Parametric investigation toward achieving an optimal magnetorheological mount

Walter Bradley Anderson

Follow this and additional works at: http://utdr.utoledo.edu/theses-dissertations

Recommended Citation
http://utdr.utoledo.edu/theses-dissertations/782

This Thesis is brought to you for free and open access by The University of Toledo Digital Repository. It has been accepted for inclusion in Theses and Dissertations by an authorized administrator of The University of Toledo Digital Repository. For more information, please see the repository's About page.
A Thesis

entitled

Parametric Investigation Toward Achieving an Optimal Magnetorheological Mount

by

Walter Bradley Anderson

Submitted to the Graduate Faculty as partial fulfillment for

The Master of Science in Mechanical Engineering

Dr. Mohammad H. Elahinia, Committee Chair

Dr. Mehdi Pourazady, Committee Member

Dr. Sorin Cioc, Committee Member

Dr. Patricia Komuniecki,
Dean to the College of Graduate Studies

The University of Toledo

May 2010
Copyright © 2010

This document is copyrighted material. Under copyright law, no parts of this document may be reproduced without the expressed permission of the author.
An Abstract of

Parametric Investigation Toward Achieving an Optimal Magnetorheological Mount

Walter Bradley Anderson

Submitted as partial fulfillment of the requirements for

The Master of Science in Mechanical Engineering

The University of Toledo

May 2010

Hybrid vehicles have been developed to increase the fuel economy of existing vehicles. Examples include hydraulic and electric hybrid vehicles. A common issue in hybrid vehicles is the additional vibration and vibration-caused noise. This vibration is primarily caused by the switching of power modes, i.e. from the conventional gasoline engine to the electric or hydraulic motor. In certain cases, this vibration, due to the wide frequency range characteristics, cannot effectively be isolated through the use of conventional isolation technologies. In this thesis, a mixed mode magnetorheological mount has been investigated to mitigate this and similar wide-frequency vibration.
In order to effectively isolate vibration, a mount should be designed for a specific application. Different engines however are required for each application; a designer therefore must alter the engine mount with the desired stiffness and damping characteristics. In this work a parametric analysis has been performed on one of the main components of the mount, the elastomeric top. The analysis and comparison to the experimental results are discussed in detail. Three different parameters were the focus of this investigation. The rubber thickness (by two means) and height were studied. It was found that varying the mount diameter (thickness) is the most influential parameter that was investigated. The initial analyses to validate the finite element model showed at most a ~5% difference. The results of this study will be instrumental for designing and optimizing different mounts including magnetorheological and hydraulic mounts.
Acknowledgements

I would first like to thank my advisor Dr. Mohammad Elahinia for giving me the opportunity to work on this project. I have a great deal of respect and admiration for him. Without his excellent guidance, deftness and tutelage, the completion of this thesis would not have been possible. I would also like to thank Dr. Walter Olson for allowing me the opportunity to work with the hydraulic hybrid team. The group’s engineering savvy is appreciable. Another well deserved thank you goes to my committee members Dr. Sorin Cioc and Dr. Mehdi Pourazady. Their expert teaching skills helped me to be able to complete this graduate degree. Also a well deserved thank you to the Mechanical, Industrial and Manufacturing Engineering department’s faculty and staff is in order for their support throughout the duration of this project.

A great thank you is due to my friends in the Dynamic and Smart Systems laboratory for their ceaseless support and friendship. There truly is a family feel within this lab.

An incredible thank you is due to my family who has supported me in many different ways throughout this wonderful journey. My mother, Linda Anderson and father, Gregg Anderson has always been there to back me in my scholastic ambitions. I would also like to say thank you to Alexandria Shinn for her support. Their encouragement and aid will not be forgotten.
# Table of Contents

Abstract ............................................................................................................................. iii
Acknowledgements ............................................................................................................ iii
Table of Contents ............................................................................................................... iii
List of Figures ..................................................................................................................... v
List of Tables ...................................................................................................................... x
Nomenclature ..................................................................................................................... xi
Chapter One ........................................................................................................................ 1
Introduction .......................................................................................................................... 1
  1.1. General Overview .................................................................................................... 1
  1.2. Statement of the problem ......................................................................................... 2
  1.3. Outline ...................................................................................................................... 2
  1.4. Approach .................................................................................................................. 3
  1.5. Contribution ............................................................................................................. 3
Chapter Two ........................................................................................................................ 5
Literature Review ................................................................................................................. 5
  2.1. Hybrid Vehicles ....................................................................................................... 6
  2.2. Elastomeric Mounts ................................................................................................. 9
  2.3. Hydraulic mounts ................................................................................................. 10
List of Figures

Figure 1 Parallel HHV configuration [9]............................................................................ 6
Figure 2 Series HHY configuration [10] .............................................................................. 7
Figure 3 Fundamental vibration harmonics of a 9 cylinder P/M rotating at 1000rpm [4].. 8
Figure 4 Engine and mount schematic [13]......................................................................... 9
Figure 5 Typical rubber engine mount [14]......................................................................... 9
Figure 6 Dynamic stiffness of the prototype rubber top [1].................................................. 10
Figure 7 Typical hydraulic mount (section view)................................................................. 11
Figure 8 Dynamic stiffness curves for a hydraulic mount and a rubber mount [15]........... 12
Figure 9 Hydraulic mount ideal dynamic stiffness curve [15]............................................ 12
Figure 10 Schematic of an active mount [16]................................................................. 13
Figure 11 Experimental results with and without control of the active mount [16]......... 14
Figure 12 Semi-active engine mount with a detailed view of the variable inertia track and
decoupler orifice area [17]............................................................................................ 14
Figure 13 Transmissibility plots comparing the mount as an active mount and as a passive
mount, shown in the left pane is the high frequency range; in the right is the low
frequency range [17].................................................................................................... 15
Figure 14 Working modes of MR fluid ............................................................................. 17
Figure 15 Particle arrangements for MRE [21]................................................................. 18
Figure 16 Images of MR fluid from an SEM, upper photo shows the random distribution
and the lower photo shows the particles aligning to a magnetic field [22]..................... 19
Figure 17 Schematic of the cylindrical rectilinear damper [23] ................................................... 20
Figure 18 Experimental set-up of a squeeze type mount [24] ....................................................... 20
Figure 19 Experimental hysteresis loops increasing (left) and transmissibility reduction (right) through the use of squeeze mode [24] ........................................................................ 21
Figure 20 Section view of the proposed damper ........................................................................... 22
Figure 21 Schematic of the experimental test set up [26] ............................................................. 22
Figure 22 Experimental hysteresis loops for the proposed SMD [26] ......................................... 23
Figure 23 Energy dissipation as a function of frequency for the varying magnetic field strengths [26] .................................................................................................................... 23
Figure 24 Engineering schematic and the equivalent model [27] ................................................. 24
Figure 25 Transmissibility results (-•- experiment, ---- model) [27] ............................................. 24
Figure 26 Flow and shear mode MR mount [28] ............................................................................ 25
Figure 27 Schematic of the experimental set up [28] ................................................................... 26
Figure 28 Acceleration plots comparing the rubber mount to the mixed mode mount. Plot ‘a’ corresponds to position 4 (p4), b to position 1 (p1), c to position 2 (p2) and d to position 3 (p3) [28]. ....................................................................................................................... 27
Figure 29 Schematic of the proposed damper [29] ...................................................................... 28
Figure 30 Prototype MRPG damper [29] .................................................................................... 29
Figure 31 Experimental hysteresis loops for the damper [29] .................................................. 29
Figure 32 Schematic of the proposed MR damper: 1- Cylinder cover 2- Guides 3-Inner cylinder 4- Outside cylinder 5-Coil 6- Piston rod 7-Hole 8-Gasbag 9-Sleeve 10-Coil protector 11-Wire 12-Seal ring [30] ....................................................................................... 30
Figure 33 Experimental damping force versus displacement [30] ........................................... 30
Figure 34 Schematic of the commercial damper made by LORD Corp. [31] .................. 31
Figure 35 Engineering schematic for the damper [31] ..................................................... 31
Figure 36 Experimental results (black solid lines) compared to model results (red dotted lines) for the commercial damper [31] .............................................................................. 32
Figure 37 CAD model of the prototype mount [1] ........................................................... 35
Figure 38 The middle assembly in an exploded view [1] ................................................ 37
Figure 39 The inner magnetic coil responsible for squeeze [1] ........................................ 38
Figure 40 Outer magnetic coil responsible for flow mode [1] ............................................. 38
Figure 41 Upper portion of the mount .............................................................................. 39
Figure 42 Housing and rubber bottom .............................................................................. 39
Figure 43 Flow mode dynamic stiffness for displacement excitation of 0.2mm [1] ......... 42
Figure 44 Squeeze mode dynamic stiffness, displacement excitation of 0.2mm [1] ...... 43
Figure 45 Different working modes working envelopes, (a) shows the envelope for flow mode only, (b) shows the working envelope for squeeze mode only and (c) shows the combined mode working envelope [1] ................................................................. 43
Figure 46 Example axisymmetric element ....................................................................... 48
Figure 47 CAD model of the rubber top ........................................................................... 52
Figure 48 Section view of the rubber mount ..................................................................... 52
Figure 49 Nomenclature of the rubber top ........................................................................ 53
Figure 50 Geometric parameters that were altered for the analysis .................................. 55
Figure 51 Euler-Bernoulli beam example ....................................................................... 56
Figure 52 Relationship between hardness and Young's modulus, the solid line is the theoretical calculation, the broken line is the empirical relation and the circles are the experimental measurements [33] .................................................................................................................. 57

