilateral was higher in intensity. This translates into more blur inside and outside the target for the non-iterated bilateral.

Some artifacts are present in Figure 29.B (5×5 Median, 9×9 Bilateral 3 Iterations, $\sigma_d = 2$, $\sigma_r = 2.5$HU). These can be reduced by using a larger pre-filter. The results are displayed in Figure 29 A and B.
Artifact Suppression

Figure 29

Using larger pre-filter reduced the noise more than the iterated smaller pre-filter, Figure 29.A.

As a final example the bilateral filter is compared to the median filter. The results are displayed in Table VIII. The spatial Gaussian filter is also displayed for reference.

Table VIII

<table>
<thead>
<tr>
<th>Filter</th>
<th>Pre-Filter</th>
<th>Range</th>
<th>Kernel Size</th>
<th>Group 1 Target 1</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian, D=3</td>
<td>None</td>
<td>N/A</td>
<td>15x15</td>
<td>0.68</td>
<td>1</td>
</tr>
<tr>
<td>Median</td>
<td>None</td>
<td>N/A</td>
<td>15x15</td>
<td>0.64</td>
<td>1</td>
</tr>
<tr>
<td>Med/Bilat, D=2</td>
<td>Median, 7x7</td>
<td>2.5</td>
<td>9x9</td>
<td>0.71</td>
<td>3</td>
</tr>
<tr>
<td>Bilateral, D=5</td>
<td>None</td>
<td>100</td>
<td>15x15</td>
<td>0.67</td>
<td>1</td>
</tr>
</tbody>
</table>
From Table VIII, the bilateral without a pre-filter or iterations must have a range value of 100 before it can match the noise reduction of the other filters. With a range value of 100 the response of the bilateral would effectively be a spatial Gaussian filter. To compare edges, Figure 30A-D shows selected images from Table VIII.
For comparable noise reduction the bilateral filter preserved more edges than the median filter. Figure 31 is the result of subtracting the previous two images, the $15 \times 15$ median was subtracted from the iterative bilateral Figure 30 A and B.

The targets in group two are the second lowest in contrast from the background and are displayed in Figure 32.
The iterative technique is useful for reducing noise and at the same time maintaining edges. For the median filter or spatial Gaussian filter to produce the same amount of noise reduction as the iterative bilateral, their kernel sizes had to be larger and it is advantageous to use smaller kernel sizes. Smaller objects can be effectively smoothed out of an image with large kernel sizes. Thus, the iterative bilateral is a sound technique for filtering low contrast images.

When selecting a range value consideration should be given to the noise level and what structures in an image are desired to be preserved. When the range is low with respect to the noise level the noise reduction will be minimal. If set too high the filter’s response is similar to a spatial Gaussian filter. In general, for maximum edge preservation, the range value should be set as low as possible.

The techniques demonstrated are ways of lowering the noise level so that a lower range value can be used. In general knowledge of how the range value effects noise reduction
should be characterized. It was also shown that the bilateral filter can be compared to the spatial box filter for equivalent smoothing. The noise reduction for the box filter is approximately $2 \cdot \frac{1}{\sqrt{N}}$ for the soft tissue reconstructed images. Being able to relate the bilateral filter to the spatial box filter through the range value allows one to understand, with a given noise level, how well a particular range value will fair. If the range must be kept low, to preserve low contrast structures, the median pre-filter and/or iterative techniques can help achieve increased noise reduction and at the same time maintain edges. Using the lowest contrast object for the calculation of the range is advantageous in that all other structures will be preserved.