Figure 53 Bose Electroforce® machine ............................................................................ 58

Figure 54 Upper left is the original cross section of the mount, in the upper right: the mount with 2mm removed, bottom: 2mm added ............................................................................................................. 61

Figure 55 Parameter A revolved cut example .................................................................. 62

Figure 56 Parameter A revolved additions ....................................................................... 63

Figure 57 Stiffness plot for Dimension A alterations ....................................................... 64

Figure 58 Sketch of the mount with parameter B highlighted ......................................... 65

Figure 59 Stiffness plot for dimension B alterations ....................................................... 66

Figure 60 Picture explaining unexpected results in Parameter B ................................. 67

Figure 61 Upper left is the unaltered mount, upper right is the maximum altered for material removal and the bottom is the maximum addition of material ...................... 68

Figure 62 Stiffness plot for dimension C variations ....................................................... 69

Figure 63 Loaded tube .................................................................................................. 72

Figure 64 Boundary conditions for the analysis ............................................................ 73

Figure 65 Shear strain plot ......................................................................................... 74

Figure 66 Von Mises stress plot for the mount top ...................................................... 74

Figure 67 Normal (z direction) Piola-Kirchhoff stress plot ........................................... 75

Figure 68 Radial (r direction) Piola-Kirchhoff stress plot ............................................. 76

Figure 69 Shear Piola-Kirchhoff stress plot ................................................................ 76

Figure 70 Total Piola-Kirchhoff stress plot ................................................................... 77
Figure 71 Point that was chosen for the stress strain plot................................................. 77
Figure 72 Stress strain plot for the point................................................................. 78
List of Tables

Table 1 Explanation of control systems ................................................................. 24
Table 2 Summary of experimental results compared to FEA .................................. 59
Table 3 Parameter alterations in table form .......................................................... 60
Table 4 Alterations for Parameter C in table form .............................................. 67
# Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area of the element</td>
</tr>
<tr>
<td>Ai</td>
<td>Cross sectional area of the inertia track</td>
</tr>
<tr>
<td>Ap</td>
<td>Effective piston area</td>
</tr>
<tr>
<td>b</td>
<td>Damping of the rubber</td>
</tr>
<tr>
<td>C1</td>
<td>Compliance of the upper chamber</td>
</tr>
<tr>
<td>C2</td>
<td>Compliance of the lower chamber</td>
</tr>
<tr>
<td>Csq</td>
<td>Damping of the squeeze mode</td>
</tr>
<tr>
<td>CAD</td>
<td>Computer aided design</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational fluid dynamics</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree(s) of freedom</td>
</tr>
<tr>
<td>E</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>EOM</td>
<td>Equation of motion</td>
</tr>
<tr>
<td>ER</td>
<td>Electrorheological</td>
</tr>
<tr>
<td>Fsq</td>
<td>Force of the squeeze mode</td>
</tr>
<tr>
<td>FE</td>
<td>Finite element</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite element analysis</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite element methods</td>
</tr>
<tr>
<td>H</td>
<td>Magnetic field</td>
</tr>
<tr>
<td>HHV</td>
<td>Hydraulic hybrid vehicle</td>
</tr>
<tr>
<td>Hz</td>
<td>Hertz (cycles per second)</td>
</tr>
<tr>
<td>Ii</td>
<td>Inertance of the fluid in the inertia track</td>
</tr>
<tr>
<td>kr</td>
<td>Stiffness of the rubber</td>
</tr>
<tr>
<td>m</td>
<td>Meter</td>
</tr>
<tr>
<td>mm</td>
<td>Millimeter</td>
</tr>
<tr>
<td>M</td>
<td>Mass of the vibrating body</td>
</tr>
<tr>
<td>MMR</td>
<td>Multiaxial magnetorheological</td>
</tr>
<tr>
<td>MR</td>
<td>Magnetorheological</td>
</tr>
<tr>
<td>MRE</td>
<td>Magnetorheological elastomer</td>
</tr>
<tr>
<td>MRPG</td>
<td>Magnetorheological polymer gel</td>
</tr>
<tr>
<td>N</td>
<td>Newton</td>
</tr>
<tr>
<td>NVH</td>
<td>Noise, vibration and harshness</td>
</tr>
<tr>
<td>OEM</td>
<td>Original equipment manufacturer</td>
</tr>
<tr>
<td>P/M</td>
<td>Pump/motor</td>
</tr>
<tr>
<td>Pa</td>
<td>Pascal (N/m²)</td>
</tr>
<tr>
<td>Paper space</td>
<td>Domain of a 3D modeling software where the technical drawing is generated</td>
</tr>
<tr>
<td>psi</td>
<td>Pounds per square inch</td>
</tr>
<tr>
<td>Rr</td>
<td>Resistance of the fluid against the wall in the inertia track</td>
</tr>
<tr>
<td>rpm</td>
<td>Rotations per minute</td>
</tr>
<tr>
<td>SDOF</td>
<td>Single degree of freedom</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning electron microscope</td>
</tr>
<tr>
<td>SMD</td>
<td>Squeeze mode damper</td>
</tr>
<tr>
<td>V</td>
<td>Volts</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Volume</td>
</tr>
<tr>
<td>Vacuum filled</td>
<td>A procedure to degas fluid and fill an evacuated mount</td>
</tr>
<tr>
<td>W</td>
<td>Work done by external forces</td>
</tr>
<tr>
<td>( K_{\text{dyn}} )</td>
<td>Dynamic stiffness</td>
</tr>
<tr>
<td>( F_T )</td>
<td>Transmitted force</td>
</tr>
<tr>
<td>( x_{\text{ex}} )</td>
<td>Excitation displacement</td>
</tr>
<tr>
<td>( x_i )</td>
<td>Position of the fluid in the inertia track</td>
</tr>
<tr>
<td>( \dot{x}_i )</td>
<td>Velocity of the fluid in the inertia track</td>
</tr>
<tr>
<td>( L/h )</td>
<td>Ratio that is defined by the orifice geometry</td>
</tr>
<tr>
<td>( \ddot{x} )</td>
<td>Input (excitation) acceleration</td>
</tr>
<tr>
<td>( \dot{x} )</td>
<td>Input (excitation) velocity</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Stress</td>
</tr>
<tr>
<td>( \tau_y )</td>
<td>Yield stress</td>
</tr>
<tr>
<td>( \dot{\gamma} )</td>
<td>Fluid shear rate</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Plastic viscosity</td>
</tr>
<tr>
<td>( \Pi )</td>
<td>Total potential energy</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>Total strain energy</td>
</tr>
<tr>
<td>( N )</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>( N_{i,j,k} )</td>
<td>Shape function for node i or j or k</td>
</tr>
<tr>
<td>( a_i )</td>
<td>Parameter to write the shape function in a more compact form</td>
</tr>
<tr>
<td>( b_i )</td>
<td>Parameter to write the shape function in a more compact form</td>
</tr>
<tr>
<td>( c_i )</td>
<td>Parameter to write the shape function in a more compact form</td>
</tr>
<tr>
<td>( \bar{r} )</td>
<td>Average radius needed for simplification</td>
</tr>
<tr>
<td>( \bar{z} )</td>
<td>Average height needed for simplification</td>
</tr>
</tbody>
</table>

\[
\begin{bmatrix} \varepsilon \end{bmatrix} \quad \text{Strain matrix}
\]
\[
\begin{bmatrix} D \end{bmatrix} \quad \text{Material property matrix}
\]
\[
\begin{bmatrix} B \end{bmatrix} \quad \text{The partial derivative of the shape function matrix}
\]
\[
\begin{bmatrix} \partial \end{bmatrix} \quad \text{Partial derivative matrix}
\]
\[
\begin{bmatrix} N \end{bmatrix} \quad \text{Shape function matrix}
\]
\[
\{d\} \quad \text{Degree of freedom vector}
\]
\[
\begin{bmatrix} K \end{bmatrix} \quad \text{Stiffness matrix}
\]
\[
\begin{bmatrix} \bar{B} \end{bmatrix} \quad \text{Modified the partial derivative of the shape function matrix}
\]
Chapter One

Introduction

1.1. General Overview

Smart materials have gained the attention of researchers throughout the past several decades. Smart materials are a special class of materials that change their properties in different situations. Examples of smart materials are shape memory alloys, electrorheological fluid, piezoelectric materials, magnetorheological fluid, etc. Magnetorheological (MR) fluids are the subject of this research. MR fluid is a fluid that changes its behavior in the presence of a magnetic field. The fluid alters its behavior from that of a Newtonian fluid to that of a Bingham plastic, without and with the presence of a magnetic field, respectively. Jacob Rabinow invented magnetorheological fluid in the 1940’s and the research field has been rapidly growing ever since [1]. There are existing commercial applications of MR fluid devices, such as LORD Corporation’s MR Magneride™ damper [2]. There is only one known commercial MR mount and its application is limited to performance sports cars [3]. There is a need for a more general MR mount. This is due to the increased popularity of hybrid vehicles. Coupled with that is the need for a design basis for the different applications.
1.2. **Statement of the problem**

There is a need for better noise, vibration and harshness (NVH) mitigation associated with the implementation of hybrid vehicles. The NVH issues are associated with and are caused by the power modes switching, e.g. switching from the conventional gasoline engine to the electric motor or hydraulic. This is due to the different operating conditions of each power source and the fact that they are coupled together. This vibration happens over a wide range of frequencies [4]. Due to this, conventional vibration isolation techniques are rendered unacceptable. There is need for a design basis for the optimal mount for the specific application. The objective of this study is to determine the static stiffness alterations by way of altering passive design parameters for the elastomeric top of the mount.

1.3. **Outline**

This thesis provides the background for the mount and the steps taken to perform the analysis.

Chapter 2 presents an extensive literature review on MR fluid and applications. It includes information relating to the modeling of the fluid and different applications for MR fluid. Also included in this chapter is an overview of the classic mount solutions, i.e. elastomeric and hydraulic.

Chapter 3 presents the prototype mount that was studied and a qualitative explanation of the mount’s dynamic response. Plots of the response will be presented and briefly discussed. For more information about the prototype mount, please refer to [1].

Chapter 4 presents the modeling of the prototype mount. The response modeling of the mount is done in a finite element modeling package, COMSOL Multiphysics®.
Chapter 5 presents the results of the modeling and the finite element analysis (FEA). The effect of the parametric alterations will be extensively discussed. The chapter will be summarized and the conclusions of the results will be presented.

Chapter 6 presents concepts for future work and a brief discussion of other mount concepts.

1.4. Approach

The effects of design parameters on the stiffness of an MR mount are investigated. Hardness tests of the elastomer were performed to develop the material properties. To this end a finite element model of the elastomeric parts of the mount is developed. The simulation was then compared to experimental static deflections. This model was then used to calculate the static characteristics of the mount.

1.5. Contribution

The following are the publications that have been a result of this work.


Chapter Two

Literature Review

This chapter presents a literature review of the problem (hybrid vehicle vibration), the basis of the proposed solution: magnetorheological (MR) fluid, some of its applications and a fundamental description of the existing mount types. Since the development of the automobile, vibration isolation has been a concern for engineers. The advancements in engine technology and power trains while maintaining passenger comfort as a priority has caused engineers to develop clever solutions to the vibration problem associated with them. With the current trend for hybridization of vehicles and the push for more ‘green’ transportation, these classic solutions have been found inadequate.

Primarily, the classic mount solutions can be categorized as elastomeric (rubber) or hydraulic (fluid). Elastomeric mounts can be considered to have a singular value for stiffness and damping. Hydraulic mounts are characterized by their dynamic response, the dynamic stiffness curve. These will be discussed in detail in future sections.

MR fluid is a fluid that changes from a Newtonian fluid to a Bingham plastic in the presence of a magnetic field. MR fluid has three different working modes: squeeze, shear and flow. For the prototype mount in question, squeeze and flow modes are those
that are utilized. More extensive descriptions of MR fluid will be found in the following sections.

2.1. Hybrid Vehicles

Hybrid vehicles are now commercially available through several manufacturers [5] [6] [7]. A new type of hybrid vehicle that is under development is a hydraulic hybrid [4] [8]. For the sake of brevity only the hydraulic hybrid vehicle (HHV) will be discussed. There are two types of hydraulic hybrids: parallel and series. The configuration of each can be seen in Figure 1 & Figure 2.

Figure 1 Parallel HHV configuration [9]
Figure 2 Series HHY configuration [10]

In the parallel configuration the motor is still mechanically linked to the wheels and the hydraulic components assist the engine. During breaking, the wheels drive the pump/motor (P/M), in pump mode to charge the high pressure accumulator (called regenerative braking). The accumulator is charged by pumping hydraulic oil from the low pressure accumulator into the high pressure accumulator where a pressurized bag is located and filled with an inert gas, such as nitrogen. When an accumulator is charged, or pressurized, it is called the high pressure accumulator. The high pressure accumulator then stores the pressurized oil as a potential energy source and will deliver it upon need. When the car accelerates, the P/M works in motor mode and takes the energy stored in the high pressure accumulator, delivers it to the low pressure accumulator, and helps drive the wheels. In the series configuration, the motor is completely decoupled from the wheels and only operates to drive the P/M to store energy in the high pressure accumulator. This way, the engine only operates close to its peak efficiency. Therefore, engine operation would only occur when all of the energy is used from the regenerative braking and more energy is needed to propel the vehicle.

Inherent to the HHV is a vibration profile originating from the (P/M) [4]. The P/M is the hydraulic equivalent of an electric generator/motor. This vibration occurs over a wide range of frequencies, see Figure 3.
The P/M is generally directly coupled to the engine creating a treacherous vibration profile. This is due to the fact that a conventional engine, having a conflicting vibration profile intrinsically [11], is coupled with the P/M. The conflicting vibration profile is categorized as having a large displacement at low frequencies and having a small displacement at high frequencies, i.e. high damping is needed for low frequencies and small damping is needed for high frequencies [11].

It should also be noted that some vehicle manufacturers have taken an alternate approach to fuel management and increased mileage. The concept is generally known as a cylinder on demand. With this approach the engine only uses the amount of cylinders that is demanded by the driver. Typically, a 6 cylinder engine can be run on 6, 4 or 3 cylinders, depending on the situation. One can easily see the dynamic balancing for such an engine would be challenging.
2.2. Elastomeric Mounts

Elastomeric, or rubber, mounts were originally developed to address the engine vibration. Elastomeric mounts are the simplest way to provide some vibration isolation between the engine and chassis. The purpose of a mount is to statically support a payload and dynamically isolate vibration transmission [12]. A series of mounts are needed to support an engine, for some applications this can be simplified as seen in Figure 4.

![Figure 4 Engine and mount schematic [13]](image)

A typical elastomeric engine mount can be seen in Figure 5.

![Figure 5 Typical rubber engine mount [14]](image)

The dynamic stiffness of a rubber mount is a fairly singular value. This has been seen experimentally in [1], see Figure 6.
Here the dynamic stiffness is defined as below:

\[ K_{\text{dyn}} = \frac{F_T}{x_{\text{ex}}} \]  

\text{Equation 1}

Where:

- \( K_{\text{dyn}} \) is the dynamic stiffness
- \( F_T \) is the transmitted force
- \( x_{\text{ex}} \) is the displacement of the excitation

An obvious advantage to an elastomeric mount is cost. It is simply a block of rubber with two threaded rod extensions. There is the advantage of no moving parts, so mechanical failure is much more limited. Also, the mount is easy to install. The disadvantage is the isolation performance. The singular value for damping and stiffness yield a harsh vibration for the engine operating conditions.

\textbf{2.3. Hydraulic mounts}
Hydraulic mounts were invented to address the conflicting requirements of the engine vibration. A typical hydraulic mount (the cross section) can be seen in Figure 7.

![Figure 7 Typical hydraulic mount (section view)](image)

The hydraulic mount is filled with oil (not shown) that fills the upper chamber, enclosed by the top rubber, and the lower chamber, enclosed by the bottom rubber. The decoupler is designed such that it is the same density as the carrier oil. If the excitation (originating from the vibrating body) is a large displacement, then the decoupler seats in its ‘cage’ and forces the oil to flow through the inertia track. This portion of the mount is named as such because the fluid in the inertia track has inertance and when forced to flow, absorbs more energy, hence leading to higher damping. If the excitation is a small displacement, then the decoupler simply oscillates back and forth in its cage allowing the
fluid to flow around it. This leads to less energy absorption and lower damping. It should be noted that the dominating parameters of the mount are the effective piston area, the top compliance chamber and the bottom chamber compliance. The effective piston area is defined as the volume change of the upper chamber divided by the input displacement. The compliance is the inverse of the stiffness.

Generally, a hydraulic mount is characterized by its dynamic stiffness curve. Ahn et al. investigated how to improve the performance of a hydraulic mount. A typical dynamic stiffness curve for a hydraulic mount compared to a rubber mount can be seen in Figure 8.

![Figure 8 Dynamic stiffness curves for a hydraulic mount and a rubber mount [15]](image)

The purpose of the research by Ahn et al. was to lower the notch frequency without increasing the resonance peak, see Figure 9.

![Figure 9 Hydraulic mount ideal dynamic stiffness curve [15]](image)
Ahn et al. did not show any experimental results though; they only offer different simulation techniques to determine optimal mount parameters to lower the notch without raising the peak.