**Example 2: 100mAs/4mm Soft Tissue Reconstructed Low Contrast Slice**

When filtering in high noise environments, such as the 100mAs low contrast slice, the iterative bilateral filter and or a pre-filter will be necessary. The noise level for the 100mAs slice is 9.06HU. The value of $\frac{\sigma_r}{\sigma_{\text{noise}}}$ is 0.46, with a $\sigma_r$ value of 4.23HU. The range value was increased due the high noise. A $9 \times 9$ median pre-filter will reduce the noise level to roughly 2.5HU. The bilateral with a range value of 4.23HU can then be applied with iterations. The results are displayed in Figure 33 A and B.
The image in Figure 33.A shows some globule artifacts. Because the soft tissue reconstruction kernel has a fair amount of smoothing, the bilateral filter will preserve these globules. Therefore, a larger pre-filter and multiple iterations should be used to smooth away the globules.

b) Bone Reconstructed Images

Figure 34 displays the results of the noise reduction for the theoretical and actual box filter. The theoretical is based on: \( \frac{\sigma(x)}{\sigma} = 1/\sqrt{N} \). Where \( \sigma(x) \) is the noise of the processed image.
After a kernel size of $3 \times 3$, the box filter for bone kernel reconstructed images begins to respond closer to theory. This may be due to the fact that with negatively correlated noise as found in the bone reconstruction kernel, the high center spike from the reconstruction process raises the average for small kernel sizes. As the kernel sizes become larger there are more pixels in the neighborhood to average out the high intensity spike. On the other hand, for the median filter, the midpoint between the high center intensity spike and the low may be higher. Thus the noise becomes preserved with these two filters.

Figure 35 shows the plots of the box equivalent vs. $\sigma_r/\sigma_{\text{noise}}$. 
For the phantom the range values of interest are from approximately 1-5HU. These range values would allow very little smoothing for the bone reconstructed images. Thus, the noise should be reduced prior to applying the bilateral and iterations of the bilateral will be needed to use those range values.

Contrast for target groups two and three in Figure 36 are quite comprised by the high noise.
A large box filter will be first applied and the range value increased from 2.5HU. The difference between the targets in group three, the lowest contrast targets, and the background is 4.23HU. Using 4.23HU for the value of $\sigma_\text{r}$ would assign an approximate range weighting value of 0.6 to any background pixels which have a difference from the center pixel of 4.23HU. The bilateral kernel size will also be increased to an $11 \times 11$.

The results are displayed in Figure 37 A-C. The 400mAs soft tissue reconstructed image is included in Figure 37 along with a median filtered image for comparison.
Bone Reconstruction

A
9 × 9 box pre-filter, 11 × 11
Bilateral $\sigma_d = 3$, $\sigma_r = 4.23$HU, 2
Two Bilateral Iterations
Noise: 0.73HU

Soft Tissue Reconstruction

B
400mAs/4mm 7 × 7 Median Pre-Filter, 9 × 9 Bilateral $\sigma_d = 2$
$\sigma_r = 2.5$HU, 3 Bilateral Iterations
Noise: 0.76HU
The image in Figure 37.A was subtracted from the image in 37.B. The result is displayed in Figure 38.
Note that the soft tissue reconstructed image, Figure 37.B blurred slightly less than the bone reconstructed image. There was little change for target groups two and three.

Figure 39 shows the first target of group one for both images.
There was slightly more blur with the bone reconstructed image. Relatively speaking, the noise reduction was far more significant with the bone reconstructed image than the soft tissue reconstructed image. Recall the soft tissue kernel reconstruction positively correlates the noise which means there is greater smoothing in the original image. The bone kernel negatively correlates the noise and there is less smoothing in the original reconstructed image, but greater noise reduction with the image post-processing.

In summary, for high noise environments like the 100mAs soft tissue kernel and the 400mAs bone reconstructed images, the bilateral filter can reduce noise significantly and yet preserve edges. Depending on the value of $\sigma_r/\sigma_{\text{noise}}$ a pre-filter should be used. The median filter works well as pre-filter and showed negligible edge degradation compared to the median filter alone. Iterating the bilateral filter is also a viable technique which does not degrade edges.
Choosing an appropriate range value will ensure edge preservation. The range value was chosen based on delineating CT phantom targets from their background. For global edge preservation the range value should be set with the lowest contrast object in mind. All objects of higher contrast will be preserved this way.