Togashi & Ichiryu examined the possibility of an active hydraulic mount. The schematic of the mount can be seen in Figure 10.

![Figure 10 Schematic of an active mount [16]](image)

The experimental results are impressive. The mount has a bellows and reservoir filled with oil. Submerged within the oil is a device using a permanent magnet and a voice coil motor that acts as a solenoid valve. This mount was designed for use with diesel trucks. The reason that the diesel truck was targeted is that there is enough available space for the active mount, as it tends to be larger. Active mounts can be considered not as vibration absorption but force cancelation. The transmitted force is almost completely nullified, see Figure 11.
Although these results are impressive, there is quite a cost for this. Power requirements are usually very large for active mounts. Also, with the use of an active mount, because energy is introduced into the system, it can be driven to instability.

Foumani & Khajepour researched a more traditional modified engine mount. This mount is called semi-active. This is because sensors are needed to tell the controller how to alter the mount such that the optimal performance is achieved. No extra energy is being added to the system for this type of mount and is therefore more inherently stable. In Foumani & Khajepour’s research the inertia track length and decoupler orifice area were varied through the use of a worm gear, see Figure 12.

The rotating upper port rotates around the inertia track effectively altering the length of the track. As the plate rotates around, the holes in the center change how they ‘match up’ with the orifice holes beneath the plate effectively altering the decoupler
orifice area. Good results can be observed between the passive mount and the active mount, see the transmissibility plots in Figure 13.

![Transmissibility plots comparing the mount as an active mount and as a passive mount, shown in the left pane is the high frequency range; in the right is the low frequency range](image)

Figure 13 Transmissibility plots comparing the mount as an active mount and as a passive mount, shown in the left pane is the high frequency range; in the right is the low frequency range [17]

One of the major concerns in this research is the use of the worm gear to adjust the rotating plate. It should be very difficult to create good seals that will have an appreciable fatigue life in a vibrating environment, if the plate is driven from outside the mount. If the plate is driven from within the mount, there would need to be servo-controls in the mount, submerged in the fluid. One can see that this could be quite problematic. Either way, there is the possibility for mechanical failure. If the mount were to fail, it would then have very poor performance at any frequency other than the one at which it failed.

**2.4. Magnetorheological fluid**

**2.4.1. Fundamentals of magnetorheological fluid**

Magnetorheological (MR) fluid is a smart material that changes its behavior in the presence of a magnetic field. The fluid is similar to a ferrofluid, but the particle size is
much smaller. MR fluid has micron sized ferrous particles suspended in a carrier fluid, such as glycol. Electorheological (ER) fluid should also be mentioned. Similar properties can be observed in ER fluid as MR fluid, the difference being that rather than a magnetic field, an electrical field is used to excite the fluid. ER fluid has several major downfalls compared to MR fluid. First, MR fluid generally has a 10 times higher yield strength when compared to ER fluid [18]. Also, the power requirements for ER fluid are much greater than those for MR fluid. In a research done by Wu et al. the electric field requirement was extremely large, \( \sim 2 \times 10^6 \text{ V/m} \) [19]. MR fluid, in the absence of a magnetic field behaves like a Newtonian liquid. However, in the presence of a magnetic field the fluid behaves like a Bingham plastic. The behavior of a Bingham plastic is defined as follows [20]:

\[
\tau = \tau_y(H) + \eta \dot{\gamma} \quad \text{for } \tau < \tau_y
\]

Equation 2

Where:

\( \tau \) is the stress
\( \tau_y \) is the yield stress
\( H \) is the magnetic field
\( \dot{\gamma} \) is the fluid shear rate
\( \eta \) is the plastic viscosity

For MR fluid, there are three working modes: flow, shear and squeeze. The working modes can be qualitatively seen in Figure 14.
In flow mode, the two retaining plates are stationary and the fluid is forced to flow perpendicular to the direction of the magnetic field. In shear mode, one retaining plate moves relative to the other in a direction perpendicular to the direction of the magnetic field. In squeeze mode, one retaining plate moves relative to the other in a direction parallel to the direction of the magnetic field. The working modes that are utilized in the prototype mount are flow and squeeze. It is worth mentioning that MR fluid is only able to help remove energy from the system. This implies that at most, its application is limited to a semi-active device. A semi-active device is one that only removes energy from a system. It uses sensors such that it can change its performance to account for the different input conditions.

It should be noted that magnetorheological behavior is not limited to fluids. There are MR elastomers (MRE) as well. This is currently a topic for groundbreaking research. When the magnetic particles are suspended in an elastomer an active device can be realized through the use of polar or magnetically hard particles. Also, the behavior
changes depending upon if the elastomer was cured in the presence of a magnetic field or not. This can be qualitatively seen in Figure 15.

![Figure 15 Particle arrangements for MRE [21]](image)

With this new technology MRE can be made into an active mount. When hard magnetic particles are used a significant amount (~12mm) of motion can be seen from the elastomer [21].

### 2.4.2. Micromechanical modeling of MR fluid

It is important to note that while the macromechanical response of MR fluid is typically of interest, research has been done to study the micro structure of the fluid. In the research conducted by Jolly et al. they state that the ferrous particles are typically in the 3-5 micron size range [22]. They also provide some scanning electron microscope (SEM) images of MR fluid in its Newtonian state and its Bingham plastic state, see Figure 16.
Also provided in this research is a model for the magnetic permeability of chains consisting of spherical particles. For the model proposed, the chain structure is assumed. Good results between the proposed model and experimental results were shown.

Jang et al. researched the magnetomechanical behavior of a cylindrical rectilinear MR damper. Rich mathematics describing the particle interactions are given throughout the paper. A schematic of the damper can be seen in Figure 17.

Figure 16 Images of MR fluid from an SEM, upper photo shows the random distribution and the lower photo shows the particles aligning to a magnetic field [22]
2.4.3. Applications of MR fluid

Ha et al. studied the dynamic properties of a squeeze mode MR mount. They were able to show a reduction in transmissibility with their prototype mount. The experimental set-up can be seen in Figure 18.

Figure 18 Experimental set-up of a squeeze type mount [24]
They were able to show an effective reduction in the transmissibility and an effective increase in the hysteresis loop; the area inside the loop represents the amount of energy absorbed, see Figure 19.

![Figure 19 Experimental hysteresis loops increasing (left) and transmissibility reduction (right) through the use of squeeze mode [24]](image)

The purpose of this study was to determine the effect of an increased magnetic field on MR fluid in squeeze mode. The important thing to note with regards to this research is that there was not a controller implemented; only current on or off, i.e. a bang-bang controller. Also note the frequency content of the study. For typical engine mount applications, the frequency range of concern is 0-200 Hz [25].

Wang et al. also studied a squeeze mode damper [26]. The proposed damper here is rather nonconventional. The proposed damper is MR fluid encapsulated in elastomer directly. A section view of their proposed mount can be seen in Figure 20.
The black portion is the encapsulating rubber and the blue portion represents the enclosed MR fluid.

This squeeze mode damper (SMD) was then experimentally tested. A schematic of the experimental test set up can be seen in Figure 21.

The SMD was tested on an Instron® machine. The hysteresis loops were then presented. It was shown that more energy is absorbed in the higher magnetic field condition, see Figure 22.
It should be noted that the SMD was only tested up to 10 Hz. However, the authors were able to show that the energy dissipation was almost independent of frequency for the range tested, see Figure 23.

Liu et al. researched an MR damper for use in a structural application. They proposed a very novel concept for controlling the stiffness of the system. The model can be seen in Figure 24.
The novelty of this system is that by altering the damping coefficient of $c_2$, the stiffness can be made either equal to the stiffness of spring 1 or the stiffness of spring 1 and spring 2 in series. The transmissibility results are quite appreciable, see Figure 25.

Here, the different lines represent the different control methodologies, see Table 1 for the explanation of each.

<table>
<thead>
<tr>
<th>Name</th>
<th>Damper 1</th>
<th>Damper 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft system</td>
<td>Off</td>
<td>Off</td>
</tr>
<tr>
<td>Low damping</td>
<td>Off</td>
<td>On</td>
</tr>
<tr>
<td>-------------------</td>
<td>--------</td>
<td>-----</td>
</tr>
<tr>
<td>High damping</td>
<td>On</td>
<td>On</td>
</tr>
<tr>
<td>D on-off (soft)</td>
<td>On-off</td>
<td>Off</td>
</tr>
<tr>
<td>D on-off (stiff)</td>
<td>On-off</td>
<td>On</td>
</tr>
<tr>
<td>S on-off (low)</td>
<td>Off</td>
<td>On-off</td>
</tr>
<tr>
<td>S on-off (high)</td>
<td>On</td>
<td>On-off</td>
</tr>
<tr>
<td>D + S on-off</td>
<td>On-off</td>
<td>On-off</td>
</tr>
</tbody>
</table>

It is to be expected that the D + S on-off control system would yield the best transmissibility results. Also, note that the frequency range is quite low due to the research being tailored to a structural application.

Hong et al. researched an MR mount utilizing two working modes. A schematic of the mount can be seen in Figure 26.

![Flow and shear mode MR mount](image)

**Figure 26 Flow and shear mode MR mount [28]**

It is worth noting that the use of the two working modes is questionable due to the definition of each mode. Recall Figure 14 and the definitions of each mode. Flow mode is defined by the retaining plates remaining stationary, and shear mode is defined by
relative retaining plate motion. The mount was experimentally evaluated; the set up can be seen in Figure 27.

![Figure 27 Schematic of the experimental set up [28]](image)

In their research the proposed mount was compared experimentally to a pure elastomeric mount. Measured acceleration and displacement plots were given for the experimental results. An example of the acceleration plots can be seen in Figure 28.
Figure 28 Acceleration plots comparing the rubber mount to the mixed mode mount. Plot ‘a’ corresponds to position 4 (p4), b to position 1 (p1), c to position 2 (p2) and d to position 3 (p3) [28].
For the point of interest, that is most affected by the MR mount (as seen in Figure 28), p4 (plot a), it can be seen that there is a drop in the acceleration from the rubber mount to the controlled MR mount.

Evrensel et al. researched a low force damper for use in a washing machine application [29]. This is a new type of MR damper because it utilizes MR polymer gel (MRPG). MRPG is different than MR fluid in because MRPG has a lower settling out rate, other than this the behavior is essentially the same. A schematic of the proposed damper can be seen in Figure 29.

![Figure 29 Schematic of the proposed damper [29]](image)

In Figure 29, the shaded area represents the MRPG, the white areas represent the other portions of the mount. In an effort to reduce manufacturing costs: as many portions of the mount were made of plastic as possible. The manufactured mount can be seen in Figure 30.
Experimental hysteresis loops for the experimental damper show good results and coincides with other research in the fact that as the magnetic field intensity is increased, the amount of energy absorbed increased as well; see Figure 31.

Chen & Chen researched an MR damper for use in military vehicles. An interesting design concept is utilized in this damper; rather than the use of a floating piston (one with holes in it) and a gasbag is utilized. This eliminates the need for seals and the need to have a moving electromagnetic coil fixed to the aforementioned piston. A schematic of the proposed damper can be seen in Figure 32.
Another point of interest in this research is that Chen & Chen analyzed the damper using a computational fluid dynamics (CFD) method to study the flow of the MR fluid within the damper. The amount of damping force was one of the key design aspects for this mount. Chen & Chen stated that they were able to generate high damping forces, see Figure 33.

Spaggiari & Dragoni researched an MR damper as well, but this design had a gasbag (accumulator) and a floating cylinder. It should be noted that this research aimed
at developing a model to match a commercially available MR damper. A schematic of the commercial damper can be seen in Figure 34.

![Figure 34 Schematic of the commercial damper made by LORD Corp. [31]](image)

The model that was used to replicate the behavior was an adaptation of the Bouc-Wen model. The engineering schematic can be seen in Figure 35.

![Figure 35 Engineering schematic for the damper [31]](image)

The Bouc-Wen model is simply a mathematical model based on an evolutionary variable evolving with a differential low, used for any hysteretic behavior. The model
was then coded into the MATLAB/Simulink® environment and simulated. Very good correlation was observed, see Figure 36.

Figure 36 Experimental results (black solid lines) compared to model results (red dotted lines) for the commercial damper [31]
The authors were able to show that control was achievable based solely off of current as an input. This is very different because typically to control an MR device, stroke, frequency, current and speed are the necessary input parameters.

In summary, there are many different applications for MR fluid. MR dampers are much more common than MR mounts. This is due to the ease of modeling and the ease of manufacture. Active mounts can be shown to have excellent results but with a high cost; the cost of space and energy consumption. Also, because active mounts are not considered vibration absorbers, but force cancellers, it does raise the question: “Where does the cancelation force push off?” The experimental results of the different MR systems clearly show that they are advantageous over the classical mount solutions.
Chapter Three

The Prototype Mixed Mode MR Mount

This research is a continuation of previous research on the MR mount at the Dynamic and Smart Systems Laboratory at the University of Toledo. As part of the previous research, a mathematical model was developed and the prototype mount was manufactured and experimentally tested. The model was then adjusted to match the experimental results. The aim of this chapter is to introduce the mount and its capabilities.

3.1. The Prototype mount

First the prototype mount will be introduced in the form of a computer aided design (CAD) model. This is because the experimental results are more easily understood with an idea of the physical system. In addition, the colored images allow for easier explanation of the mount.

3.2. The mount CAD model

The mount was first constructed as a CAD model and then manufactured. Due to the similarity of an MR mount and a hydraulic mount, two portions of an original equipment manufacturer (OEM) hydraulic mount were utilized, namely the rubber top and rubber bottom. The CAD model of the prototype mount can be seen in Figure 37.
The mount has two OEM portions, the upper and lower rubber; ① & ②, respectively. The inner magnetic coil, responsible for the activation of squeeze mode is denoted by ③. To help direct the magnetic field, highly magnetically permeable 1018 steel was used for the inner coil housing, ④. The outer magnetic coil, used for activating the fluid in flow mode is denoted by ⑤. The housing for the outer magnetic coil is made of the same material as the inner magnetic coil house and is denoted by ⑥. The annular gap where flow mode is activated is denoted by ⑦. The outer housing for the mount is
denoted by ⑧. The ring to clamp the rubber top to the housing is denoted by ⑨. The screw coming from the rubber top was modified such that the mount can be vacuum filled, this is denoted by ⑩. The other retaining plate needed for squeeze mode (the squeeze plate) is denoted by ⑪. The bottom post where the mount would be mounted to the chassis is denoted by ⑫.

There is also color coordination to Figure 37 as well. The rubber or plastic portions are shown in green. The magnetic coils are shown in red. The magnetically permeable material is shown in pink. The magnetically impermeable material is shown in black or grey.

The exploded view of the entire middle assembly can be seen in Figure 38.
The color coding is the same for Figure 38 as for Figure 37.

### 3.3. Pictures of the mount

The inner magnetic coil, responsible for the activation of squeeze mode can be seen in Figure 39.
The outer magnetic coil, responsible for flow mode can be seen in Figure 40.

The rubber top, closing collar and squeeze plate can be seen in Figure 41.
The housing and bottom rubber can be seen in Figure 42.
3.4. Mount experimental results

The model for the mount was developed using Newton’s Second Law and the continuity equation. These equations were then programmed into the MATLAB/Simulink® environment and simulated. For the sake of brevity, the equations will not be derived here, only the experimental results will be shown, for more information on the equation development, please refer to [1]. The equations of motion for the mount are defined as follows:

\[ I_i A_i \ddot{x}_i + R_i A_i \dot{x}_i + A_i \left( \frac{1}{C_1} + \frac{1}{C_2} \right) x_i = \frac{A_p}{C_1} x + 2 \frac{L}{h} \tau_y(H) \text{sign}(\dot{x}_i) \]  \hspace{1cm} \text{Equation 3}

\[ M \ddot{x} + b \dot{x} + \left( k_r + \frac{A_p^2}{C_1} \right) x + C_{sq} \ddot{x} + F_{sq} = \frac{A_i A_p}{C_1} x_i \]  \hspace{1cm} \text{Equation 4}

Where:

- \( I_i \) is the inerance of the fluid in the inertia track
- \( A_i \) is the cross sectional area of the inertia track
- \( R_i \) is the resistance of the fluid against the wall in the inertia track
- \( \dot{x}_i \) is the velocity of the fluid in the inertia track
- \( C_1 \) is the compliance of the upper chamber
- \( C_2 \) is the compliance of the lower chamber
- \( x_i \) is the position of the fluid in the inertia track
- \( A_p \) is the effective piston area
- \( \frac{L}{h} \) is a ratio defined by the orifice geometry
- \( \tau_y(H) \) is the field dependent yield stress
M is the mass of the vibrating body
\( \ddot{x} \) is the excitation acceleration
\( b_r \) is the damping constant of the rubber
\( \dot{x} \) is the velocity of the excitation
\( k_r \) is the stiffness of the rubber
\( C_{sq} \) is the damping effect of the squeeze mode
\( F_{sq} \) is the force from the squeeze mode

Equation 5 & Equation 6 were programmed into the MATLAB/Simulink® environment and simulated before the mount was constructed. After the testing, the experimental results were input into the ‘grey box’ solver in MATLAB® called the Identification Toolbox. This was used to extract the proper values for the equations such that the mounts behavior could be predicted accurately.

For flow mode testing with zero field, the dynamic stiffness is very similar to that of a passive hydraulic mount. When the magnetic field is applied, the damping is increased until saturation when the mount behaves like a pure elastomeric mount. This is because the MR fluid develops such a high yield strength that flow between the upper chamber and lower chamber is no longer possible with the input excitation. Figure 43 shows the dynamic stiffness curve for flow mode with different current inputs.
It should be noted that there is a drop off in the dynamic stiffness above ~80 Hz. This is due to the filling procedure at the time of testing. For that procedure air was allowed to enter the mount with the MR fluid. This was problematic because air is compressible and MR fluid is incompressible (pressure inside the mount stays below 70psi). For the equations of motion, the MR fluid is considered incompressible. Because of the mixing of fluid, the mount behavior then becomes unpredictable. Since the first round of testing the trapped air problem has been eliminated.