The type of reconstruction kernel will influence how well noise reduction will occur. For the soft tissue reconstructed images it was observed that, for the box filter, \( \sigma(\bar{x})/\sigma = 2 \cdot 1/\sqrt{N} \). This is because there is a considerable amount of smoothing from the reconstruction process. On the other hand the bone reconstructed images have less smoothing from the reconstruction process. Thus, noise reduction will be greater with the pre-filter for bone reconstructed images.

c) High Resolution Slices

The high resolution slices were not pre-filtered as they are considerably higher in intensity than background. The bone reconstructed 400mAs/4mm resolution slice was used. The noise level was measured as 20.52HU. The bars are closely spaced so a small kernel size will be used. A \( 3 \times 3 \) kernel with a domain=0.5 will be used on the bone reconstructed 100mAs image.

For the bone kernel the maximum smoothing for the \( 3 \times 3 \) kernel occurs when \( \sigma_r/\sigma_{noise} = 2.5 \), see Figure 35. To find the range value: \( \sigma_r/20.52 = 2.5; \sigma_r = 51.3 \)HU. The results are displayed in Figure 40 A-C.
400mAs/4mm Bone Reconstructed, Bilateral Filtered

A
3 × 3 Bilateral/Box Domain
\( \sigma_r = 12.8 \text{HU} \),
Noise: 11.55HU

400mAs/4mm Bone Reconstructed, Spatial Gaussian Filtered

B
3 × 3 Spatial Gaussian \( \sigma_d = 0.5 \)
Noise: 16.57HU
Figure 40. C shows that the bilateral box domain filter blurred somewhat less than the spatial Gaussian filter. A bilateral with a box domain was chosen to maximize noise reduction. Figure 40 A-C indicates that the bilateral filter can preserve high resolution structures as well as the spatial Gaussian filter. In addition the bilateral filtered image had greater noise reduction.

In summary, the bilateral filter was applied to CT phantom images. The noise in CT is correlated, multiplicative noise. It was shown that the bilateral can successfully be deployed in this type of noise environment. Applying a pre-filter can improve the results of the bilateral filter. In addition reprocessing the image with many passes of the bilateral improved results as well.
d) Clinical CT Images

The clinical image used in this section is shown in Figure 41.

The image is of a gun shot wound with associated bleeding. Measurements were made of the average CT number in the wound area and the surrounding area. The wound area was approximately 65HU and the surrounding tissue about 32HU. The wound region had minimal distance of 2mm. The field of view is 240mm and the matrix size is $512 \times 512$. The pixel size was calculated to be: $240\text{mm}/512 = 0.47\text{mm}$. Thus a $3 \times 3$ filtering kernel was selected for the processing of this image.

To separate the wound area from background a range value of about 5-8HU is used, $(65\text{HU} - 32\text{HU})/4 = 8.25\text{HU}$. This will assign weighting factors of approximately zero for
any pixels that have a difference of 8.25HU from the center pixel. The value of \( \sigma_r/\sigma_{\text{noise}} \) is 8.25/3.5 = 2.36. For the soft tissue reconstruction filter a bilateral box domain filter would with a range of 8.25HU would be approximately equivalent to a 1.5 \times 1.5 box filter, see Figure 24. The CT numbers of the wound area and surrounding area vary. Thus, to be conservative, range values below 8.25HU are used for less noisy images. The techniques used for the CT phantom analysis will be applied to the clinical images. Figure 42 A and B shows the 5mm Head Smooth reconstructed image.

![Original Unfiltered Image, \( \sigma_{\text{noise}} = 4.89 \)HU](image)

Head Smooth 5mm Slice, Bilateral Filtered
The subtracted image in Figure 42.C shows that there is negligible edge blur after the bilateral filter was applied. The benefit of using the bilateral filter in this image was reduced noise and insignificant edge blur.
Next the 5mm bone sharp reconstructed image is analyzed. The results are shown in Figure 43 A-C.