Squeeze mode is shown to be equivalent to adding an elastic spring in the mount. With an increase in supplied current, there is an increase in dynamic stiffness, see Figure 44. The noise observed in Figure 44 comes from the sensor noise inherent to the testing machine.
With the combination of these two working modes, the dynamic stiffness curve develops a large envelope. This is important because the advantage of MR fluid is the dynamic response of the fluid, so if unexpected events occur, the mount is able to respond quickly with the proper stiffness such that the lowest transmissibility is realized. The envelope is shown in Figure 45.

Figure 45 Different working modes working envelopes, (a) shows the envelope for flow mode only, (b) shows the working envelope for squeeze mode only and (c) shows the combined mode working envelope [1]
It is also interesting that the combination of the modes can be seen to simply be the addition of the two modes, i.e. superposition. This implies that the modes are independent.
Chapter Four

Modeling

4.1. Finite element modeling

Finite element (FE) modeling was first thought of by Hrennikoff and publically published in 1941 [32]. Finite element modeling has gained a great deal of attention in the engineering field over the past 50 years. This is because the exact solutions for the governing differential equations for engineering problems do not lend themselves to complex geometry and multi-body systems. FE modeling has also gained a great deal of attention because engineers can rapidly run different simulations, which is far less expensive than the counterpart, experimental prototype testing. It should be noted that the finite element method (FEM) is just an approximation and does need to be verified experimentally. Engineers CANNOT model a system in an FEM package, generate a simulation, and expect the physical system to behave EXACTLY the same. The contribution of this research is the parametric analysis of the prototype MR mount rubber top. To reduce the amount of calculations needed and reduce the size of the matrices, the geometry of the rubber top was considered and an axisymmetric approach was adopted. The FEM package that was utilized was COMSOL Multiphysics 3.5a. The principle of minimum total potential energy was used as the way to establish equilibrium. Mathematically, this is defined as follows:
\[ \Pi = \Lambda - W \]  
Equation 7

Where:

\( \Pi \) is the total potential energy

\( \Lambda \) is the total strain energy

\( W \) is the work done by external forces

This equation is easily satisfied, the partial derivative of \( \Pi \) with respect to a distal value for structural mechanics problems is equal to zero \( \left( \frac{\partial \Pi(D)}{\partial D} = 0 \right) \). The total strain energy can be found through a volumetric integration which can be seen mathematically below.

\[ \Lambda = \frac{1}{2} \int \int \int [\varepsilon]^T [D][\varepsilon] dV \]  
Equation 8

Where:

\( [\varepsilon] \) is the strain matrix

\( [D] \) is the material property matrix

\[
[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix}
1-\nu & \nu & \nu & 0 \\
\nu & 1-\nu & \nu & 0 \\
\nu & \nu & 1-\nu & 0 \\
0 & 0 & 0 & \frac{1-2\nu}{2}
\end{bmatrix}
\]

Where:

\( \nu \) is poisson’s ratio

\( E \) is Young’s modulus

The strain matrix can be defined as follows:
\[
\{\varepsilon\} = [B]\{d\} 
\]

Equation 9

Where:

\[
\{d\} \text{ is a vector for the unknown degrees of freedom (} \{d\} = \begin{bmatrix} r_i \\ z_i \\ r_j \\ z_j \\ r_k \\ z_k \end{bmatrix} \text{)}
\]

\[
[B] \text{ is the partial derivatives of the shape functions, i.e. } [B] = [\partial][N]
\]

Where:

\[
[N] \text{ is the shape function matrix}
\]

\[
[\partial] \text{ is the derivative matrix}
\]

More explanation about these matrices will be given later. Substituting Equation 9 into Equation 8 the following is obtained:

\[
\Lambda = \frac{1}{2} \{d\}^T \left( \iiint [B]^T [D][B] dv \right) \{d\}
\]

The value enclosed in the parenthesis is actually the stiffness matrix and can be defined as:

\[
[K] = \iiint [B]^T [D][B] dv
\]

In which the property matrix [D] is constant and can be removed from the integration. If and only if the [B] matrix is constant, it can be removed as well, yielding:

\[
[K] = [B]^T [D][B] \cdot \nu
\]

Where:

\[
\nu \text{ is volume.}
\]
However, for axisymmetric triangular elements, the [B] matrix is not constant as will be shown in the following derivation.

Shape functions are expressions for each node of an element. Because triangular elements were used in this analysis, this will pertain to triangular elements, see Figure 46.

![Figure 46 Example axisymmetric element](image)

Each node is denoted by a separate letter, either i, j or k. All of the forces, stresses, etc. are distributed to the nodes. The shape functions are functions such that at their own node, they equal one, but equal zero at every other node. For axisymmetric triangular elements the shape functions are defined as follows, however these must be modified, this will be explained in the following derivation:

\[ N_i = \frac{1}{2A}[a_i + b_j r + c_k z] \]

Where:
The derivative matrix \([\partial]\) is defined as follows:

\[
[\partial] = \begin{bmatrix}
\frac{\partial}{\partial r} & 0 \\
1/r & 0 \\
0 & \frac{\partial}{\partial z} \\
\frac{\partial}{\partial z} & \frac{\partial}{\partial r}
\end{bmatrix}
\]

Therefore it can be seen that the \([B]\) matrix will be defined as follows:

\[
[B] = \frac{1}{2A} \begin{bmatrix}
b_i & 0 & b_j & 0 & b_k & 0 \\
2AN_j/r & 0 & 2AN_j/r & 0 & 2AN_j/r & 0 \\
0 & c_i & 0 & c_j & 0 & c_k \\
c_i & b_i & c_j & b_j & c_k & b_k
\end{bmatrix}
\]

As can be seen, the \([B]\) matrix is not a constant matrix. This would mean that for each element an actual integration would have to be performed. This is extremely computationally expensive and unacceptable. Due to this an approximate matrix is defined, \([\bar{B}]\) which represents the average value of the \([B]\) matrix. The following values are defined to define \([\bar{B}]\):

\[
\bar{r} = (r_i + r_j + r_k)/3 \\
\bar{z} = (z_i + z_j + z_k)/3
\]

With these changes, the stiffness matrix can be defined without integration:

\[
[K] \equiv 2\pi A \bar{r} [\bar{B}] [D] [\bar{B}] \quad \text{Equation 10}
\]

With this approximation, the shape functions are defined as follows:

\[
N_i = \frac{1}{2A} \left[ a_i + b_i \bar{r} + c_i \bar{z} \right]
\]
Where:

\[ a_i = r_jz_k - z_jr_k \]
\[ b_i = z_j - z_k \]
\[ c_i = r_k - r_j \]
\[ N_j = \frac{1}{2A} [a_j + b_j\bar{r} + c_j\bar{z}] \]

Where:

\[ a_j = r_kz_i - z_kr_i \]
\[ b_j = z_k - z_i \]
\[ c_i = r_i - r_k \]
\[ N_k = \frac{1}{2A} [a_k + b_k\bar{r} + c_k\bar{z}] \]

Where:

\[ a_k = r_i z_j - z_i r_j \]
\[ b_k = z_k - z_i \]
\[ c_k = r_j - r_i \]

So, the shape function, \([N]\) matrix in totality is as follows:

\[
[N] = \begin{bmatrix}
N_i & 0 & N_j & 0 & N_k & 0 \\
0 & N_i & 0 & N_j & 0 & N_k \\
0 & 0 & N_i & 0 & N_j & 0 \\
\end{bmatrix}
\]

Equation 11

Now the fully developed \([B]\) is defined as follows:
\[
\begin{pmatrix}
 b_i & 0 & b_j & 0 & b_k & 0 \\
 0 & c_i & 0 & c_j & 0 & c_k \\
 \frac{a_i}{\bar{r}} + b_i + \frac{c_i \bar{z}}{\bar{r}} & 0 & \frac{a_j}{\bar{r}} + b_j + \frac{c_j \bar{z}}{\bar{r}} & 0 & \frac{a_k}{\bar{r}} + b_k + \frac{c_k \bar{z}}{\bar{r}} & 0 \\
 c_i & b_i & c_j & b_j & c_k & b_k 
\end{pmatrix}
\]  

Equation 12

The finite element formulation that is presented above can be found in many FE textbooks. It is derived here to re-emphasize that the stiffness matrix, what is directly related to the displacement of the mount is an approximation for triangular axisymmetric elements. This also partially explains the difference between the experimental results and the FE results found in Section 5.1.

4.2. Three Dimensional CAD modeling

Before the finite element analysis could be completed a model of the rubber top must be generated. The rubber top was generated in a CAD modeling package, SolidWorks®. A rubber top was cut in half, measurements were taken and the model was generated based on these measurements. This model is shown in Figure 47.
The cross section of the mount is very important because the design of the encompassed rubber is paramount to the response, see Figure 48.

The 3D mount was then taken into paper space such that a 2D model could be created. This 2D model was then saved as a DXF file and imported into the COMSOL Multiphysics® environment where the FEA was completed.

The nomenclature of the mount can be seen in Figure 49.
The apple is the upper metallic (aluminum) portion of the mount, this was simplified and the screw to attach the filler neck was not added. The outer ring is made of steel and serves the purpose of containing the rubber and directing its deformation. The shoulder is the portion that is clamped down to the housing using the retaining collar.
Chapter Five

Results and discussion

The parametric analysis was carried out and the results obtained. In this chapter they will be presented and discussed. Only some FE plots will be presented. The value that was chosen as the baseline for comparison is a pseudo-stiffness. This was determined by dividing the applied load by the maximum negative deflection (Stiffness = Load/Deflection).

The intended application of the mount is to isolate pump/motor (P/M) components from the chassis in hydraulic hybrid vehicles. This analysis was performed because different P/M and engine configurations have different isolation requirements. The objective therefore was to correlate variation in the elastomeric components to static stiffness of the mount. Also, the deflection due to the weight of the payload is important. These two things are paramount design considerations. Additional analysis was conducted to determine how several parameters altered the static response of an OEM elastomeric top.

5.1. Parameter description and comparison to experiment
Three different parameters were altered to observe how the mount response is affected. The parameters that were altered can be seen in Figure 50.

These parameters were not chosen arbitrarily. Parameters A & B were chosen because if the mount were to represent an Euler-Bernoulli beam, as seen in Figure 51, these would be characteristic dimensions 1 & 3. It is known that a beam with these dimensions define the stiffness.
As seen in Figure 51, the characteristic parameter 2 cannot be related to the elastomeric top due to the mount axisymmetry. It is also important to note that in Euler-Bernoulli beam theory, shear is neglected because bending stress is dominant. This is primarily due to the fact that the characteristic parameter 3 is much greater than the other two characteristic parameters. This is not the case for the elastomeric top, shear is the dominate force. This will be explained further in the remainder of this chapter.

For the material properties for the metal portions, cast iron was used. In the analysis, no plastic yielding occurs in the apple or the outer ring and the deflections are minimal. This coincides with physical testing so the assumption is valid. For the rubber, the Shore hardness testing of a specimen cut from the rubber top was done per ASTM D-2240. The hardness was found to be 54. The hardness was then translated into a Young’s modulus using the theory of elasticity [33]. The translation is an approximation, as can be seen in Figure 52.
As can be seen in Figure 52, there is a slight variance when translating between hardness and Young’s modulus. The hardness is translated according the equation below.

\[
E = \frac{0.0981(56 + 7.66sh)}{0.137505(254 - 2.54sh)}
\]

Equation 13

Where:

- \(E\) is Young’s modulus in Pascals [Pa]
- \(sh\) is the Shore A hardness

Several experiments were done on the physical rubber top to choose the correct properties for the FEA. These tests were carried out on a Bose Electroforce® machine, see Figure 53.
This machine is similar to an Instron® machine, but rather than using hydraulics to develop the force, an electromagnetic actuator with high frequency capability is used. Five different experimental case studies were done for a comparison analysis between the FE model and the experimental results. The experiments and analysis were conducted as follows:

1. The mount was securely placed in the Bose® machine
2. The load applied to the load sensor due to the mount’s weight was noted
3. Displacement was applied until a near whole value of force was achieved

4. The displacement and force were noted

5. Simulations for the elastomeric top were completed

The FEA results compared to the experiment are summarized in Table 2.

<table>
<thead>
<tr>
<th>Study</th>
<th>Load (N)</th>
<th>Displacement in experiment (mm)</th>
<th>Displacement in FEA (mm)</th>
<th>Percent difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>999.5</td>
<td>4.333</td>
<td>4.334</td>
<td>0.023</td>
</tr>
<tr>
<td>Case 2</td>
<td>1499.5</td>
<td>6.777</td>
<td>6.682</td>
<td>1.402</td>
</tr>
<tr>
<td>Case 3</td>
<td>1200.5</td>
<td>5.471</td>
<td>5.249</td>
<td>4.058</td>
</tr>
<tr>
<td>Case 4</td>
<td>799.5</td>
<td>3.637</td>
<td>3.448</td>
<td>5.197</td>
</tr>
<tr>
<td>Case 5</td>
<td>502.5</td>
<td>2.28</td>
<td>2.155</td>
<td>5.482</td>
</tr>
</tbody>
</table>

The 999.5N case will be referred to as the 1kN case from now on for the reader’s ease. The 1kN case was selected for further study due to the accuracy of the solution. It should be noted that the Shore A hardness used in the simulation was 43.5, NOT 54. This is due to unavailable material for experimental tests to properly determine the rubber material properties for the hyperelastic model. Also, it should be mentioned that the rubber’s age, as well as its work life plays a VERY influential roll on the hardness. The rubber specimen that was tested was very aged and had NOT been cycled in many years.

**5.2. Parameter A analysis**

Parameter A was varied by 0.1mm increments in the positive and negative direction up to 0.5mm, a 1mm and a 2mm analysis were performed as well, see Table 3.
Here and throughout this chapter, the term add or subtract refers to the amount of elastomeric material in the rubber top.

<table>
<thead>
<tr>
<th>Add</th>
<th>Remove</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1mm</td>
<td>0.1mm</td>
</tr>
<tr>
<td>0.2mm</td>
<td>0.2mm</td>
</tr>
<tr>
<td>0.3mm</td>
<td>0.3mm</td>
</tr>
<tr>
<td>0.4mm</td>
<td>0.4mm</td>
</tr>
<tr>
<td>0.5mm</td>
<td>0.5mm</td>
</tr>
<tr>
<td>1.0mm</td>
<td>1.0mm</td>
</tr>
<tr>
<td>2.0mm</td>
<td>2.0mm</td>
</tr>
</tbody>
</table>

Parameter A had an initial value of 33.4mm. The amount of material variation was chosen to find a trend in the parametric alterations. Other cases of material alterations we performed, but they will not be mentioned here. If the material is altered too much, the outer ring will deflect and this is not desirable. The fatigue life of the metal then becomes a limiting factor and a design consideration. The extreme cases of the alteration and the control case can be seen in Figure 54.
Figure 54 Upper left is the original cross section of the mount, in the upper right: the mount with 2mm removed, bottom: 2mm added

For the removal of material for Parameter A, a revolved cut was used, see Figure 55. Shown in Figure 55, is the removal of 0.1mm of material. That is the dimension that was altered in the manner aforementioned. The other dimension, namely 2.137mm, has no significant meaning; it was simply assigned based off of what was needed such that the side of the trapezoid remained parallel to the outer ring.
This was done such that the small side of the trapezoid is the dimension that was varied. One of the sides was made parallel with the outer ring such that it would not begin to ‘cut into’ the outer ring and such that the material variation was limited to Parameter A. However, this caused a non-uniform material removal. This also means that the cross section was not altered uniformly with respect to the control case. The effects of this can be seen in the bi-linear stiffness plot Figure 57.

The addition of material was done in a slightly different manner. Here, a largely trapezoidal section was not added, see Figure 56.
Due to this, the stiffness change is much more linear as was expected, see Figure 57. In Figure 56, the case of the addition of 0.2mm is shown. This was the driving dimension that altered the amount of material added. This value was changed in the manner that was aforementioned.

The value that was chosen as the baseline for comparison is a pseudo-stiffness. This was determined by taking the applied load and dividing it by the maximum negative deflection. The results of this alteration can be seen in Figure 57.
The stiffness can be seen to alter linearly for the addition, and linearly once the baseline is established for the material removal, as was explained. The bi-linear stiffness change was expected and is explained in the General discussion section.

5.3. Parameter B analysis

Parameter B was altered in the same manner as parameter A, see Table 3. The difference was that the parameter was varied in the revolved sketch itself, rather than an addition or removal of material. The parameter in question, as defined in the sketch, can be seen in Figure 58.
The initial value is for this parameter is 24.566mm. Shown in Figure 58, is the case where 0.1mm of material was removed. The results of the different simulations with respect to the pseudo-stiffness can be seen in Figure 59.
An almost pure linear relationship can be seen for Parameter B alterations. For this plot however, the linear portion stabilizes after alteration. The portion of the plot that does not fit with the rest of the data points is the unaltered response, at 0mm. This portion of the response was not expected. The reason for these unexpected results is the sketch had to be slightly altered from the original such that the parameter could be altered consistently, see Figure 60. As stated in the Figure, the side of the outer ring and the apple are not parallel in the OEM part (differ by 3.14°). To achieve a consistent parametric analysis though, the sketch lines were made parallel and the simulations were then ran. This explains the linear response after alteration of Parameter B.
5.4. Parameter C analysis

Parameter C was altered in the same manner. For these alterations the methodology was slightly different for each direction of alteration. This is qualitatively shown in Figure 61, and numerically in Table 4.