**A**
Unfiltered, 5mm Bone Sharp, $\sigma_{\text{noise}} = 38.52$HU

**B**
Bone Sharp 5mm Slice, $\sigma_{\text{noise}} = 8.68$HU
3x3 Box Pre-Filter, 3x3 Bilateral, $\sigma_d = 0.8, \sigma_r = 8$, 2 Iterations
The bullet fragments are more discernable with the filtered bone sharp kernel. A box pre-filter was first passed over the image to reduce the high noise spikes. In addition the range value was increased to 8HU to further suppress noise spikes. Figure 43.C shows that there was very little edge degradation after filtering the image.

Finally the 2mm head smooth and bone sharp reconstructed images are filtered. Figure 44 A and B show the head smooth reconstructed image.
Unfiltered 2mm Head Smooth Reconstructed, $\sigma_{\text{noise}} = 9.18$HU

2mm Head Smooth Reconstructed Bilateral Filtered

$3 \times 3$ Median Pre-Filter, $3 \times 3$ Bilateral, $\sigma_d = 0.8, \sigma_r = 5, \sigma_{\text{noise}} = 5.73$HU

Figure 44

The 2mm bone sharp reconstructed image is displayed in Figure 45 A and B.
Unfiltered 2mm Bone Sharp Reconstructed

(A) Unfiltered 2mm Bone Sharp Reconstructed, $\sigma_{\text{noise}} = 72.56$HU

Unfiltered 2mm Bone Sharp Reconstructed Bilateral Filtered

(B) 2mm Bone Sharp Reconstructed
3×3 Box Pre-Filter, 3×3 Bilateral $\sigma_d = 0.8$, $\sigma_r = 10$HU, $\sigma_{\text{noise}} = 17.19$HU

Figure 45
For the image in Figure 45.B the range value was increased to 10HU. This was done to decrease spikes and reduce the noise. When the range value was 9HU, with all the other parameters the same, the noise was slightly higher, 18.52HU.

The clinical images presented demonstrate the utility of the bilateral filter on clinical CT images. The noise level in these images, especially the bone reconstructed images, was at a level that the range values required to separate the injured area from the normal area would have little noise reduction. Thus, a pre-filter and/or iterations of the bilateral were applied. A box filter and higher range value was used for the high resolution images. This assisted in decreasing the noise spikes.

When applying the bilateral filter to CT images consideration should be given to the reconstruction kernel. A soft tissue reconstruction kernel will have less noise and less resolution. Thus, if a pre-filter is necessary the median pre-filter is preferable over linear smoothing filters. The median filter typically maintains edges better than linear smoothing filters. In addition increasing the range value may blur the edges further due to the reduced resolution.

The bone reconstructed images will have better resolution and less contrast. A pre-filter will have greater noise reduction as the bone kernel smoothes less than the soft tissue kernel. Using a box filter as the pre-filter is possible since the bone reconstructed images will have better defined edges. The examples presented illustrate that the bilateral filter can be successfully used with CT images.
Summary of Results & Discussion

The object of this paper was to examine the parameters of the bilateral filter. In order to understand the impact they have on the overall response of the filter they were separated. The domain component was studied by using the spatial Gaussian filter. The range component was studied by combining it with a box domain. The noise reduction for the spatial Gaussian filter and bilateral box domain filter could be related through the box spatial filter. In doing so the noise reduction of the bilateral filter could be better predicted.

The total dependencies of the bilateral are: kernel size, domain value, range value and noise level. The kernel size and domain value were studied with the spatial Gaussian filter. By relating the spatial Gaussian filter to the box filter predicting the noise reduction was simplified. It was also shown that the spatial Gaussian filter preserved edges better than the box filter for equivalent noise reduction. Thus, it was concluded that the spatial Gaussian was the best linear smoothing candidate to combine with the bilateral filter. In addition a Gaussian distribution is desirable for the range component for similar reasons.

The range value was chosen by delineating objects from background or each other. In general, if the difference between two objects was taken as the range value, a range weighting factor of approximately 0.6 would be assigned to any pixels that have that
difference to the center pixel. If the difference is divided by four, a weighting factor of approximately zero would be assigned. This provides the best object delineation.

The range value will be limited by the noise level. In general if $\sigma_r / \sigma_{\text{noise}}$ is less than one there will be little to no smoothing. If the range value is too small the range can be increased so that noise is reduced to a level that the preferred range value can be used. In addition a pre-filter such as the median filter can be used to reduce the noise.

For the computer generated phantom with Gaussian noise, the CT phantom and CT clinical images the same general guidelines were used. More specifically:

1. The kernel size and domain value are determined by the size of structures in the image. Large values may blur smaller objects from an image. The domain value should be set according to the kernel size. A small kernel and large domain value may effectively become a box filter, i.e. the Gaussian distribution becomes truncated. A large kernel size and small domain value will result in an effective kernel size smaller than what was specified.

2. Choosing the range value can be done by examining object contrast. To ensure that all structures in an image are preserved, the range value should be calculated with the lowest contrast object.
3. The range value will be limited by the noise level of an image. If $\sigma_r/\sigma_{\text{noise}}$ is small, little to no smoothing will occur or the noise may become more speckled due to outliers. The range value can be increased to overcome image noise. However, increasing the range value too much will result in greater blurring. A pre-filter can also be used. The pre-filter can be used to reduce the noise to a level where $\sigma_r/\sigma_{\text{noise}}$ becomes large enough that the preferred range value can be used. The value of $\sigma_r/\sigma_{\text{noise}}$ will be effected by the type of noise. It varied with CT reconstruction type.

The examples presented have shown that the bilateral filter is versatile in various noise environments. When the methods described are used, the response of the filter can be made more straightforward and more predictable.
Conclusions

The papers discussed in the Literature chapter confirm that the bilateral filter has proven to be a feasible non-linear smoothing filter. Several improvements have been implemented and the theoretical underpinnings of the bilateral filter have been studied. Like most non-linear filters, predicting how the bilateral filter will respond with different types of images is complex. The goal of this paper was to make the response of the bilateral filter more predictable and straightforward by examining the domain and range components.

From this study the following conclusions can be drawn:

1. The bilateral filter could be studied by separating the domain and range components. A link to the box filter, with respect to noise reduction, could be made for each component. The domain and range components could be linked through the box filter. This made the noise reduction of the bilateral filter more predictable.

2. The noise dependency of the bilateral filter was simplified by the ratio \( \frac{\sigma_{\text{range}}}{\sigma_{\text{noise}}} \). This ratio can then be linked to a box equivalent size for a bilateral filter with a box domain. In doing so the impact the range value has on noise reduction could be simplified.
3. The range value can be effectively determined by object contrast. The best edge preservation for all objects in an image occurs when the lowest contrast object is used for the range value calculation.

4. The techniques of using a pre-filter and iterating the bilateral filter increased noise reduction and preserved edges. They can allow the use of a low range value even when $\frac{\sigma_{\text{range}}}{\sigma_{\text{noise}}}$ is small.

It was shown that the bilateral filter can be successfully used for smoothing clinical CT images. Based on the general guidelines discussed, the bilateral filter reduced image noise and at the same time preserved edges. This important quality makes it a strong candidate for medical image processing. Preserving edges is crucial for identifying anatomy and pathology. The guidelines offered by this paper have shown that the bilateral filter has the capacity to not smooth over important features yet reduce noise in uniform areas. This however is dependent on the domain and range values selected.

The CT images examined in this paper showed good results with the bilateral filter. A further study may include dose reduction guidelines. Reducing the mAs reduces patient dose. However, it also increases image noise. This study has shown that significant improvement can be made with high noise CT images.


100