Table 4 Alterations for Parameter C in table form

<table>
<thead>
<tr>
<th>Add</th>
<th>Remove</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1mm</td>
<td>0.1mm</td>
</tr>
<tr>
<td>0.2mm</td>
<td>0.2mm</td>
</tr>
<tr>
<td>0.3mm</td>
<td>0.3mm</td>
</tr>
<tr>
<td>0.4mm</td>
<td>0.4mm</td>
</tr>
<tr>
<td>0.5mm</td>
<td>0.5mm</td>
</tr>
<tr>
<td>N/A</td>
<td>1.0mm</td>
</tr>
<tr>
<td>N/A</td>
<td>2.0mm</td>
</tr>
</tbody>
</table>
To avoid oddities in the response, the removal of material was made such that a continuous curve was generated; see the upper right picture in Figure 61. The stiffness change is the most linear, see Figure 62. This is to be expected. Because smooth alterations of the material were made, the response should be linear. As stated earlier, this will be discussed in the General discussion section. It should also be noted that the slope of the line in Figure 62 does not match the other 2. This is because this plot was defined by Parameter C, where increasing this dimension, decreases the amount of elastomer.
Figure 62 Stiffness plot for dimension C variations

For this parameter of the material alteration, addition of rubber specifically, was not as extreme as in the other cases, as seen in Table 4. Only 0.5mm of material was added. More material was not able to be added because of the geometric constraint. Also, the deflection of the outer ring is minimal so the addition of material did not ‘stiffen’ the mount. This alteration is similar to the variance seen with Parameter A, but with less effect.

5.5. General discussion

The stiffness can be seen to have varied in near linear fashions for each parameter, for the non-linear or bi-linear sections, the causes were explained. Parameter B was shown to be the most influential. Here the stiffness variation is the largest. This is expected because varying this parameter effectively increases or decreases the diameter of the mount. Space considerations may not lend this dimension to an easily varied one.
A designer therefore may have to use the other two parameters to achieve the desired stiffness.

Due to limited material for testing, the Neo-Hookean hyperelastic material model was used in the finite element code to represent the rubber. The Neo-Hookean model is a special case of the Mooney-Rivlin model. There are very large deformations involved in the tests and therefore the infinitesimal strain theory does not apply. Infinitesimal strain theory is what is used for most engineering problems. Most metals have a very small amount of strain in the elastic region (0.2%) [34]. A design is usually only concerned with the elastic deformation, and with the deformation being so small, small strain theory is applicable. The stress strain relationship, for large deformations, in tensor notation is defined below [34]:

$$
\begin{align*}
I^{ij} &= 2 \frac{\partial W}{\partial I_1} \delta^{ij} + 4 \frac{\partial W}{\partial I_2} \left[ \delta^{ij} (1 + \gamma_{rr}) - \delta^{ir} \delta^{js} \gamma_{rs} \right] \\
&+ 2h \left[ \delta^{ij} (1 + 2\gamma_{rr}) - \delta^{ir} \delta^{js} \gamma_{rs} + 2\epsilon^{inn} \epsilon^{jrs} \gamma_{mr} \gamma_{ns} \right]
\end{align*}
$$

Equation 14

Where:

- $I^{ij}$ is the Kirchhoff stress tensor
- $h$ is the hydrostatic pressure
- $\delta^{ij}$ is the Kronecker delta ($\delta^{ij} = 1$ if $i=j$ and $\delta^{ij} = 0$ if $i \neq j$)
- $\epsilon^{xxx}$ is the strain for a given direction
- $\gamma_{xx}$ is the shear strain for a given direction
- $W$ is the elastic potential

For the Mooney-Rivlin model the elastic potential takes the form:
\[ W = C_1(I_1 - 3) + C_2(I_2 - 3) \]  

Equation 15

Where:

\[ C_1 = C_{10} \text{ (an experimental material property)} \]
\[ C_2 = C_{01} \text{ (an experimental material property)} \]
\[ I_1 \text{ is the first invariant } (2\gamma_{ii} + 3) \]
\[ I_2 \text{ is the second invariant } (2(\gamma_{ij}\gamma_{ji} - \gamma_{ij}\gamma_{ji}) + 4\gamma_{ii} + 3) \]

The Neo-Hookean model is a simplification of the Mooney-Rivlin model where \( C_1 \) is set equal to \( \frac{1}{2} G \) (\( C_1 = \frac{1}{2} G \)) and \( C_2 \) is set equal to zero (\( C_2 = 0 \)). For the case of simple shear (which this is), the stress tensor has a linear relationship with the shear strain, as seen below:

\[ t^{12} = G\gamma \]
\[ t^{11} - t^{22} = G\gamma^2 \]
\[ t^{22} = t^{33} \]

This phenomenon explains the observed linear changes for the different parameters. A simplification of the mount can be seen in Figure 63, or visualize a revolved Figure 51. Here the green arrows represent a fixed surface, the magenta arrows represent a load and it can be thought that there is a roller constraint on the inner diameter (not shown).
Another engineering value that is often of concern is stress. The stress that is normally discussed is the Von Mises stress. However, the Von Mises stress does not apply here because they are derived from infinitesimal strain theories. As mentioned before, this approach does not accurately capture the physics of the mount. For this reason, the Piola-Kirchhoff stress tensor must be discussed. The Piola-Kirchhoff stress tensor is composed of the Jacobian and the deformation gradient. The 2nd Piola-Kirchhoff stress tensor is often discussed rather than the 1st Piola-Kirchhoff stress tensor due to its symmetry. The 2nd Piola-Kirchhoff stress tensor is defined below [35]:

\[ S = JF^{-1} \cdot \sigma \cdot F^{-T} \]  

\[ \text{Equation 16} \]

Where:

\[ S \] is the 2nd Piola-Kirchhoff stress tensor

\[ J \] is the Jacobian \( (J = \det(F)) \)

\[ F \] is the deformation gradient

\[ \sigma \] is the Cauchy stress tensor
The stress is calculated in curvilinear Lagrangian coordinates. Using Lagrangian coordinates rather than Cartesian the large deformations can be accounted for and the stresses appropriately written. It should be noted that formulation of nonlinear elasticity is still disputed in literature [35].

The boundary conditions for the mount were such that a pressure was applied to the top of the apple and the shoulder was clamped or fix, this can be seen in Figure 64.

![Figure 64 Boundary conditions for the analysis](image)

Mesh optimization was completed for this model. 39,786 degrees of freedom (DOF) was found to give the best results. This was extremely computationally expensive. 11,453 DOF was found to be the optimal mesh size. This only yielded 0.001mm difference from the ~40,000 DOF model and was about 5x faster.

Typically the division between small (infinitesimal) strain theory and the theory of finite deformations is 10-15% strain. The strain in the rubber top is in this range, see Figure 65.
As can be seen from the figure, the strain is quite substantial. Beyond this, the Von Mises stress plot does not make sense for the rubber top, see Figure 66.
It should be noted that the scale on the stress plot had to be altered dramatically to see any contours in the rubber. The reason that the outer ring is white is that the stress is so high compared to the rubber that it is off the altered scale and therefore is not shown.

The 2nd Piola-Kirchhoff stress plots can be seen in the following figures.

![Figure 67 Normal (z direction) Piola-Kirchhoff stress plot](image-url)
Figure 68 Radial (r direction) Piola-Kirchhoff stress plot

Figure 69 Shear Piola-Kirchhoff stress plot
Also of interest is a stress strain plot. The non-linearity of the rubber is substantial within this figure. The point that was chosen for the stress strain plot in Figure 72 can be seen in Figure 71.
Figure 72 Stress strain plot for the point

This plot is one of the more nonlinear plots; some of the points have a fairly linear response. This does not change the fact that finite deformation theory must be used.
Chapter Six

Future Work

The analyses performed here provided the fundamental basis for a designer to engineer a proper mount. The different parameters were shown to vary linearly, which was to be expected because the mathematics surrounding the Neo-Hookean material coincides with this. The next steps in this analysis is to physically vary the mount top as was done in the CAD models and run the experiments to verify the simulations. Also it would be interesting to create an appropriate section of rubber to experimentally test and develop the needed constants for the Mooney-Rivlin model.

Additionally, the transient responses should be looked at. The hysteretic behavior of the rubber top will change with these parameter alterations. Ergo, dynamic simulations should be the next step for continuing this research.

Other mount possibilities should also be pursued, such that the mount presented in [1], a wide-bandwidth MR mount. With this mount, the different orifices (\(\mathbb{O}\) in Figure 37) can be controlled individually. Having this freedom allows one to be able to place the notch frequency in different locations. Another interesting concept is a multi-axis MR mount. With this mount, vibration coming from different axes can be isolated simultaneously.
Essentially there are still limitless possibilities for vibration isolation through the use of MR fluid. Due to the rheological behavior, many engineering designs that were not thought possible now are.
Works Cited


83

